Assignment-1

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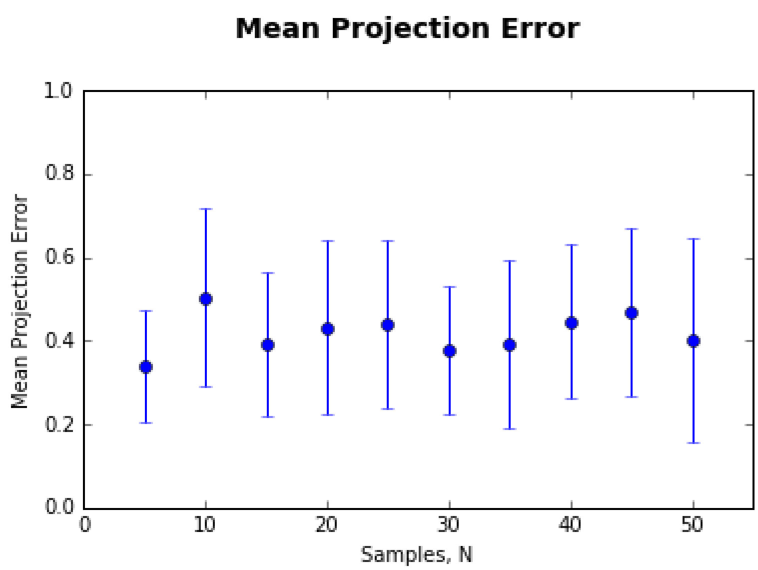
Ques 1.

A. Code description - Two files of Question1_A and Question1_B ipython notebooks are provided. The camera calibration toolbox of opencv python is used. Both the codes needs to be run in same folder as dataset. Set1 folder contains chosen images and Set2 folder contains first 20 images.
B. The notebook Question1_A generates the Intrinsic parameters, Extrinsic parameters and distortion parameters.Extrinsic parameters are also reported.
C. The notebook Question1_B randomly samples of images are picked up. For each sample, standard deviation is calculated. Plot of the same is shown below.

$$camera\ matrix = \begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$Distortion\ coefficients = (k_1 \quad k_2 \quad p_1 \quad p_2 \quad k_3)$$

