LEUPHANA UNIVERSITY

PROBABILISTIC MODELING: SUMMER SEMESTER 2021

Determinants of Bitcoin Price: Bayesian Structural Time Series Approach

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Abbreviations

BSTS Bayesian structural time series.

BTC Bitcoin.

LL Local level.

LLT Local linear trend.

MAPE Mean absolute percentage error.

MCMC Markov chain Monte Carlo.

MSE Mean squared error.

VIX CBOE Volatility Index.

1 Introduction[1]

Probabilistic time-series models are becoming popular in the forecasting field as they help to make optimal decisions under uncertainty. Bayesian-based probability and time series methods allow us to adapt the models to uncertainty and make better predictions.

Recently cryptocurrency price prediction has attracted extensive attention from investors, researchers, regulators, and the media. The most discussed cryptocurrency is Bitcoin, a digital currency that one can buy, sell and exchange directly, without an intermediary like a bank. Each Bitcoin transaction is recorded publicly, this ledger is called a blockchain, and copies of that ledger are distributed among computers all over the world, automatically updating with every transaction. All bitcoin transactions are verified by a massive amount of computing power, they are also hard to reverse and difficult to fake. Bitcoin isn't backed by the government or any issuing institution,and because it rejects a centralized financial system, is criticized by authorities, since they can't control illegal cash movements or money laundering. (Franko, 2014)

Another prominent characteristic of Bitcoin is that it doesn't possess classical functions of money, since it lacks intrinsic value, which means it is not backed by gold or any other hard asset; on top of that, it is characterized by high volatility behavior, i.e. the price changes rapidly and unpredictably. (Halaburda, 2016)

Poyser's original paper (2018) explores the association between Bitcoin's market price and a set of internal and external factors by employing the Bayesian structural time series approach (BSTS). The idea behind Bayesian structural time series (BSTS) is to create a superposition of layers such as cycles, trend, and explanatory variables that are allowed to vary stochastically over time, additionally, it is possible to perform a variable selection through the application of the Spike and Slab method. This study differentiates among several attractiveness sources and employs a method that provides a flexible analytic framework, decomposing each of the components of the time series, applies variable selection, and dynamically examines the behavior of the explanatory variables. (Poyser, 2018).

The objective of our report is to extend the model, based on the algorithm implemented in the research paper, to focus on different time series forecasting models utilizing Bayesian approach, and to compare model performance. The initial notion of the exact replication of the original paper was not implemented because of the different nature of the data, i.e. another time range, introduction of shocks, as well as different approaches to data collection and interpretation.

[1] written by Valeriia Chyhirova

2 Summary of the Original Paper[2]

This research is based on the study of Poyser (2018). The selected paper explores various financial, social and macroeconomic factors determining the Bitcoin price. As regressors for the BTC price level, Poyser employs Google Trends data from 27 countries as proxy for public interest, Bitcoin Platform statistics as well as macro-financial variables. The data is collected for the period from January 2013 to May 2017 (Poyser, 2018).

In the paper the author utilises Scott and Varian (2013) and Brodersen et al., 2015 Bayesian Structural Time Series (BSTS) methodology. Following the methodology, the author builds six BSTS models, two basic ones, Local level (LL) and Local linear trend (LLT), and four models with regressors, both time-invariant and time-variant ones. The prediction accuracy of the model specifications reported in the paper are presented in the table 1. The paper's best model specifications with respect to reported one step ahead prediction errors are Local linear trend (LLT) and Local linear trend with time-invariant regressors (LLTTI).

Table 1: One step ahead prediction accuracy according to the different specification

Model	sMAPE	MAE	MSE
Local level (LL)	3.146	12.749	506.992
Local level with time-invariant regressors (LLTI)	4.874	12.139	457.65
Local level with time-variant regressors (LLTV)	4.181	14.588	702.588
Local linear trend (LLT)	2.97	12.026	499.041
Local linear trend with time-invariant regressors (LLTTI)	4.134	11.782	455.146
Local linear trend with time-variant regressors (LLTTV)	3.825	12.73	540.861

Having conducted the analysis, the author concludes that the Bitcoin price responds to some of the financial drivers, namely, the gold price, S&P 500, USD-EURO and YUAN-USD exchange rates. He also finds that the Bitcoin price level is affected by Google Trends of 13 selected countries including Bolivia, Venezuela, Brazil, Colombia, Nigeria, United States, Russia, Thailand as well as several European countries. However, none of the internal factors represented by Bitcoin platform statistics has a significant impact on the Bitcoin price formation. (Poyser, 2018).

3 Model Description and Implementation

3.1 Bayesian Structural Time Series[3]

Time series data can be investigated by different approaches. Alternative models have their own both advantages and disadvantages. When the focus is to discover determinants of a time series variable with other variables, we are left with a bunch of models that can

[2]written by Viktoriia Trokhova
[3]written by Osman Deger

be employed, to name a few: ARDL (auto regressive distributed lags), VAR (vector autoregressive), VECM (vector error correction model). In this study, Bayesian Structural Time Series (BSTS) model which is also known as State Space model will be employed. The choice of this model over the others is due to its certain advantages. In order to stress the advantages, first we introduce the set of equations of the BSTS methodology.

$$y_t = \mu_t + \sum_{i=1}^{k} \lambda_{it}\bar{\omega}_{it} + \sum_{i=1}^{k} \beta_{jt}x_{jt} + \iota_t + \epsilon_{y,t}, \quad \epsilon_{y,t} \sim N(0, \sigma^2_{\epsilon_y}) \tag{1}$$

$$\mu_{t+1} = \mu_t + \nu_t + \epsilon_{\mu_t}, \quad \epsilon_{\mu_t} \sim N(0, \sigma^2_{\epsilon_\mu}) \tag{2}$$

$$\nu_{t+1} = \nu_t + \epsilon_{\nu_t}, \quad \epsilon_{\nu_t} \sim N(0, \sigma^2_{\epsilon_\nu}) \tag{3}$$

$$\lambda_{i,t+1} = \lambda_{i,t} + \epsilon_{\lambda_i,t}, \quad \epsilon_{\lambda_i,t} \sim N(0, \sigma^2_{\epsilon_\lambda}) \tag{4}$$

$$\beta_{j,t+1} = \beta_{j,t} + \epsilon_{\beta_j,t}, \quad \epsilon_{\beta_j,t} \sim N(0, \sigma^2_{\epsilon_\beta}) \tag{5}$$

$$\iota_{1,t+1} = -\iota_{1,t} - \iota_{2,t} - \iota_{3,t} + \epsilon_{\iota,t}, \quad \epsilon_{\iota,t} \sim N(0, \sigma^2_{\epsilon_\iota}) \tag{6}$$

$$\iota_{2,t+1} = \iota_{1,t} \tag{7}$$

$$\iota_{3,t+1} = \iota_{2,t} \tag{8}$$

Equation 1 named as "Observation Equation", is the main structural equation to construct the BSTS framework. Left hand side of the equation is the variable under scope which is price of Bitcoin for this study. On the right hand side, μ_t stands for the level and trend component of the model. The block of $\sum_{i=1}^{k} \lambda_{it}\bar{\omega}_{it}$ is included to capture possible shocks or interventions to the investigated variable such as the effect of tweets of Elon Musk on the price of Bitcoin. Here, ω_{it} is the set of intervention variables, and λ_{it} is the set of parameters of the intervention variables. Next, $\sum_{i=1}^{k} \beta_{jt}x_{jt}$ is the regression component of the equation. In this study we have various regressors grouped as external and internal drivers. All of those explanatory variables fall under the set of x_{jt}. Furthermore, β_{jt} is the set of coefficients of each explanatory variable. Finally, ι_t is a possible component to account for seasonality. Our variable of interest, price of bitcoin, does not have the pattern of seasonality, hence following the author, this part will be dropped in implementation section. As the equation states, there is the assumption of normality on error term with constant variance.

