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MT15046

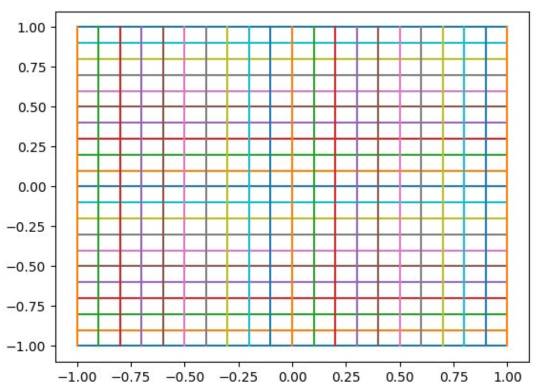
Projective Geometry

Assignment 4

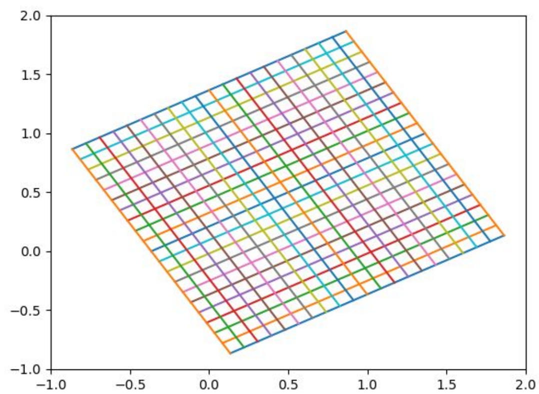
Ques 1 -

Plots

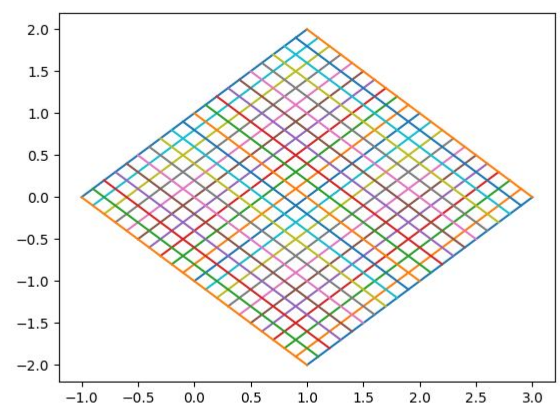
Initial plot



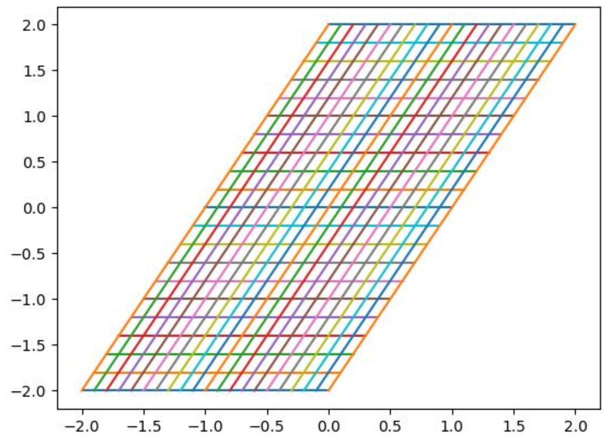
a) Matrix 1



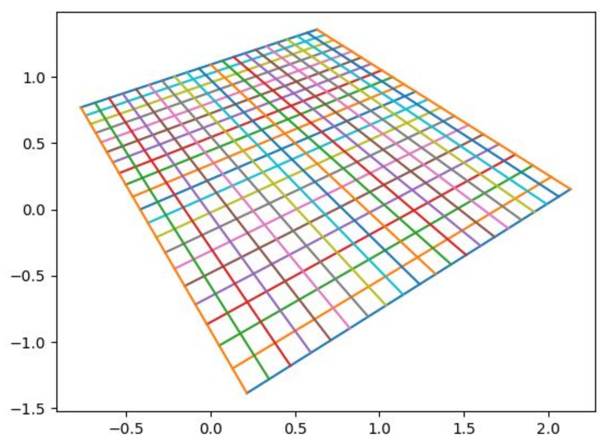
b) Matrix 2



c) Matrix 3



d) Matrix 4



b)

- length between points

Matrix 1 will preserve the property, because M1 is a euclidean transform. It just rotates and translates the images.

