The logarithm of z is a complex number w, such that $e^w = z$ and $Ln(z) = w$. In my theory, I represent z as the complex number $z = \left(\frac{n-1}{n+1}\right) + i\left(\frac{\sqrt{4n}}{n+1}\right)$, which is the same as Euler's $e^{it} = cos(t) + i\ sin(t)$, where $t = acos(x),\ and\ x = \left(\frac{n-1}{n+1}\right) = 1 - \left(\frac{2}{n+1}\right)$, $and\ n = (1,2,3 \ldots \infty)$ Taking it a step further with $d = (1,2,3, \ldots n)$, you get a complex number $z_{nd} = x + iy$ and its conjugate such that,

$x = 1 - \left(d\left(\frac{2}{n+1}\right)\right)$ = n+1 divisions (d) of the unit circle

$iy = \left(\frac{\sqrt{4d(n+1-d)}}{n+1}\right)$ = secant lines of the n+1 unit circle divisions (d)

Equation 1.1 Unit circle: $z_{nd} = 1 - \left(d\left(\frac{2}{n+1}\right)\right) + i\left(\frac{\sqrt{4d(n+1-d)}}{n+1}\right)$

This basically encodes $n + 1$ as nodes of a standing wave which is a harmonic or overtone of 1 i.e. fundamental frequency. Now, project z_{nd} out from the unit circle concentrically, to go from a radius of 1 to a radius of $(n + 1)/2$.

