General map of reliability (The summary of Kang's seminar in CAS)

1. The origin of reliability

1.1 The reliability in World War 2

In order to infer the failure probability of rocket launch during the period of World War 2, Robert Lusser[1] provided the probability measure, which is the early development of reliability. Based on the classical probability theory[2], the relevant researches about reliability also keep to the following principles:

- Axiom 1. The probability of an event is a non-negative real number:

$$P(E) \in \mathbb{R}, P(E) \geq 0 \qquad \forall E \in F \tag{1}$$

  where, F is the event space. It follows that $P(E)$ is always finite.

- Axiom 2. The probability that at least one of the elementary events in the entire sample space occur is 1, which obeys the assumption of unit measure:

$$P(\Omega) = 1 \tag{2}$$

- Axiom 3. Relying on the assumption of σ-additivity[4], any countable sequence of disjoint sets (synonymous with mutually exclusive events) $E_1, E_2, \dots$ statisfies:

$$P(\bigcup_{i=1}^{\infty} E_i) = \sum_{i=1}^{\infty} P(E_i) \tag{3}$$

  that is to, for the independent events, the probability of either event occurring is the sum of the probability of each event occurring.

From the Kolmogorov axioms, one can deduce the chain rule[5] (also called the general product rule), which permits the calculation of any member of the joint distribution of a set of random variables, and is also useful for Bayesian Networks.

Especially for the mutually exclusive events, we will have:

$$P(\prod_{i=1}^{\infty} E_i) = \prod_{i=1}^{\infty} P(E_i) \tag{4}$$

According to the principles mentioned above of probability, the researchers built the theory frame of reliabililty in the early stage, includes:

- Utilize the probability to represent the reliability of each unit or system;
- Unit reliability = 1 - failure probability;

- The failure probability can be acquired by statistical methods;
- The system reliability calculated by the product of each unit (cascade model);
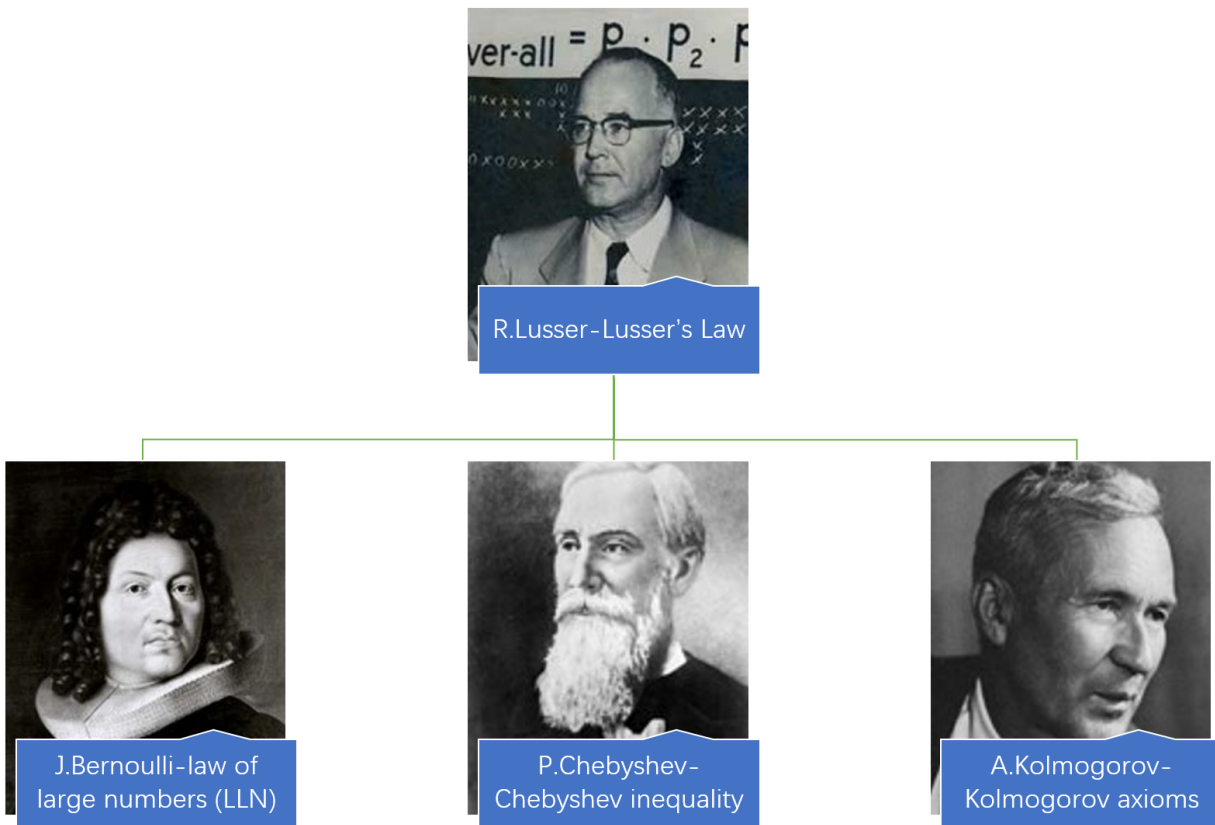

