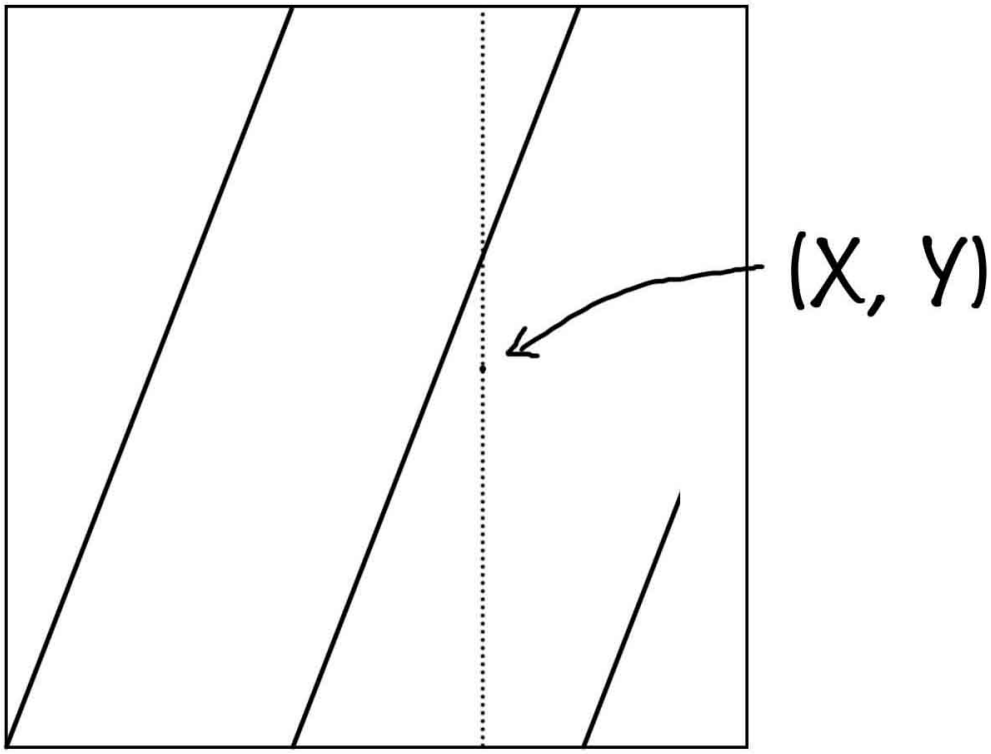
Handout: How close can you go without a collision?

Given any arbitrary point on our donut, lets call it the one represented by (X, Y) in our unit square, we know that our line $y = mx$ might not hit it, but if m is **irrational**, how close can we get?

1. Can we guarantee that $y = mx$ gets within, say, $1/5$ of (X, Y)?



   Let's break this into steps:

2. Show that there must be two points on the donut with first coordinate X and which are on the line $y = mx$, which are *also* within $1/5$ of each other? [Hint: What happens when the line in the above picture continues out the right of the box, again and again?]
3. How can we use the two points in (2) to find a point within $1/5$ of (X, Y)?
4. Was there anything special about $1/5$? Could we do this same argument for as small a piece as we'd like? $1/1000$ or $1/10000000$?
5. How close can we get to an arbitrary point (X, Y) with our line $y = mx$?