Aryabhatta Knowledge University (AKU)Computer Science and Engineering (CSE)Mathematics-IIISolved Exam Paper 2019

Q1 a) If y=A cos (log x) + B sin (log x), Show that x2yn+2 +(2n+1) * yn+1 + (n2 + 1) yn=0 where yn= dny/dxn

If $y = A\cos(\log x) + B\sin(\log x)$ ------------(1)

Differentiating (1) w.r.t x, we get

$y_1 = -a\sin(\log x)1/x + b\cos(\log x)1/x$

$xy_1 = -a\sin(\log x) + b\cos(\log x)$ ------------(2)

Diff 2 again w.r.t x, then we get

$xy_2 + y_1 = -a\cos(\log x)1/x - b\sin(\log x)1/x$

$x^2y_2 + xy_1 = -[a\cos(\log x) + b\sin(\log x)]$

$x^2y_2 + xy_1 = -y$

$y_2x^2 + y_1x + y = 0$ -------------------------(3)

Diff 3 by Leibnitzle theorem n times, we get

$[\, y_{n+2}x^2 + {}^nc_1\, y_{n+1}2x + {}^nc_2\, y_n.2 \,] + [\, y_{n+1}x + {}^nc_1\, y_{n-1}] + y_n = 0$

$x^2y_{n+2} + 2nxy_{n+1} + xy_{n+1} + n(n-1)y_n + ny_n + y_n = 0$

$x^2y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$

Q2 a) Show that the following function is continuous at the point (0,0):

f(x,y) $\dfrac{2x^3+3y^3}{x^2+y^2}$, (x,y) != (0,0)

0 , (x,y) = (0,0)

Answer:

$$f(x,y) = \begin{cases} \dfrac{2x^3+3y^3}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

$$0 \le \left| \frac{2x^3+3y^3}{x^2+y^2} \right| \le \frac{2|x^3|}{x^2+y^2} + \frac{3|y^3|}{x^2+y^2} = 0$$

$$0 \le \frac{2|x^3|}{x^2} + \frac{3|y^3|}{y^2} = 2|x| + 3|y|$$

lt(x,y)-->(0,0) $2|x| + 3|y| = 0$

∴ by the squeez theorem,, we conclude that

lt(x,y)-->(0,0) f(x,y) = (0,0) that is => lt(x,y)-->(0,0) $\frac{2x^3+3y^3}{x^2+y^2} = 0$

∴ f is continous at (0,0)

Q2 b) If z(x+y) = x2 + y2 show that

$$\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$$

solution:

z(x+y) = x2 + y2

$$z = \frac{x^2+y^2}{x+y}$$

$$\frac{\partial z}{\partial x} = \frac{(x+y)2x-(x^2+y^2).1}{(x+y)^2} = \frac{x^2+2xy-y^2}{(x+y)^2}$$

$\frac{\partial z}{\partial y} = \frac{(x+y)2y-(x^2+y^2).1}{(x+y)^2} = \frac{-x^2+2xy+y^2}{(x+y)^2}$

$\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right) = \frac{x^2+2xy-y^2}{(x+y)^2} = \frac{-x^2+2xy+y^2}{(x+y)^2} = \frac{2(x-y)}{(x+y)}$

$\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)2 = \frac{4(x-y)^2}{(x+y)^2}$ --1

$4\left(1-\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right) = 4\left[1 - \frac{x^2+2xy-y^2}{(x+y)^2} - \frac{-x^2+2xy+y^2}{(x+y)^2}\right]$

$= \frac{4(x-y)^2}{(x+y)^2}$ --2

From 1 & 2

$\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)2 = 4\left(1-\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right)$

Q3 a)Transform the equation

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x^2} = 0$ into polar coordinates.

