Task: Fields of Trees

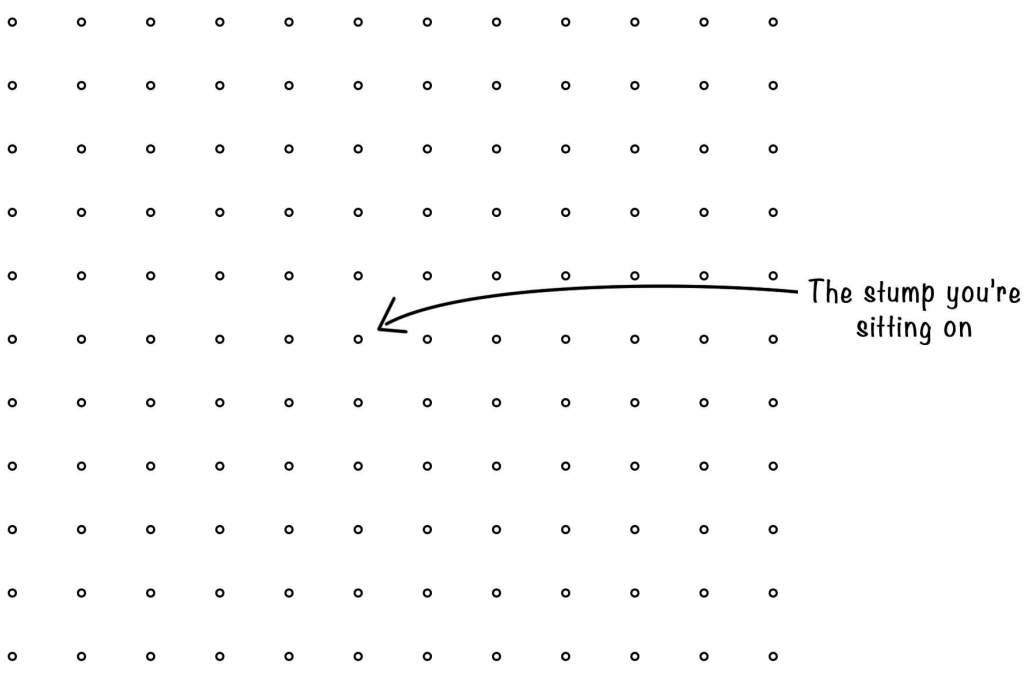
The difference between reality and an imaginary forest.

Set Up:

Suppose you're in a forest of trees planted on a perfect grid, and that the tree in the middle of the forest has been chopped to a stump and that you're sitting on that.

Challenge:

1. Is there a direction that you can look to see out of this forest?
   (hand out Forest Handout).



Answer: this depends on how thick our trees are and how far out the forest extends.

2. What if the trees get thinner and thinner, but the forest went on for further and further? That is, what if the trees were infinitely thin, but the forest went on forever in every direction? Is it possible to find a direction to look in that doesn't eventually bump into a tree?

   Answer: this is a subtle question! Rather than answering it though, guide thinking instead. Try asking:

   - What would stop a sightline?
     [Answer: a tree.]
   - What would it mean for a sightline to hit a tree?
     [Answer: the sightline runs through the origin, $(0, 0)$, and (A, B) where A and B are integers.]
   - Does that tell us something about the nature of the sightline slope?
     [Answer: yes, the slope would have to be rational, ie. a fraction.]
   - Are there any other slopes?
     [Answer: yes, every irrational slope would miss all the trees!]