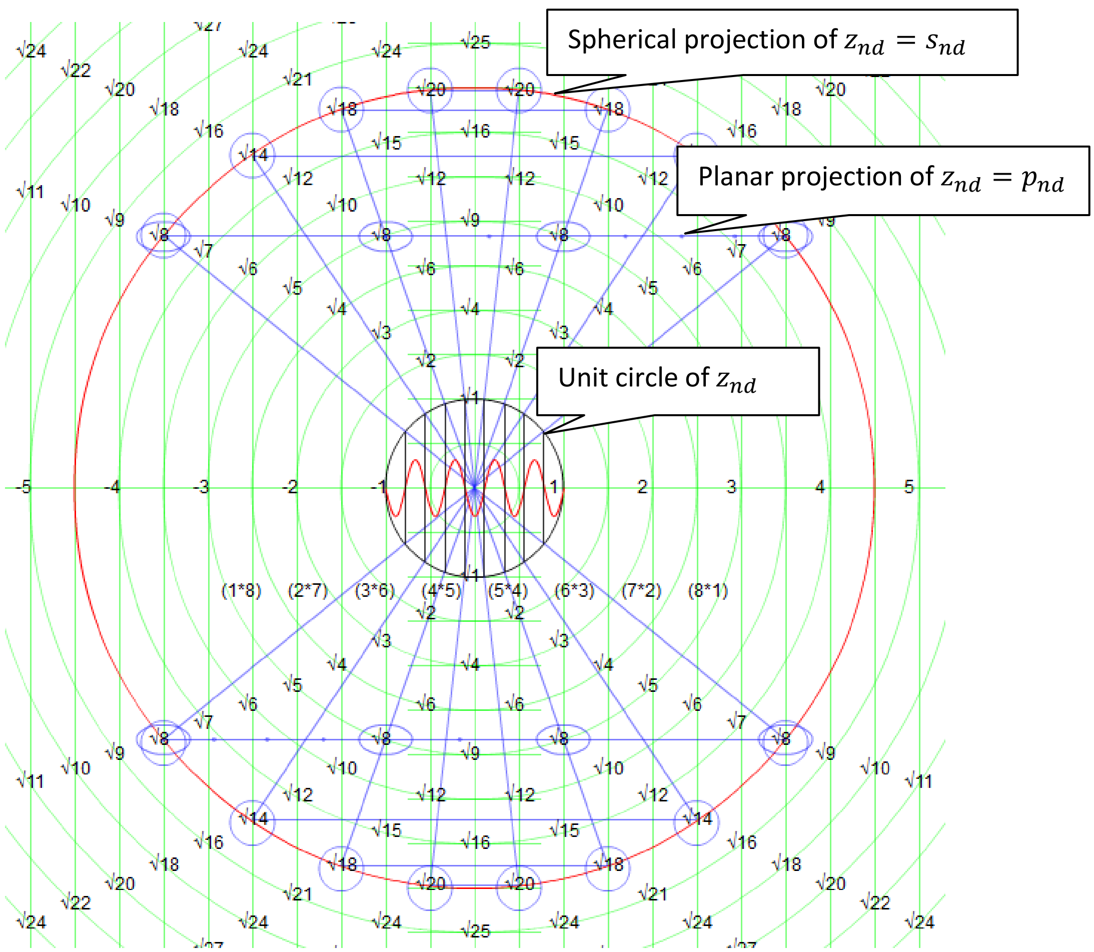

Figure 1.1 $where\ n = 8$. Notice the parabolic shape of the square root of integer multiples of d.

This spherical projection s of z_{nd} transforms $x + iy$ as follows:

$$x = \left(\frac{n+1}{2}\right) - d$$

$$iy = \sqrt{d(n+1-d)}$$

Equation 1.2 spherical projections: $s_{nd} \; = \; \left(\frac{n+1}{2}\right) - \; d \; + \; i\sqrt{d(n+1-d)}$

This transformation helps define a kind of propagating $wavefront$. The s_{nd} points fall on spherical $wavefront$ and the iy values fall on planar $wavefront$. These plane $wavefronts$ carry some interesting properties, particularly when $iy = \sqrt{n}$. The collection of points or $locus$, propagates at a $\sqrt{n}$ ratio to n, containing points at:

$$x = (n - d^2)/2d$$

$$iy = \sqrt{n}$$

Equation 1.3 planar projections: $p_{nd} \; = \; (n - d^2)/2d + \; i\sqrt{n}$

So as n goes to infinity, p_{nd} carries a set of points that define the parabolas created by the square root of integer multiples of d as mentioned in **Figure 1.1** These parabolas are of the standard form "$y^2 = 4px$" such that "$p = \left(\frac{d}{2}\right)$ $and \; x = x + \left(\frac{d}{2}\right)$." Following the geometric "conic section" definition of a parabola, we get a visual representation of the traditional cone's side (triangle) and top (circle) view.

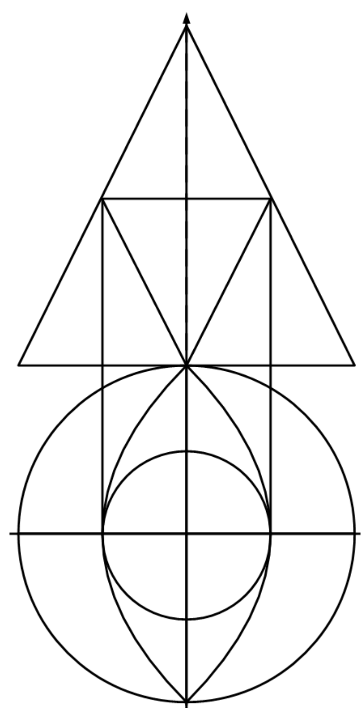

Figure 1.2 Conic section parabolas. $y^2 = 4\left(\frac{d}{2}\right)(x + \left(\frac{d}{2}\right))$

So to reiterate the equations thus far we have with $n = (1,2,3 \ldots \infty)$ and $d = (1,2,3, \ldots n)$:

- Equation 1.1 Unit circle encoding of n: $z_{nd} = 1 - \left(d\left(\frac{2}{n+1}\right)\right) + i\left(\frac{\sqrt{4d(n+1-d)}}{n+1}\right)$
- Equation 1.2 Spherical projections of z_{nd}: $s_{nd} = \left(\frac{n+1}{2}\right) - d + i\sqrt{d(n+1-d)}$
- Equation 1.3 Planar projections of z_{nd}: $p_{nd} = (n - d^2)/2d + i\sqrt{n}$
- Figure 1.2 Conic section parabolas. $iy^2 = 4\left(\frac{d}{2}\right)(x + \left(\frac{d}{2}\right))$

Let's focus on the $s_{nd} = x + iy$ equation right now. When s_{nd} is a Gaussian integer it is also a Pythagorean triple. These s_{nd} Gaussian integers follow the equation:

$$x = \left(\frac{dn^2 + d}{2}\right) - d$$

$$iy = nd$$

$$* \; when \; d \; is \; an \; odd \; integer \; then \; n \; must \; be \; also$$

