Qn. Set Code-1

Semester: 5^{th}
Programme: B.Tech
Branch: CSE, IT, CSCE, E&CSE

AUTUMN END SEMESTER EXAMINATION-2022

5^{th} Semester B.Tech

DESIGN AND ANALYSIS OF ALGORITHMS

CS2012

(For 2021 (L.E), 2020 & Previous Admitted Batches)

Time: 3 Hours | Full Marks: 50

Answer any SIX questions.
Question paper consists of four SECTIONS i.e. A, B, C and D.
Section A is compulsory.
Attempt minimum one question each from Sections B, C, D.
The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable and all parts of a question should be answered at one place only.

SECTION-A

1. Answer the following questions. [1 × 10]

(a) Suppose we are sorting an array of eight integers using heapsort, and we have just finished some heapify (either maxheapify or minheapify) operations. The array now looks like this: 16 14 15 10 12 27 28 How many heapify operations have been performed on root of heap?

(b) Find the worst case time complexity of quick sort and its recurrence.

(a) Time complexity is O(n^2) and recurrence is T(n) = T(n-2) + O(n)

(b) Time complexity is O(n^2) and recurrence is T(n) = T(n-1) + O(n)

(c) Time complexity is O(nLogn) and recurrence is T(n) = 2T(n/2)

(d) Time complexity is O(nLogn) and recurrence is T(n) = T(n/10) + T(9n/10) + O(n)

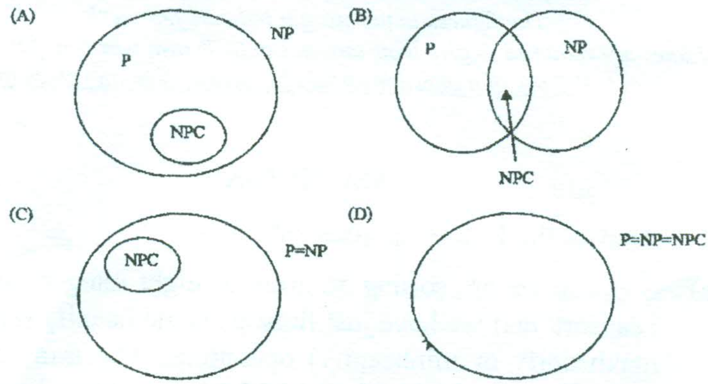
(c) Solve the following recurrence relation?

$T(n) = 7T(n/2) + 3n^2 + 2$

(d) Sort the following functions in the decreasing order of their asymptotic (big-O) complexity:

$f1(n) = n^{\sqrt{n}}$, $f2(n) = 2^n$, $f3(n) = (1.000001)^n$, $f4(n) = n^{(10)} * 2^{(n/2)}$

(e) Suppose a polynomial time algorithm is discovered that correctly computes the largest clique in a given graph. In this scenario, which one of the following represents the correct Venn diagram of the complexity classes P, NP and NP Complete (NPC)?



(f) Compute the minimum number of scalar multiplications required to multiply four matrices having dimensions 20 x 15, 15 x 30, 30 x 5 and 5 x 40
(a) 6050
(b) 7500
(c) 7750
(d) 12000

(g) Let us consider, two sequences "QPQRR" and "PQPRQRP". Determine the LCS of these sequences.
(a) QPRR
(b) PQRR
(c) QPQR
(d) All of the above

(h) Let us consider two problem A and B. The problem B is NP complete. The problem A reduces to problem B in polynomial time. Determine which of the following statement is correct?
(a) If A can be solved in polynomial time then B can also be solved in polynomial time
(b) A is NP complete problem
(c) A is NP hard problem
(d) A is in NP but not in NP complete

(i) Let us consider a file consists of six characters such as A, B, C, D, E, and F having probabilities of 1/2, 1/4, 1/8, 1/16, 1/32, and 1/32 respectively. Determine which of the following codes (huffman) for the letters A, B, C, D, E, and F?
(a) 0, 10, 110, 1110, 11110, 11111
(b) 11, 10, 01, 001, 0001, 0000
(c) 11, 10, 011, 010, 001, 000
(d) 110, 100, 010, 000, 001, 111

(j) Determine which of the following algorithm is based on the principle of dynamic programming.
(a) Floyd Warshll Algorithm for all pairs shortest path
(b) Dijkstra Algorithm for single source shortest paths
(c) Fractional Knapsack problem
(d) Prim's Minimum Spanning Tree

SECTION-B

2. (a) Find an optimal number of Scalar multiplication required of a MATRIX-CHAIN product whose sequence of dimensions are< 5,10,3,12,5,50,6>. Write the recursive procedure of matrix chain multiplication and its time complexity. [4]

(b) Use a dynamic programming algorithm to find the Longest Common Subsequence between the following two sequences: X = ababaabaa and Y = aababaab. Write the recursive procedure of LCS and its time complexity. [4]

