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1 Introduction

Due to recent *climate change* related issues critical floods on urban areas have started becoming more and more common. Hence, a software able to estimate the likelihood of such an event could be a useful tool in order to prevent the most dramatic scenarios.

For this reason we proposed the implementation of a *Bayesian Network* to model the likelihood of floods in various municipalities of the Italian Veneto region. The structure of the network was inspired by the paper *Assessing urban flood disaster risk using Bayesian network model and GIS applications* [1], although quite deeply modified for didactic reasons.

The project was developed in Python using the pgmpy library [2] and it is available at the repository [3].

2 Bayesian Network construction

The Bayesian Network is composed by nine nodes which are listed below and represented in Figure 1:

• Flood:

It describes the likelihood of particularly serious floods on a yearly basis. The nodes *Road Density*, *Slope* and *Rainfall amount* are strongly causally related with it;

• Road density:

It illustrates the density of paved surfaces in a given area discretized in three levels: *dense*, *medium* and *sparse*. Intuitively the higher the road density the higher the chances of a serious flood. The nodes *Population density* and *Per unit GDP* are causally related with it;

• Slope:

It explains whether the considered area is either *steep* or *flat*. Obviously a flat area has more chances to be flooded. It depends on the node *Elevation*;

• Rainfall Amount:

It summarizes the average amount of water present on the ground in a year which of course increases the chances of flooding. The possible values are *huge*, *medium* and *little*. It is influenced by the nodes *Rainfall frequency* and *River Density*;

• Population density:

It shows the density of the population of an area and it can be *dense*, *medium* or *sparse*. The node *Per unit GDP* is related with it. A densely populated area requires a higher presence of roads;

- **Per unit GDP:** It describes the average richness of people in an area divided into the categories: *high, medium* and *low*;
- Elevation: It illustrates the elevation of the territory in exam. It can be high, medium or low.
- Rainfall frequency: It explains the average rainfall frequency observed in a year. It is discretized in the values frequent, medium and rare;
- River density: It shows the density of rivers or other water elements in an area. It can be either dense or sparse.

The Conditional Probability Distribution (CPD) tables were defined through a qualitative approach, by inspecting the data of the websites [4, 5, 6, 7, 8].

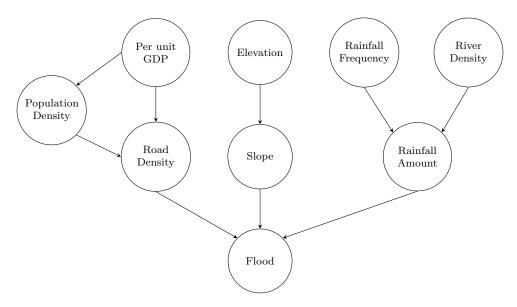


Figure 1: Bayesian Network

3 Theory concepts

In this section some theoretical concepts such as *independence*, trails of causal influence, Markov blanket and inference are explained through their application to our network.

3.1 Independence and trail of influence

Through the use of the method local_independencies we showed all the local independencies between the nodes given their parents as evidence (Table 1).

Per unit GDP \(\text{Slope}\), Rainfall amount, Elevation, Rainfall frequency, River density \(\) Per unit GDP

Road density \(\text{L}\) Rainfall frequency, Rainfall amount, Elevation, Slope, River density \(\) Population density, Per unit GDP

Elevation \(\text{L}\) Population density, Rainfall amount, Road density, Per unit GDP, Rainfall frequency, River density

Slope \(\text{L}\) Population density, Rainfall amount, Road density, Per unit GDP, Rainfall frequency, River density \(\text{Elevation}\)

Rainfall frequency \(\text{L}\) Population density, Elevation, Road density, Per unit GDP, Slope, River density

River density \(\text{L}\) Population density, Rainfall frequency, Elevation, Road density, Per unit GDP, Slope

Rainfall amount \(\text{L}\) Population density, Elevation, Road density, Per unit GDP, Slope \(\text{R}\) Rainfall frequency, River density

Flood \(\text{L}\) Population density, Elevation, Per unit GDP, Rainfall frequency, River density \(\text{R}\) Rainfall amount, Road density, Slope

Table 1: Independencies of the variables given their parents as evidence

Then we tested some other independencies checking active trails of influence between nodes using the method active_trail_nodes. For example we queried the method for the active trail of the node River density given the evidence Rainfall amount, which activates the *V-structure* and trail of influence between the River density and the Rainfall frequency nodes, making them dependent on each other.

