[Total No. of Questions - 9] [Total No. of Printed Pages - 4]

MAY-24-1045

MCA-6104 (Discrete Mathematical Structures)

MCA-1st CBCS/NEP

Time : 3 Hours

Max. Marks : 60

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note: Attempt five questions in all, selecting one question each from Section A, B, C and D. Section E is compulsory.

SECTION - A

1. Prove the validity of the following argument using rules of inference:

"If today is Tuesday, then I have a test in mathematics or physics. If my physics sir is sick then I do not have test in physics. Today is Tuesday and my physics sir is sick. Therefore, I have a test in mathematics." (12)

2. Find $\cup_{i=1}^{\infty} A_i$ and $\cap_{i=1}^{\infty} A_i$ if for every positive integer i,

a. $A_i=[-i, i]$, that is, the set of real numbers x with $-i \leq x \leq i$. (6)

b. $A_i = [i, \infty]$, that is, the set of real numbers x with $x \geq i$. (6)

SECTION - B

3. a. How many numbers must be selected from the set {1, 2, 3, 4, 5, 6} to guarantee that at least one pair of these numbers add up to 7? [Use Pigeonhole Principle] (6)

b. A bag contains 2 white balls, 3 black balls and 4 red balls. In how many ways can 3 balls be drawn from the bag if at least one black ball is to be included in the draw? (6)

4. Suppose C_n, the average number of companies made by the quick sort algorithm, when sorting n elements in random order, satisfy the following recurrence relation;

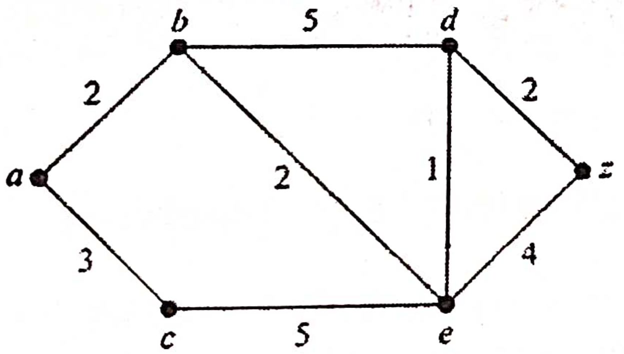
$$C_n = -3C_{n-1} - 3C_{n-2} - C_{n-3}$$

with the initial conditions, $c_0 = 1$, $c_1 = -2$ and $c_2 = -1$.

Solve the recurrence relation to find an explicit formula for C_n. (12)

SECTION - C

5. Find a shortest path between a and z in the given weighted graph. Also, find the length of the shortest path. (12)

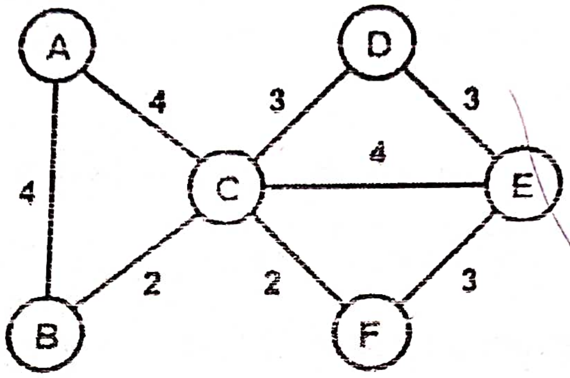


6. Suppose there are seven final assessment tests to be scheduled in VIT-AP university. Suppose the courses are numbered 1 through 7. Suppose that the following pairs of courses have common students: 1 and 2, 1 and 3, 1 and 4, 1 and 7, 2 and 3, 2 and 4, 2 and 5, 2 and 7, 3 and 4, 3 and 6, 3 and 7, 4 and 5, 4 and 6, 5 and 6, 5 and 7, and 6 and 7. Using graph colouring, find that how can the final exams at a university be scheduled so that no student has two exams at the same time? (12)

[P.T.O.]

SECTION - D

7. Six blocks A, B, C, D, E, F of a University are separated by a distance as given by the graph below:



Find the minimal distance connecting all the blocks using Both Kruskal's and Prim's Algorithm. (12)

8. Let $\mathbb{R}$ be a group of all real numbers under addition and $\mathbb{R}^+$ be a group of all positive real numbers under multiplication. Check whether the mapping, f: $\mathbb{R}$ to $\mathbb{R}^+$, defined by $f(x) = 2x$, for all x in $\mathbb{R}$ is homomorphism or not? (12)

SECTION - E (Compulsory)

9. (i) Translate the statement, "All tools are in the correct place and are in excellent condition." into logical expressions using predicates, quantifiers, and logical connectives.

(ii) Let f be the function from {a, b, c, d} to {1, 2, 3,4} with f(a) = 4, f(b) – 2, f(c) = 1, and f(d) = 3. Is f one-one and onto function?

(iii) A student can choose a computer project from one of three lists. The three lists contain 23, 15, and 19 possible projects, respectively. No project is on more than one list. How many possible projects are there to choose from, using basic counting principle?

(iv) Give a recursive definition of a^n, where a is a nonzero real number and n is a non-negative integer.

(v) Define a binary tree with an example.

(vi) Give two reasons why the set of odd integers under addition is not a group. (6×2=12)