Solution:

we have x = rcosθ , y = rsinθ

r2 = x2 + y2 , θ = tan−1y/x

$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial r}.\frac{\partial r}{\partial x} + \frac{\partial u}{\partial \theta}.\frac{\partial \theta}{\partial x} => \frac{\partial u}{\partial x}(\cos\theta) - \frac{\partial u \sin\theta}{\partial\theta \; r}$

$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right)$

$= \left(\cos\theta\frac{\partial}{\partial r} - \frac{\sin\theta}{r}.\frac{\partial}{\partial\theta}\right)\left(\cos\theta\frac{\partial u}{\partial r} - \frac{\sin\theta\partial u}{r\;\partial\theta}\right)$

$= \cos\theta\frac{\partial}{\partial x}\left(\cos\theta\frac{\partial u}{\partial r} - \frac{\sin\theta}{r}\left(\frac{\partial u}{\partial\theta}\right)\right) - \frac{\sin\theta}{r}.\frac{\partial}{\partial\theta}\left(\cos\theta\frac{\partial u}{\partial r} - \frac{\sin\theta}{r}\right)$

$= \cos\theta\left(\cos\theta\frac{\partial^2 u}{\partial r^2} + \frac{\sin\theta\partial u}{r^2\;\partial\theta} - \frac{\sin\theta}{r}.\frac{\partial^2 u}{\partial r\partial\theta}\right) - \frac{\sin\theta}{r}\left(-\sin\theta\frac{\partial u}{\partial r} + \right.$

$\left.\cos\theta\frac{\partial^2 u}{\partial r\partial\theta} - \frac{\cos\theta\partial u}{r\;\partial\theta} - \frac{\sin\theta\partial^2 u}{r\;\partial\theta^2}\right)$

$= \cos 2\theta\frac{\partial^2 u}{\partial r^2} + \frac{\sin\theta\cos\theta\partial u}{r^2\;\partial\theta} - \frac{\sin\theta\cos\theta}{r}\frac{\partial^2 u}{\partial r\partial\theta} + \frac{\sin^2\theta\partial u}{r\;\partial r} -$

$\frac{\sin\theta\cos\theta}{r}\frac{\partial^2 u}{\partial\theta\partial u} + \frac{\sin\theta\cos\theta\partial u}{r^2\;\partial\theta} + \frac{\sin^2\theta\partial^2 u}{r^2\;\partial\theta^2}$

$\frac{sin^2\theta\partial^2 u}{r^2\ \partial\theta^2}$ ---(1)

$= \frac{\partial u}{\partial y} = \frac{\partial u}{\partial r}\frac{\partial r}{\partial y} + \frac{\partial u}{\partial \theta}\frac{\partial \theta}{\partial y}$

$= \frac{\partial u}{\partial r}$ y/r + $\frac{\partial u}{\partial \theta}\frac{x}{(x^2+y^2)}$

$= \frac{\partial u}{\partial r}$ sinθ + $\frac{\partial u}{\partial \theta}\frac{cos\theta}{r}$

$= \frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}(\frac{\partial u}{\partial y})$

$=$ (sinθ $\frac{\partial}{\partial u} + \frac{cos\theta\ \partial}{r\ \ \partial\theta}$)(sinθ$\frac{\partial u}{\partial r} + \frac{cos\theta\partial u}{r\ \ \partial\theta}$)

$=$ sinθ$\frac{\partial}{\partial r}$(sinθ $\frac{\partial u}{\partial r} + \frac{cos\theta\partial u}{r\ \ \partial\theta}$) + $\frac{cos\theta\ \partial}{r\ \ \partial\theta}$(sinθ$\frac{\partial u}{\partial r} + \frac{cos\theta\partial u}{r\ \ \partial\theta}$)

$=$ sinθ[sinθ$\frac{\partial^2 u}{\partial r^2} - \frac{cos\theta\partial u}{r^2\ \ \partial\theta} + \frac{cos\theta}{r} + \frac{\partial^2 u}{\partial r\partial\theta}$] + $\frac{cos\theta}{r}$[cosθ [$\frac{\partial u}{\partial r}$+

sinθ$\frac{\partial^2 u}{\partial r\partial\theta} - \frac{sin\theta\partial u}{r\ \ \partial\theta} + \frac{cos\theta\partial^2 u}{r\ \ \partial\theta^2}$]

$=$ sin2θ$\frac{\partial^2 u}{\partial r^2} - \frac{sin\theta cos\theta\partial u}{r^2\ \ \ \partial\theta} + \frac{sin\theta cos\theta\ \partial^2 u}{r\ \ \ \ \partial r\partial\theta} + \frac{cos^2\theta}{r}(\frac{\partial u}{\partial r}) + \frac{sin\theta cos\theta}{r}$

