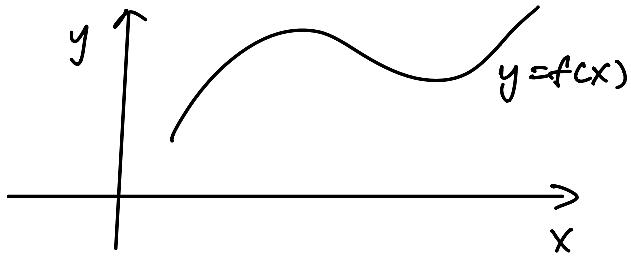
## 1.1.1 Review and Introduction

$y = f(x)$




derivative :

$$\frac{dy}{dx}(x) = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

if this limit exists

min/max of $f$ occurs where $f' = 0$ (or on boundary of domain)

**Integration**

$$\int_a^b f(x)\,dx = \text{"area under the graph"} = \lim_{n\to\infty} \sum_{i=1}^{n} \underbrace{f(x_i^*)}_{\substack{\text{height}\\ \text{of } i\text{th}\\ \text{rectangle}}} \underbrace{\Delta x}_{\text{width}}$$

if $f > 0$

. . . . . .

**Fundamental Theorem of Calculus**

① $\int_a^b f'(x)\,dx = f(b) - f(a)$

② $\frac{d}{dx} \int_a^x f(t)\,dt = f(x)$

$$L = \int_a^b \sqrt{1+f'(x)^2}\, dx$$ length of graph

$$A = \int_a^b 2\pi \cdot f(x) \sqrt{1+f'(x)^2}\, dx$$ area of surf of revo

In this course:

- More general curves and surfaces in 2d 3d
- Func of 2/3 Var
- Par Deri.
- Integration in 2 or 3 dim
- Fundamental Theorem of Line Integrals.

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1.1.2:

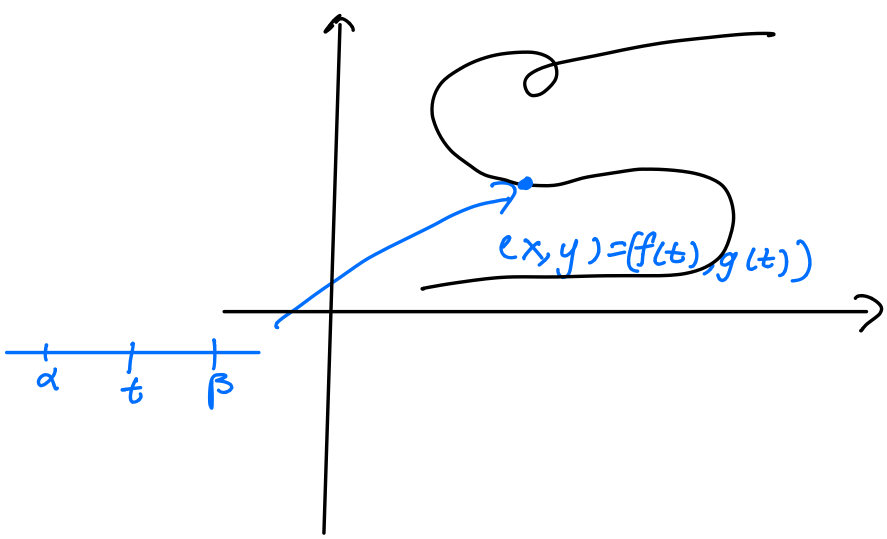
<u>Parametrized Curve</u>

$x = f(t)$

$y = g(t)$

$\alpha \leq t \leq \beta$

parameter (time)