Fig 1. The origiaon of reliability

1.2 DoD AGREE - The report about reliability

Based on the excellent works of R.Lusser, DoD published the AGREE report in 1957. From this report, the definition of reliability was elucidated formally and sophisticatedly:

- Reliability is definied as the probability that a system will perform its intended function for a specified interval within a given time and under specified conditions;
- The reliability can be calculated by the formula: $R(t) = P(T > t)$, noted t is the regular time, and T is the longevity of products;
- Construct the index system, such as, MTBS, failure rate, etc;
- Determine the method of reliability allocation and PFA;
- Design the Reliability Acceptance Test, a statistics test method;

So as to practical:

- Provide the measures about the reliability design, like damping, heat-dissipated;
- Provide the redundant design for critical parts;
- Indicate that necessary measures should be taken to ensure the reliability requirements of purchasing parts;

- Indicate that enhance the failure feedback to improve the product design to realize the sustained growth of reliability;

Above all, the early development of reliability relies on probability, statistics, the law of large numbers as well. And as for the engineering application, it can be concluded that the pattern "index selection-index predication-experimental confirmation-failure analysis-design improvement" has been determined.

2. The development of reliability

With the technology developed rapidly, the reliability has been attracted more and more attention both in industry, military as well as aerospace field, and the reliability theory also made a great leap, which developed 4 branches: statistics-based, physical-based, logical model and reliability design.

2.1 Statistics-based method

The validity of the statistics-based reliability prediction method is guaranteed by LLB and probability axioms, which applies the frequency to evaluate the probability. Thus, we are allowed to count the failure events and predict the failure probability.

Subsystem	Parts	Failure percentage%
Electrical equipment	Integrated circuit	77.5
	Cord	12.5
	Joint	10.0
Flippers	Others	63.8
	Fender	21.8
	Hydraulic cylinder	14.4
Hydraulic system	Power equipment	65.5
	Hydraulic piping	20.2
	Accessories	14.3
Scalable system	Position limit switch	90.3
	Cylinder	9.7

Fig 2. Statistical Analysis of Failure Data - Failures Percentage of Product Components

In addition, the researchers also defined various indexes to represent the reliability:

- Failure rate (λ)[7]: the frequency with which an engineered system or component fails, expressed in failures per unit of time;
- Mean time between faiures (MTBR, $1/\lambda$): the predicted elapsed time between inherent failures of a mechanical or electronic system, during normal system operation. While about the non-repairable system, we apply mean time of failure (MTTF) to measurement;

- Mean time to repair (MTTR): represents the average time required to repair a failed component or device[8]. As far as i'm concerned, it is the total corrective maintenance time for failures divided by the total number of corrective maintenance actions for failures during a given period[9];
- Reliable Life: the life for which the unit/item will be functioning successfully with the reliability of R;
- The time between overhauls (TBO or TBOH): the manufacturer's recommended number of running hours or calendar time before an aircraft engine or other component requires overhaul[10];
- Availability[11]: The degree to which a system or equipment is in a specified operable and committable state at the start of a mission, when the mission is called for at an unknown, i.e, a random, time. Simply put, availability is the proportion of time a system is in a functioning condition. This is often described as a mission capable rate. Mathematically, this is expressed as 100% minus unavailability. And it also can be explained as the ratio of the total time a functional unit is capable of being used during a given interval to the length of the interval;

In order to shed the lights on the product failure, the researchers also design numerous statistical test methods for the design and manufacturing process, such as Environmental Stress Screening (ESS)[12] Reliability Acceptance Test, Reliability Growth Test[13], Accelerated Life Test (ALT) [14], Accelerated Degradation Test (ADT), etc. And according to a huge amount of repeat experiment results, researchers are able to design empirical formulas to represent the failure rate distinctly:

Type		Failure model
Monolithic bipolar and MOS digital circuits, PLA and PAL circuits		$\lambda_p = \pi_Q[C_1\pi_T\pi_V + (C_2 + C_3)\pi_E]\pi_L$
Monolithic bipolar and MOS analog circuits		$\lambda_p = \pi_Q[C_1\pi_T\pi_V + (C_2 + C_3)\pi_E]\pi_L$
Monolithic bipolar and microprocessor		$\lambda_p = \pi_Q[C_1\pi_T\pi_V + (C_2 + C_3)\pi_E]\pi_L$
Memory	SRAM, DRAM, ROM, CCD	$\lambda_p = \pi_Q[C_1\pi_T\pi_V + (C_2 + C_3)\pi_E]\pi_L$
	PROM	$\lambda_p = \pi_Q[C_1\pi_T\pi_V\pi_{PT} + (C_2 + C_3)\pi_E]\pi_L$
	UVEPROM, EEPROM	$\lambda_p = \pi_Q[C_1\pi_T\pi_V\pi_{CYC} + (C_2 + C_3)\pi_E]\pi_L$