$(\frac{\partial^2 u}{\partial r\partial\theta}) - \frac{sin\theta cos\theta\partial u}{r^2\ \ \ \partial\theta} + (\frac{\partial^2 u}{\partial\theta^2})$

$=$ sin2θ$\frac{\partial^2 u}{\partial r^2}$ − 2$\frac{sin\theta cos\theta\partial u}{r^2\ \ \ \partial\theta}$+2$\frac{sin\theta cos\theta\ \partial^2 u}{r\ \ \ \ \partial r\partial\theta} + \frac{cos^2\theta}{r}(\frac{\partial u}{\partial r})$ +

$(\frac{\partial^2 u}{\partial\theta^2})$ --(2)

By adding 1 & 2

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ = (sin2θ+cos2θ)$\frac{\partial^2 u}{\partial r^2}$+(sin2θ+cos2θ)1/r$\frac{\partial u}{\partial r}$+ (sin2θ+cos2θ)1/r2$\frac{\partial^2 u}{\partial\theta^2}$

$=$ ($\frac{\partial^2 u}{\partial r^2}$+ 1/r$\frac{\partial u}{\partial r}$+1/r2$\frac{\partial^2 u}{\partial\theta^2}$) ans

Q4 Find the extreme values of f(x,y ,z) = 2x+3y+z, such that x2+y2=5 and x+z=1

f(x,y,z) = 2x + 3y + z ---------------------------(1)

f(x,y) = (x2 + y2) - 5 ------------------(2)

y(x,z) = x+z-1 ------------------(3)

Lagranges Multipliers Equations are

$\frac{\partial f}{\partial x}+ \lambda \frac{\partial \phi}{\partial x}+ m \frac{\partial \Psi}{\partial x}= 0$

$\frac{\partial f}{\partial y}+ \lambda \frac{\partial \phi}{\partial y}+ m \frac{\partial \Psi}{\partial y}= 0$

$\frac{\partial f}{\partial z}+ \lambda \frac{\partial \phi}{\partial z}+ m \frac{\partial \Psi}{\partial z}= 0$

$2 + \lambda(2x) + m(1) = 0$ ------------(4)

$3 + \lambda(2y) + m(0) = 0$ ------------(5)

$1 + \lambda(0) + m(1) = 0$ ------------(6) => $m = -1$

putting the values of m in (4) and (5), we get

$2 + 2\lambda x - 1 = 0 \Rightarrow 2\lambda x = -1,\ x = -\frac{1}{2\lambda}$

$3 + 2\lambda y = 0 \Rightarrow 2\lambda y = -3,\ y = -\frac{3}{2\lambda}$

putting the values of x,y in x2 + y2 = 5 , we get

$\frac{1}{4\lambda^2}+ \frac{9}{4\lambda^2}= 5 \Rightarrow \frac{10}{4\lambda^2}= 5$

$2\lambda 2 = 1 \Rightarrow \lambda = +- \frac{1}{\sqrt{2}}$

we know that , $x = -\frac{1}{2\lambda} = +- \frac{\sqrt{2}}{2} = +- \frac{1}{\sqrt{2}}$

$y = -\frac{3}{2\lambda} = +- \frac{3\sqrt{2}}{2} +- \frac{3}{\sqrt{2}}$

From (3) , x+z =1 or z = 1-x

$z = 1 +- \frac{1}{\sqrt{2}}$

putting $x = \frac{1}{\sqrt{2}}$, $y = \frac{3}{\sqrt{2}}$ and $z = 1 - \frac{1}{\sqrt{2}}$ in equation (1), we get

$f = \frac{2}{\sqrt{2}}+ \frac{9}{\sqrt{2}}+ 1 - \frac{1}{\sqrt{2}}= \frac{10}{\sqrt{2}}+ 1 = 5Ö2 + 1$

putting $x = -\frac{1}{\sqrt{2}}$, $y = -\frac{3}{\sqrt{2}}$ and $z = 1 + \frac{1}{\sqrt{2}}$ in equation (1), we get