Equation 1.4 Gaussian Integers: $g_{nd} = \left(\frac{dn^2+d}{2}\right) - d + i\,nd$

The Pythagorean triples $a^2 + b^2 = c^2$, are derived from g_{nd} by $a = iy$ and $b = x$, such that $c - b = d$. It's fairly obvious to see that when $d = 1$ and n is odd, then g_{nd} is a primitive Pythagorean triple. If $d = 1$ and n is $even$, then $2 * g_{nd}$ is a Pythagorean triple. It's no wonder then, that a scatter plot of Pythagorean triples faintly outline the same parabolic shape shown in **Figure 1.2.**

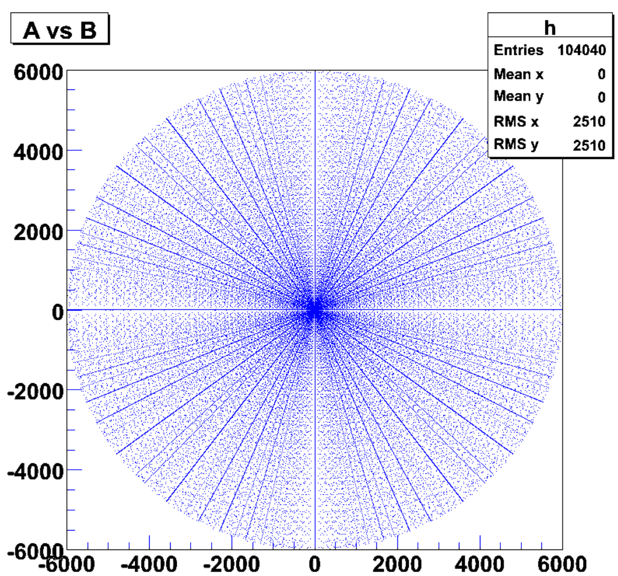

Figure 1.3 A scatter plot of the legs (*a*,*b*) of the Pythagorean triples with *c* less than 6000. Negative values are included to illustrate the parabolic patterns in the plot more clearly.