λ_p	Failure rate, 10^{-6}/h	π_T	Temperature stress coefficient	C_1	Circuit complexity coefficient
π_E	Environmental coefficient	π_V	Voltage stress coefficient	C_2	Circuit complexity coefficient
π_Q	Quality coefficient	π_{PT}	Programmable process coefficient of PROM circuits	C_3	Package complexity coefficient
π_L	Maturity coefficient	π_{CYC}	Read/Write circulation rate coefficient of EEPROM circuits		

Fig 3. Failure model of electronic components

While, about the statistical method to predict the failure probability has four essential defects:

1. In real engineering applications, the failure happens is rare, which is small sample events, thus it doesn't meet the requirements of LLB, and the prediction accuracy will be affected dramatically;

2. Contribute to the statistics requirement, only when the failure happens can we collect the data. Thus, beforehand reliability prediction is not allowed;

3. The statistical models only give us a number about the failure probability, which can not provide any clue about the improvement direction. In consequence, it is useless for engineering practice;

4. Boil down to the cascade model and probability production principle, the product failure rate will be extremely small. However, it is not consistent with the fact;

By the way, about the small sample estimation is always a difficulty in statistics and machine learning, which is not satisfied the LLB (While, about the massive data statistics, the sampling strategy will be an art, how can we take sample resample to reflect the overall distribution and realize unbiased estimate). The valid solve strategies are to introduce more external information and merge abundant Priori knowledge. For instance, the Bayes method relies on the posterior probability to amend the prior probability continuously, so the estimation will be better and better. As for other methods are the same idea. About the dilemma that probability tends to zero caused by a continued product, which is similar to the Vanishing Gradient Problem in Back Propagation algorithm. Hence, whether we can referent the ResNet idea, using skip product method to alleviate this problem. Or can we apply the attention mechanism provided in the machine translation, select the core parts to calculate the relibility? Besides, although it seems reasonable for the cascade model to calculate the continued product, in the real situation, it is impossible. Tracing to its root, the assumption of independence for each unit is quite strictly. It means the failure probability of a model is not equal to the failure probability product of each part. Therefore, we can not usage this formula simply, especially for the parts level. Or we have to design a more rational formula for the failure probability calculation, at least aiming the weak link and taking the maximum value for all the units is a feasible strategy.

2.2 Physical method

To overcome the above challenges, researchers attempt to figure out the mechanism of failures, to research the physical model of parts, material, process, etc.

Therefore, the failure time model has been proposed, which also named the causality model:

1	Gate oxide breakdown E model $$TF = C \cdot \exp\left(-\frac{A \cdot V_g}{t_c}\right) \cdot \exp\left(\frac{E_a}{T_g}\right)$$ Life span	Design parameter	t_c	The oxide layer thickness of chip
			V_g	Gate voltage of oxide layer
		Material parameter	C	Breakdown factor of E model
			A	Field acceleration factor of E model
			E_a	Activation energy of breakdown
		Load parameter	T_g	Work temperature of chip
2	Archard abrasion life model $$N = \frac{h_s HA}{\mu W_a L_m}$$ Abrasion time	Threshold parameter	h_s	Maximum allowable of abrasion
		Material parameter	H	Material hardness
			μ	Coefficient of kinetic fiction
			A	Contact area of wear surface
		Load parameter	W_a	Positive pressure of contact surface

Fig 4. Failure time model

Furthermore, the working state of products can not be the same in the whole life, consider the performance degradation of the product lifecycle, experts further provided the performance degradation model:

General type of performance degradation model

$$P = f(t, \boldsymbol{x}, \dots), P_{th}$$

P	Performance parameter vector	$\boldsymbol{x}$	Input parameter vector
t	time	P_{th}	Performance parameter threshold

Shock and vibration stress model (first order vibration)