3.2 Markov Blanket

The pgmpy library offers the method get_markov_blanket that returns the nodes which compose the Markov blanket of the queried variable. Therefore we checked the active trails of all the variables given their Markov blankets as evidence and we observed that no trail of influence was active (Examples in Figure 3).

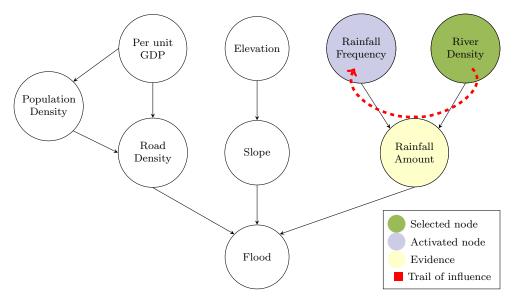
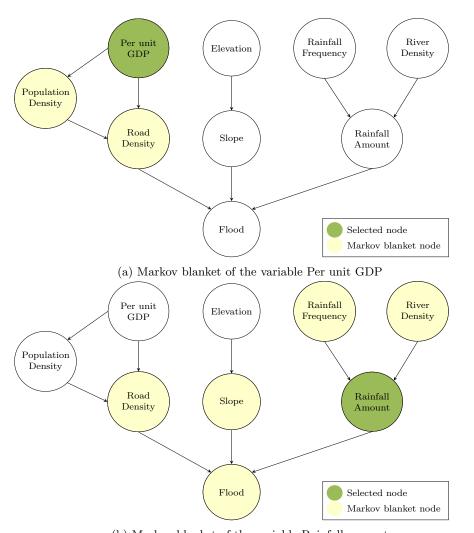


Figure 2: Example of an activated v-structure



(b) Markov blanket of the variable Rainfall amount

Figure 3: Examples of Markov blankets of specific nodes of the network

3.3 Exact inference

Afterwards, we decided to compute the *exact inference* of specific values using the VariableElimination class provided by the pgmpy library. To begin, the likelihood of Flood has been computed and the results are shown in Table 2.

Flood	phi(Flood)
Yes	0.0785
No	0.9215

Table 2: Exact inference of Flood

Next, starting from the worst case scenario we tested the chances of serious floods and then we performed other queries in order to determine which of the causes is more determinant. In addition we executed some "reverse" queries to estimate causes taking as evidence the effect, such as the probability distribution of Slope given the effect Flood (Table 3).

Slope	phi(Slope)
Steep	0.0736
Flat	0.9264

Table 3: Exact inference of Slope given the presence of Flood $(P(Slope \mid Flood = Yes))$

3.4 Approximate Inference

At this point we computed the approximate inference with three different procedures: Rejection sampling, Weighted likelihood method and Markov chain Monte Carlo method. In general all the three methods obtain very good precision in estimating the probabilities of a variable given no evidence. The results were very accurate since the network is not so complex and it has a limited number of nodes and states. Different results are obtained when probabilities are computed given a piece of evidence and the number of samples is limited. Each evaluation test has been carried out while fixing the RNG seed at a value of 50.

3.4.1 Rejection Sampling

To implement this procedure we used methods of the ApproximateInference class provided by pgmpy. Among other evaluations, we considered the worst case scenario, namely the probability of Flood given frequent rainfalls and a flat area with high Road and River density. We computed the probability with 1,000 and 40 samples. The accuracy in the second case is, as expected, lower, as shown in table 4.

Flood	$ ext{phi}(ext{Flood})$	Flood	$ ext{ phi(Flood)}$	Flood	phi(Flood)
Yes	0.1608	Yes	0.1840	Yes	0.2500
No	0.8392	No	0.8160	No	0.7500
(a) Exact inference		(b) R.S. (1,000 samples)		(c) R.S. (40 samples)	

Table 4: Comparison of P(Flood | Road density = Dense, River Density = Dense, Slope = Flat, Rainfall Frequency = Frequent) by exact inference and rejection sampling with a different numbers of samples

3.4.2 Weighted Likelihood Method

To apply this procedure we slightly modified the ApproximateInference and BayesianNetwork classes by extending them since the pgmpy library doesn't offer a handy method to compute it. Repeating

the same experiment the results are better than the previous case when a lower number of samples is used (Table 5).