- angle between lines

Matrix 1 and Matrix 2 will preserve the property, because M1,M2 are euclidean and similarity transform for the images. Similarity transform scales,rotates,translates(reverse order) the image. So there is no change in angle between lines.

- maps parallel lines to parallel lines.

M1,M2,M3 will preserve the property, because they are euclidean, similarity, affine transforms. So the parallel lines remain parallel. The final images are reported, so each property can be seen.

c) M1 - euclidean

M2 - Similarity

M3- Affine

M4 - Projective

The decomposition of M4 in Similarity,Affine,Projective =

$H = H_s * H_a * H_p$.

$$= \begin{bmatrix} sR & t \\ 0^T & 1 \end{bmatrix} \begin{bmatrix} K & 0 \\ 0^T & 1 \end{bmatrix} \begin{bmatrix} I & 0 \\ v^T & v \end{bmatrix} = \begin{bmatrix} A & t \\ v^T & v \end{bmatrix}$$

$H_s \quad H_A \quad H_P$

$A = sRK + tv^T$

we know that $\det K = 1$

$s > 0$ & $v \neq 0$

$$H = \begin{pmatrix} \sqrt{3} & -1 & 1 \\ 1 & \sqrt{3} & 1 \\ 1/4 & 1/2 & 2 \end{pmatrix}$$

, we know $V_T = [1/4 \ 1/2$

$$t = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} \sqrt{3} & -1 \\ 1 & \sqrt{3} \end{bmatrix}$$

$$tv^T = \begin{bmatrix} 1/4 & 1/2 \\ 1/4 & 1/2 \end{bmatrix}$$

$A - tv^T = sRK$.

$$\begin{bmatrix}\sqrt{3} & -1\\ 1 & \sqrt{3}\end{bmatrix} - \begin{bmatrix}1/4 & 1/2\\ 1/4 & 1/2\end{bmatrix} = sRK$$

$$\begin{bmatrix}1.4 & -1.5\\ 0.75 & 1.2\end{bmatrix} = sRK$$

$$\text{Det}\begin{pmatrix}1.4 & -1.5\\ 0.75 & 1.2\end{pmatrix} = \det(sRK)$$

$$2.9 = s^2\, \underbrace{|R|}_{1}\overbrace{|K|}^{1}$$

$$s = \sqrt{2.9}$$

$$|R\cdot K| = 1$$

$$\left|\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}K_1 & K_2\\ 0 & K_3\end{pmatrix}\right| = 1$$

Date: ....

$$\begin{bmatrix} sR & t \\ 0^T & 1 \end{bmatrix} \begin{bmatrix} k_1 & k_2 & 0 \\ 0 & k_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1/4 & 1/2 & 2 \end{bmatrix} = \begin{bmatrix} \sqrt{3} & -1 & 1 \\ 1 & \sqrt{3} & 1 \\ 1/4 & 1/2 & 2 \end{bmatrix}$$

Solving them, using online calculator

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$s = \sqrt{2.9}$$

$$H_s \begin{bmatrix} \sqrt{2.9}\cos(28.5) & -\sqrt{2.9}\sin(28.5) & 1 \\ \sqrt{2.9}\sin(28.5) & \sqrt{2.9}\cos(28.5) & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$H_A \begin{bmatrix} 1.8 & -0.5 & 0 \\ 0 & 0.4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$H_p \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1/4 & 1/2 & 2 \end{bmatrix}$$

Ques 2)

Affine rectification -

Method

Parallel lines remain parallel after affine distortion.

Logic → Apply transformation matrix H_1 that maps vanishing lines back into line at infinity $\vec{l}_\infty$.

Method → Take 2 pairs of parallel lines $\vec{l}^{(1)}$, $\vec{l}^{(2)}$, $\vec{m}^{(1)}$, $\vec{m}^{(2)}$

$\vec{l}_1 \parallel \vec{l}_2$ and $\vec{m}_1 \parallel \vec{m}_2$

These two sets of parallel lines will intersect at points $\vec{p}^{(1)}$ & $\vec{p}^{(2)}$.