$$Z_{allow} = \frac{0.00665B}{Ch\sqrt{L}}$$

→ Time irrelevance

$$Z_{max} = \frac{386G_{in}P_F}{f_n^{1.5}} \cdot R_{xy}$$

$Z_{max} > Z_{allow}$, failure happened

Z_{allow}	Ideal maximum displacement
Z_{max}	Practical maximum displacement

Input parameter

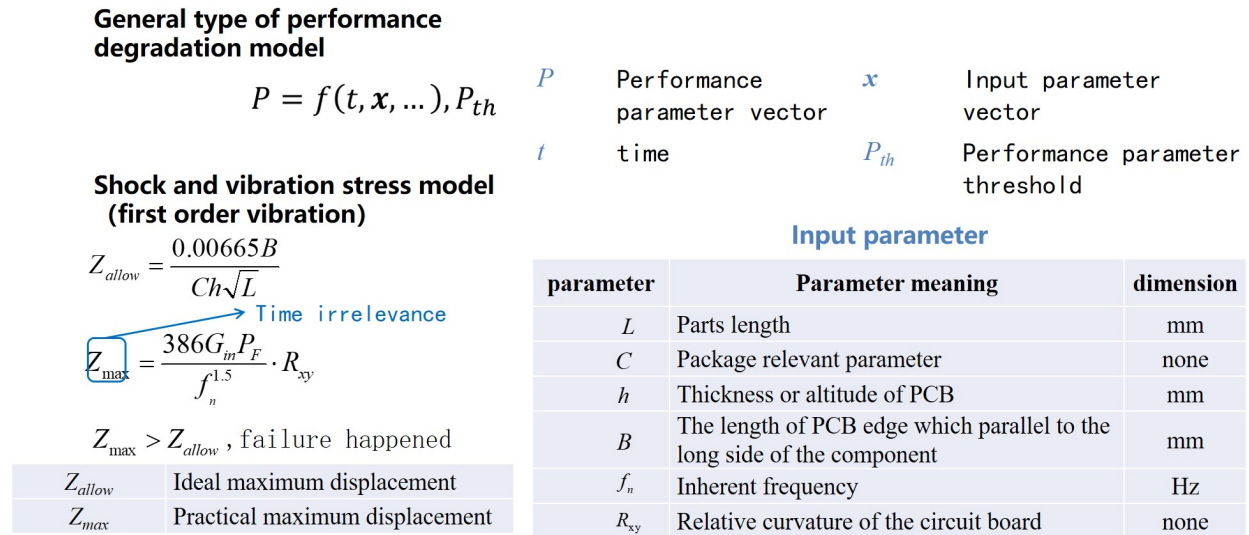| parameter	Parameter meaning	dimension
L	Parts length	mm
C	Package relevant parameter	none
h	Thickness or altitude of PCB	mm
B	The length of PCB edge which parallel to the long side of the component	mm
f_n	Inherent frequency	Hz
R_{xy}	Relative curvature of the circuit board	none

Fig 5. Performance degradation model

The physical model could analysis the essential reason for failures, and stimulate to acquire the reliability index through the parameter randomization. Thus, the drawbacks of the statistical model, such as a small sample, hard to improve, can be relieved effectively.
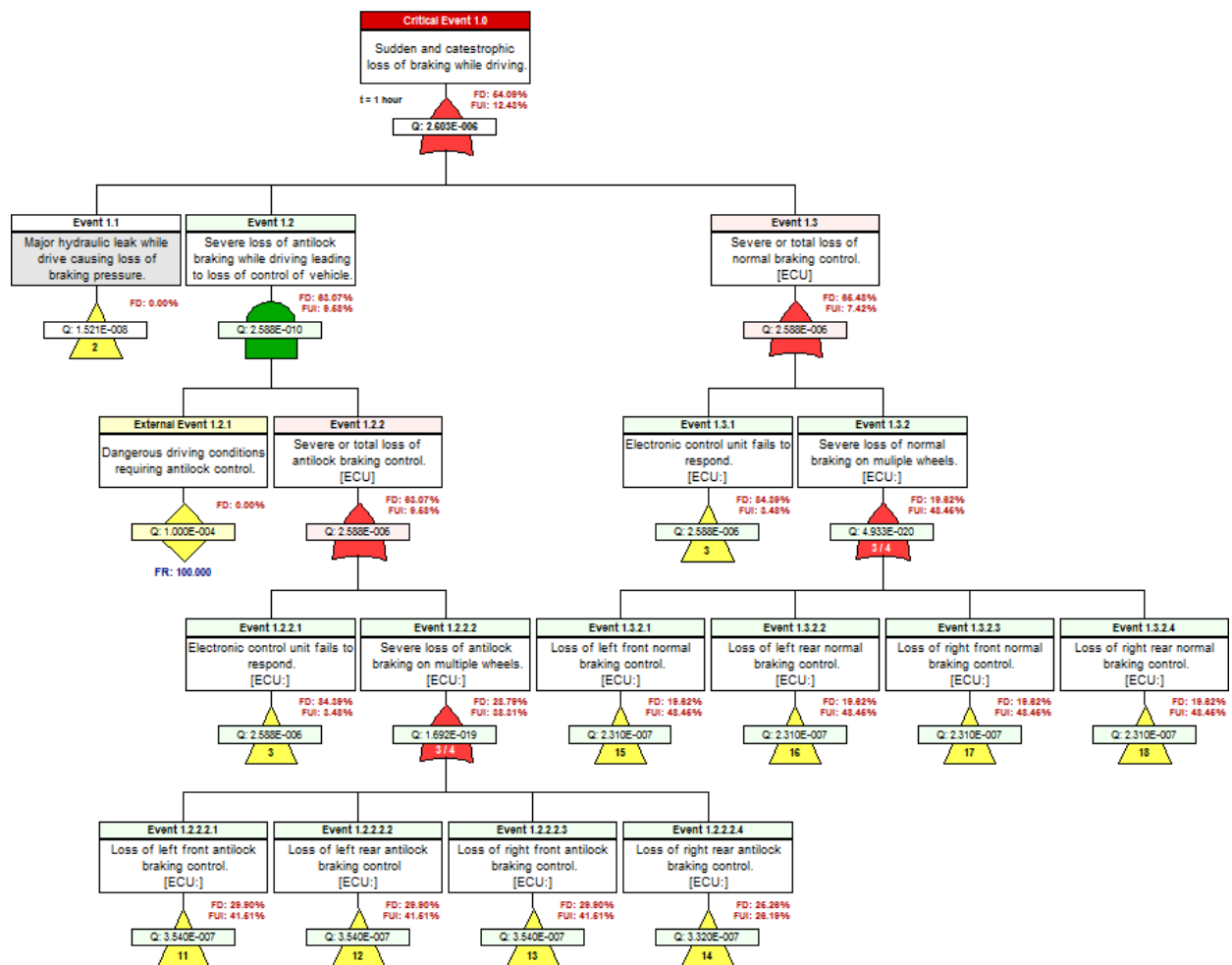
Nevertheless：