This observation equation alone already reveals the exciting advantages of state space modelling. The model allows to form different setups making it flexible. For instance, local level model, which will be discussed in the next section in more detail, includes only the first term on the right hand side of the equation, in other words, ignores all possible effective explanatory variables, seasonality and shocks. Hence, it is possible to form a very simple time series model. Being able to construct different types of setups to explore the variable is one of the biggest advantages of BSTS model.

Moreover, the observation equation also states that coefficients can be time variant. Unlike other time series models, with BSTS it is possible to allow the coefficients to change over time leading them to have a dynamic structure. As Metcalfe states, time series data is prone to sudden changes and this makes us to focus on models which allows parameters to adapt over time (Cowpertwait & Metcalfe, 2009).

As another advantage of BSTS; unlike the other alternative time series models, BSTS does not require the series to be stationary. The required assumption is on the error term that is following Gaussian pattern with homoscedasticity.

Furthermore, this model allows each component in observation equation to be defined uniquely. Equations 4-10 are called "State Equations". These equations define each component of observation equation. In a quick glance, it can be seen that these equations have random walk nature. They can be defined in another form as well which is another advantage of BSTS models. With BSTS, we can already obtain some basic time series models with arranging the equation. As Scott and Varian (2013) state: assuming no shock or regressor or seasonality component along with nu_t to be zero, Equation 1 and 2 together will form a random walk estimation for Bitcoin price which would suggest best prediction as most recent past value. Furthermore, if the variance of the error term in equation 2 is assumed to be null, making the process stable along time, the best prediction of bitcoin price will be average of its past values (Scott & Varian, 2013).

Last three equations in the list of state equations are related to the seasonality which we will not be using in this study. Other equations will be utilized according to the construction of the model. The author had 6 different models in his paper ranging from the simple local level model to time variant dynamic coefficients model. Following the author, in this study we also employ different models. The specific setups will be explained in detail in model estimation section.

3.2 Posterior Inference and Prediction[4]

3.2.1 Kalman Filter

Kalman filter is the key tool for analysing and investigating the State Space Models.

The algorithm works traditionally in a three-step process: Prediction is to forecast the future values given the past information; filtering is to produce the best estimate of the current state given current and the past observation and smoothing is to obtain the best estimate of the past values, given the record of the observations including current observation. In the prediction step, the Kalman filter produces recursively estimates of the current state variables given the past observations $p(\alpha_t|y_{1:t-1})$, along with their uncertainties. Once the outcome of the next measurement (necessarily corrupted with some amount of error, including random noise) is observed $p(y_t|\theta_t)$, the well known analytic form of the Bayes's Theorem leads to the output of filtering step $p(\theta_t|y_{1:t}$ using a standard set of recursive calculations. In this step, we earn the advantages of the the assumptions that all components of the models are Gaussian distributed to gain the two Gaussian distributions: the predicted $p(\theta_{t+1}|y_{1:n})$ and $p(\theta_t|y_{1:t})$. In fact, the mean of posterior distribution $p(\theta_t|y_{1:t})$ is updated using a weighted average, with more weight being given to estimates with higher certainty.

As the algorithm is recursive, it can run in real time, using only the present input measurements and the previously calculated state and its uncertainty matrix; no additional past information is required. The Kalman filter estimates state at each time point, given the preceding observations and the observation at that time.

One more important computational requirement arising in Kalmal filter approach for solving State Space Model is that one needs to simulate the states from the posterior distribution. A common and powerful sampling method that is known widely is Markov chain Monter Carlo (MCMC). In this project work, MCMC is used as the simulation-based method following the work from Poyser (2018) and Scott and Varian (2014)

3.2.2 Variable Selection

From the above section about Bayesian Structural Time Series it is known that these models are powerful as they allow us to model an extensive range of possibilities including different components. The components could be chosen flexibly for trend, seasonality to different external regressors in either static or dynamic regression models. Equation (1) contains the set of external variables, for which the term $\sum_{i=1}^{k} \beta_{jt} x_{jt}$ stands. The complexity of Kalman filter algorithm is linearly proportional in the length of the data and quadratic in the size of the latent state space (Scott and Varian (2014)); hence, it is crucial task to decrease the number explanatory regressors to improve the implementation time. Moreover, selecting the most important covariates bring us a deeper understanding about the causal impact of the different regressors on the underlying predictor. This certainly

[4] written by Huyen Priet-Nguyen

also plays an important role for the business drivers to extract the information from outside, while there is not much inside information at hand. Thus, variable selection becomes more common in the recent econometric researches.

Spike and Slab is a hierarchical Bayesian model, where the spike refers to a center of mass concentrated around or nearly to zero, while the slab is represented as a wide (high variance) normally distributed prior (Poyser (2018)). Spike and Slab then is a natural way of Bayesian inference, which present the sparsity in choosing regression coefficients. In George and McCulloch (1993) the mixture of two normal distribution is presented:

$$p(\beta_j, \gamma_j) = p(\gamma_j)p(\beta_j)p(\beta_j|\gamma_j), \tag{9}$$

where

$$p(\beta_j)p(\beta_j|\gamma_j) \sim (1 - \gamma_j)N(0, \tau_j^2) + \gamma_j N(0, c_j\tau_j^2)$$

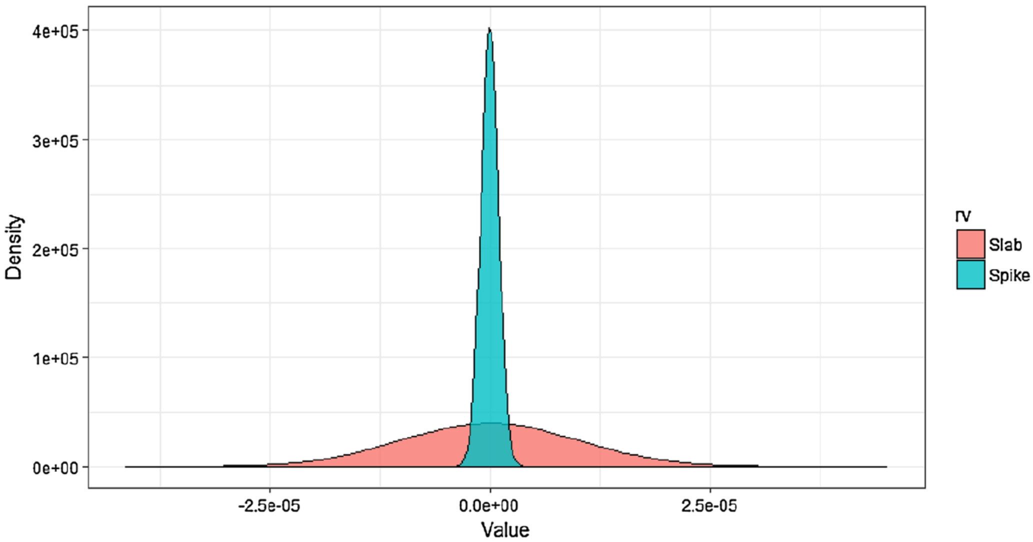


Figure 1: Example of Spike and Slab distribution.

Insert different values for τ_j and c_j, we obtain the Slab and Spike distribution for β_j and γ_j. Figure 1 illustrates the Spike and Slab priors.

