60

$.60n \rightarrow$ lose \$1

$.30n \rightarrow$ win \$1

$.08n \rightarrow$ " \$2

$.02n \rightarrow$ win \$3

Total gain

$.60n \times (-1) + .30n(1) + .08n \times (2) + .02n(3) = -.08n$

$.30n \rightarrow$ win \$1

$.08n \rightarrow$ " \$2

$.02n \rightarrow$ win \$3

Total gain

$$\frac{.60n\times(-1) + .30n(1) + .08n\times(2) + .02n(3)}{n} = \frac{-.08n}{n}$$

$$.60\times(-1) + .30\times 1 + .08\times 2 + .02\times 3 = -.08$$

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not a fair game

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Prob. Values of the X

<u>not a fair</u> game

- Let X denote the gain in one play
- Then -0.08 is the expected value of X
- $E[X], \mu_X,$ or μ is the expected value of X

$\boxed{E[X],\ \mu,\ \mu_X}$

$\boxed{E(X)}$

Exa

x	-1	1	2	3	
$P_X(x)$	0.60	0.30	0.08	0.02	1.00
$x \cdot P_X(x)$	-0.60	0.30	0.16	0.06	-0.08

- Let X denote the gain in one play
- Then -0.08 is the expected value of X
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Expectation

$E[X]$, μ, μ_X

$\boxed{E(X)}$

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$$\boxed{E[X],\ \mu,\ \mu_X}$$

$$\boxed{E(X)}$$

Expectation

$$E[X] = \sum_{x \in S_X} x \cdot P_X(x)$$

x	-1	1	2	3	

hotel at which the conference is held offers a bargain rate of a dollars per day per person if reservations are made 45 or more days in advance, but charges a cancellation fee of 2a dollars per person. The department is not certain how many professors will go. However, from past experience it is known that the probability of the attendance of i professors is 1/5 for i = 8, 9, 10, 11, and 12. If the regular rate of the hotel is 2a dollars per day per person, should the department make any reservations? If so, how many?

Charge \$a Cancellation fee 2a

$X_i \triangleq$ Radom Variable rep. the cost if i professors attend of the attendance of i professors is $\boxed{1/5}$ for i = $\boxed{8, 9, 10, 11, \text{ and } 12.}$ If the regular rate of the hotel is 2a dollars per day per person, should the department make any reservations? If so, how many?

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Charge \$a Cancellation fee 2a

$X_i \triangleq$ Radom Variable rep. the cost if the reservation is made for i professor.

$E[X_B] = 5($

$= 60a$

$$E(X_B) = (40a)\frac{1}{5} + (50a)\frac{1}{5} + (60a)\frac{1}{5} + (70a)\frac{1}{5} + (80a)\frac{1}{5} = 60a$$

60

$$E(X_9) = (42a)\frac{1}{5} + (45a)\frac{1}{5} + (55a)\frac{1}{5} + (65a)\frac{1}{5} + (75a)\frac{1}{5} = 56.4a.$$

$$E(X_{10}) = \boxed{(44a)\frac{1}{5}} + (47a)\frac{1}{5} + (50a)\frac{1}{5} + (60a)\frac{1}{5} + (70a)\frac{1}{5} = 54.2a.$$

$$E(X_{11}) = (46a)\frac{1}{5} + (49a)\frac{1}{5} + (52a)\frac{1}{5} + (55a)\frac{1}{5} + (65a)\frac{1}{5} = 53.4a.$$

$$E(X_{12}) = (48a)\frac{1}{5} + (51a)\frac{1}{5} + (54a)\frac{1}{5} + (57a)\frac{1}{5} + (60a)\frac{1}{5} = 54a.$$

X_{11} has the smallest expected value. Therefore, making 11 reservation is the most reasonable policy

$Y = g(X)$. $S_Y = \{1, 4, 9\}$

$$P_X(x) = \begin{cases} .90 & x = 1 \\ .08 & x = 2 \\ .02 & x = 3 \\ 0 & o/w \end{cases}$$

and the random variable Y is called a derived random variable.

What will be the PMF of Y?