* $\vec{l} = (l_1\ l_2\ l_3)^T$ connecting $\vec{p}_1$ $\vec{p}_2$

(vanishing line)

$$\vec{p}_1 = \vec{l}^{(1)} \times \vec{l}^{(2)}$$

$$\vec{p}_2 = \vec{m}^{(1)} \times \vec{m}^{(2)}$$

$$\vec{l} = \vec{p}^{(1)} \times \vec{p}^{(2)}$$

$$H_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ l_1 & l_2 & l_3 \end{bmatrix}$$

$$\vec{l}_\infty = (0,0,1)^T$$

$$H_1^{-T} = \begin{bmatrix} 1 & 0 & -l_1/l_3 \\ 0 & 1 & -l_2/l_3 \\ 0 & 0 & 1/l_3 \end{bmatrix}$$

$$H_1^{-T}\vec{l} = (0,0,1)^T$$

Mapping to infinity.

* Choose 2 sets of lines which are parallel
- Find vanishing line $\vec{l} = (l_1, l_2, l_3)^T$
- Form H_1 & Apply H_1 to image.

$$X_a = H_1 X_c$$

Code - Matlab file

Metric rectification-

Method

Start with affine rectified image X_a.

$$\underbrace{H_2 = \begin{pmatrix} A & \vec{t} \\ 0 & 1 \end{pmatrix}}_{\text{Affine Transform matrix}}$$

* Start with 2 pairs of orthogonal lines

$\vec{l} \perp \vec{m}$.

& we have
$$\vec{l'} = H_2^{-T}\vec{l}$$
$$\vec{m'} = H_2^{-T}\vec{m}$$

They are orthogonal so,

$$(l_1/l_3 \, , \, l_2/l_3)(m_1/m_3 \, , \, m_2/m_3)^T = 0$$

$$l_1 m_1 + l_2 m_2 = \vec{l}^T C_\infty^* \vec{m} = 0$$

$$C_\infty^* = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$C_\infty^{*'} = H_2 C_\infty^* H_2^T$$

$$\vec{l}^T C_\infty^* \vec{m} = \vec{l}'^T H_2 H_2^{-1} C_\infty^{*'} H_2^{-T} H_2^T \vec{m}'$$

$$= \vec{l}'^T C_\infty'^* \vec{m}'$$

$$= 0$$

$$\vec{l}'^T \vec{C_\infty^*} \vec{m}' = \vec{l}'^T H_2 C_\infty^* H_2^T \vec{m}'$$

$$= \vec{l}'^T \begin{pmatrix} A & \vec{t} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} I & \vec{0} \\ \vec{0} & 1 \end{pmatrix} \begin{pmatrix} A^T & \vec{0} \\ \vec{t}^T & 1 \end{pmatrix} \vec{m}'$$

$$= \vec{l}'^T \begin{pmatrix} AA^T & \vec{0} \\ \vec{0} & 0 \end{pmatrix} \vec{m}'$$

$$(l_1', l_2') AA^T (m_1' \; m_2')^T = 0$$

$$S = AA^T, \quad S = \begin{pmatrix} s_{11} & s_{12} \\ s_{12} & 1 \end{pmatrix}$$

$$(l_1' m_1', \; l_1' m_2' + l_2' m_1') \begin{bmatrix} s_{11} \\ s_{12} \end{bmatrix} = -l_2' m_2'$$

Pair of orthogonal lines is needed.

$\vec{l_1} \perp \vec{m_1}$ $\quad$ $\vec{l_2} \perp \vec{m_2}$ to obtain S

Bonus (Theory).

We use metric rectification using C_∞^*.

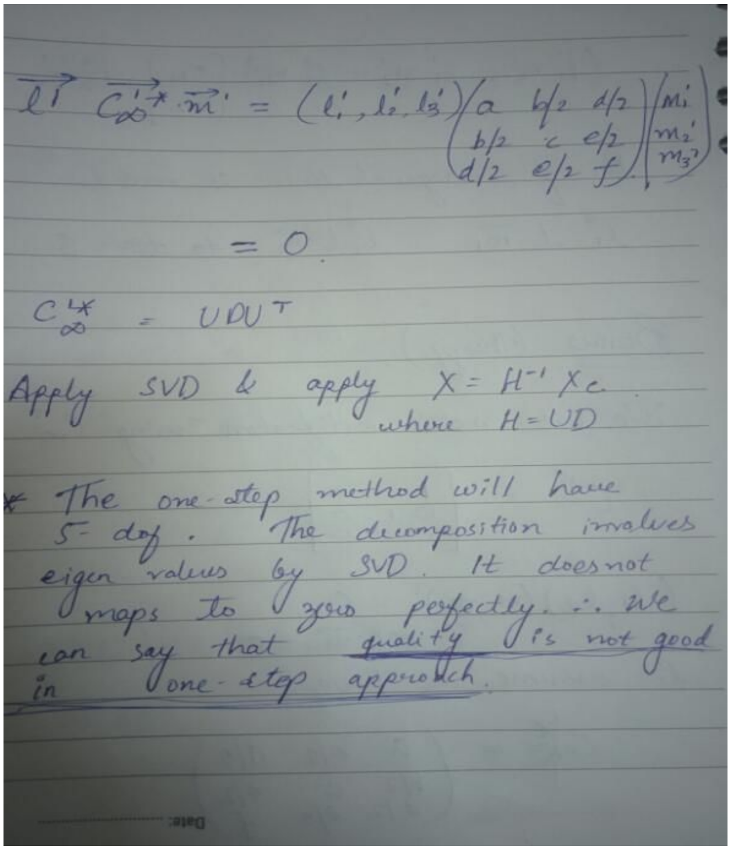
$$C_\infty^* = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$(l_1, l_2)(m_1, m_2)^T = 0$$