Microsoft Excel - New prime .xlsx

	E	F	G	H	J	K	L	M	O	P	Q	R	T	U	V	W	Y	Z	AA	AB
1	1	1	0	1	3	2	0	2	5	3	0	3	7	4	0	4	9	5	0	5
2	4	2	1.5	2.5	9	4	3	5	14	6	4.5	7.5	19	8	6	10	24	10	7.5	12.5
3	9	3	4	5	19	6	8	10	29	9	12	15	39	12	16	20	49	15	20	25
4	16	4	7.5	8.5	33	8	15	17	50	12	22.5	25.5	67	16	30	34	84	20	37.5	42.5
5	25	5	12	13	51	10	24	26	77	15	36	39	103	20	48	52	129	25	60	65
6	36	6	17.5	18.5	73	12	35	37	110	18	52.5	55.5	147	24	70	74	184	30	87.5	92.5
7	49	7	24	25	99	14	48	50	149	21	72	75	199	28	96	100	249	35	120	125
8	64	8	31.5	32.5	129	16	63	65	194	24	94.5	97.5	259	32	126	130	324	40	157.5	162.5
9	81	9	40	41	163	18	80	82	245	27	120	123	327	36	160	164	409	45	200	205
10	100	10	49.5	50.5	201	20	99	101	302	30	148.5	151.5	403	40	198	202	504	50	247.5	252.5
11	121	11	60	61	243	22	120	122	365	33	180	183	487	44	240	244	609	55	300	305
12	144	12	71.5	72.5	289	24	143	145	434	36	214.5	217.5	579	48	286	290	724	60	357.5	362.5
13	169	13	84	85	339	26	168	170	509	39	252	255	679	52	336	340	849	65	420	425
14	196	14	97.5	98.5	393	28	195	197	590	42	292.5	295.5	787	56	390	394	984	70	487.5	492.5
15	225	15	112	113	451	30	224	226	677	45	336	339	903	60	448	452	1129	75	560	565
16	256	16	127.5	128.5	513	32	255	257	770	48	382.5	385.5	1027	64	510	514	1284	80	637.5	642.5
17	289	17	144	145	579	34	288	290	869	51	432	435	1159	68	576	580	1449	85	720	725
18	324	18	161.5	162.5	649	36	323	325	974	54	484.5	487.5	1299	72	646	650	1624	90	807.5	812.5
19	361	19	180	181	723	38	360	362	1085	57	540	543	1447	76	720	724	1809	95	900	905
20	400	20	199.5	200.5	801	40	399	401	1202	60	598.5	601.5	1603	80	798	802	2004	100	997.5	1002.5
21	441	21	220	221	883	42	440	442	1325	63	660	663	1767	84	880	884	2209	105	1100	1105
22	484	22	241.5	242.5	969	44	483	485	1454	66	724.5	727.5	1939	88	966	970	2424	110	1207.5	1212.5
23	529	23	264	265	1059	46	528	530	1589	69	792	795	2119	92	1056	1060	2649	115	1320	1325
24	576	24	287.5	288.5	1153	48	575	577	1730	72	862.5	865.5	2307	96	1150	1154	2884	120	1437.5	1442.5
25	625	25	312	313	1251	50	624	626	1877	75	936	939	2503	100	1248	1252	3129	125	1560	1565
26	676	26	337.5	338.5	1353	52	675	677	2030	78	1012.5	1015.5	2707	104	1350	1354	3384	130	1687.5	1692.5
27	729	27	364	365	1459	54	728	730	2189	81	1092	1095	2919	108	1456	1460	3649	135	1820	1825
28	784	28	391.5	392.5	1569	56	783	785	2354	84	1174.5	1177.5	3139	112	1566	1570	3924	140	1957.5	1962.5
29	841	29	420	421	1683	58	840	842	2525	87	1260	1263	3367	116	1680	1684	4209	145	2100	2105
30	900	30	449.5	450.5	1801	60	899	901	2702	90	1348.5	1351.5	3603	120	1798	1802	4504	150	2247.5	2252.5
31	961	31	480	481	1923	62	960	962	2885	93	1440	1443	3847	124	1920	1924	4809	155	2400	2405
32	1024	32	511.5	512.5	2049	64	1023	1025	3074	96	1534.5	1537.5	4099	128	2046	2050	5124	160	2557.5	2562.5
33	1089	33	544	545	2179	66	1088	1090	3269	99	1632	1635	4359	132	2176	2180	5449	165	2720	2725
34	1156	34	577.5	578.5	2313	68	1155	1157	3470	102	1732.5	1735.5	4627	136	2310	2314	5784	170	2887.5	2892.5
35	1225	35	612	613	2451	70	1224	1226	3677	105	1836	1839	4903	140	2448	2452	6129	175	3060	3065
36	1296	36	647.5	648.5	2593	72	1295	1297	3890	108	1942.5	1945.5	5187	144	2590	2594	6484	180	3237.5	3242.5
37	1369	37	684	685	2739	74	1368	1370	4109	111	2052	2055	5479	148	2736	2740	6849	185	3420	3425
38	1444	38	721.5	722.5	2889	76	1443	1445	4334	114	2164.5	2167.5	5779	152	2886	2890	7224	190	3607.5	3612.5
39	1521	39	760	761	3043	78	1520	1522	4565	117	2280	2283	6087	156	3040	3044	7609	195	3800	3805
40	1600	40	799.5	800.5	3201	80	1599	1601	4802	120	2398.5	2401.5	6403	160	3198	3202	8004	200	3997.5	4002.5
41	1681	41	840	841	3363	82	1680	1682	5045	123	2520	2523	6727	164	3360	3364	8409	205	4200	4205
42	1764	42	881.5	882.5	3529	84	1763	1765	5294	126	2644.5	2647.5	7059	168	3526	3530	8824	210	4407.5	4412.5

Sheet20 Sheet21 Sheet22 Sheet23 Sheet24 Sheet25 Sheet16 Sheet26 Sheet27 Sheet28 Sheet29 Sheet30 Sheet31 Sheet32 Sheet33

Figure 1.4 Gaussian Integers: $g_{nd} = \left(\frac{dn^2+d}{2}\right) - d + i\,nd$