In the Bayessian inference for Spike and Slab regression, Bayes's Theorem formula is used to determine the posterior distribution based on the Spike and Slab posterior.

$$P(\gamma|\beta) \propto P(\beta|\gamma)P(\gamma) \tag{10}$$

A popular choice for Spike is that γ follows the product of independent Bernoulli distributions with inclusion probability 0.5, this is known as uninformative prior. However, in our next implementation Section, some experiments for including Prior Expectations into the Spike and Slab regression is proceeded in order to compare with the uninformative prior approach. These prior beliefs could come from an outside study or a previous version of the model. In our case, it comes from the previous uninformative approach, and we assume that it bring us some first understanding about the model. The latter approach output more improved results in terms of some accuracy errors of the models.

Finally, a MCMC sampling is also carried out in order to draw the prior distribution to get the estimation of the inclusion probabilities of different regressors in the model.

3.3 Data[5]

Poyser's paper estimates Bitcoin's price drivers with two main branches of explanatory variables included in the model: internal variables that regulate supply and demand (Bitcoin currency statistics), and external ones (macroeconomic and financial variables, as well as sentiment analysis), overall 55 variables.

A new time period was used for our research: from January 2017 to June 2021, as opposed to January 2013 to May 2017 used in the paper, since more recent data gives us a possibility to make more accurate predictions. On the other hand, Bitcoin price behaviour in the new data period is characterised by significantly higher volatility and fluctuations.

Figure 2: Bitcoin price in USD, 2013-2021

Bitcoin statistics such as circulation, transaction volume, hash rate, mining difficulty and price in USD were scraped from Blockchain.info website through API with respect to the website terms and conditions. The list of external variables was also extended by our team: we have added silver price along with exchange currency rates of Chinese YUAN, UK pound sterling and Russian ruble to the US dollar.

[5]written by Valeriia Chyhirova

We have followed the author's approach to select attractiveness variables based on Google search Trends by selected countries. However, we included *bitcoin* not as just a search term, but as a topic, to include misspellings and searches of the term in other languages, since it represents an interest in the topic more accurately. This finding is backed by relevant research on sentiment analysis of Bitcoin price (Yelowitz & Wilson, 2015). As opposed to Poyser paper (2018), countries were chosen based on the volume data: more than 80 BTC per week on average are bought with relevant currencies. Later in the research we have encountered a multicollinearity issue regarding country trends. The popularity of Bitcoin worldwide rises simultaneously with each individual country trend, resulting in high intercorrelations among independent variables. Hence arises the multicollinearity that inflates the variances of coefficients resulting in misinterpretations of variables' significance. To address this, series with more than 0.8 correlation were dropped, resulting in the final list of 7 countries.

The final addition to the model from our side is the introduction of shock-like variables. Since the time period we have chosen for our analysis, and especially the window of 2020-2021 years, is characterised by higher price volatility and fluctuations, we assumed it could be influenced with unexpected and unpredictable events that affected an economy as a whole, an Bitcoin price in particular. We investigated how Elon Musk (an entrepreneur and business magnate) and his Twitter activity, as well as COVID-19 pandemic, affect the fluctuations of Bitcoin price. The data was taken from @elonmusk Twitter platform and Johns Hopkins Coronavirus Resource Center, respectively (JHI, 2021). Corona variable consists of daily new cases data, starting from January 2020, and zeros prior to that date. Four of Musk's Twitter events are taken into account. According to Ante, 2021, the effect of his tweets is almost immediate, and makes the highest impact in some hours of the same day. This shock is designed as a variable that lies within the closed interval [0,1], being 1 at the day of shock, and decaying to 0 in several days along with the effect of the tweets.

3.4 Software[6]

We utilized *bsts* package from Scott (2014), a tool for time series regression using dynamic linear models fit in R environment. The package was built to use Bayesian posterior sampling, and uses Markov chain Monte Carlo (MCMC) to sample from the posterior distribution of a Bayesian structural time series model. This function can be used either with or without contemporaneous predictor variables (in a time series regression). The state specification is passed as an argument to *bsts*, along with the data and the desired number of MCMC iterations. The *bsts* package is open source.

Fitting the model under a spike-and-slab prior with the set prior inclusion probability of each predictor was run using the *BoomSpikeSlab* R package (Scott 2010), which is similar to *bsts*, but with only a regression component and no time series. The marginal posterior inclusion probabilities are also produced by *BoomSpikeSlab*.

[6]written by Valeriia Chyhirova

4 Model Estimation and Discussion of the Results

4.1 Static models without regressors and comparisons[7]

Model 1: Local Level Model The simplest model out of all models that are investigated in this study is called Local Level. It ignores all possible explanatory variables, seasonality, shocks and even trend. Hence the observation and state equation takes the form as the following:

$$y_t = \mu_t + \epsilon_{y,t}, \quad \epsilon_{y,t} \sim N(0, \sigma^2_{\epsilon_y}) \tag{11}$$

$$\mu_{t+1} = \mu_t + \epsilon_{\mu_t}, \quad \epsilon_{\mu_t} \sim N(0, \sigma^2_{\epsilon_\mu}) \tag{12}$$

The state equation for μ_t is now a random walk which makes the process for bitcoin as determined by the level which is dynamic by a random walk set up. When this model is exercised with our data for 10000 iterations we obtain the following output.

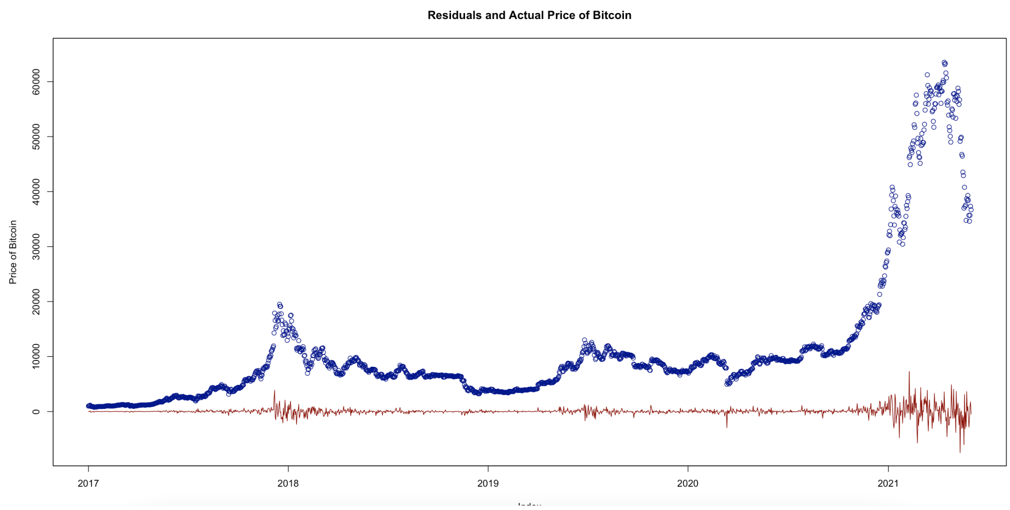


Figure 3: Residual of LL and Actual Price of Bitcoin

When we take a closer look on the residuals of the model, we observe that Local Level (LL) model tends to have larger errors during high volatility periods such as 2018 and after 2021. The calculated mean absolute percentage error (MAPE) for this model is 2.85 percent.

Model 2: Local Linear Trend Model The second model to be considered is also without aid of other explanatory variables. This time the trend component is included in the model.

