Qn. Set Code-1

Semester: 4th
Programme: B.Tech
Branch: CSSE

SPRING END SEMESTER EXAMINATION-2023

4th Semester B.Tech

DESIGN & ANALYSIS OF ALGORITHMS

CS-2012

(For 2022 (L.E), 2021 & Previous Admitted Batches)

Time: 3 Hours | Full Marks: 50

Answer any SIX questions.
Question paper consists of four SECTIONS i.e. A, B, C and D.
Section A is compulsory.
Attempt minimum one question each from Sections B, C, D.
The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable and all parts of a question should be answered at one place only.

SECTION-A

1. Answer the following questions. [1 × 10]

(a) Write a recurrence for the running time of fun(n) defined below, and find the solution of that recurrence. Assume that addition can be done in constant time.

```
int fun(int n)
{
   int x;
   if (n ==1)
      return 1;
   else
   {
     x = fun(n-1);
     return x+x;
   }
}
```

fun(5) returns _____ value.

(b) Is $n! = O(n^n)$? Justify your answer.

(c) State true or False.

i) In a connected, weighted graph, every lowest weight edge is always in some minimum spanning tree.

ii) If a problem X can be reduced to a known NP-hard problem, then X must be NP-hard.

(d) How will you check for a graph's acyclic with DFS and BFS?

(e) Given a big array, how to efficiently find k'th largest element in it?

(f) Compare insertion sort with quick sort.

(g) Choose the correct answer.
Given a sorted array of integers, what can be the minimum worst case time complexity to find ceiling of a number x in a given array (x may or may not be an element in the array)? The Ceiling of an element x is the smallest element present in array which is greater than or equal to x. Ceiling is not present if x is greater than the maximum element present in array. For example, if the given array is {10, 20, 30, 40, 50, 60} and x = 42, then output should be 50.
i) O(loglog n) ii) O(n log n) iii) O(log n) iv) $O(n^2)$
Justify your answer.

(h) What can be the best data structure to be used to find 10 maximum numbers from a big file containing billions of numbers? What is the worst case time complexity of this problem w.r.t the data structure used?

(i) Define optimal storage on tapes problem.

(j) Match the following:

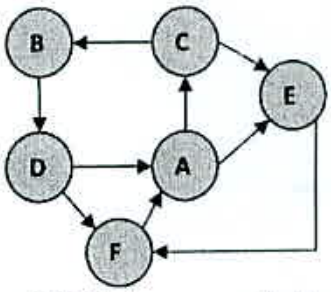
(P) Prim's algorithm for minimum spanning tree	(A) Backtracking
(Q) Floyd-Warshall algorithm for all pairs shortest paths	(B) Greedy method
(R) Merge sort	(C) Dynamic programming
(S) Sum of Subset problem	(D) Divide and conquer

SECTION-B

2. (a) Write a Θ(nlogn) algorithm to determine whether or not the elements of an array are unique. Analyze its overall-time complexity. [4]

(b) Find a solution to the recurrence [4]
$T(n) = 4T(n/2) + \Theta(n)$, $T(1)=1$

3. (a) Consider the following graph: [4]



a) Compute the DFS tree and draw the tree edges, forward edges, back edges and cross edges.
b) Write the order in which the vertices were reached for the first (i.e. pushed into the stack)
c) Write the order in which the vertices became dead ends (i.e. popped from the stack)

(b) Describe aΘ(n log n) time algorithm that, given a set S of n integers and another integer x, determines whether or not there exist two elements in S whose sum is exactly x. [4]

SECTION-C

4. (a) Use a dynamic programming algorithm to find the Longest Common Sub-sequence between the following two sequences: [4]

X = ababaabaa

Y = aababaabb

(b) Find an optimal solution to the fractional knapsack instance n=7, W=15. (v1, v2, v3, v4, v5, v6, v7) = (5, 15, 10, 7, 6, 20, 3) and (w1, w2, w3, w4, w5, w6, w7) = (2, 3, 4, 6, 1, 4, 1), where n is the number of items, W is the knapsack capacity that thief can carry, v_i stands for value or profit w_i stands for weight of the i^{th} element. [4]

5. (a) The operation HEAP-DELETE(A, i) deletes the item in node i from heap A. Give an implementation of HEAP-DELETE that runs in O(log n) time for an n-element max-heap. [4]

(b) State and explain union by rank and find-path compression algorithms of Dis-joint Data Structures with suitable examples. [4]

6. (a) Construct a Huffman code for the following data (show all the steps): [4]

Symbol	A	B	C	D
Frequency	0.1	0.1	0.4	0.3

How many bits are needed to encode a string containing 10 A's, 5 B's, 15 C's and 2 D's using this code. Compare this code with another code where each character is encoded with fixed two bits. Which code is better?

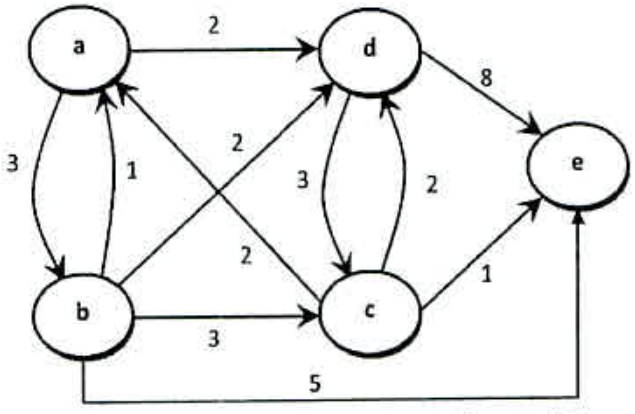
(b) Assume that you are given a chain of matrices < A1 A2 A3 A4 >, with dimensions 2x3, 3x4, 4x5 and 5x6 respectively. Compute the optimal number of multiplications required to calculate the chain product and also indicate what the optimal order of multiplication should be using parentheses. [4]

SECTION-D

7. (a) Devise a "Binary Search" algorithm that splits the set into two sets, one of which is twice the size of the other. How does this algorithm compare with standard binary search? [4]

(b) Insertion sort can be expressed as a recursive procedure as follows. In order to sort A[1…n], we recursively sort A[1..n-1] and then insert A[n] into the sorted array A[1..n-1]. Write the procedure and a recurrence for the running time of this recursive version of insertion sort. [4]

8. (a) Use suitable shortest path algorithm to find out shortest path from vertex 'a' to all other vertices. Show all the steps. [4]



(b) Describe P, NP and NPC class of problems through suitable examples. [4]

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