



SPRING END SEMESTER EXAMINATION-2019

4th Semester B.Tech & B.Tech Dual Degree

DESIGN AND ANALYSIS OF ALGORITHMS

CS-2008

(For 2018(L.E) & 2017 Admitted Batches)

Time: 3 Hours

Full Marks: 50

Answer any SIX questions.

Question paper consists of four sections-A, B, C, D.

Section A is compulsory.

Attempt minimum one question each from Sections B, C, D.

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable and all parts of a question should be answered at one place only.

SECTION-A

1. Answer the following questions. [1 × 10]
- (a) Rank the following functions by order of their growth in increasing sequence?
 $\log n$, $\log \sqrt{n}$, \sqrt{n} , $n \log \sqrt{n}$, $n!$, $2n$
- (b) Find out the complexity of the given function $\sum_{k=1}^n \log k$.
- (c) What is the maximum number of elements will not participate at 5th level of partitioning of an n-length array in Quick-Sort?
A. $n/4$ B. $2^4 - 1$ C. $2^5 - 1$ D. $2^4 + 1$
- (d) What is the worst case time complexity of Insertion-Sort where position of the data to be inserted into the sorted array is calculated using Linear-Search?
(A) n
(B) $n \log n$
(C) n^2
(D) $n^2 \log n$
- (e) Let s be a sorted array of n integers. Let $t(n)$ denotes the time taken for the most efficient algorithm to determine

if there are two elements with sum less than 1000 in s.
Which of the following statements is true?

- a) $t(n)$ is $O(1)$ b) $t(n)$ is $O(n \log n)$ c) $t(n)$ is $O(n)$
d) $t(n)$ is $O(n^2)$

- (f) Let $Z = \langle z_1, \dots, z_k \rangle$ be any Longest-Common-Subsequence of the sequences X and Y . Which of the following is (are) true?

- A. If $x_m = y_n$, then $z_k = x_m = y_n$ and Z is an LCS of X_m and Y_n .
B. If $x_m \neq y_n$, then $z_k \neq x_m$ and Z is an LCS of X_{m-1} and Y .
C. If $x_m \neq y_n$, then $z_k \neq y_n$ and Z is an LCS of X_{m-1} and Y .
D. If $x_m \neq y_n$, then $z_k \neq x_m$ and Z is an LCS of X and Y_{n-1} .

- (g) Differentiate between Divide-and-Conquer and Dynamic-programming approach.

- (h) Match the following Algorithms and its Complexities

- | | |
|-------------------|---------------------------------|
| (A) $O(\log n)$ | (a) Finding max-min in an array |
| (B) $O(n)$ | (b) Heap-sort |
| (C) $O(n \log n)$ | (c) Binary search |
| (D) $O(n^2)$ | (d) Insertion sort |

- (i) Which of the following statement is true about adjacency-list representation?

- Space complexity for both directed and undirected graphs is $O(V^2)$.
- Space complexity for directed graph is $O(V)$ and Space complexity for undirected graphs $O(E)$
- Space complexity for directed graph is $O(V^2)$ and Space complexity for undirected graphs $O(V+E)$
- Space complexity for directed graph is $O(V+E)$ and Space complexity for undirected graphs $O(V^2)$
- Space complexity for both directed graph and undirected graphs is $O(V+E)$

- (j) Which of the following is an NP-Complete Problem?

- A. 2CNF B. Euler tour C. Hamiltonian Cycle D. Shortest Path

SECTION-B

2. (a) What is the significance of asymptotic notations? Define different asymptotic notations used in algorithm analysis.

[4]

- (b) State and explain master's method, and use the method to give tight asymptotic bounds for the recurrence [4]

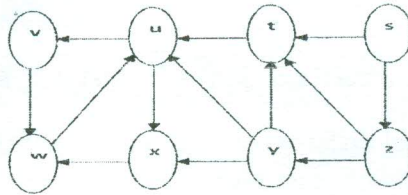
$$T(n) = 4T(n/2) + n^3.$$

3. (a) Solve the recurrence $T(n) = \sum_{i=1}^n T(i) + 1$, for $n \geq 2$. [4]
- (b) Define and differentiate between P, NP and NP-complete problems with examples. [4]

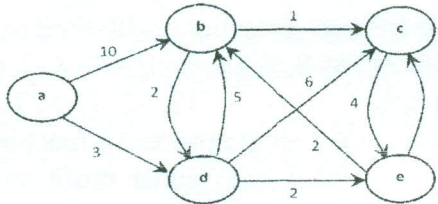
SECTION-C

4. (a) Given 10 different jobs along with their start time (s_i) and finish time (f_i) as $S_i = \langle 1, 2, 3, 4, 7, 8, 9, 9, 11, 12 \rangle$ and $f_i = \langle 3, 5, 4, 7, 10, 9, 11, 13, 12, 14 \rangle$. These jobs are to be scheduled on a single processor machine and all these jobs are associated with equal profit values. Write an efficient procedure to generate a schedule of these jobs on the machine to obtain maximum profit. [4]
- (b) Write the algorithm for MAX-HEAPIFY(A, i). Consider the array $A = \{27, 17, 3, 16, 13, 10, 1, 5, 7, 12, 4, 8, 9, 0\}$ to be used for constructing a Heap and answer the following questions. [4]
- i. Whether the Heap satisfies the Max-Heap property at each internal node.
- ii. If it is not then fix the Max-Heap property at those positions where it is not satisfying Max-Heap property.
5. (a) Write the algorithms to find maximum and minimum of an array using (i) Straightforward approach and (ii) Divide-And-Conquer approach. Find out the number of basic operations performed by these approaches and make a comparison analysis. [4]
- (b) State and explain the Longest Common Subsequence problem. Determine an LCS of the given two sequences $\langle a, b, b, a, b, a, b, a \rangle$ and $\langle b, a, b, a, a, b, a, b \rangle$. [4]

6. (a) Traverse the following graph by DFS technique with 's' as source vertex. Draw the DFS tree/forest and mention the DFS sequence. [4]



- (b) Use suitable shortest path algorithm to find out shortest path between a to c and a to e. [4]



SECTION-D

7. (a) Write a Quick-Sort algorithm that randomly selects an element as pivot element and derive the average case time complexity of this algorithm. [4]
(b) Devise a "Binary Search" algorithm that splits the set into two sets, one of which is twice the size of the other. How does this algorithm compare with standard binary search? [4]
8. (a) Given a sorted array with n distinct elements, we are required to find 3 such distinct elements whose sum is equal to 0. Design an efficient algorithm that implements the above mentioned requirements and analyze its time complexity. [4]
(b) Suppose a file to be transferred through the network contains the following characters with their number of occurrences as $\langle a: 15, b: 25, c: 5, d: 35, e: 20 \rangle$. Determine an efficient strategy that can minimize the total cost of transferring that file of 100 characters. Find out the total cost of transfer if transferring cost for 1-bit of data is 4 units. [4]

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