$P_Y(y) =$

$P[Y = 9] =$

$Y = g(X)$. $S_Y = \{1, 4, 9\}$

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$P[Y = 9] = P[X = 3] = .02$

P

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$P[Y=9] = P[X=3] = .02$

$P[Y=4] = .08$

$P[Y=1] = P[X=-1] + P[X=1] = .90$

$Y = g(X)$. $S_Y = \{1, 4, 9\}$ $P_X(x) = \begin{cases} .90 & x=1 \\ .08 & x=2 \\ .02 & x=3 \\ 0 & o/w \end{cases}$

and the random variable Y is called a derived random variable.

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$$P_Y(y) = \begin{cases} .90, & y=1 \\ .08, & y=4 \\ .02, & y=9 \\ 0, & o/w \end{cases}$$

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What will be the PMF of Y?

$$P_Y(y) = \begin{cases} .08, & y=4 \\ .02, & y=9 \\ 0, & o/w \end{cases}$$

$$E[Y] = 1 \times .9 + 4 \times .08 + 9 \times .02$$

$$P[Y=9] = P[X=3] = .02$$

$$P[Y=4] = .08$$

$$P[Y=1] = P[X=-1] + P[X=1] = .90$$

$$E[Y] = 1 \times .60 + 1 \times .30 + 4 \times .08 + 9 \times .02$$

$$E[Y] = \sum_{x} g(x) \cdot P_X(x)$$

For a discrete random variable Y, if there exists another random variable X, where $Y = g(X)$, then the PMF of X is