[7] written by Osman Deger

Hence, we have the following set up for Local Linear Trend (LLT) model:

$$y_t = \mu_t + \epsilon_{y,t}, \quad \epsilon_{y,t} \sim N(0, \sigma^2_{\epsilon_y}) \tag{13}$$

$$\mu_{t+1} = \mu_t + \nu_t + \epsilon_{\mu_t}, \quad \epsilon_{\mu_t} \sim N(0, \sigma^2_{\epsilon_\mu}) \tag{14}$$

$$\nu_{t+1} = \nu_t + \epsilon_{\nu_t}, \quad \epsilon_{\nu_t} \sim N(0, \sigma^2_{\epsilon_\nu}) \tag{15}$$

The state equation for μ_t under this scenario shaped by a random walk and a trend component ν_t which has its own random walk nature. The trend is allowed to change by time. The following output is obtained when the same number of iterations exercised.

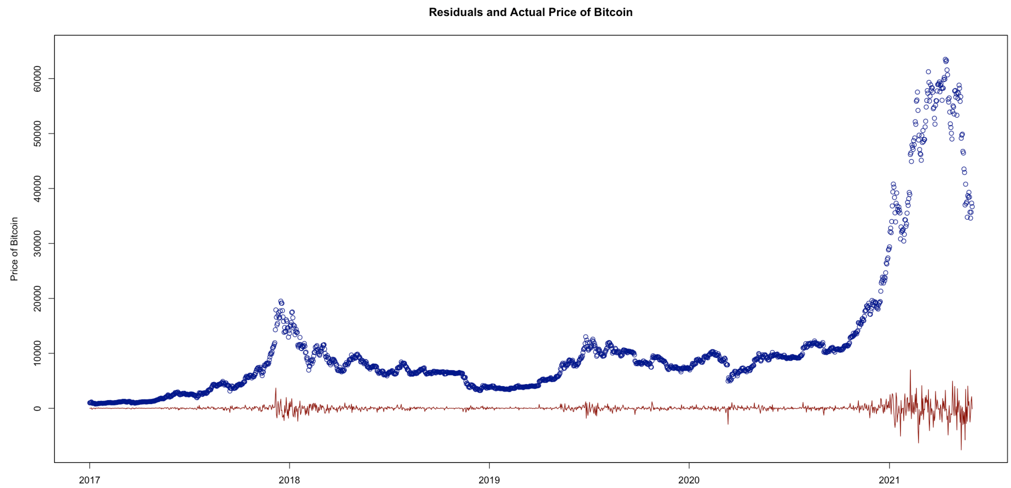
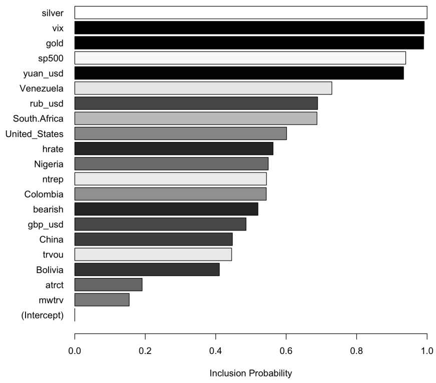


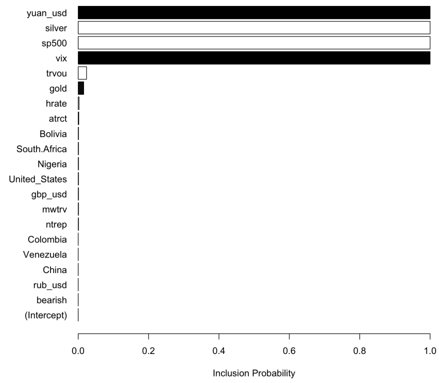
Figure 4: Residual of LLT and Actual Price of Bitcoin

LLT model seems to perform very similar to the previous model. In the absence of other possible explanatory variables, Bitcoin price can be predicted with a low error during low fluctuation periods. The calculated error measure (MAPE) for this model is 2.86 percent, almost the same with basic LL model.

4.2 Static models with regressors and their comparison[8]



(a) Uninformative Prior. MAPE: 3.32. MSE: 554028.4



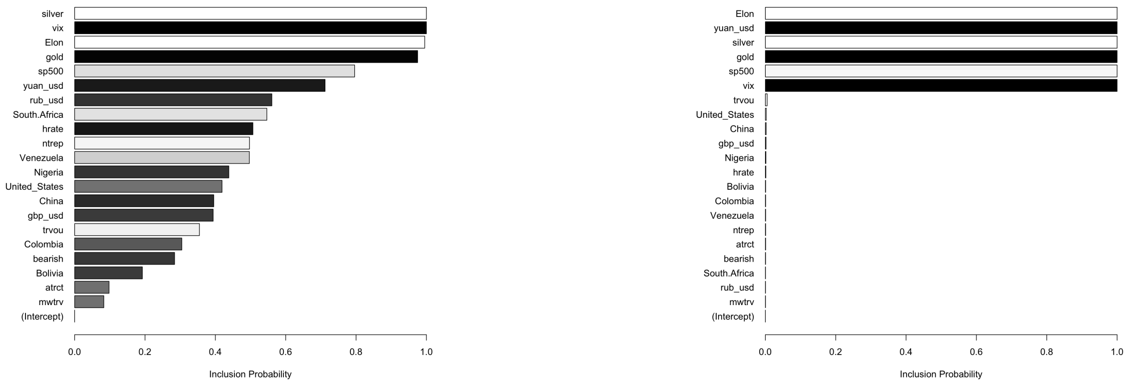
(b) With Prior. MAPE: 3.16. MSE: 557562.9

Figure 5: LL with regressor without Shock

Model 3: Local Level (LL) with regressors (without Shock) This model is run with 20 different regressors after choosing them carefully based on the collinearity elimination. The two alternative experiments are conducted: one without priors, that is called uninformative priors and one with prior by including the prior expectations as well as the inclusion prior probabilities, which are obtained from the first run with the uninformative priors. The Mean absolute percentage error (MAPE) (which is known as the mean absolute percentage error) of the prior approach shows a better result with 3.16% while the one of uninformative is 3.32%. In contrast, the Mean squared error (MSE) of the uninformative prior has a better result. The most influential regressors on Bitcoin price are CBOE Volatility Index (VIX), S&P 500, Gold, Silver.

Model 4: LL with regressors (with Shock - Elon Musk) Bitcoin price experiences the dramatic change in the year of 2021. Some of the events that we could think of as the reasons for this change could be the Corona pandemic and Elon Musk's tweets. We have run Elon Musk twitter's shock design first and in the model 4, Spike and Slab regression results in the significant inclusion probability of this Shock. Again, we check the two approaches: uninformative prior and the other is with prior. A similar result to the model 3 without shock, the approach with uninformative prior has a better mean square error MSE but worse mean absolute percentage error MAPE.

[8]written by Huyen Priet-Nguyen



(a) Uninformative Prior. MAPE: 3.46 MSE 549278.4

(b) With Prior. MAPE: 3.26 MSE: 552369.5

Figure 6: LL with regressors (with Shock - Elon Musk)

Comparison of MSE in Models 3, 4 An alternative option to investigate the advantage of designing the shock from Elon Musk is the comparison of MSE between Model 3 with local level setting with regressors without the shock and Model 4 - Local level with regressors and with shock. In the Table 2 the one with the shock shows a better mean square error MSE in both cases of prior approaches.

Table 2: Comparison of the error MSE of Model 3 and 4

	Uninformative Priors	With Priors
LL with regs. without Shock	554028.4	557562.9
LL with regs. with Shock	549278.4	552369.5

An observation of the impact of shock: A comparison between LL and LL with Shock (Elon Musk) A visual observation of the impact of the shock design is shown in the Figure 7 comparing the residuals of the Local level model with the local level model with shock from Elon Musk twitter. We take a close view into the first half of the year 2021. It is easy to see that the both residuals fluctuate dramatically, however, the ones without shock (represented by blue dots) are more spread out than the red line, which stands for the residual of the local level with shock. This explains why we get the better MSE for the model with shock.

