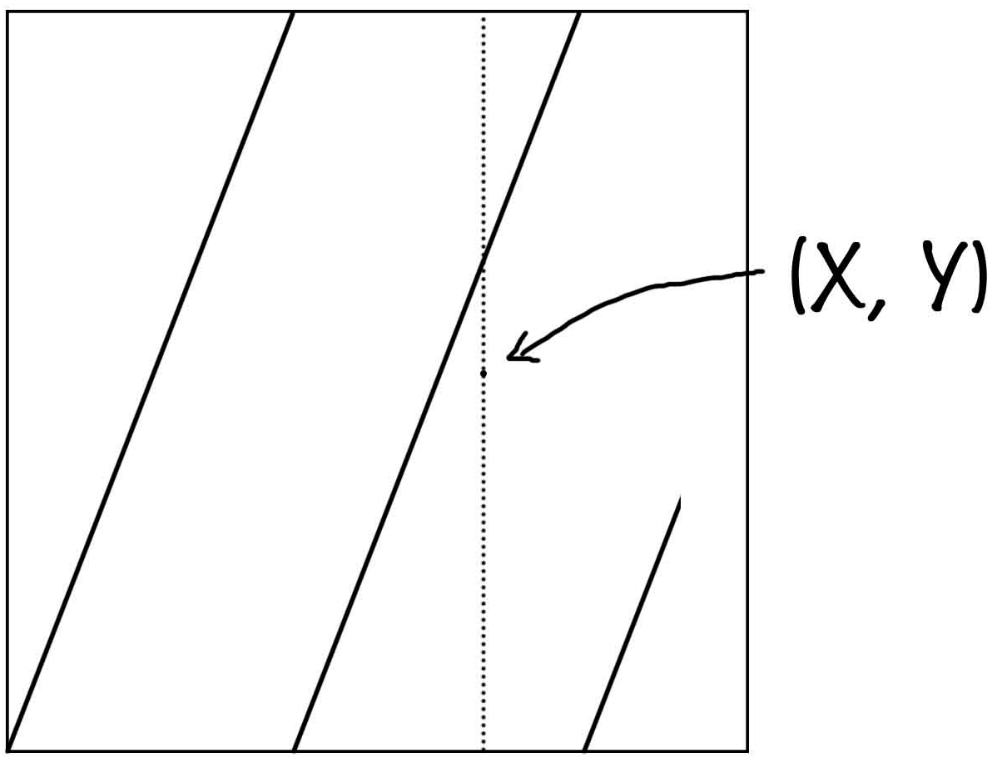
Task: How close can you go without a collision?

Given any arbitrary point on our donut, let's call it the one represented by (X, Y) in our unit square, we know that our line $y = mx$ might not hit it, but if m is **irrational**, how close can we get?

How can we visually guarantee that our line $y = mx$ must hit *many* points on the donut with X as the first coordinate? And how can we use these points to gaurantee $y = mx$ gets within, say, $1/5$ of (X, Y) on the donut, even if it never hits it? The following image is helpful:



Talking Points: Encourage students to think about ideas from the fourth Task, ie. divvy up the vertical line $x = X$ into fifths.

Some useful stepping stones include:

1. Is there a way to guarantee two points on the donut and the line $y = mx$ that *also* have first coordinate X and which are within $1/5$ of each other?

Answer: Divide $x = X$ into fifths and then trace $y = mx$ until it crosses $x = X$ six times!

2. How can we use the two points in (1) to find a point within $1/5$ of (X, Y) on the donut?

   Answer: leapfrog up or down until we get two points either side of (X, Y).

3. Was there anything special about $1/5$? Could we do this same argument for as small a piece as we'd like? $1/1000$ or $1/10000000$?

   Answer: nope, nothing special about 5.

4. How close can we get to an arbitrary point (X, Y) with our line $y = mx$?

   Answer: As close as we like!