1. How can we construct the physical model, it's difficult to figure out the intrinsic mechanism and determine the key factors which affect the item most.
2. In addition, it is a common method to specify parameters obey the Gaussian distribution, while how can we ensure the correctness of this precondition. Whether can we utilize the mixture gaussian to build the model?

2.3 Logic model

Through decompose the system or module into the independent item/unit to rebuild it, and determine the logic relation between each part, then analysis the failure probability. The typical logic model includes a failure tree model and probability graph model, showing as follow:

Types of reliability models

Fig 6. Reliability frameworks
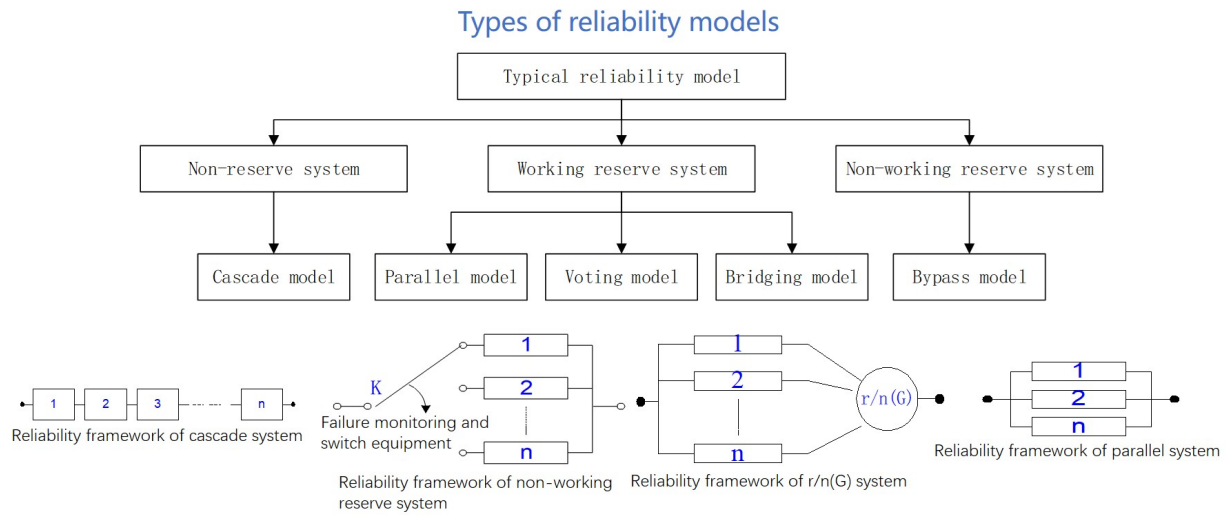
FTA (Fault tree analysis)[15] is a valid method for reliability evaluation. FTA is a top-down, deductive failure analysis in which an undesired state of a system is analyzed using Boolean logic to combine a series of lower-level events. By understanding how systems can fail, to identify the best ways to reduce risk or to determine (or get a feeling for) event rates of a safety accident or a particular system level (functional) failure.

Fig 7. Fault tree analysis

Logic model considers units work cause system work and units failure cause system failure, so that we can calculate the failure probability relying on the logic relation. While, since it also assumes the item/unit independent, the logic model faces probability ebbs quickly too.

2.4 Reliability design

In spite of we can not bring the failure mechanism to the light, we are allowed to enhance the reliability in the design stage, such as derating design, protective design (thermal design, electromagnetic compatibility design, damping, anti-radiation design) for electronic products, safety factor design, and stress intensity design for mechanical products. This strategy can ensure the products' reliability effectively, especially for engineering applications.