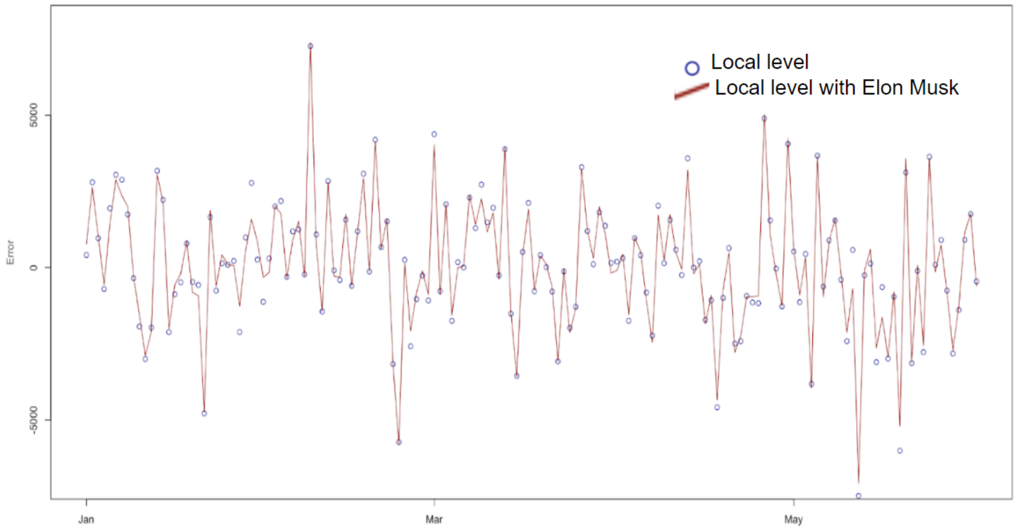
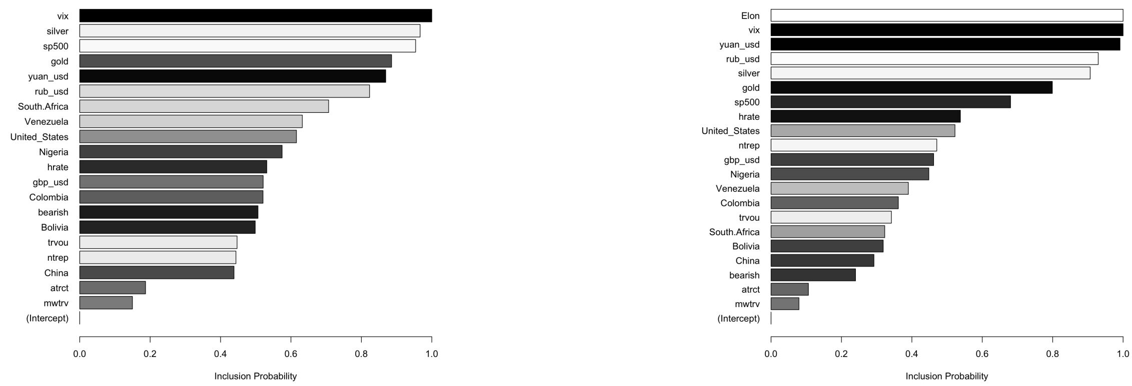


Figure 7: Residual of LL and LL with regressors and shock

Models 5, 6: Local Linear Trend (LLT) with regressors (without and with Shock) A brief comparison is made for these two Models. Similar to the Local Level Models with regressors, the one with priors approaches always show a better MAPE but a worse mean square error MSE. In contrast, the ones with shock have better mean square error MSE but worse mean absolute percentage error MAPE.



(a) LLT with regs. without Shock, uninformative Prior: MAPE: 3.4 MSE: 555176.4

(b) LLT with regs. with Shock, uninformative Prior: MAPE: 3.46 MSE: 549278.4

Figure 8: Local Linear Trend (LLT) with regressors (without and with Shock)

Comparison between Simple SSM without regressors and Shock and SSM with regressor and Shock As we see in the Table 3, the simple local level model has better MAPE than the more complicated model with regressors and shocks. The reason behind utilizing the State Space Model with regressors and shocks, and not using the simple Local Level model to have the best MAPE value, is that we need to analyze the causal impacts that play very important role in capturing the relationships between the regressors, shocks and the dependent variable (e.i Bitcoin price in our case). By applying Spike and Slab regression, the inclusion probabilities and the expectations of the regressor coefficients are determined in the right table. These relationships are revealed and we can truly understand how do the models with regressors and shocks work. Bayesian Structural Time Series using regressors and shock generally provide the possibility to incorporate outside information when we cannot extract information from our own single time series data.

Table 3: Comparison of the errors of LL, and LL with regressors and shock

	MAPE(%)	MSE
Local Level (LL)	2.85	577539.7
LL with regs. and shock	3.26	552369.5

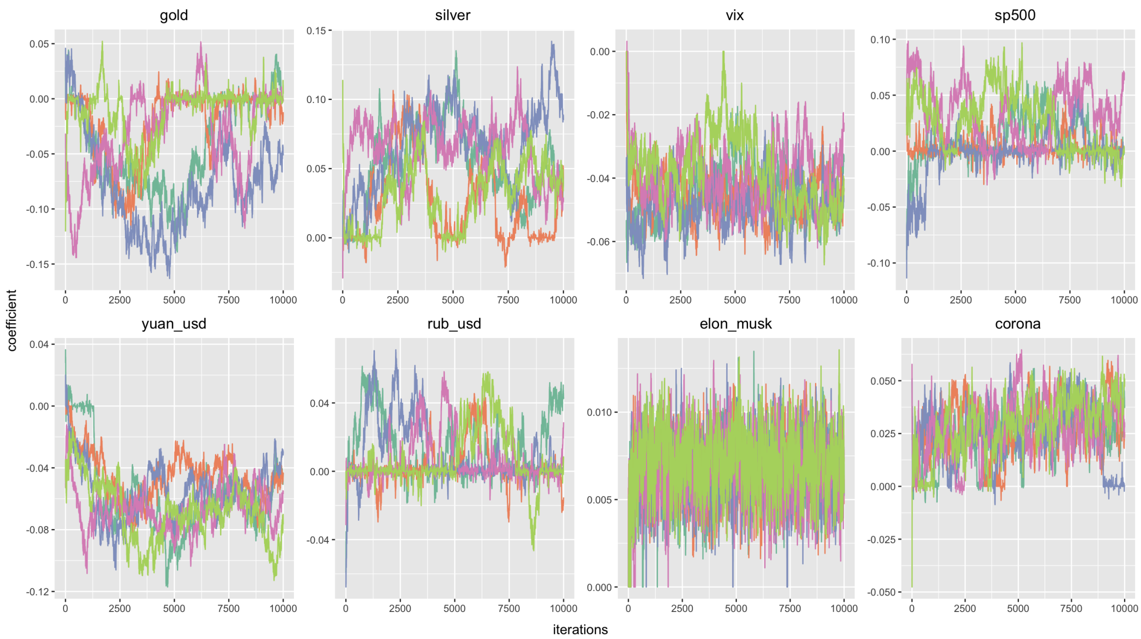


Figure 9: MCMC for most relevant time-invariant regressors

4.2.1 MCMC Sampling[9]

In this research MCMC simulations with 10.000 iterations, 5 random seeds and 10% burn-in period have been run for all variables. Figure 9. shows the results of MCMC simulations for 8 most relevant regressors, the inclusion probability of which surpassed 45% across multiple seeds. It can be denoted that there is a group of variables with relatively stable positive or negative coefficients. This group contains YUAN-USD exchange rate, volatility index, corona and Elon Musk. At the same time the coefficients for gold, silver, S&P 500 and RUB-USD exchange rate exhibit relatively high fluctuations which may impede the interpretation of their impact on Bitcoin price. As for the sign of the obtained coeffi-

[9]written by Viktoriia Trokhova

cients, silver price, RUB-USD exchange rate, S&P 500, Elon Musk and Corona have, for the most part, positive values, while YUAN-USD exchange rate, VIX and gold negative ones. MCMC coefficients for the rest variables exhibit less stability and mostly oscillate around zero.

