Chapter 14.7 Maximum and Minimum Value

- Idea: Given a graph, how to find minima and maxima.

- For a function of two var:
  - local maxima: if $f(x, y) \leq f(a, b)$ when (x, y) is near (a, b), $f(a, b)$ is a local max. value.
  - local minima: if $f(x, y) \geq f(a, b)$ when (x, y) is near (a, b), $f(a, b)$ is a local min. value.

- Theorem: If f has a local max/min at (a, b), and the 1st-order PD exists, then $f_x(a, b) = 0$ and $f_y(a, b = 0$.
So gradient is a $\vec{0}$.

* Second Derivative Test:

Suppose 2nd PD of f are continuous on a disk with center (a,b), and suppose $f_x(a,b)=0$, $f_y(a,b)=0$.

Let $D = D(a,b) = f_{xx}(a,b) f_{yy}(a,b) - [f_{xy}(a,b)]^2$

(a) If $D>0$ and $f_{xx}(a,b)>0$, then $f(a,b)$ is a local min.

(b) If $D>0$ and $f_{xx}(a,b)<0$, then $f(a,b)$ is a local max.

(c) If $D<0$, then $f(a,b)$ is not local max or min, it's a saddle point

If $D=0$, no information given

$$D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = f_{xx}f_{yy} - (f_{xy})^2$$

Extreme Value Theorem for 2D:

If f is continuous on a closed, bounded set D in $\mathbb{R}^2$, then f attains an abs. max val $f(x_1,y_1)$ and an abs. min. val. $f(x_2,y_2)$ at some points (x_1,y_1) an (x_2,y_2) in D.

bounded set contained within some disk

closed set contains all boundary points.

To find the absolute max. and min. values of a continuous function f on a closed, bounded set D:

① Find the values of f at the critical points of f in D

② Find the extreme values of f on the boundary of D

③ The largest values from steps 1 and 2 is abs. max val. Smallest is abs. min. val.