Derating design

Thermal design

Safety factor design

Stress-Intensity design

Fig 8. reliability design

All in all, whether the products can work properly and achieve the specific function or not, it is determined by the task requirements as well as user definition, which is reflected in the specific performance indexes. Hence, comparing to the performance parameter margins, we can adjust the system status. And the environment test and reliability enhanced test is just to confirm the performance margins. That is the positive thinking to solve the problem, rather than count the failure times and give the failure probability.

But:

1. It is also obscure and arbitrary for experts to determine the value of margin;

2. The performance margin can not be reflected the reliability measurement easily, which only reflect whether the failure will arise, and how likely it will arise;

3. As for stress-intensity design, the distribution of stress and intensity are also be denoted as Gaussian distribution, which has not solide theoretical supports, and may have some problems in practice.

3. Some solutions about the above challenges

Generally speaking, owing much to the inherent uncertain and cognition uncertain[16] that lead the aforementioned challenges. In order to overcome these problems and to make the prediction more accuracy, two major classes of methods have been proposed:

- Inaccuracy probability reliability measurements: Bayes theory, Evidence theory, Interval analysis theory;
- Fuzzy reliability;

3.1 Bayes reliability

Based on Bayes theory, the posterior distribution can be represented like this:

$$p(\theta|y) = \frac{f(y|\theta)p(\theta)}{m(y)} \tag{5}$$

where, $p(\theta|y)$ denotes as posterior probability distribution, $f(y|\theta)$ denotes as likelihood function, $p(\theta)$ denotes as prior distribution, and $m(y)$ denotes as sample density function. Thus, we can adjust the prior distribution and rely on a small amount of failure data to estimate the reliability [17].
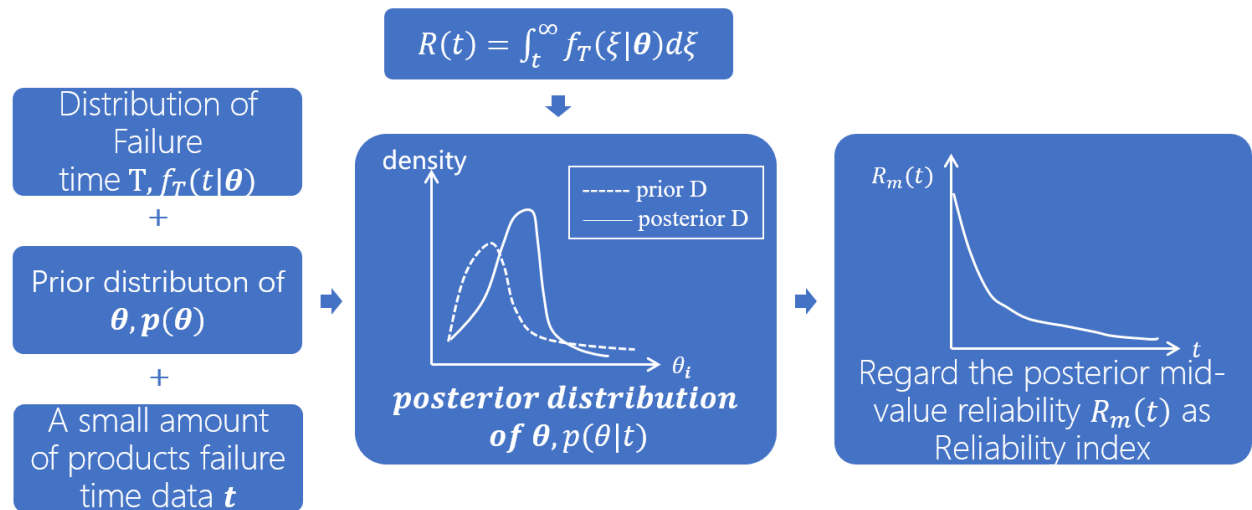