$f = 2\frac{1}{\sqrt{2}}+ 3\left(\frac{-3}{\sqrt{2}}\right)+ \left(1 + \frac{1}{\sqrt{2}}\right) = -\frac{2}{\sqrt{2}}- \frac{9}{\sqrt{2}}+1 + \frac{1}{\sqrt{2}}$

f = 1 – 6 Ö2 ans

Q5 Evaluate òòs F.n.ds where F = 4xz i^ + y2ĵ + yzk^ and S is surface of the cube bounded by x=0, y=1, z=0 , z=1, by using Gauss divergence theorem

s.no	surface		ds	
1	OABC	-k	dxdy	z= 0
2	DEFG	k	dxdy	z= 1
3	OAFG	-ĵ	dxdz	y= 0
4	BCDE	ĵ	dxdz	y=1
5	ABEF	i^	dydz	x=1
6	OCDG	-i^	dydz	x=0

òòs F¯ n^ ds = òòOABC F¯ n^ ds + òòDEFG F¯ n^ ds + òòOAFG F¯ n^ ds + òòBCDE F¯ n^ ds + òòABEF F¯ n^ ds +

òòOCDG F¯ n^ ds ---------------------------------------(1)

òòOABC F¯ n^ ds = òòOABC (4xz i^ + y2ĵ + yzk^)(-k)dxdy

= $\int_0^1\int_0^1 -yzdxdy = 0$(as z = 0)

òòDEFG (4xz i^ + y2ĵ + yzk^)(k)dxdy = òòDEFG yzdxdy

$\int_0^1\int_0^1 y(1)dxdy$ = $\int_0^1 dx\left[\frac{y^2}{2}\right]_{0-1}$= [x]0-1 = 1/2

òòOAFG (4xz i^ + y2ĵ + yzk^)(-j^)dxdz = òòOAFG y2dxdz = 6 (as y =0)

òòBCDE(4xz i^ + y2ĵ + yzk^)(j^)dxdz = òòBCDE (-y2)dxdz

-$\int_0^1 dx\int_0^1 dz$= (x)0-1(z)0-1 = -1 (as y = 1)

òòABEF (4xz i^ + y2ĵ + yzk^).i^ dydz = òò4xzdydz

=$\int_0^1\int_0^1 4(1)zdydz$=> 4(y)0-1 $\left(\frac{z^2}{2}\right)_{0-1}$ = 4 (1) $\left(\frac{1}{2}\right)$ = 2 òòOCDG(4xz i^ + y2ĵ + yzk^)(-i^) dydz = $\int_0 \int_0 4(1)zdydz$ = 0 (as x = 0)

on putting these values in (1), we get

òòs F¯ n^ ds = 0 + 1/2 + 6 -1 + 2 + 0 => 3/2

Q 6 (a) Evaluate $\frac{\partial}{\partial\theta}${A*{ B * C }}

	i^	ĵ	k^
(B*C) =	cosq	-sinq	-3
	2	3	-1

= i^ (sinq + 9) – ĵ (-cosq + 6) + k^ (3cosq + 2sinq)

	i^	ĵ	k^
A*(B * C) =	sinq	cosq	Q
	(sinq + 9)	(cosq - 6)	(3cosq + 2sinq)

= i^[cosq(3cosq + 2sinq) - q(cosq – 6)]

- ĵ[sinq(3cosq + 2sinq) - q(sinq + 9)]

+ k^[sinq(cosq – 6) - cosq (sinq + 9)]

at q = 0

=> i^[cos0(3cos0 + 2sin0) - 0(cos0 – 6)]

- ĵ[sin0(3cos0 + 2sin0) - 0(sin0 + 9)]

- ĵ[sin0(3cos0 + 2sin0) - 0(sin0 + 9)]

=> 3i^ - 9k^ ans

Q 6 A particle moves along the curve x=t3 + 1, y=t2, z= 2t+5 where t is the time. Find the components of the velocity and acceleration at t=1 in the direction i+j+3k

x = t3 + 1 , y = t3 , z = 2t + 5

¯r = xi^ + yĵ + zk^

¯r = (t3 + 1)i^ + (t3)ĵ + (2t + 5)k^

velocity = $\frac{dr}{dt}$ = 3t2 i^ + 2tĵ + 2k^

when t = 1, we have , $\frac{dr}{dt}$= 3 i^ + 2ĵ + 2k^

unit velocity along (i^ + ĵ + 3k^) = (i^ + ĵ + 3k^) / Ö (1 + 1 + 9)

= $\frac{1}{\sqrt{11}}$(i^ + ĵ + 3k^)

component of velocity (3 i^ + 2ĵ + 2k^) along (i^ + ĵ + 3k^)