$*\ when\ d\ is\ an\ odd\ integer\ then\ n\ must\ be\ also$

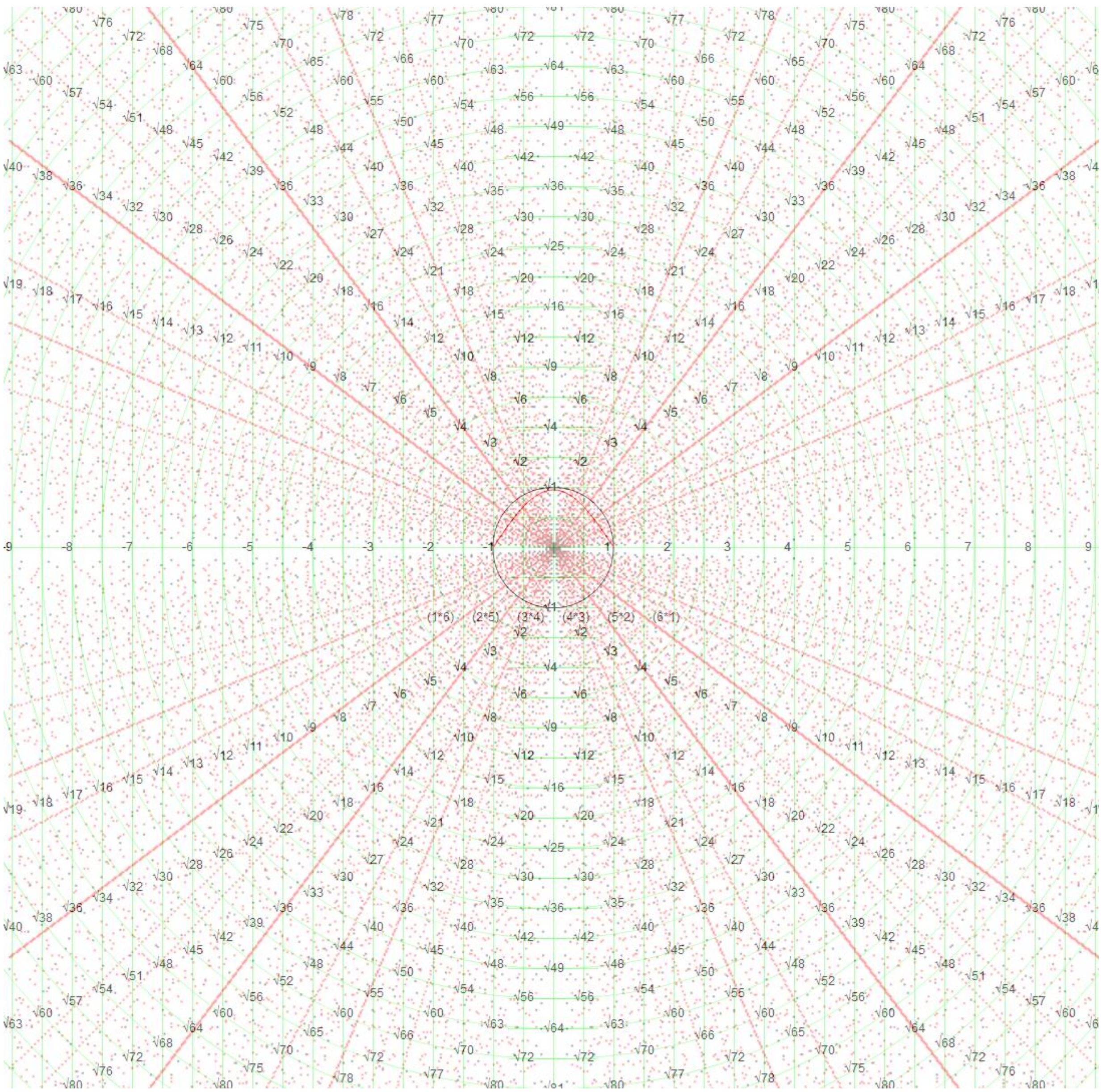

Figure 1.4 The Pythagorean triples scatter plot overlay of **Equation 1.2** spherical projections$s_{nd} = \left(\frac{n+1}{2}\right) - d + i\sqrt{d(n+1-d)}$. Notice the dark lines of the scatter plot highlight square number vectors.

Now let's take another look at **Equation 1.3** planar projections $p_{nd} = \frac{(n-d^2)}{2d} + i\sqrt{n}$. As n goes to infinity, p_{nd} carries a $locus$ that defines the parabolas created by the square root of multiples of d. When $\frac{(n-d^2)}{2d} = 0\ mod$ .5 then d is a factor of n so if n is prime then $d = 1\ or\ n$.