Fig 9. Bayes Theory

3.2 Evidence theory

Proposed by A.Dempster and G.Shafer, evidence theory calculates the Belief and Plausibility to determine the probability interval[18]. To be specific, according to the weighted sum of failure assignment function for the variable interval to determine the failure probability interval[19].
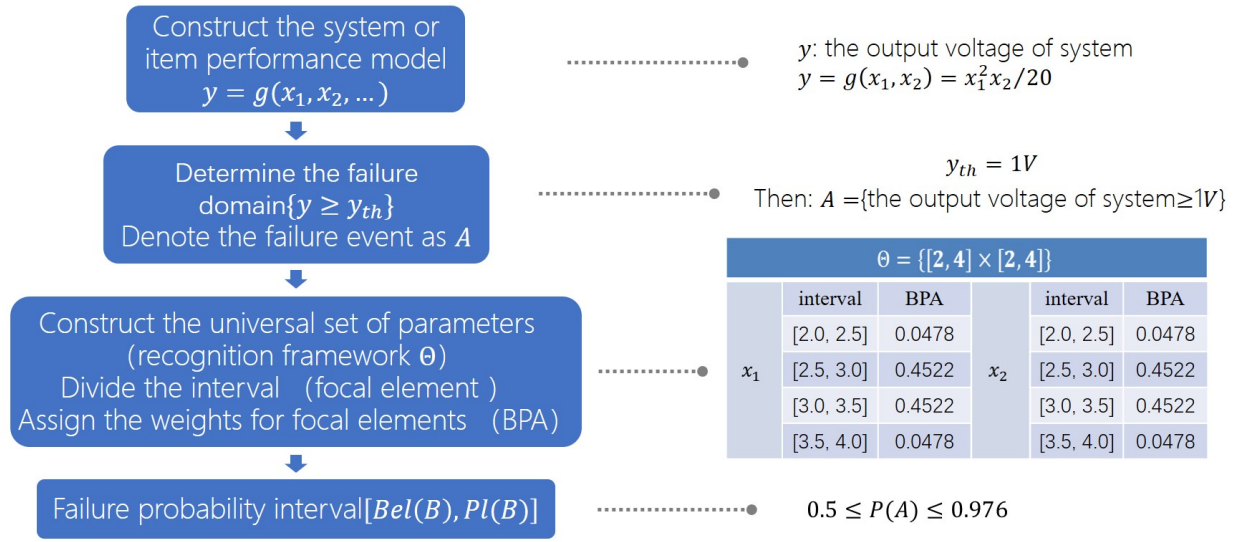
Fig 10. Evidence Theory

The figure flow:
- Construct the system or item performance model $y = g(x_1, x_2, ...)$ ⟶ y: the output voltage of system $y = g(x_1, x_2) = x_1^2 x_2 / 20$
- Determine the failure domain $\{y \geq y_{th}\}$ Denote the failure event as A ⟶ $y_{th} = 1V$ Then: A = {the output voltage of system ≥ 1V}
- Construct the universal set of parameters (recognition framework Θ) Divide the interval (focal element) Assign the weights for focal elements (BPA) ⟶ table below
- Failure probability interval $[Bel(B), Pl(B)]$ ⟶ $0.5 \leq P(A) \leq 0.976$

Θ = {[2, 4] × [2, 4]}

	interval	BPA		interval	BPA
x_1	[2.0, 2.5]	0.0478	x_2	[2.0, 2.5]	0.0478
	[2.5, 3.0]	0.4522		[2.5, 3.0]	0.4522
	[3.0, 3.5]	0.4522		[3.0, 3.5]	0.4522
	[3.5, 4.0]	0.0478		[3.5, 4.0]	0.0478

Through introduce more expert knowledge or additional information to improve the prediction accuracy, while how to merge and utilize this supplementary information effectively, it is a question. What's more, transfer the point estimation to interval estimation, it seems more reliable for reliability evaluation, while how to balance the interval size, precision, and confidence is also a challenge.

3.3 Interval analysis

The thought of interval analysis is quite plain, that is considering apply interval to represent the uncertainty[20]. Briefly speaking, this method realizes the transfer from input parameters interval to output failure probability interval based on interval algorithm or optimization algorithm.

The flow:
- input parametes interval $x_L \leq x \leq x_U$ ⟶ model $y = f(x)$ ⟶ output failure probability interval $y_L \leq y \leq y_U$
- interval algorithm or optimization algorithm