$$\vec{l}^T C_\infty^* \vec{m} = 0$$

Let assume we have

$$C_\infty'^* = \begin{pmatrix} a & b/2 & d/2 \\ b/2 & c & e/2 \\ d/2 & e/2 & f \end{pmatrix}$$



$$\vec{l'}\, \vec{C'^{*}_{\infty}}\, \vec{m'} = (l'_1, l'_2, l'_3) \begin{pmatrix} a & b/2 & d/2 \\ b/2 & c & e/2 \\ d/2 & e/2 & f \end{pmatrix} \begin{pmatrix} m'_1 \\ m'_2 \\ m'_3 \end{pmatrix}$$

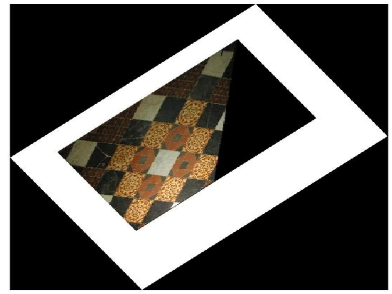
$$= 0.$$

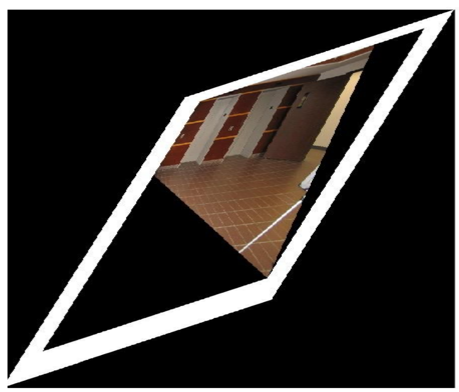
$$C'^{*}_{\infty} = UDU^T$$

Apply SVD & apply $X = H^{-1} X_c$.
where $H = UD$

* The one-step method will have 5-dof. The decomposition involves eigen values by SVD. It does not maps to zero perfectly. ∴ We can say that quality is not good in one-step approach.

Results

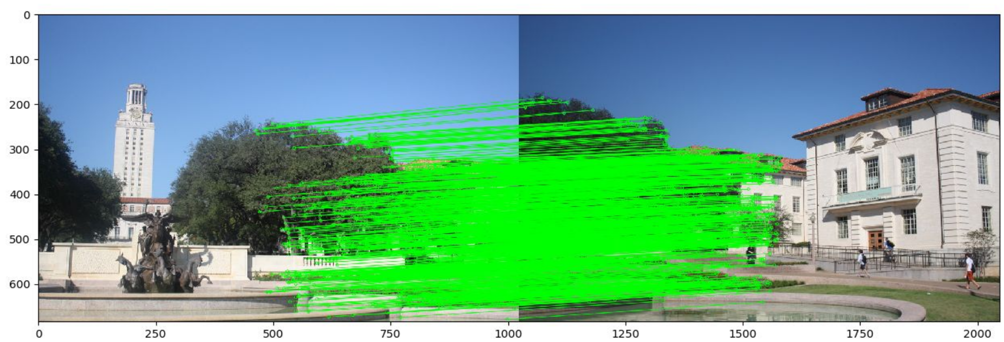



Ques 3)

Initial Image

FEATURE MATCHING



FINAL HOMOGRAPH


0
100
200
300
400
500
600
700
800
0
200
400
600
800
1000
1200
1400
1600