Assignment 1

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1 Part A - Three People Crossing

Three art collectors, Alice, Bob and Carol are on the western bank of a river and want to get across. Each has recently purchased an artwork. Alice has a painting worth \$20,000, Bob has a vase worth \$12,000, and Carol has a sculpture worth \$8,000. The only boat has a capacity of two people, or one person and an artwork. They don't trust each other, but agree on a plan of travel such that the people in any particular location are never with art that is worth more than they collectively own.

1.a Notation

To illustrate this problem we will use A, B, and C to represent Alice, Bob, and Carol respectively. The art work will be represented by the cost value, in the thousands, respective to the order of the people. For example, if B and C's artwork are together on one section of the bank, the notation will be:

BC - \$12 and \$8

To represent the river as a whole we will use two square braces "[ ]". Left of this first square brace will represent the left bank and to the right of the second will be the right bank. In between will represent the river and identify anything in between the braces as on the boat.

Left of the river [On the River] Right of the River

1.b Initial & Goal State

With this notation established, the initial state will be:

ABC - $20, $12, $8 [ ] - - -

and the goal state:

- - - [ ] ABC - $20, $12, $8

1.c Allowable States

The allowable states are as follows:

- A - $12, $8 [ ] BC - $20
- BC - $12, $8 [ ] A - $20
- AC - $20, $8 [ ] B - $12
- AB -$20, $12 [ ] C - $8
- BC - $20 [ ] A - $12, $8
- A - $20 [ ] BC - $12, $8
- B - $12 [ ] AC - $20, $8
- C - $8 [ ] AB -$20, $12

1.d Solution

1. ABC - $20, $12, $8 [ ] - - -
2. AC - $20, $8 [B - $12] - - -
3. AC - $20, $8 [ ] B - $12
4. AC - $20, $8 [B] $12
5. ABC - $20, $8 [ ] $12
6. BC - $20 [A - $8] $12
7. BC - $20 [ ] A - $12, $8

8. BC - \$20 [A] \$12, \$ 8
9. ABC - \$20 [ ] \$12, \$8
10. A - \$20 [BC] \$12, \$8
11. A - \$20 [ ] BC - \$12, \$8
12. A - \$20 [C - \$8] B - \$12
13. AC - \$20, \$8 [ ] B \$12
14. C - \$8 [A - \$20] B - \$12
15. C - \$8 [ ] AB - \$20, \$12
16. C - \$8 [B - \$12] A - \$20
17. BC - \$12, \$8 [ ] A - \$20
18. \$12, \$8 [BC] A- \$20
19. \$12, \$8 [ ] ABC - \$20
20. \$12, \$8 [A] BC - \$20
21. A - \$12, \$8 [ ] BC - \$20
22. \$12 [A - \$8] BC - \$20
23. \$12 [ ] ABC - \$20, \$8
24. \$12 [B] AC - \$20, \$8
25. B - \$12 [ ] AC - \$20, \$8
26. - - - [B - \$12] AC - \$20, \$8
27. - - - [ ] ABC - \$20, \$12, \$8

1.e Additional People

Just as everyone gets to the eastern bank, Dave and Elisha arrive on the western bank wanting to cross. Dave has a statue worth $15000 and Elisha has glassware worth $7000.
Assuming the same constrains apply, it would not matter who we send back across the river because the problem is no longer solvable with this number of people.

1.f What must happen for the problem to be solvable?

In order for it to be solveable the boat must be large enough to hold the majority of people. In this case we would need a boat with a capacity of 3 since we have 5 people.

2 Part B - String Substitutions

Fred has a text file containing a string, for each move he can make any of the following substitutions: 1. B with AAA 2. AC with AAAA 3. AAA with BC 4. AAAAAB with AA

2.a Is it possible to turn AAA into AAABC? How, or why not?

Yes, it is possible to turn the string AAA into AAABC. The substitutions for this are as follows:

1. $\underline{\text{AAA}}$: AAA $\rightarrow$ BC
2. $\underline{\text{B}}\text{C}$: B $\rightarrow$ AAA
3. $\text{AA}\underline{\text{AC}}$: AC $\rightarrow$ AAAA
4. $\text{AAA}\underline{\text{AAA}}$: AAA $\rightarrow$ BC
5. AAABC

2.b Is it possible to turn a string of eight As into a string of 29 As? How, or why not?

Yes, it is possible to turn a string of eight As into a string of 29 As. The basic sequence for this is as follows:

1. AAAAA$\underline{\text{AAA}}$: AAA $\rightarrow$ BC
2. AAAAA$\underline{\text{B}}$C: B $\rightarrow$ AAA
3. AAAAAAAA$\underline{\text{AC}}$: AC $\rightarrow$ AAAA

The sequence adds three As to the string each time it is run. Completing this sequence seven times will create a string of 29 As.
This can also be seen algebraically. If x is equal to the number of times this sequence is run, we have:

$$29 = 8 + 3x \tag{1}$$

Solving this gives that x = 7. Therefore, the sequence must be run 7 times.

2.c Is it possible to turn a string of eleven As into a string of 31 As? How, or why not?

Yes, it is possible to turn a string of eleven As into a string of 31 As. This can be done using the same sequence from Section 2.2:

1. AAAAA$\underline{\text{AAA}}$: AAA $\rightarrow$ BC
2. AAAAA$\underline{\text{B}}$C: B $\rightarrow$ AAA
3. AAAAAAAA$\underline{\text{AC}}$: AC $\rightarrow$ AAAA

Since this sequence adds 3 As to the string each time it is run, we can run the sequence 7 times to get 31 As.

$$31 = 11 + 3x \tag{2}$$

Solving this gives that x = 7. Therefore, the sequence must be run 7 times.

3 Part C - Determinging O(g), $\Omega(g)$, or $\Theta(g)$

3.a

$f = \Theta(g)$

3.b

$f = O(g)$

3.c

$f = \Omega(g)$

3.d

$f = O(g)$

3.e

$f = \Theta(g)$

3.f

$f = \Omega(g)$

3.g

$f = O(g)$

3.h

$f = \Omega(g)$

3.i

$f = O(g)$

3.j

$f = O(g)$

3.k

$f = \Theta(g)$

3.l

$f = O(g)$

3.m

$f = \Theta(g)$

3.n

$f = \Theta(g)$

3.o

$f = \Omega(g)$

3.p

$f = O(g)$

3.q

$f = O(g)$

4 Show that, if c is a positive real number, then $g(n) = 1 + c + c^2 + ... + c^n$ is:

4.a $\Theta(1)$ if $c < 1$

With a geometric series $a + ar^2 + ar^3 + ... + ar^{n-1} = \frac{a}{1-r}$ when $r < 1$
So when $c < 1$, $g(n) = \frac{1}{1-c}$
$\therefore g(n) = \Theta(1)$ because $\frac{1}{1-c}$ is a constant.

4.b $\Theta(n)$ if $c = 1$

If $c = 1 \implies g(n) = n + 1$
$\lim_{n\to\infty} g(n) = \infty$ and grows at the rate of n
$\therefore g(n) = \Theta(n)$

4.c $\Theta(c^n)$ if $c > 1$

With a geometric series $a + ar^2 + ar^3 + ... + ar^{n-1} = \frac{a(r^n-1)}{r-1}$ when $r \neq 1$
If $c > 1 \implies g(n) = \frac{c^n-1}{c-1}$
$\lim_{n\to\infty} g(n) = \infty$ with $c^n >> c$
$\therefore g(n) = \Theta(c^n)$