The flow:
- Performance parameter model $y = f(x_1, x_2, ...)$ + Distribution feature parameter interval of expert-based, $[\mu_{iL}, \mu_{iU}], [\sigma_{iL}, \sigma_{iU}], ...$
- the box of probability, y — $F_Y(y)$ — method: Cartesian product, Optimization algorithm
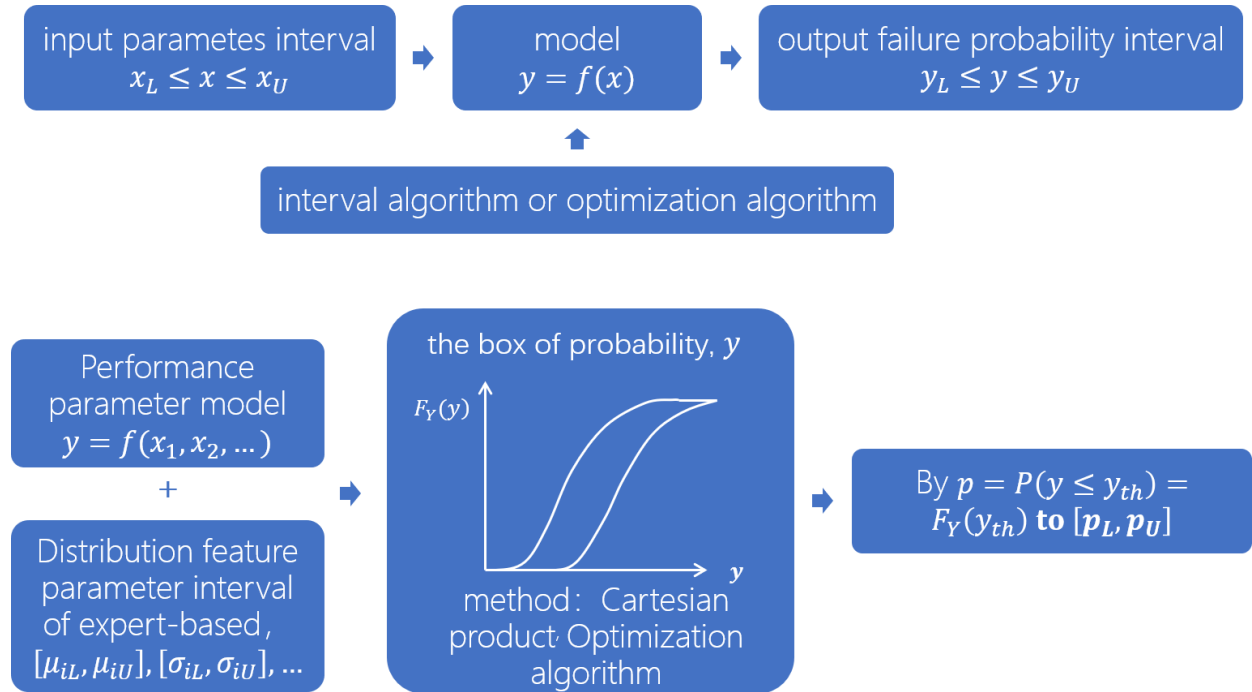- By $p = P(y \leq y_{th}) = F_Y(y_{th})$ to $[p_L, p_U]$

Fig 11. Interval analysis Theory

While, the evidence method is still calculate the probability based on the cascade model. Thus, the problem of probability vanishes, and expansion is existing yet[22].

3.4 Fuzzy reliability

In the realistic world, almost everything is not blake or white, there is a middle area of gray. That's to say anything has a sort of uncertainty. We can not rely on the probability of measurement of this status. For instance, about the age, the stature, what's the age can be considered as younger or elder exactly, it is challenging to offer a standard answer. And the fuzzy theory is able to represent this uncertainty based on membership functions[23].

About the reliability, fuzzy reliability points out that the system only has two states, work, and failure, in the meantime, consider the failure time T is a fuzzy variable and define the possibility of $t > T$ to represent the reliability:

$$R(t) = \Pi(T \geq t) \tag{6}$$

While, there is a nature paradox between axiom 1 and axiom 2 of possibility theory.

$$
\begin{aligned}
&\therefore \begin{cases} axiom2 : \Pi(\Gamma) = \Pi(\Lambda_1, \Lambda_2) = 1 \\ axiom3 : \Pi(\Gamma) = max(\Pi(\Lambda_1), \Pi(\Lambda_2)) = 1 \end{cases} \\
&\therefore \exists\, \Lambda_i = 1, (i = 1, 2) \\
&\because \Lambda_1, \Lambda_2 \text{ are exclusive events} \\
&\therefore \begin{cases} \Lambda_1 < 1 \implies \Lambda_2 = 1 \\ \Lambda_2 < 1 \implies \Lambda_1 = 1 \end{cases}
\end{aligned} \tag{7}
$$

Hence, possibility theory does not meet the normative, and it will not be suitable for reliability calculation[24].

3.5 Brief summary

Totally speaking, the early development of reliability is closed to the probability theory. It is applying the failure probability to measure the system reliability. While the fundamental of probability is a statistic, and in a realistic world, the failure happened are small sample events. Thus, it's difficult for us to acquire an accurate prediction, and only when the failures arise can we perform statistical analysis. In addition, the probability calculation of the cascade model faces the probability of vanishing problems. What's more, the failure probability is merely a number, which can not provide any suggestions about the improvement of the system. Attribute to the above defects. Researchers struggle to construct the physical model to analysis the cause of failures and based on the experiments as well as stimulation to predict the reliability and improve the system at the same time. While limited to cognitive uncertainty and inherent uncertainty, it is hard to determine the core factors and build comprehensive models. As for the logic model, it is a sophisticated method to analysis failure pattern and failure reasons, but about the reliability prediction, it is also powerless. In engineering practice, the technicists apply reliability design (derating design, protective design, safety factor design, and stress intensity design) to ensure the reliability of the products, it is safe to say that it is a compelling work for tasks, but the problem still exists.

How to increase the reliability accuracy, there are three main strategies, include Bayes theory, interval estimation, and fuzzy theory. Must say the Bayes method is a powerful theoretical framework through introduce the posteriori information and external data to amend the prior knowledge and improve the accuracy. Fuzzy theory is also a genius thought, but it still has a little nibs. Well, belief reliability seems to provide a reasonable solution very recently.

4. Belief reliability

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