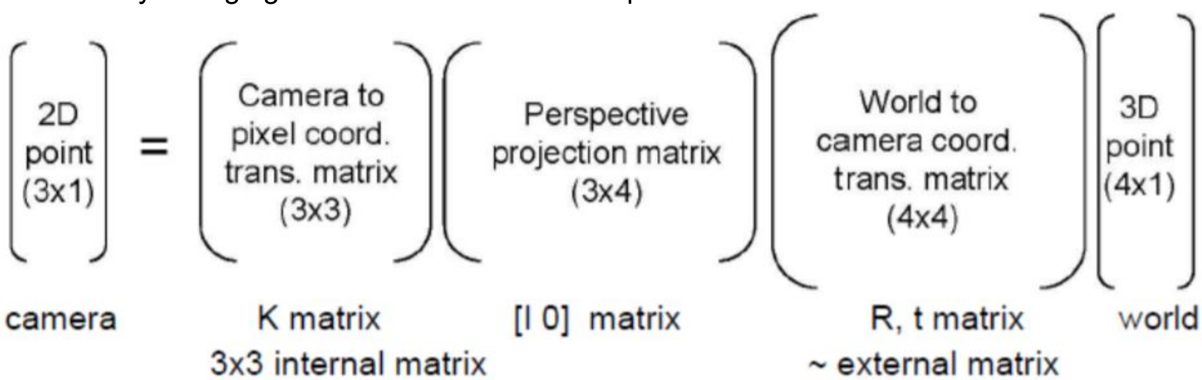


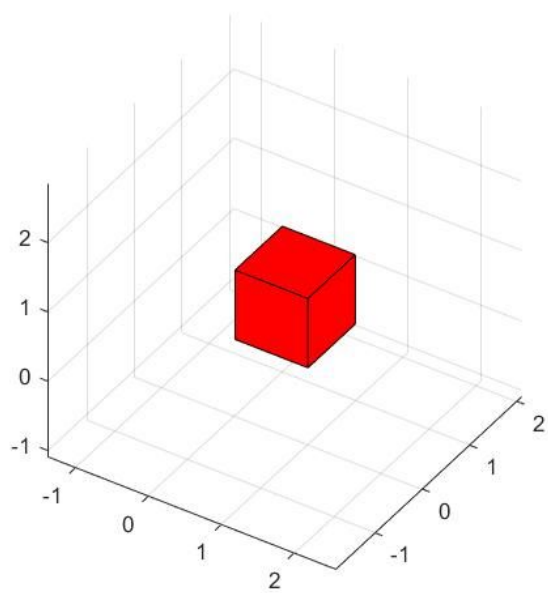
Notebooks are attached for the same. Dataset is kept in the system.

Ques 2.

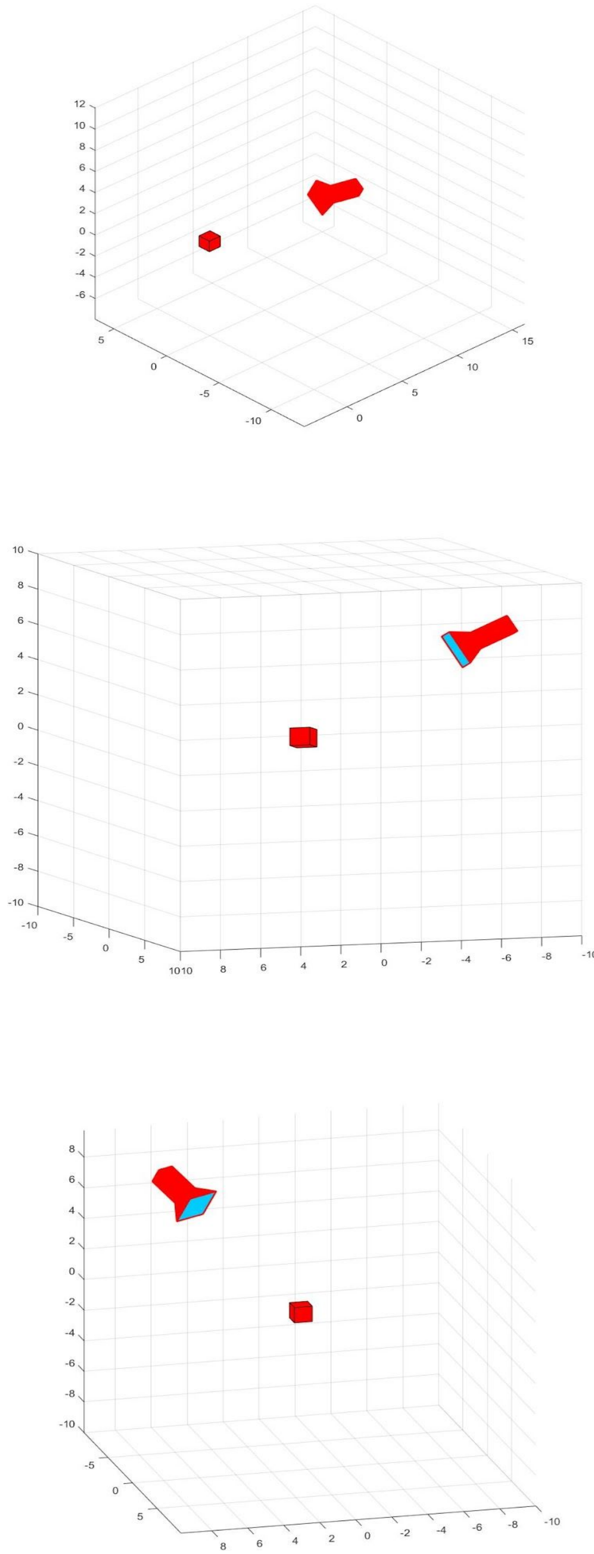
Code is made in following order.

1. The transformation and code is explained in Question2.m code.
2. The 2D image and 3D scenes are shown below.
3. Transformation matrix is calculated in code itself.
4. After transformation matrix, camera coordinates are found out.
5. Scenes are generated.
6. Normalized image coordinates are taken out. Likewise pixel coordinates can be taken out by changing values of camera intrinsic parameters.