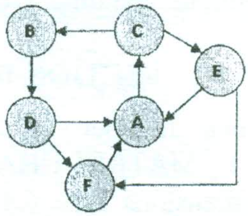
3. (a) What will be optimal Huffman code for the following set of symbol having given frequencies: A:14, B:19, C:40, E: 15, f,20. Draw the decode tree for both fixed and variable length encoding scheme for the above data. Explain which method compress more amount of data. [4]

(b) Find an optimal solution to the knapsack instance n=7, W=15. (v1, v2, v3, v4, v5, v6, v7) = (5, 15, 10, 7, 6, 20, 3) and (w1, w2, w3, w4, w5, w6, w7) = (2, 3, 5, 6, 1, 4, 1), where n is the number of items, W is the knapsack capacity that thief can carry, v_i stands for value or profit w_i stands for weight of the i^{th} element. [4]

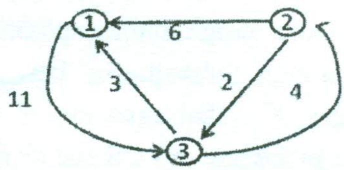
SECTION-C

4. (a) Consider the following graph and solve the followings [4]

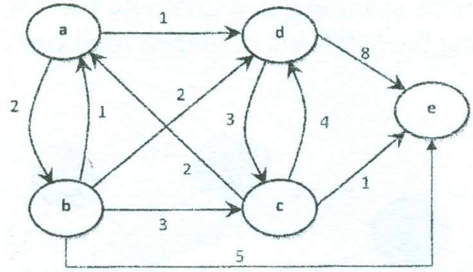
   a) Compute the DFS tree and draw the tree edges, forward edges, back edges and cross edges.
   b) Write the order in which the vertices were reached for the first (i.e.pushed into the stack)
   c) Write the order in which the vertices became dead ends (i.e. popped from the stack)



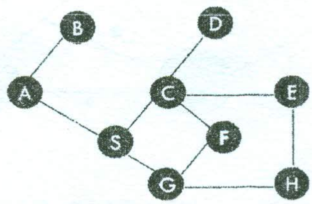
(b) Find all pair shortest path using Floyd Warshall algorithm for the following graph. [4]



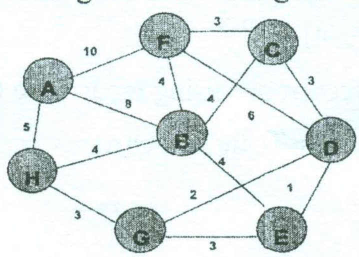
5. (a) Use suitable shortest path algorithm to find out shortest path from vertex 'a' to all other vertices. Show all the steps. [4]



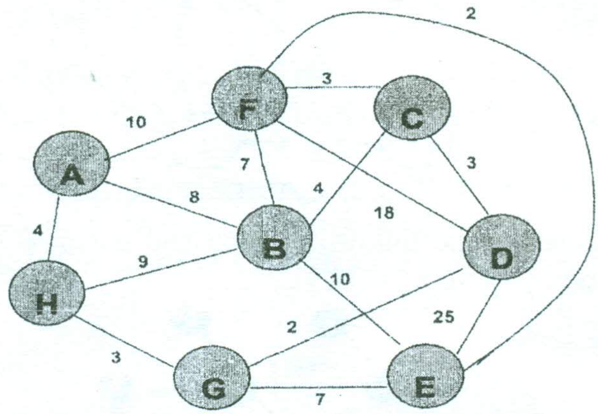
(b) Consider the following graph and compute the BFS tree. [4]



6. (a) A weighted graph G = (V, E)below has the sets V = {A, B, C, D, E, F, G, H} of vertices and connected with certain number of edges as per the below diagram. Apply Kruskal algorithm on this graph to build its minimum spanning tree. Perform all steps of the main Kruskal's for-loop for the edges in increasing cost: (i) indicate the disjoint Sets (ii) If the edge is added, list all sets of vertices after their merging (iii) Finally, list all the edges forming the MST and give their total cost. [4]



(b) A weighted graph G = (V, E) below has the sets V = {A, B, C, D, E, F, G, H} of vertices and connected with certain number of edges as per the below diagram. Apply Prim's algorithm on this graph to build its minimum spanning tree.Compute the list all the edges forming the MST and give their total cost. [4]



SECTION-D

7. (a) Assuming $n = 3^m$ with the integer $m = \log_3 n$, compute the time complexity of $T(n)$ by solving the recurrence $T(n) = 3T(n/3) + 6$ with the base condition $T(1) = 0$. [4]

(b) Solve the recurrence using recurrence tree method [4]

$$T(n) = 3T(n/4) + \Theta(n^2)$$

8. (a) Solve the following recurrence relation using Master's theorem- [4]

$$T(n) = 2T(n/4) + n^{0.51}$$

(b) Solve the recurrence using recurrence tree method [4]

$$T(n) = T(n/3) + T(2n/3) + O(n)$$

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