

Now previously we learned the multiple recursion but the question is where is the use of the concept of **Multiple Recursion**

→ The straightforward answer is **Subsequence** and **Dynamic Programming**

Subsequence

Subsequence means follow the particular order it can be **Contiguous or non-Contiguous**

[Subarray = that is Contiguous Not Non-Contiguous ✓
Subsequence = that is both Contiguous or Non-Contiguous ✓

Example we have array and print all the Subsequences with maintain the order

arr = [3, 1, 2]

Important Point

→ [3, 1, 2]

→ that is also a subsequence but not follow the order of the array

Subsequence are

that are all follow the array order

[3
1
2
3 1
1 2
3 2
3 1 2]

→ Contiguous Subsequence

→ Non-Contiguous Subsequences

→ Contiguous

[] → Also that is the Subsequence because we can not take any element from the array

How can we do this problem

→ Now this problem we have very popular algorithm that is known as

Power Set (Using this algorithm we find all the Subsequence of the array)

[Total 8 Subsequence of this array possible]

✓ → But here we solved the using the concept of the Recursion

Let's Understand the Pattern

→ Now we want an principle of **taking and Not-taking** because on every index we have

⊙ Option

→ Taking that index
→ Not take that index

Example

array	Subsequence
[3 1 2]	
X X X	= []
✓ X X	= 3
X ✓ X	= 1
X X ✓	= 2
✓ ✓ X	= 3 1
✓ X ✓	= 3 2
X ✓ ✓	= 1 2
✓ ✓ ✓	= 3 1 2

where

X → not taking
✓ → taking

Now I observed that every index we have tuple of option take or not take that is the situation

✓✓✓ = 3 1 2

Now I observed that every index we have tuple of option take or not take that is the situation behind this permutation

```

myfunction(index, mylist) {
  if (index == n) {
    print(mylist);
    return;
  }
  mylist.add(mylist[index]);
  myfunction(index+1, mylist);
  mylist.remove(mylist[index]);
  myfunction(index+1, mylist);
}
  
```

Base Case
 Add the element / Subsequence into the mylist
 taking the case
 Not taking case

Let's understand the Sudo Code and Dry Run

arr = [3, 1, 2]
n = 3



Again same process follow cases function again and again and print the subsequence just similar previously we done it

Region of mylist.remove()

because while exit that exit out add what if it EM not that add subsequence if that exit add what are find

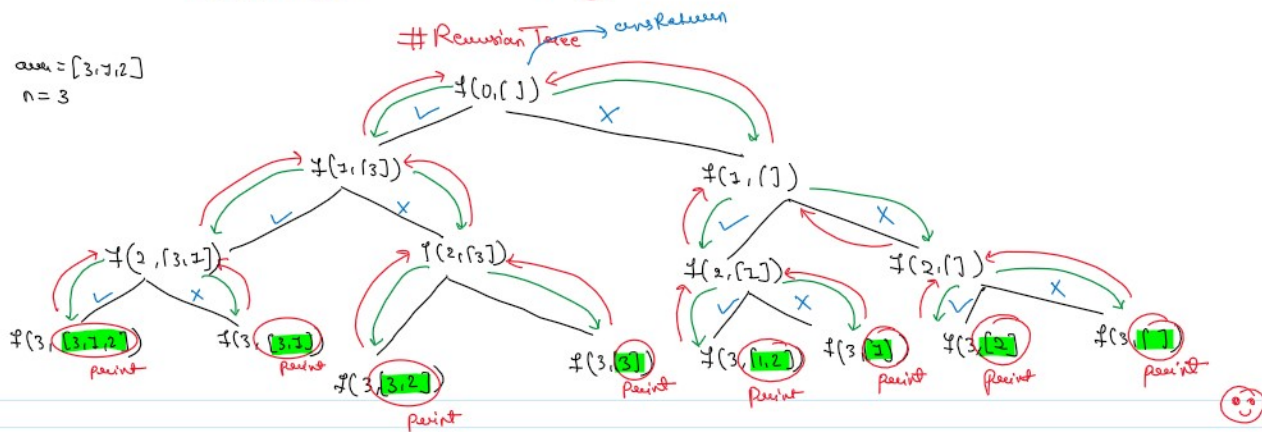
now what if I don't want
2nd & 3rd subsequence if yes
it's not add it's not find

myFunction(index+1, mylist);

myFunction(index+1, mylist);

what is the Region behind of Remaining
the last element

arr = [3, 1, 2]
n = 3



Time Complexity = ?

for every function we have tuple of option
take or not take = 2^n (Exponential)
 $O(2^n)$

Space Complexity = $O(N)$

because we store the one element
that is our subsequence

Implementation Python

```
class RecursionOnSubsequence:

    def printAllSubsequences(self, index, arr, myans):

        if index >= len(arr):
            print(myans)
            return

        myans.append(arr[index])
        self.printAllSubsequences(index+1, arr, myans)
        myans.remove(arr[index])
        self.printAllSubsequences(index+1, arr, myans)

    # reverse order
    def reverseOrd(self, index, arr, myans):

        if index >= len(arr):
            print(myans)
            return

        self.reverseOrd(index + 1, arr, myans)
        myans.append(arr[index])
        self.reverseOrd(index+1, arr, myans)
        myans.remove(arr[index])

ans = RecursionOnSubsequence()
arr = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
print(ans.printAllSubsequences(0, arr, []))

print('Reverse')
print(ans.reverseOrd(0, arr, []))
```

Implementation Java

```
import java.util.*;

public class L6_Recursion_on_Subsequences {
    public static void main(String[] args) {

        int[] arr = {1, 2, 3};
        ArrayList<Integer> myans = new ArrayList<>();
        printAllSubsequences(0, arr, myans);

        System.out.println("For Reverse Printing");
        reversePrint(0, arr, myans);

    }

    // for reverse printing
    private static void reversePrint(int index, int[] arr,
        ArrayList<Integer> myans) {

        if(index == arr.length){
            System.out.println(myans);
            return;
        }

        reversePrint(index+1, arr, myans);
        myans.add(arr[index]);
        reversePrint(index+1, arr, myans);
        myans.remove(myans.size()-1);

    }

    private static void printAllSubsequences(int index, int[]
        arr, ArrayList<Integer> myans) {

        if(index == arr.length){
            System.out.println(myans);
            return;
        }

        myans.add(arr[index]);
        printAllSubsequences(index+1, arr, myans);
        myans.remove(myans.size()-1);
        printAllSubsequences(index+1, arr, myans);

    }

}
```

```
printAllSubsequences(index+1,arr,myans);
```

```
}
```

```
}
```