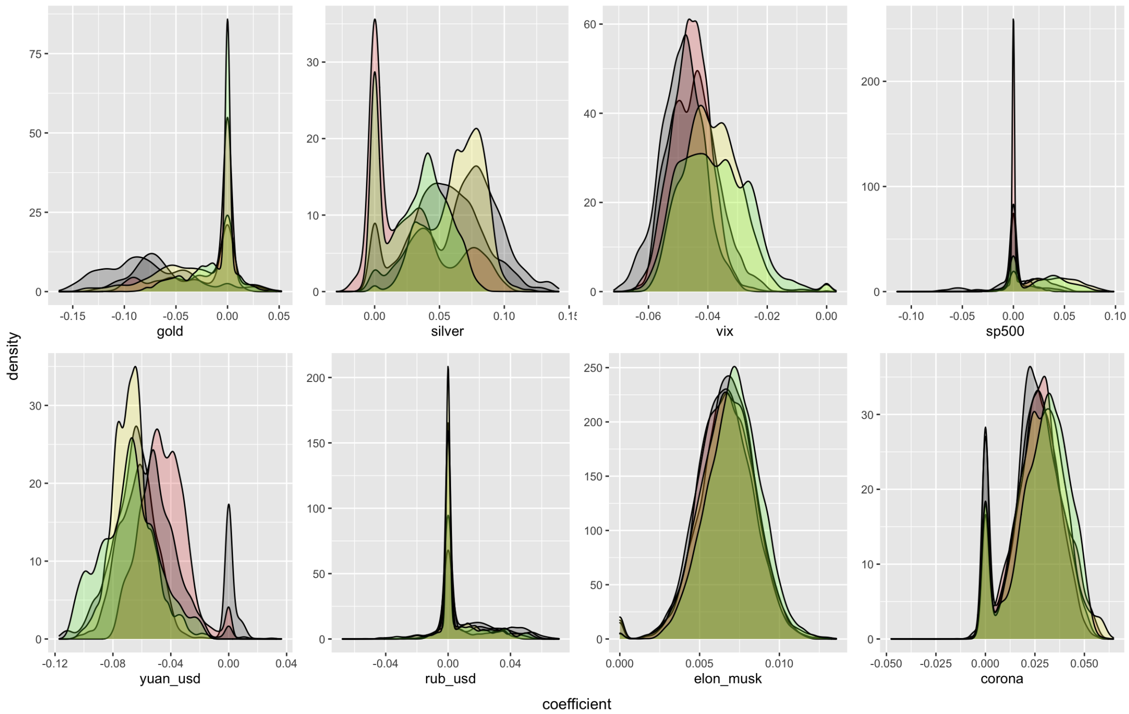


Figure 10: Density plots for most relevant time-invariant regressors

4.2.2 Density plots

Figure 10. shows density of the regressors with high inclusion probabilities. The density plots are a good representation of a spike and slab regression. The more the plot resembles a normal distribution or a slab, the higher inclusion probability it has, the more the density plot look like a spike, the less relevant it is for the model. According to the plots, it can be denoted that such variables as Elon Musk, VIX, YUAN-USD exchange rate and silver are expected to have a more significant impact on Bitcoin price than S&P 500, RUB-USD exchange rate and gold price.

4.3 Dynamic Models[10]

The dynamic model allows the regressors' coefficients to vary over time. First, we have built a dynamic model with all possible regressors. Then we have searched for a more parsimonious model taking into account the inclusion probabilities of variables in the model with time-invariant regressors. The following regressors yielded high inclusion probabilities across multiple seeds: volatility index, gold price, silver price, RUB-USD exchange

[10]written by Viktoriia Trokhova

rate, YUAN-USD exchange rate, S&P 500. For all models 10.000 MCMC iterations with burn-in period of 10% were run.

Table 4: One step ahead prediction accuracy according to the different dynamic model specifications

Model	sMAPE	MAE	MSE
Local level with all time-variant regressors without shock (LLTV)	3.608	395.924	785.919
Local level with all time-variant regressors with shock (LLTVS)	3.618	406.812	903.261
Local level with most relevant time-variant regressors without shock (LLRTV)	3.164	371.917	786.147
Local level with most relevant time-variant regressors with shock (LLRTVS)	3.165	386.277	876.949
Local linear trend with all time-variant regressors withiout shock (LLTTV)	3.491	381.634	773.779
Local linear trend with all time-variant regressors with shock (LLTTVS)	3.490	380.305	798.273
Local linear trend with most relevant time-variant regressors without shock (LLTRTV)	3.089	358.808	767.760
Local linear trend with most relevant time-variant regressors with shock (LLTRTVS)	3.068	364.522	784.775

Local linear trend model with most relevant time-variant regressors with shocks is the best dynamic model specification based on sMAPE. It is followed by Local linear trend model with most relevant time-variant regressors without shocks, which exhibits the lowest MAE and MSE errors. On the whole, local linear trend proves to be a better specification for a dynamic model than a local level one. What is more, models with only 8 most relevant regressors result in lower errors than models with all regressors.

4.4 Interpretation of static and dynamic coefficients[11]

The results of the local linear trend model with time-invariant regressors [12] are presented in the tables 5 and 7. (see Appendix). Table 7 shows time-invariant statistics of the standardized coefficients with an uninformative prior and table 5 time-invariant statistics of the standardized coefficients with a Spike and Slab prior. Table 5. provides more accurate than table 7 estimation of marginal posterior means and 95% highest density intervals (HDI) since it already incorporates the results of the uninformative approach. Before running the regression we have scaled all the variables by taking their sample means and dividing them by their standard deviation following the approach in original paper. Thus, we interpret the obtained coefficients as an expected standard deviation in the response variable associated with one standard deviation change in the covariate. (Poyser, 2018).

In the table 5 there are 8 regressors including shock variables with the inclusion probability of 100%. The results of the dynamic model with the selected most relevant regressors are depicted in the figure 11. It shows marginal probability distributions of relevant determinants of Bitcoin price with 95% (HDI).