$$P_Y(y) = \sum_{x:g(x)=y} P_X(x)$$

$E[X + Y] = ?$ $E[X] + E[Y]$ E[

~~R 4~~

RV → Expectation of sum of RV = Sum of their individual expectation

$E[aX + b] = ?$

$E[aX+b] = aE[X]+b$

n-th Moment of a Random Variable

n-th Moment of a Random Variable

$Y = X^n$

$E[Y]$

1

$$E[X^n] = \sum_x x^n P_X(x)$$

$n=0, \quad E[X^0] = 1$

$n=1, \quad E[X] \rightarrow$ expected value

n

n-th Moment of a Random Variable

1

$Y = X^n$

$E[Y]$

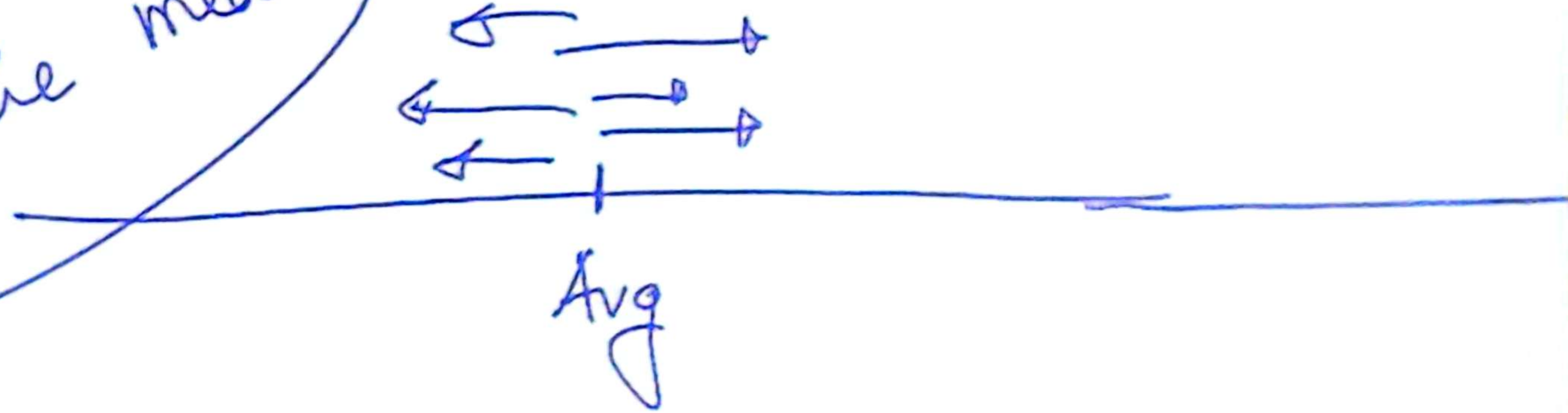
$$E[X^n] = \sum_x x^n P_X(x)$$

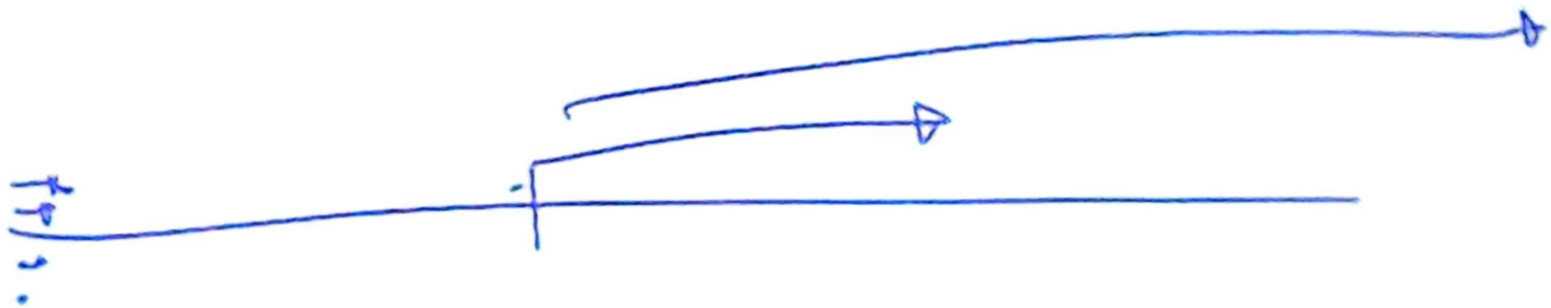
$n=0, \quad E[X^0] = 1$

$n=1, \quad E[X] \rightarrow$ expected value

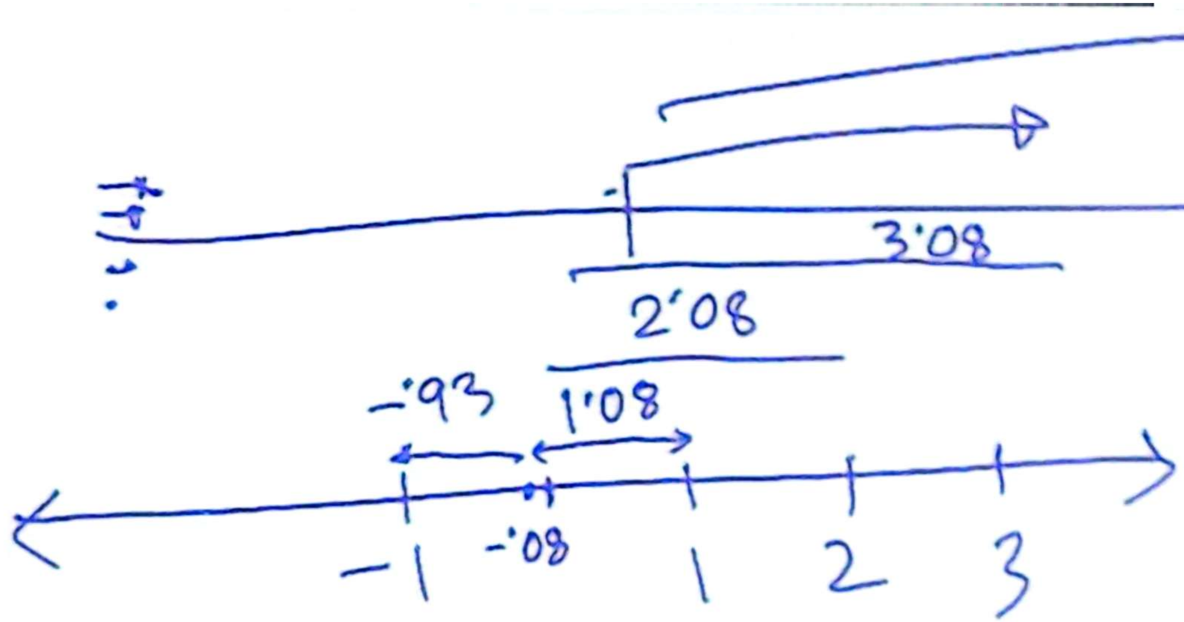
$n=2, \quad E[X^2] = \sum_x x^n \cdot P_X(x)$