2-D view of cube

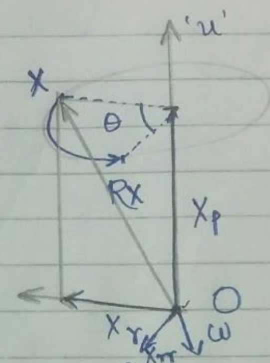


3-D views of camera and cube

Q→3.

There is a vector 'x'.
We rotate the vector 'X' anti-clockwise about axis 'u' (unit vector).

$Rx \equiv$ New vector coordinates.
$R \equiv$ rotation matrix.



$$X_p = (u \cdot x)\, u \quad \text{——— ①}$$

Vector projection = dot product of vector x with unit vector 'u' in direction of 'u'.

Now, $X_r = X - X_p$ ——— ②

Let's create vector 'ω'
$\omega = u \times x$ ——— ③
ω has same length as X_r

Perform 2D rotation of vector X_r in plane $X_r\,\omega$.

$X_{rr} = X_r \cos\theta + \omega \sin\theta$. ——— ④

Finally $Rx = X_{rr} + X_p$
Putting values ~~in~~ of ①, ②, ③, ④ in above equation

$$Rx = (x - (u\cdot x)u)\cos\theta + (u \times x)\sin\theta + (u\cdot x)u$$

$$Rx = x\cos\theta - (u\cdot x)u\cdot\cos\theta + (u\times x)\sin\theta + (u\cdot x)u$$

$$Rx = x\cos\theta + (u\cdot x)u(1-\cos\theta) + (u\times x)\sin\theta.$$

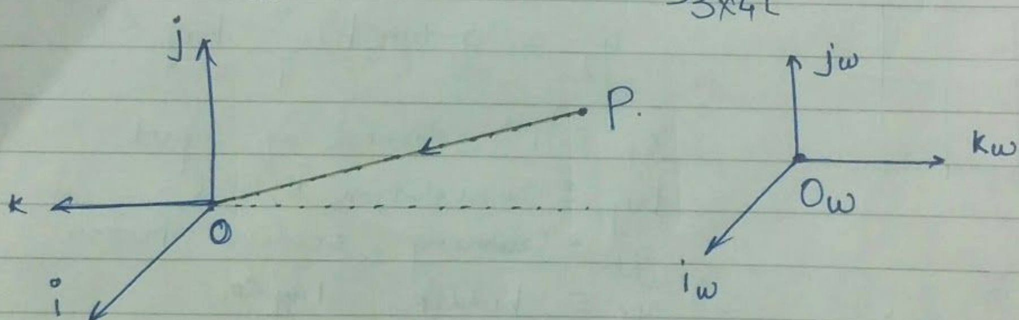
Q.4. Let 'O' denote homogenous coordinate vector of optical center of camera in world reference frame.

'M' is perpestive projection matrix.

'M' includes world to camera transformation.

$$\therefore \quad O = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$M = \begin{bmatrix} \text{Perspective projection} \\ \text{matrix} \\ \\ \end{bmatrix}_{3\times4} \begin{bmatrix} \text{World to camera} \\ \text{trans matrix} \\ \\ \end{bmatrix}_{4\times4}$$



World to camera transformation matrix includes rotation and translation

$$MO = \begin{bmatrix} \text{Perspective projection} \\ \text{matrix} \\ \\ \end{bmatrix}_{3\times4} \begin{bmatrix} \text{World to} \\ \text{camera} \\ \\ \end{bmatrix}_{4\times4} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

No rotation takes place. Just the translation.

By assumption, we take.

$$\begin{bmatrix} \text{World to} \\ \text{camera} \\ \text{trans. matrix} \end{bmatrix}_{4\times4} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4\times4}.$$

Translating to camera coordinates, optical center.

$$\begin{bmatrix} 1 & 0 & 0 & -x \\ 0 & 1 & 0 & -y \\ 0 & 0 & 1 & -z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$MO = \begin{bmatrix} \text{Perspective Proj.} \\ \text{Matrix} \end{bmatrix}_{3\times4} \begin{bmatrix} 1 & 0 & 0 & -x \\ 0 & 1 & 0 & -y \\ 0 & 0 & 1 & -z \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4\times4} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}_{4\times1}$$

$I|0$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$MO = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Intuition

This is obvious because the image is <u>not</u> uniquely defined at optical center.