The highest marginal posterior mean is exhibited by YUAN-USD exchange rate. One standard deviation change in YUAN-USD exchange rate is linked to -0.065 (-0.099, -0.035) SD

[11]written by Viktoriia Trokhova

[12]due to the fact that local level and local linear trend model with time-invariant regressors provide similar time-invariant statistics of the standardized coefficients, only the results of the latter specification are reported

Table 5: Time-invariant statistics of the standardized coefficients with Spike and Slab prior

Variable	Mean	2.5%	97.5%	Inclusion probability
Intercept	0.000	0.000	0.000	0.000
Confirmation time	0.000	0.000	0.000	0.001
Exchange trade volume	0.000	0.000	0.000	0.003
Hash rate	0.000	0.000	0.000	0.004
Trans excl. popular	0.000	0.000	0.000	0.002
My wallet trans.	0.000	0.000	0.000	0.000
VIX	-0.042	-0.059	-0.022	1.000
S&P 500	0.017	-0.031	0.086	1.000
Gold price	-0.040	-0.110	0.025	1.000
Silver price	0.042	-0.025	0.084	1.000
Bearish	0.000	0.000	0.000	0.000
YUAN-USD exchange rate	-0.065	-0.099	-0.035	1.000
GBP-USD exchange rate	0.000	0.000	0.000	0.002
RUB-USD exchange rate	0.033	0.003	0.068	1.000
United States	0.000	0.000	0.000	0.000
China	0.000	0.000	0.000	0.001
Nigeria	-0.001	-0.020	0.000	0.001
South Africa	0.002	0.000	0.028	0.023
Venezuela	0.000	0.000	0.000	0.001
Colombia	0.000	0.000	0.000	0.001
Bolivia	0.000	0.000	0.000	0.000
Corona	0.027	0.006	0.049	1.000
Elon Musk	0.007	0.003	0.010	1.000

change in BTC price. When we observe the dynamic model results in figure 11, we see that effect of YUAN was deviating between negative and positive until first half of 2020. Later the effect has become solidly negative. At the same time, the coefficient of RUB-USD exchange rate, though being negative for a certain time, has become positive at the end of the analysed period. One change in its standard deviation is associated with 0.033 (0.003, 0.068) SD change in the Bitcoin price volatility.

Another interesting finding is that gold and silver although both are precious metals as alternative investment channels have different effect on BTC price through time. The marginal posterior mean coefficients for silver is positive (0.042) and for gold is negative (-0.040). However, taking into consideration time-variant coefficients makes it clear that this is mainly true only starting from the second half of 2020. In the time period from 2017 to mid 2020 the relations between these precious metals and Bitcoin price were opposite, that is, the coefficient for gold was mostly positive and for silver mostly negative.

Marginal posterior mean of VIX, which is an index to capture investors confidence, gets negative after second half of 2020. Possibly investors felt riskiness in the middle of 2020 with the corona pandemic, and soon they adjusted their expectations. Decrease in the VIX during last 12 months with a negative effect as seen in figure 11 helps explaining why in the same period Bitcoin price has risen. The effect of S&P 500, on the other hand, as a measure of stock market performance is positive at all times.

It can be noted that the behaviour of most coefficients has changed significantly since the second half of 2020. For example, gold price and YUAN-USD exchange rate switched from positive to negative. The volatility index has also been exhibiting a downward trend for the last year and a half, while coefficients of S&P 500 and RUB-USD appear to rise up to the beginning of 2021 and stay constant afterwards. The coefficient for silver exhibits the highest volatility in the studied period. It went up at the end of 2020 and then dropped sharply in the beginning of 2021. A possible explanation for such sudden movements of dynamic coefficients could be the declaration of the corona global pandemic in March 2020.

Due to the anticipated impact of the pandemic on the Bitcoin price, we've decided to include the registered daily cases of Covid-19 as a shock into our model. The results of the dynamic regression with Corona are shown in the figure 10. As it can be observed in the plot, the relationship between Corona and Bitcoin price was positive until summer 2020, then it stayed constant until December 2021 and since that time it has been exhibiting high volatility switching from positive to negative and vice versa multiple times. A possible reason for the mostly positive impact of Corona pandemic on the Bitcoin price might be that the general uncertainty and unpredictability of the pandemic situation could make people search for alternative ways to invest and preserve their wealth in an unstable economic situation.

Mean standardized coefficient for time-invariant case of the shock variable on tweets of Elon Musk is positive (0.007). This variable is designed using 4 impactful tweets of Elon

Musk. 2 of them had positive and 2 had negative sentiment. Since the overall mean turned out to be positive, it can be interpreted as tweets that were in favor of Bitcoin had a greater effect on the BTC price compared to the negative ones. In the time-variant version of the coefficients, which is shown in figure 12, it is seen that impact is positive at all times. Hence, tweets of Elon musk tend to increase the Bitcoin price.

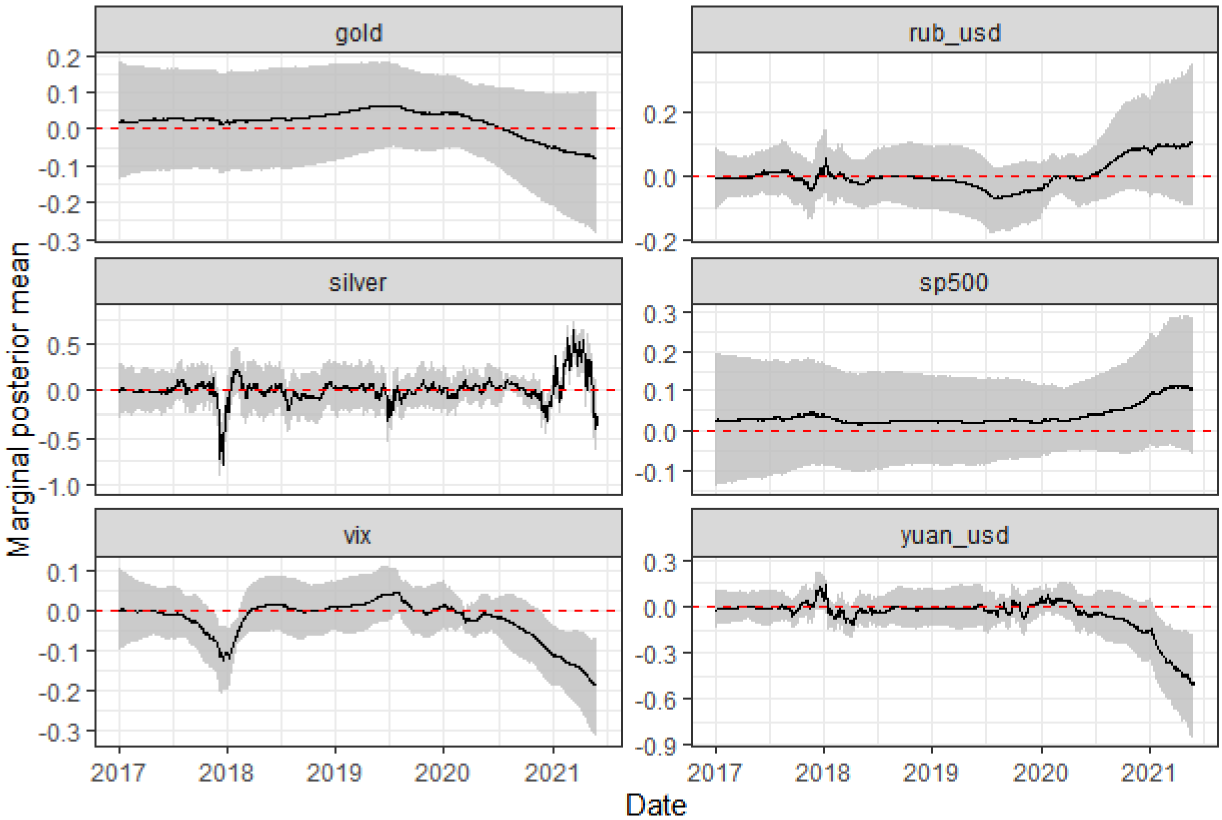


Figure 11: Time-variant standardized coefficients for most relevant regressors without shocks

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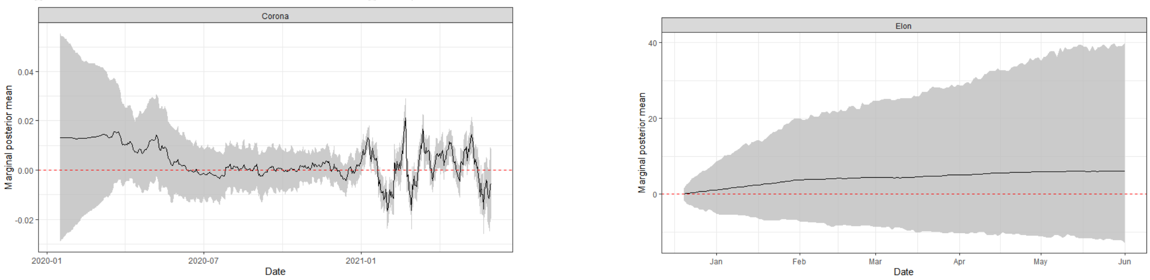

Figure 12: Time-variant standardized coefficients for Corona and Elon Musk tweets

5 Conclusion[13]

The aim of this study is to extend the research carried out by Poyser (2018) by utilising BSTS with both regressor and shock components to study the dynamics of Bitcoin price. While the period of analysis in Poyser's paper covers the time frame from 2013 to 2017, this research analyses the data from 2017 to 2021. Thus, despite the methodology used in both papers is the same, for the most part, it is not possible to compare the results directly. However, it is still feasible to draw parallels and analyse the changes in the impact of the Bitcoin's determinants on its price over the years.