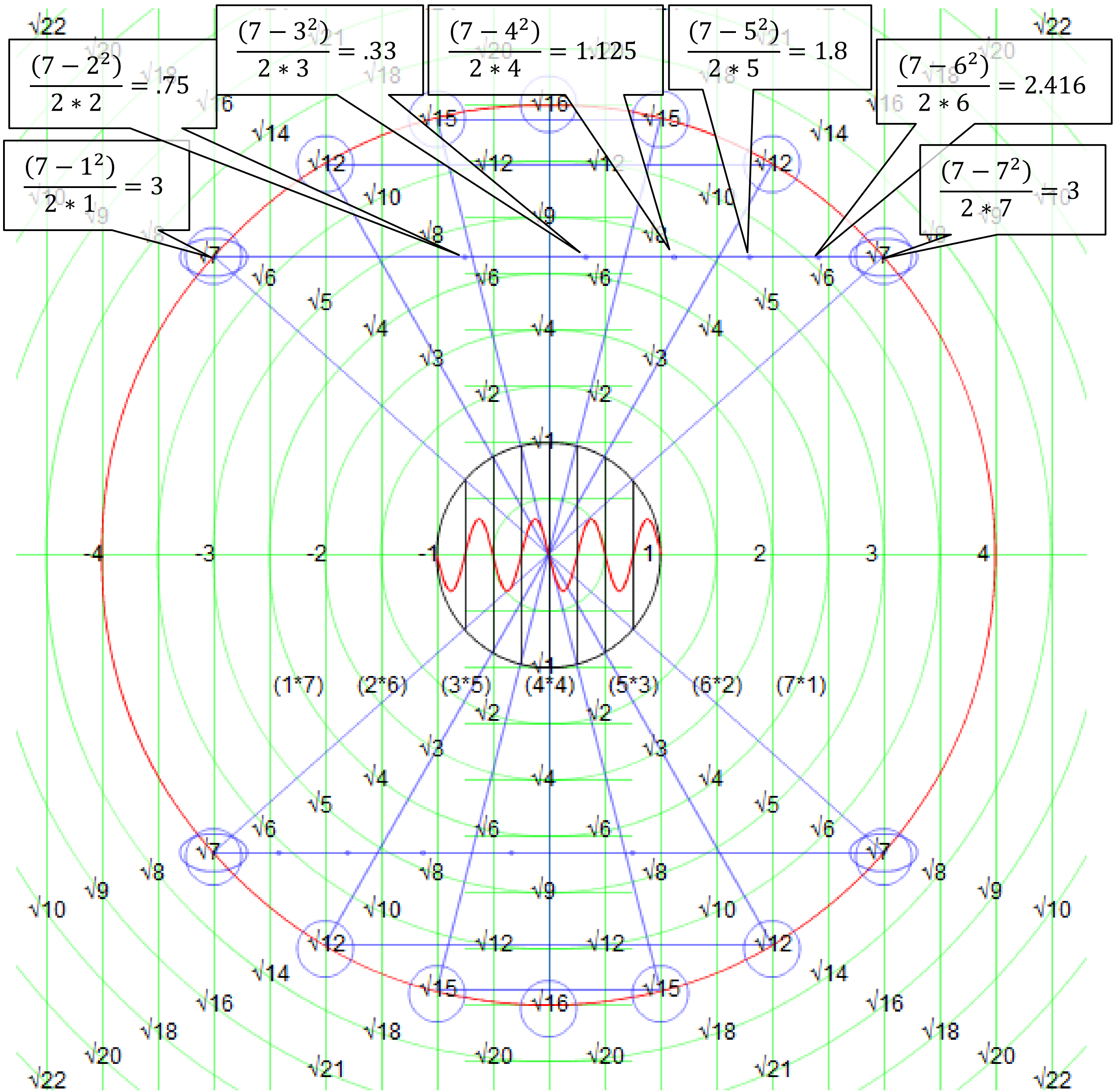

Figure 1.5 planar projections $p_{nd} = \frac{(n-d^2)}{2d} + i\sqrt{n}$ where n=7. Notice the collection of points defining the parabolas of d only intersect the lattice points at 1 and n when n is prime.

