1. Eliminate left recursion in the following grammar:

 $S \longrightarrow SX$

 $S \longrightarrow SSb$

 $S \longrightarrow XS$

S --> a

SOLUTION:

Original grammar:

$$S \longrightarrow SX$$

 $S \longrightarrow SSb$

 $S \longrightarrow XS$

S --> a

$$\alpha 1 = X$$
, $\alpha 2 = Sb$, $\beta 1 = XS$, $\beta 2 = a$

Modified grammar

$$\begin{array}{ll} S & --> XSS' \mid aS' \\ S' & --> XS' \mid SbS' \mid \epsilon \end{array}$$

2. Consider the A-rules:

$$A --> aB | b | CBB$$

There are common terminals in these firsts, so A rules FAIL the pairwise disjointness test.

Consider B-rules:

$$B \longrightarrow aB \mid ba \mid aBb$$

There are common terminals in these firsts, so B rules FAIL the pairwise disjointness test.

Consider C-rules:

$$C \longrightarrow aaA \mid b \mid caB$$

```
first(aaA) = { a }
first(b) = { b }
first(caB) = { c }
```

There are no common terminals in these firsts, so C rules PASS the disjointness test.

Overall, the grammar FAILS the disjointness tests because at least one non-terminal FAILS the disjointness tests.

```
3. (id + id) * id
(id + id( * id
(id + id) * id
```

Stack	Input		Action
0	(id + id) * id\$		S4
0(4	id + id) * id\$		S5
0(4id5	+ id) * id\$		R6 (Use GOTO [4,F]) F -> id
0(4F3	+ id) * id\$		R4 (Use GOTO [4,T]) T -> F
0(4T2	+ id) * id\$		R2 (Use GOTO [4,E]) E -> T
0(4E8	+ id) * id\$		S6
0(4E8+6	id) * id\$	S5	
0(4E8+6id5) * id\$		R6 (Use GOTO [6,F]) F -> id
0(4E8+6F3) * id\$		R4 (Use GOTO [6,T]) T -> F
0(4E8+6T9) * id\$		R1 (Use GOTO [4,E]) E -> E + T
0(4E8) * id\$		S11
0(4E8)11 * id\$	R5 (Use GOTO [0,F]) F -> (E)		
0F3	* id\$		R4 (Use GOTO [0,T]) T -> F
0T2	* id\$		S7
0T2*7	id\$		S5
0T2*7id5 \$		R6 (Use C	GOTO [7,F]) F -> id
0T2*7F10	\$		R3 (Use GOTO $[0,T]$) T -> T * F
0T2	\$		R2 (Use GOTO [0,E]) E -> T
0E1	\$		ACCEPT

```
E 2=> T

3=> T * F

6=> T * id

4=> F * id

5=> (E) * id

1=> (E + T) * id

4=> (E + F) * id

6=> (E + id) * id

2=> (T + id) * id

4=> (F + id) * id
```

```
6 = > (id + id) * id
```

(id + id(*id)

Stack	Input		Action
0	(id + id) * id		S4
0(4	id + id(*id)		S5
0(4id5	+ id(* id\$		R6 (Use GOTO [4,F]) F -> id
0(4F3	+ id(* id\$		R4 (Use GOTO [4,T]) T -> F
0(4T2	+ id(* id\$		R2 (Use GOTO [4,E]) E -> T
0(4E8	+ id(* id\$		S6
0(4E8+6	id(* id\$	S5	
0(4E8+6id5	(* id\$		ERROR; blank entry for ACTION(5,'(')