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Assignment → Parameter Estimation

Solution 1 :→ $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$

$$L(x_1, x_2 \ldots x_n) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x_i-\mu)^2}{2\sigma^2}}$$

$$= \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^n e^{-\frac{(\Sigma x_i-\mu)^2}{2\sigma^2}}$$

Taking log on both sides we get

$$h(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{-1/2\left(\frac{x_i-\theta_1}{\theta_2}\right)^2}$$

$$\ln h(\theta_1, \theta_2) = \sum_{i=1}^{n}\left[\ln\frac{1}{\sqrt{2\pi}} + \ln\frac{1}{\sqrt{\theta_2}} - \frac{1}{2}\frac{(x_i-\theta_1)^2}{\theta_2}\right]$$

$$\ln h(\theta_1, \theta_2) = \sum_{i=1}^{n}\left[-\ln\sqrt{2\pi} - \ln\sqrt{\theta_2} - \frac{(x_i-\theta_1)^2}{2\theta_2}\right]$$

$$\ln h(\theta_1, \theta_2) = -n\ln\sqrt{2\pi} - n\ln\sqrt{\theta_2} - \sum_{i=1}^{n}\frac{(x_i-\theta_1)^2}{2\theta_2}$$

Taking partial derivative w.r.t $\theta_1$

$$\frac{\partial}{\partial\theta_1}\ln h(\theta_1, \theta_2) = \sum_{i=1}^{n}\frac{2(x_i-\theta_1)}{2\theta_2}$$

$$\frac{\partial}{\partial\theta_1}\ln h(\theta_1, \theta_2) = \sum_{i=1}^{n}\frac{(x_i-\theta_1)}{\theta_2}$$

For getting maximum value →

$$\frac{\partial}{\partial\theta_1}\ln h(\theta_1, \theta_2) = 0$$

$$\Rightarrow \Sigma x_i - \Sigma\theta_1 = 0$$

$$\Sigma x_i = n\theta_1$$

$$\theta_1 = \frac{\Sigma x_i}{n} \Rightarrow \hat{\theta}_1 = \bar{x} \text{ (Mean)}$$

Taking double derivative →

$$\frac{\partial^2}{\partial\theta_1^2}\ln(\theta_1, \theta_2) = \frac{\Sigma(-1)}{\theta_2} = \frac{-n}{\theta_2} < 0$$

Since it is $<0$, so the obtained value is max.

Now taking partial derivative w.r.t $\sqrt{\theta_2}$

$$\frac{\partial}{\partial\theta}\ln h(\theta_1, \sigma) = \frac{-n}{\sigma} + \frac{\Sigma(x_i-\theta_1)^2}{\sigma^3}$$

To get max. value, $\frac{\partial}{\partial \theta} \ln h(\theta_1, \sigma^2) = 0$

$$\Rightarrow \frac{n}{\sigma} = \frac{\Sigma(x_i - \theta_1)^2}{\sigma^3}$$

$$\Rightarrow \theta_2 = \Sigma \frac{(x_i - \theta_1)^2}{n}$$

Putting $\theta_1 = \bar{x}$, $\theta_2 = \Sigma \frac{x_i - \bar{x}}{n}$ (variance)

Now, taking double derivative →

$$\frac{\partial^2}{\partial \theta_2} \ln h(\theta_1, \theta_2) = \frac{n}{\theta_2} = \frac{3}{\theta_2^2} \Sigma(x_i - \theta_1)^2$$

$$= \frac{n}{\theta_2} - \frac{3}{\theta_2^2} n\theta_2$$

$$= -\frac{2n}{\theta_2} < 0$$

Hence max. value → $\hat{\theta}_1 = x$

$$\hat{\theta}_2 = \frac{\Sigma(x_i - \theta_1)^2}{n}$$

<u>Soln. 2</u> →

for binomial distribution →

$$pmf = {}^mC_x \, \theta^x (1-\theta)^{m-x}$$

likelihood function →

$$L(\theta) = \prod_{i=1}^{n} \left[ {}^mC_{x_i} \theta^{x_i} (1-\theta)^{m-x_i} \right]$$

$$L(\theta) = \prod_{i=1}^{n} \left[ {}^mC_{x_i} \right] \theta^{\sum_{i=1}^{n} x_i} \cdot (1-\theta)^{\sum_{i=1}^{n}(m - x_i)}$$

$$L(\theta) = \left( \pi^m C_{x_i} \right) \cdot \theta^{\Sigma x_i} \cdot (1-\theta)^{nm - \Sigma x_i}$$

$$\ln h(\theta) = \sum_{i=1}^{n} \ln {}^m C_{x_i} + \Sigma x_i \ln \theta + (nm - \Sigma x_i) \cdot \ln(1-\theta)$$

Taking derivative w.r.t. $\theta$ →

$$\frac{\partial}{\partial \theta} \ln h(\theta) = \frac{\Sigma x_i}{\theta} + \frac{nm - \Sigma x_i}{1-\theta} \cdot (-1)$$

$$\frac{\partial}{\partial \theta} \ln h(\theta) = \frac{\Sigma x_i - \theta \Sigma x_i - nm\theta + \theta \Sigma x_i}{\theta(1-\theta)}$$

$$\frac{\partial}{\partial \theta} \ln h(\theta) = \frac{\Sigma x_i - \theta \Sigma x_i - nm\theta + \theta \Sigma x_i}{\theta(1-\theta)}$$

$$\frac{\partial}{\partial \theta} \ln h(\theta) = \frac{\Sigma x_i - nm\theta}{\theta(1-\theta)}$$

To get max. value, put →

$$\frac{\partial}{\partial \theta} \ln L(\theta) = 0$$

$$\Rightarrow \theta = \frac{\Sigma x_i}{nm} = \frac{\bar{x}}{m}$$

$$= \hat{\theta} = \frac{\bar{x}}{m}$$

Taking double derivative →

$$\frac{\partial^2}{\partial \theta^2} \ln L(\theta) = \frac{-\Sigma x_i}{\theta^2} = -\frac{(nm - \Sigma x_i)}{(1-\theta)^2}$$

$$\frac{\partial^2}{\partial \theta^2} \ln L(\theta) = \frac{-\Sigma x_i - \theta^2 \Sigma x_i + 2\theta \Sigma x_i - \theta^2 nm + \Sigma x_i \theta^2}{\theta^2 (1-\theta)^2}$$

$$= -\left[\frac{(\Sigma x_i - \theta^2 nm - 2\theta \Sigma x_i)}{\theta^2 (1-\theta)^2}\right]$$

$$= \frac{-nm (1-\theta)\theta}{\theta (1-\theta)^2}$$

$$\frac{\partial^2}{\partial \theta^2} \ln L(\theta) = \frac{-nm}{\theta(1-\theta)} < 0 \quad [\theta \in (0,1)]$$

So, double derivative is $<0$, so obtained value is max.

$$\therefore \boxed{\hat{\theta} = \frac{\bar{x}}{m}}$$