I would like to offer up a new type of lattice that seems to be the basis for these equations. I refer to this lattice as the Pythagorean lattice. The lattice points are created by a very simple moiré pattern derived from the intersections of evenly spaced concentric circles and lines.

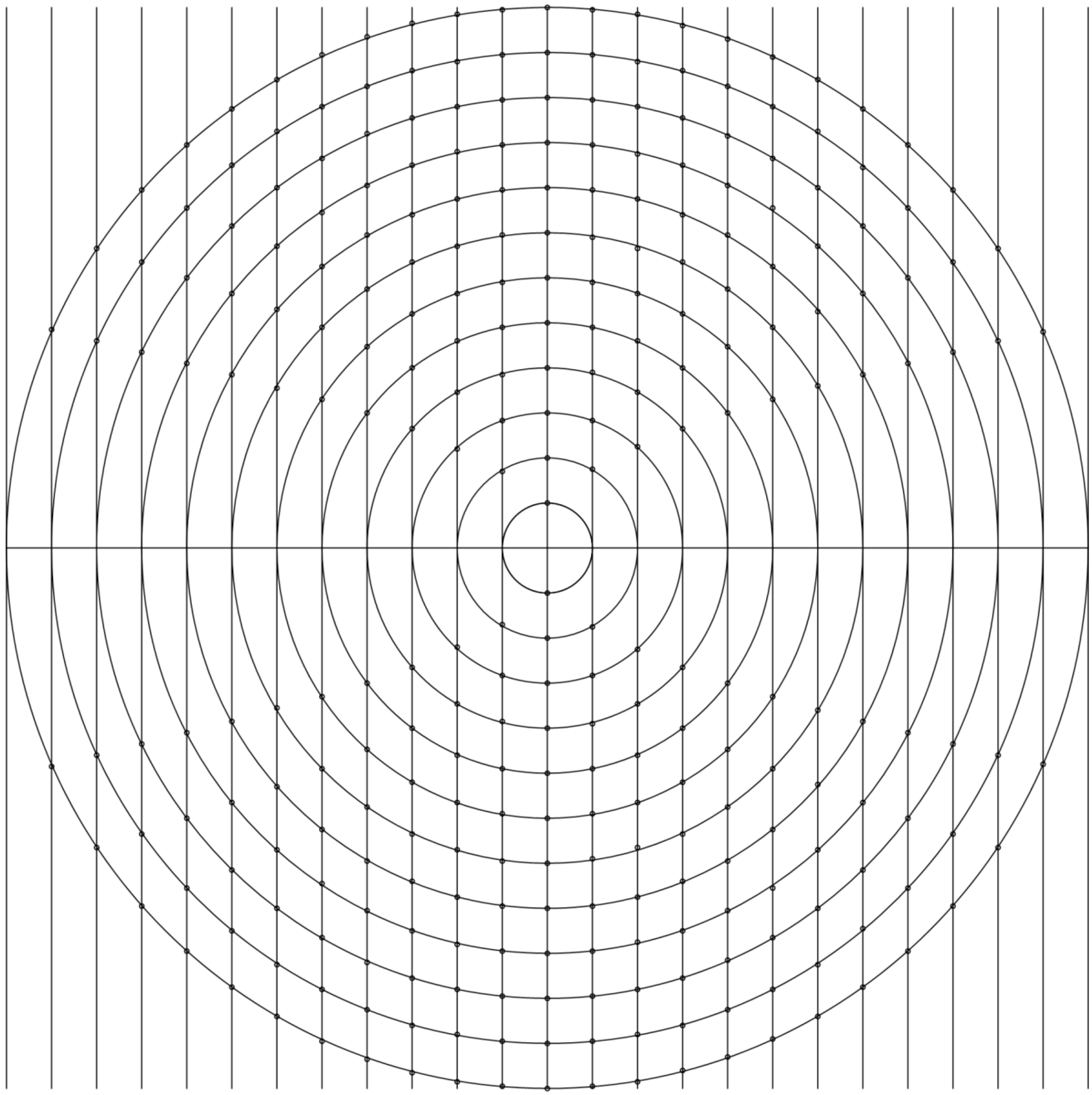

Figure 1.6 the Pythagorean Lattice

In the Pythagorean lattice the square root of primes numbers only intersect lattice points on the parabola with a vertex of ½ effectively defining all prime numbers. What is the connection to the non-trivial zeros of the Riemann zeta function?