Flood	phi(Flood)	Flood	phi(Flood)	Flood	phi(Flood)
Yes	0.1608	Yes	0.1730	Yes	0.1750
No	0.8392	No	0.8270	No	0.8250
(a) Exact inference		(b) W.L. (1,000 samples)		(c) W.L. (40 samples)	

Table 5: Comparison of P(Flood | Road density = Dense, River Density = Dense, Slope = Flat, Rainfall Frequency = Frequent) by exact inference and weighted likelihood with a different numbers of samples

3.4.3 Markov Chain Monte Carlo Method

In order to apply this algorithm we used the particular implementation of GibbsSampling offered by the pgmpy library. An interesting comparison considering the same amount of samples as the previous cases of approximate inference has been carried out. The results illustrate that not even 1,000 samples are enough to obtain a good approximation, since, after the sampling procedure, we consider just the samples that match the given evidence, resulting in the selection of a very small subset (Table 6).

Flood	phi(Flood)	Flood	phi(Flood)	Flood	phi(Flood)
Yes	0.1608	Yes	0.1627	Yes	0.2273
No	0.8392	No	0.8373	No	0.7727
(a) Ex	act inference	(b) MCMC (100,000 samples)		(c) MCMC (1,000 samples)	

Table 6: Comparison of P(Flood | Road density = Dense, River Density = Dense, Slope = Flat, Rainfall Frequency = Frequent) by exact inference and MCMC with a different numbers of samples

4 Use case

In this last part we decided to apply the Bayesian network inference to a real scenario, namely the chances of Flood in each municipality of the Italian Veneto region in the case of frequent rainfalls. The municipalities data has been collected from the following official sources [9, 10, 11, 12, 13, 14]. We computed the **Flood** chances through exact inference given as **evidence** the variables **Per unit GDP**, **Population density**, **Slope**, **River density** of each municipality and considering **frequent rainfalls** over all the areas.

$$\forall m \in \text{Municipalities}$$

 $P(\text{Flood}_m \mid \text{Per unit GDP}_m, \text{Population density}_m, \text{Slope}_m, \text{River density}_m, \text{Rainfall frequency} = \text{Frequent})$

Looking at the heatmap (Figure 4) produced by the model we can see that the probabilities of flooding in each municipality is generally low considering the fact that the chances are estimated on a yearly basis. We can see though that the likelihood is much lower if we take in exam the northern areas in contrast to the central ones. The main reasons for such results are the differences in slope and population densities of the areas. In fact the northern part is mainly mountainous, the chances of flooding only increase near Belluno since it is way denser and its surface is almost flat. More densely populated and richer areas such as some chief towns (Venezia, Verona, Vicenza, Padova, Treviso and Rovigo) have the highest likelihood of experiencing floods, whereas poorer and less populated although quietly flat areas in the southern regions show medium probabilities of being flooded. The topographic coordinates data to build the heatmap has been fetched from the repository [15].

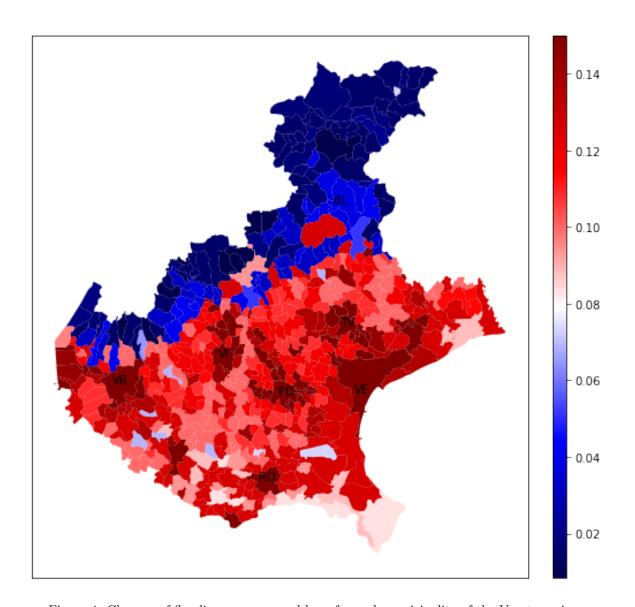


Figure 4: Chances of flooding on an annual base for each municipality of the Veneto region

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