norm

Vector and matrix norms

Syntax

```
n = norm(v)
n = norm(v,p)

n = norm(X)
n = norm(X,p)
n = norm(X,'fro')
```

Description

n = norm(v) returns the Euclidean norm of vector v. This norm is also called the 2-norm, vector magnitude, or Euclidean length. example

n = norm(v,p) returns the generalized vector p-norm. example

n = norm(X) returns the 2-norm or maximum singular value of matrix X, which is approximately max(svd(X)). example

n = norm(X,p) returns the p-norm of matrix X, where p is 1, 2, or Inf: example

- If p = 1, then n is the maximum absolute column sum of the matrix.
- If p = 2, then n is approximately max(svd(X)). This is equivalent to norm(X).
- If p = Inf, then n is the maximum absolute row sum of the matrix.

n = norm(X,'fro') returns the Frobenius norm of matrix X. example

Examples

collapse all

∨ Vector Magnitude

Create a vector and calculate the magnitude.

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```
v = [1 -2 3];
n = norm(v)
```

```
n = 3.7417
```

∨ 1-Norm of Vector

Calculate the 1-norm of a vector, which is the sum of the element magnitudes.

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```
X = [-2 3 -1];
n = norm(X,1)
```

```
n = 6
```

Euclidean Distance Between Two Points

Calculate the distance between two points as the norm of the difference between the vector elements.

Create two vectors representing the (x,y) coordinates for two points on the Euclidean plane.

```
a = [0 3];
b = [-2 1];
```

Use norm to calculate the distance between the points.

```
d = norm(b-a)
```

```
d = 2.8284
```

Geometrically, the distance between the points is equal to the magnitude of the vector that extends from one point to the other.

$$a = 0\widehat{i} + 3\widehat{j}$$
$$b = -2\widehat{i} + 1\widehat{j}$$

$$
\begin{aligned}
d_{(a,b)} &= ||b - a|| \\
&= \sqrt{(-2-0)^2 + (1-3)^2} \\
&= \sqrt{8}
\end{aligned}
$$

2-Norm of Matrix

Calculate the 2-norm of a matrix, which is the largest singular value.

```
X = [2 0 1;-1 1 0;-3 3 0];
n = norm(X)
```

```
n = 4.7234
```

Frobenius Norm of Sparse Matrix

Use `'fro'` to calculate the Frobenius norm of a sparse matrix, which calculates the 2-norm of the column vector, S(:).

```
S = sparse(1:25,1:25,1);
n = norm(S,'fro')
```

```
n = 5
```

Input Arguments

collapse all

v — **Input vector**
vector

Input vector.

Data Types: single | double
Complex Number Support: Yes

X — **Input matrix**
matrix

Input matrix.

Data Types: single | double
Complex Number Support: Yes

p — **Norm type**
2 (default) | positive integer scalar | Inf | -Inf

Norm type, specified as 2 (default), a different positive integer scalar, Inf, or -Inf. The valid values of p and what they return depend on whether the first input to norm is a matrix or vector, as shown in the table.

> **i** **Note**
>
> This table does not reflect the actual algorithms used in calculations.

p	Matrix	Vector
1	max(sum(abs(X)))	sum(abs(X))
2	max(svd(X))	sum(abs(X).^2)^(1/2)
Positive, real-valued numeric p	—	sum(abs(X).^p)^(1/p)
Inf	max(sum(abs(X')))	max(abs(X))

p	Matrix	Vector
-Inf	—	min(abs(X))

Output Arguments

⌄ **n — Matrix or vector norm**
   scalar

Matrix or vector norm, returned as a scalar. The norm gives a measure of the magnitude of the elements. By convention, norm returns NaN if the input contains NaN values.

More About

⌄ **Euclidean Norm**

The Euclidean norm (also called the vector magnitude, Euclidean length, or 2-norm) of a vector v with N elements is defined by

$$\|v\| = \sqrt{\sum_{k=1}^{N} |v_k|^2} \; .$$

⌄ **General Vector Norm**

The general definition for the p-norm of a vector v that has N elements is

$$\|v\|_p = \left[\sum_{k=1}^{N} |v_k|^p \right]^{1/p},$$

where p is any positive real value, Inf, or -Inf. Some interesting values of p are:

- If p = 1, then the resulting 1-norm is the sum of the absolute values of the vector elements.
- If p = 2, then the resulting 2-norm gives the vector magnitude or Euclidean length of the vector.
- If p = Inf, then $\|v\|_\infty = \max_i(|v(i)|)$.
- If p = -Inf, then $\|v\|_{-\infty} = \min_i(|v(i)|)$.

⌄ **Maximum Absolute Column Sum**

The maximum absolute column sum of an m-by-n matrix X (with m,n >= 2) is defined by

$$\|X\|_1 = \max_{1 \le j \le n} \left(\sum_{i=1}^{m} |a_{ij}| \right).$$

⌄ **Maximum Absolute Row Sum**

The maximum absolute row sum of an m-by-n matrix X (with m,n >= 2) is defined by

$$\|X\|_\infty = \max_{1 \le i \le m} \left(\sum_{j=1}^{n} |a_{ij}| \right).$$

⌄ Frobenius Norm

The Frobenius norm of an `m`-by-`n` matrix X (with `m,n >= 2`) is defined by

$$\|X\|_F = \sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n}|a_{ij}|^2} = \sqrt{\operatorname{trace}(X^\dagger X)}\ .$$

Tips

- Use `vecnorm` to treat a matrix or array as a collection of vectors and calculate the norm along a specified dimension. For example, `vecnorm` can calculate the norm of each column in a matrix.

Extended Capabilities

> **Tall Arrays**
Calculate with arrays that have more rows than fit in memory.

> **C/C++ Code Generation**
Generate C and C++ code using MATLAB® Coder™.

> **GPU Arrays**
Accelerate code by running on a graphics processing unit (GPU) using Parallel Computing Toolbox™.

> **Distributed Arrays**
Partition large arrays across the combined memory of your cluster using Parallel Computing Toolbox™.

See Also

cond | condest | hypot | normalize | normest | rcond | vecnorm

Introduced before R2006a