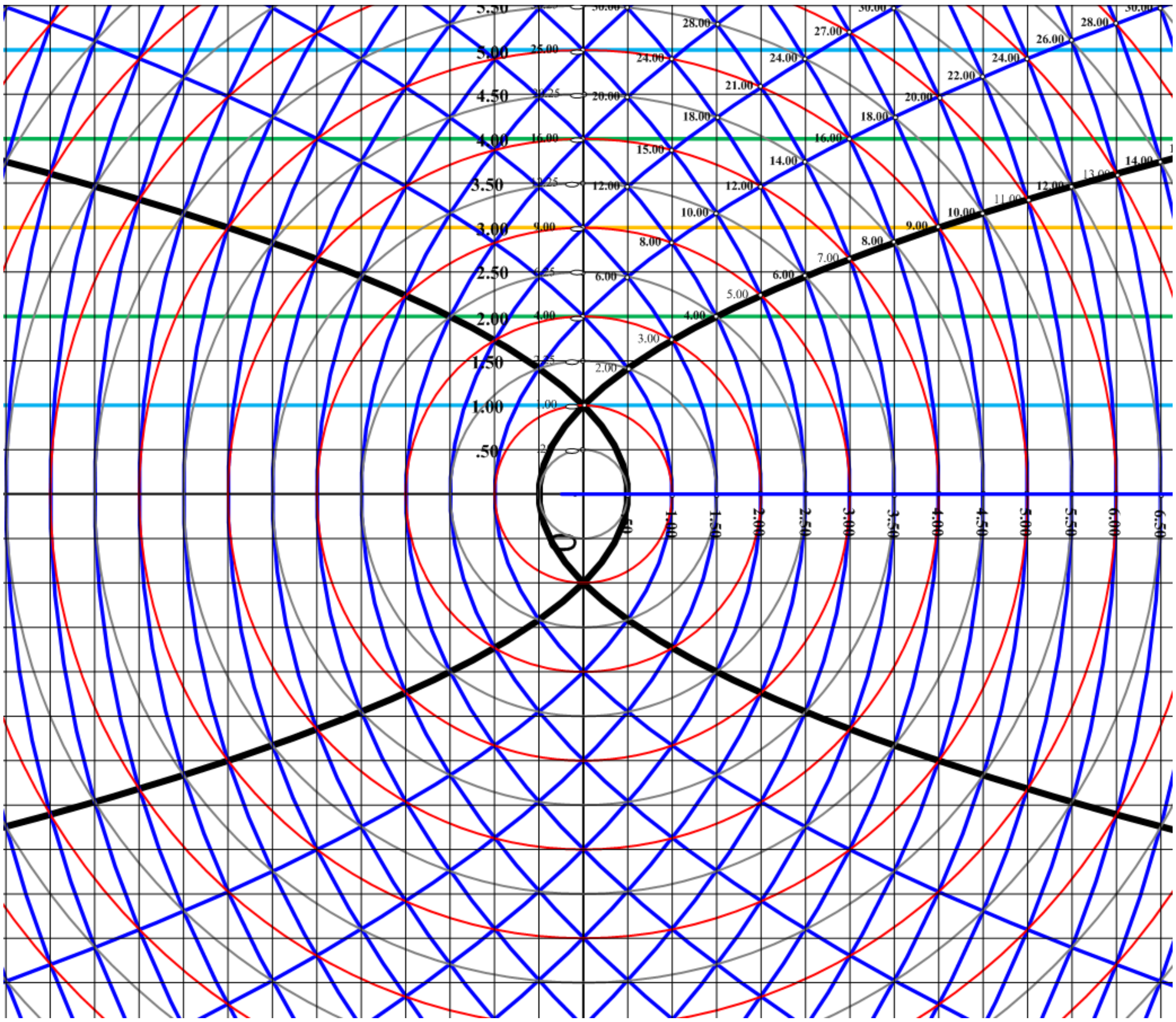

Figure 1.7 In the Pythagorean lattice the square root of prime numbers only intersect lattice points on the parabola with a vertex of ½ effectively defining all prime numbers.