Table 6: One step ahead prediction accuracy according to the different specification

Model	sMAPE	MAE	MSE
Local level (LL)	2.854	346.798	577539.7
Local level with time-invariant regressors without shock (LLTI)	3.177	351.987	558292.4
Local level with time-invariant regressors with shock (LLTIS)	3.122	351.337	549196.3
Local level with most relevant time-variant regressors without shock (LLTV)	3.164	371.917	786.147
Local level with most relevant time-variant regressors with shock (LLTRVS)	3.165	386.277	876.949
Local linear trend (LLT)	2.860	346.640	577676.1
Local linear trend with time-invariant regressors without shock (LLTTI)	3.127	352.008	557145.7
Local linear trend with time-invariant regressors with shock (LLTTIS)	3.158	354.376	555404.7
Local linear trend with most relevant time-variant regressors without shock (LLTRTV)	3.089	358.808	767.760
Local linear trend with most relevant time-variant regressors with shock (LLTTRVS)	3.068	364.522	784.775

The Bitcoin price fluctuated much more strongly in the last 4 years than it did in the period (from 2013 to 2017) considered in the initial paper. That makes a large difference for mean absolute and mean squared errors between the analysed periods of time. However, sMAPE can be used to draw parallels between our and the author's results. Table 6 summarizes the results of 10 models we have built using different specifications. Comparing the one step ahead prediction accuracy results of our models to the results of Poyser (2018) shown in the table 1, we can conlude that in both studies the basic models without regressors generate the lowest errors. In our research, however, a local level appeared to be a bit better specification than a local linear trend. The reason behind still including regressors into models is to analyse the relationship between Bitcoin price and its possible drivers over time.

Among our models with regressors LLTTRVS is the best model specification based on sMAPE. Comparing sMAPE results of LLTI and LLTTI models to the outcomes of Poyser (2018) research, it can be noted that our models exhibit a bit higher prediction accuracy. Interestingly, the set of variables with the highest inclusion probabilities obtained is quite different from the original paper. While Poyser (2018) concludes that Google searches for Bitcoin in different countries have a significant impact on Bitcoin price volatility, in our research we have not obtained the same results. Due to the multicollinearity issue, which made the inclusion probability significantly vary across different seeds, we had to select a

[13]written by Viktoriia Trokhova and Osman Deger

lesser number of countries to include into models with regressors. None of the countries from Google Trends data has exhibited a high inclusion probability. Nevertheless, the impact of Russia and China indirectly represented by RUB-USD and YUAN-USD exchange rates has proven to be significant in our models.

Three macroeconomic variables, namely, gold price, YUAN-USD exchange rate and S&P 500 have appeared to be relevant regressors in both papers. Although CBOE volatility index (VIX) has yield a low inclusion probability in the research of Poyser (2018), it appeared to be a highly relevant regressor in our study. No significant impact of the internal variables has been found neither in this research nor in the original paper. Newly added variables such as silver price, daily cases of COVID-19 and Elon Musk tweets all have appeared to have a significant positive impact on Bitcoin price.

Although we have been contributing to the original paper with new extensions, there is still a lot of room to improve the study. Due to a time constraint, unfortunately, we have to leave them for future research. Forecasting the future Bitcoin price is one of the possible improvements. Another extension is to have other financial market variables such as real estate price index, official interest rates, a designed index to take into account of price of altcoins (other cryptocurrencies) or money supply measures. In this paper we excluded possible important variables, first, to keep in line with original paper and, second, due to the fact that other variables have different time frequency or discrete nature. Furthermore, literature on Bayesian Time Series model is growing. Another research could be to implement multivariate BSTS suggested by (Qiu, Jammalamadaka, & Ning, 2018). They show in their study that Multivariate BSTS outperforms BSTS in terms of prediction accuracy. And finally, splitting the data into 2 parts such as before mid 2020 and after might allow to observe whether the structural change in the data would indicate different explanatory variables as best predictors.

The data along with the R code for our analysis can be found in the Github repository https://github.com/chy-test/pmproject

6 Reflection on Group Work[14]

We got equal input from each member of the team involved in theoretical modelling, coding, code debugging, presenting and, finally, writing the report. Our team had regular weekly meetings to exchange ideas or brainstorm new approaches. Valeriia collected the data, addressed the issues of cleaning and handling missing data, and built MCMC and density plots. Osman and Huyen designed, coded and analysed static models, as well as came up with and prepared the shock variables. Viktoriia collected volume data for countries' selection, analysed the results of MCMC sampling, built dynamic models, plotted figures and made tables of static and dynamic coefficients and interpreted them.

The difficulties we have encountered as a team were: a vast number of potential variables,

[14] written by Valeriia Chyhirova

scraping and cleaning massive amounts of data, as well as occasional bugs in the code, time constraints and digital format/Zoom fatigue. What helped: systematic revisions of models and approaches, regular meetings, a consultation with Mr. Obryan Poyser (the author of the original paper) as well as with Prof. Dr. Burkhardt Funk .

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Appendix

Figure 13: MCMC for all time-invariant regressors

Table 7: Time-invariant statistics of the standardized coefficients with an uninformative prior

Variable	Mean	2.5%	97.5%	Inclusion probability
Intercept	0.000	0.000	0.000	0.000
Confirmation time	0.000	-0.001	0.002	0.088
Exchange trade volume	0.001	0.000	0.007	0.207
Hash rate	-0.004	-0.025	0.003	0.366
Trans excl. popular	0.001	0.000	0.008	0.204
My wallet trans.	0.000	0.000	0.000	0.056
VIX	-0.049	-0.065	-0.030	1.000
S&P 500	-0.008	-0.047	0.019	0.490
Gold price	-0.055	-0.121	0.000	0.829
Silver price	0.046	-0.012	0.112	0.811
Bearish	-0.001	-0.011	0.001	0.193
YUAN-USD exchange rate	-0.063	-0.106	-0.032	1.000
GBP-USD exchange rate	-0.002	-0.020	0.009	0.297
RUB-USD exchange rate	0.036	0.000	0.070	0.924
United States	0.002	-0.007	0.024	0.314
China	0.000	-0.009	0.004	0.186
Nigeria	-0.001	-0.017	0.008	0.272
South Africa	0.003	-0.008	0.026	0.357
Venezuela	0.003	-0.005	0.021	0.363
Colombia	-0.001	-0.012	0.006	0.239
Bolivia	-0.001	-0.009	0.002	0.169
Corona	0.019	0.000	0.045	0.786
Elon Musk	0.007	0.003	0.010	0.999