= (3 i^ + 2ĵ + 2k^) . $\frac{1}{\sqrt{11}}$(i^ + ĵ + 3k^)

= $\frac{1}{\sqrt{11}}$(3+2+6) = $\frac{11}{\sqrt{11}}$= Ö11 ans

Q7 (a) Solve by the method of variation of parameters

$\frac{d^2y}{dx^2}$ + n2y = sec nx

solution: - (D2 + n2)y = sec nx

m2 + n2 = 0 => m = +- ni

cf = c1cosnx + c2sinnx

y = A'cosnx + B'sinnx ------------------(I)

by diff. Equation (1)

(-nA'sinnx + B'ncosnx = secnx) -------------(II)

Now multiplying by nsinx in equation (I) & multiplying by cosx in equation (II)

nsinnx(A'cosnx + B'sinnx = 0)

cosnx(-nA'sinnx + B'ncosnx = secnx)

A' nsinnx cosnx + B' nsin2 nx = 0

- A' nsinnx cosnx + B' ncos2 nx = 1

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nB' (sin2nx + cos2nx) = 1

B' = $\frac{1}{n}$

$\frac{dB}{dx} = \frac{1}{n}$

dB = $\frac{dx}{n}$

$\boxed{B = \frac{x}{n} + C}$

A'cosnx + B'sinnx = 0

A'cosnx + $\frac{1}{n}$sinnx = 0

A' = $-\frac{1 sinnx}{ncosnx} = -\frac{1}{n}$tan nx

òdA = $-\frac{1}{n}$ò tan nx dx + C2

A = $-\frac{1}{n^2}$log sec nx + C2

y = [$-\frac{1}{n^2}$log sec nx + C2] cos nx + ($\frac{x}{n}$ + C1)sin nx

$\boxed{Y = C1 \sin nx + C2 \cos nx + \frac{x}{n} \sin nx - \frac{1}{n^2} \cos nx. \log \sec nx}$

Q7 (b) solve $\frac{d^2y}{dx^2}$ – 2 tanx $\frac{dy}{dx}$ + 5y = sec x. ex

$\frac{d^2y}{dx^2}$ – 2 tanx $\frac{dy}{dx}$ + 5y = sin x. ex

y'' – 2 tan xy' + 5y = sin x. ex

compare with y'' + 8y' + qy = R

p = -2tanx , q = 6, R = sin x. ex

for C.F μ = e-1/2òpdx

q1 = q – $\frac{1dp}{2dx}$ – $\frac{p^2}{4}$

R1 = $\frac{R}{4}$

μ = e-1/2ò-2tanx dx = elog secx = sec x

q1 = 5 $-\frac{d}{dx}$(-2tanx)– $\frac{4tan^2x}{4}$

= 5 + sec2x – tan2x = 6

R1 = $\frac{sinxe^x}{4}$

$\frac{d^2v}{}$ ... R

(D2 + 6)v = $\frac{sinxe^x}{4}$

A.E = m2 + 6 =0 => m2 = -6 => m = +- 6i

CF = (C1cos6x + C2sin6x)

P.I = $\frac{1}{(D^2+6)}\frac{sinxe^x}{4}$ => $\frac{1}{4}\frac{1}{(D^2+6)}$(sin x. ex)

= $\frac{e^x}{4}\frac{1}{[(D+1)^2+6]}$sin x

= $\frac{e^x}{4}\frac{1}{(D^2+1+2D+6)}$sin x => $\frac{e^x}{4}\frac{1}{(-(1^2)+1+2D+6)}$sin x

= $\frac{e^x}{4}\frac{1}{(-1+1+2D+6)}$sin x => $\frac{e^x}{4}\frac{1}{(2D+6)}$sin x

= $\frac{e^x}{8}\frac{D-3}{(D^2-9)}$sin x => $\frac{e^x}{8}\frac{D-3}{(-1-9)}$sin x

= $\frac{e^x}{-80}$(D-3)sin x => $\frac{e^x}{-80}$(Dsin x − 3 sin x)

= $\frac{e^x}{-80}$(cos x − 3 sin x)

C.S = (C1 cos 6x + C2 sin6x) -$\frac{e^x}{80}$(cos x − 3 sin x