Exercise 1: Mathematical Programming

Assigned: May 04, 2021

Due: May 25, 2021, 23:59
Upload to MyStudy in the fold **Exercise 1 submission**
Submitted in a ZIP file including a PDF and Python files with the title
Exercise1-name1-name2-name3

1 Task 1

1.1 Description

The following is based on problem 12.19 from "Model Building in Mathematical Programming, Fourth Edition" by H. Paul Williams (John Wiley & Sons, 2002).

A face mask company has two factories, one at Liverpool and one at Brighton. Each factory has a monthly capacity in tons, which cannot be exceeded. In addition, it has four depots with storage facilities at Newcastle, Brimingham, London and Exeter. Each depot has a maximum monthly throughput in tons, which cannot be exceeded. The company sells its product to six customers C1, C2, ..., C6. Each customer has a monthly requirement (in tons), which must be met. Certain customers have expressed preferences for being supplied from factories or depots, which they are used to. So the prefered suppliers are given. Note that if a customer do not have preference, then it can be supplied from either a depot or the factory directly. The distribution costs are known (in €per ton delivered).

1.2 Model

Design a general mathematical model to minimizing overall cost for each month. Formally define the model and explain all of its components.

1.3 Gurobipy

Implement the model in Gurobipy and solve the problem with the data shown in Figures 1–4. And output the optimal transportation plan for each month (from which supplier to which depot or customer with quantities in tons) and the total cost.

Supplied to	Supplier					
	Liverpool factory	Brighton factory	Newcastle depot	Birmingham depot	London depot	Exeter depot
Depots						
Newcastle	0.5	–				
Birmingham	0.5	0.3				
London	1.0	0.5				
Exeter	0.2	0.2				
Customers						
C1	1.0	2.0	–	1.0	–	–
C2	–	–	1.5	0.5	1.5	–
C3	1.5	–	0.5	0.5	2.0	0.2
C4	2.0	–	1.5	1.0	–	1.5
C5	–	–	–	0.5	0.5	0.5
C6	1.0	–	1.0	–	1.5	1.5

[a] A dash indicates the impossibility of certain suppliers for certain depots or customers.

Figure 1: The distrition costs in €per ton delivered.

C1	Liverpool (factory)
C2	Newcastle (depot)
C3	No preferences
C4	No preferences
C5	Birmingham (depot)
C6	Exeter or London (depots)

Figure 2: The preferred suppliers of customers.

Liverpool	150 000 tons
Brighton	200 000 tons
Newcastle	70 000 tons
Birmingham	50 000 tons
London	100 000 tons
Exeter	40 000 tons

Figure 3: The monthly capacity of each factory and the maximum monthly throughput of each depot.

C1	50 000 tons
C2	10 000 tons
C3	40 000 tons
C4	35 000 tons
CS	60 000 tons
C6	20 000 tons

Figure 4: The monthly requirement of each customer.

1.4 Extension

Modify for model to support the following problem extension:

Due to increasing demand in Covid-19 pandemic, the company has increased the capacity in each factory and it is looking for new depots with storage facilities. There is possibility of opening new depots at Bristol and Northampton. The monthly costs (in interest charges) of possible new depots are given and the potential monthly throughputs as well. The distribution costs involving the new depots are given. Note that, the customers remain but with inceasing requirement, which should be met. Extend the model to decide which new depots should be built.

2 Task 2

2.1 Description

A garbage collection company is committed to collecting garbage in a region of 40 areas (each area has a garbage center) and taking garbage back to the depot for disposal. The company has one truck with a capacity of carrying 8 tons of garbage. We assume that all areas can be visited within one day. Some areas are residential areas. Some areas are residential, business and restaurant areas. The amount of garbage produced in each area is different. Therefore, we assume that there are two kinds of areas (areas 'every week' and 'every other week'). Garbage in the 'every week' areas needs to be collected once a week. Garbage in the 'every other week' areas is required to be collected once every other week. The route of the truck starts and ends at a depot, and it is possible to travel between all areas.

The goal is to find the optimal route for the truck on a certain day in each week (doesn't matter on which day, but the truck will only collect garbage once a week), bearing in mind that it has to:

1. Visit all the 'every week' areas,
2. Visit some of the 'every other week' areas, and
3. Work within its capacity.

On next week it must again visit the 'every week' areas, but also visit the 'every other week' farms not visited in the previous week.

2.2 Model

Write a general mathematical model (not the set partitioning model) determining two routes for the truck satisfying the constraints listed above (visit frequency of each node, truck capacity, etc.).

2.3 Extension

Assume there is a matrix of travel times between all areas and the depot and a maximum time for the route of the truck. Extend your mathematical model to support these situation. (Ignore the time it takes to load the garbage into the truck at each node)

2.4 Bonus extension

Assume now that this truck starts and ends at the depot in a given time window. Garbage center in each area has a time window for the garbage collection, i.e., the truck has to start collecting garbage in each area within the given time window. If the truck arrives too early, it has to wait. Adjust the above model to support this case.