Average fluctuation from the mean





Varia

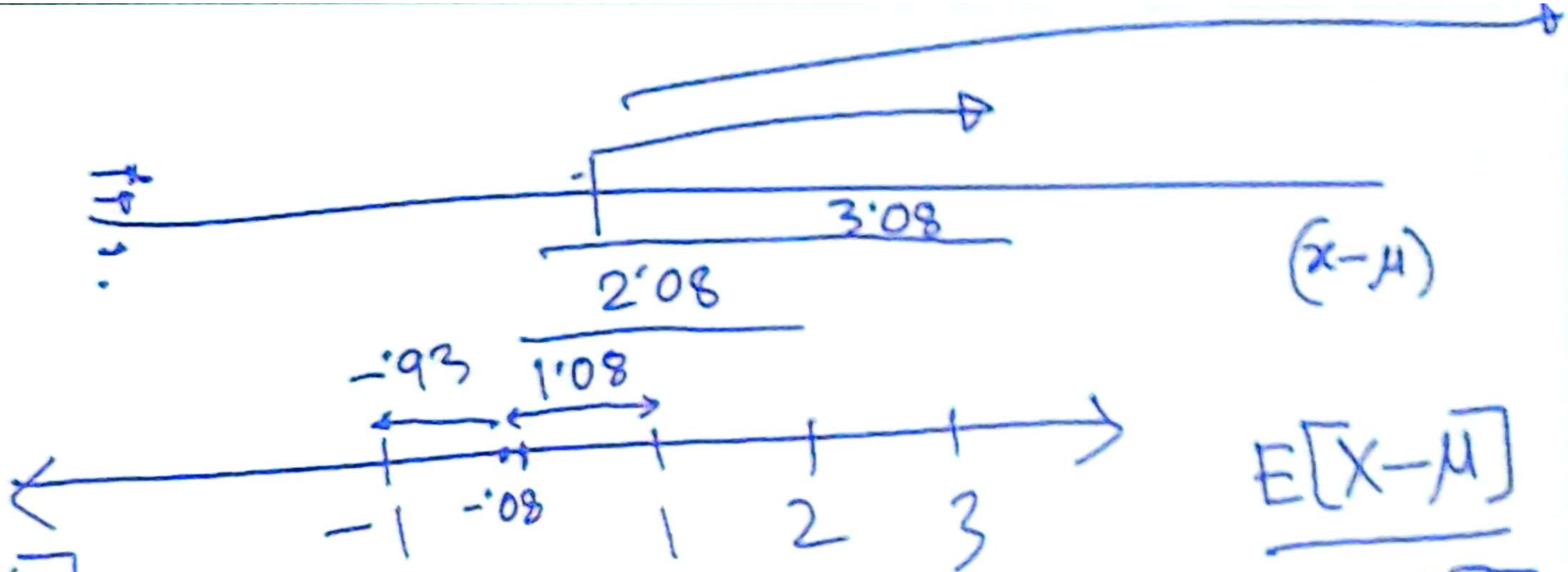


$(x-\mu)$

$$E[X-\mu]$$
$$= E[X] - E[\mu]$$
$$= \mu - \mu = 0$$

Variance: Variance measures the average magnitude of the fluctuations of a random variable from its expectations.

Varia



$E[X-\mu]$

$= E[X] - E[\mu]$

$= \mu - \mu = 0$

1. $E[|X-\mu|]$

2. $E[(X-\mu)^2]$ ✓

Variance: Variance measures the average magnitude of the fluctuations of a random variable from its expectations.

The quantity $E[(X - E[X])^2]$ is used to measure the average amount of fluctuations, and is called the variance of X

The square root of $E[(X - E[X])^2]$ is called the **standard deviation** of X.

$Var(X)$

$Var[X]$

$\underline{\underline{V[X]}}$ —

Definition 2.7. Let X be a discrete random variable with a set of possible values S_X, probability mass function $P_X(x)$, and $E(X) = \mu$. Then σ_X and $Var[X]$), called the

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$Var(X)$

$Var[X]$

$$V[X] = \sum_x (x-\mu)^2 P_X(x)$$

$Y = (X-\mu)^2$

$E[Y]$

$$= \sum_x (x-\mu)^2 P_X(x)$$

Definition 2.7. Let X be a discrete random variable with a set of possible values S_X, probability mass function $P_X(x)$, and $E(X) = \mu$. Then σ_X and $Var[X]$), called the

$$\begin{aligned}
&= E[X^2] - 2\mu E[X] + \mu^2 \\
&= E[X^2] - 2\mu^2 + \mu^2 = \boxed{E[X^2] - \mu^2} \\
&= E[X^2] - (E[X])^2.
\end{aligned}$$

$$V[X] = E[X^2] - (E[X])^2$$

2nd Moment

$Var[X] = 0$ if and only if X is a constant with probability 1.

