Handout: Two Possibilities

This worksheet deals with the following two hypotheses:

(A) if two squares of the same colours are removed from a chessboard, the rest of the chessboard cannot be covered with dominoes.

(B) If two opposite colours are removed from a chessboard, we can cover the rest of the chessboard with dominoes.

Thinking about A:

1. What happens when we try tiling a board with two white squares removed? Try it!
2. Why are there always two black squares leftover?
3. How does this say something about hypothesis (A)?

Thinking about B:

This problem is a little subtler, and a Mathematician would look for a simple place to get a hold of a problem.

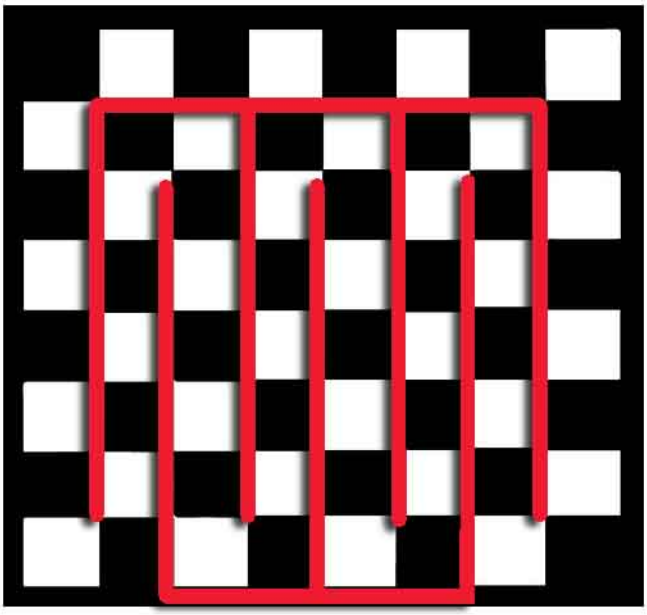
So where to start? How about a specific example?

4. What is the easiest example of two oppositely colored squares removed?
5. What if the two left-most squares on the top row were removed?
6. Find a more difficult example of two different colors?
   How about top left and top right corners removed?
7. What is the most difficult example of removing two different colored squares?

In general this sort of reasoning is great, but sometimes looking at a problem from a different perspective gives an insight that is wonderful . . .

sometimes you need to stare at it for a while . . .

8. Can you see how the following picture can be used to help find a way to cover this chessboard, regardless of which pair of black and white squares are removed?



[Hint: pick a black square and a white square and imagine them removed. Now try covering the rest of the board, staying off the red lines . . . Voila!!

Convince yourselves this will always work.]