$$\sigma_X = \sqrt{V[X]}$$

$\hookrightarrow$ standard deviation

$$Var[X] = E[X^2] - (E[X])^2$$

Proof.

$$\begin{aligned}
Var[X] &= E[(X-\mu)^2] \\
&= E[X^2 - 2\mu X + \mu^2] \\
&= E[X^2] - 2\mu E[X] + \mu^2 \\
&= E[X^2] - 2\mu^2 + \mu^2 = \boxed{E[X^2] - \mu^2} \\
&= E[X^2] - (E[X])^2
\end{aligned}$$

n-th Central Moment

$$E[(X-\mu)^n]$$

$n=0$, $E[(X-\mu)^0] = 1$

$n=1$, $E[\quad] = 0$

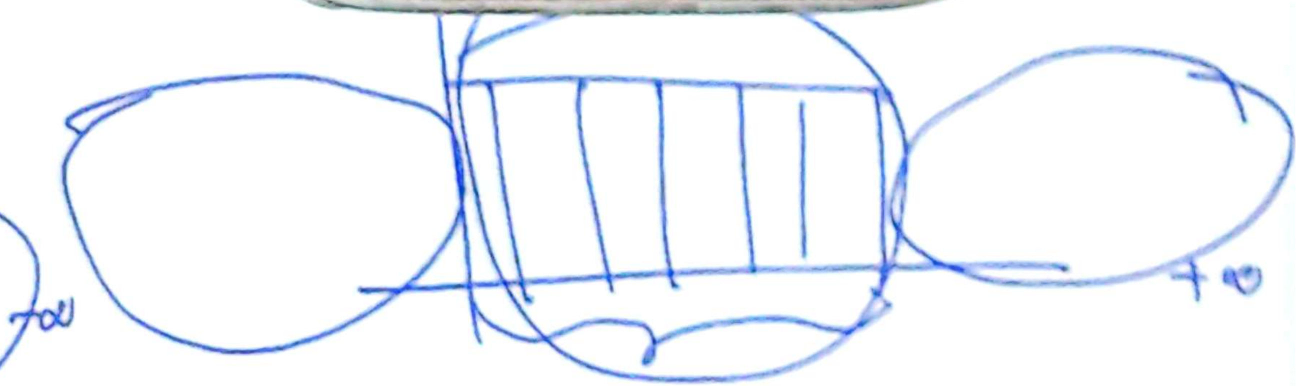
$n=2$, $E[\quad] = V[X]$

$n=3$, Skewness

$n=4$, Kurtosis

$$E[X] = \int_{-\infty}^{+\infty} x . f_X(x) dx$$

$$V[X] = \int_{-\infty}^{+\infty} (x - \mu)^2 . f_X(x) dx$$



Example: The time elapsed, in minutes, between the placement of an order of pizza and its delivery is random with pdf

$$f(x) = \begin{cases} 1/15 & \text{if } 25 < x < 40 \\ 0 & \text{otherwise} \end{cases}$$

Example: The time elapsed, in minutes, between the placement of ar pizza and its delivery is random with pdf

$$f(x) = \begin{cases} 1/15 & \text{if } 25 < x < 40 \\ 0 & \text{otherwise.} \end{cases}$$

$$E(X) = \int_{25}^{40} x \frac{1}{15} dx = 32.5.$$

$$E(X^2) = \int_{25}^{40} x^2 \frac{1}{15} dx = 1075.$$

$$V[X] = 1075 - (32.5)^2 = 18.75$$

$$\sigma_X = \sqrt{V[X]} = 4.33$$

Sketch the graph of the function

$$f(x) = \begin{cases} \dfrac{1}{2} - \dfrac{1}{4}|x - 3| & 1 \le x \le 5 \\ 0 & \text{otherwise.} \end{cases}$$

and show that it is the probability density function of a random variable X.

Find F, the distribution function of X, and show that it is continuous.

Find $E[X]$ and $V[X]$.