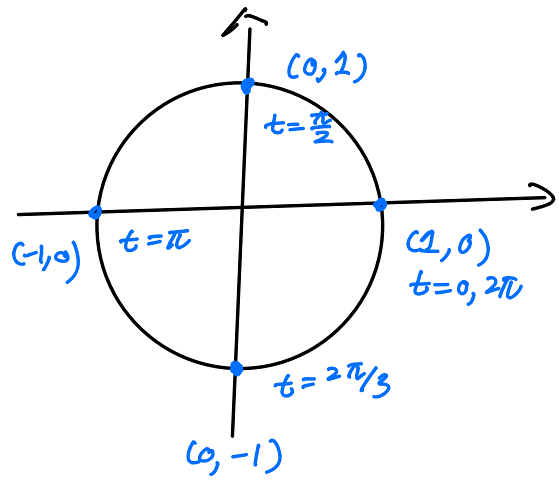


## Example 1

$x = \cos t$

$y = \sin t$

$0 \leq t \leq 2\pi$




## Example 2

$x = \cos(3t)$

$y = \sin(3t)$

$0 \leq t \leq 2\pi$

unit circle going around counter clockwise 3 times

* Parametrized Curve : more than a curve

it's := curve + parametrization (time table)

"parametric curves"

## Methods of sketching a parametrized curve

① Plot points and connect the dots

$x = t^3 - 4t, \; y = t, \; -3 \leq t \leq 3$

| t | x | y |
|---|---|---|
| -3 | -15 | -3 |
| ⋮ | ⋮ | ⋮ |
| 3 | 15 | 3 |

. . . .

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1.1.4

Slope of a parametrized curve $x = f(t)$, $y = g(t)$,
$\alpha \leq t \leq \beta$

Heuristic $\quad \text{slope} = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{g'(t)}{f'(t)}$ if $f'(t) \neq 0$

If $f'(t) = 0$, and $g'(t) \neq 0$, slope $= \infty$ (tangent line to the curve is vertical).

If $f'(t) \neq 0$, then locally the curve is a graph $y = h(x)$

$y = h(x)$

$g(t) = h(f(t))$

$g'(t) = \frac{dh}{dx} f'(t) \Rightarrow \text{slope} = \frac{g'(t)}{f'(t)}$

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1.1.5 Cycloid

- decompose motion into 2 parts:

1. know how the center moves
2. know how the point moves

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1.1.6 Area under a parametrized curve without vertical tangent

$x = f(t)$

$y = g(t)$

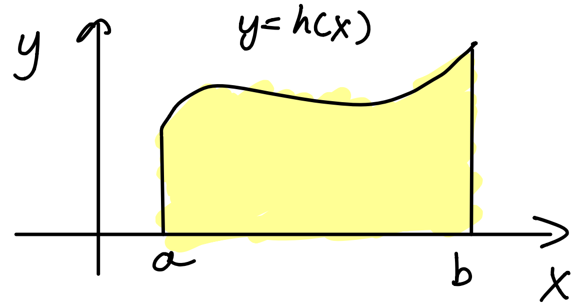
$\alpha \leq t \leq \beta$

$\text{Area} = \int_a^b h(x)\,dx \implies$ rewrite in $g$ & $f$

Substitution:

$dx = f'(t)\,dt$

$h(x) = g(t)$




$= \pm \int_\alpha^\beta g(t) f'(t)\,dt$

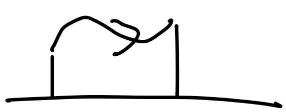
$+$ if $a = x(\alpha),\ b = x(\beta)$

$-$ if $a = x(\beta),\ b = x(\alpha)$

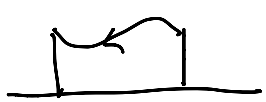
$+$ when curve goes to the right

$-$ when curve goes to the left

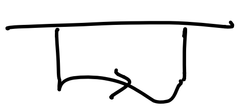
Signs:



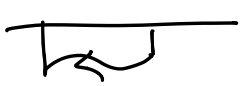
$+\int_\alpha^\beta y\,x'(t)\,dt$



$-\int_\alpha^\beta y\,x'(t)\,dt$



$-\int_\alpha^\beta y\,x'\,dt$



$+\int_\alpha^\beta y\,x'\,dt$

## 1.1.7. Length of a parametrized curve

for curve s.t. $x=f(t)$, $y=g(t)$, $\alpha \leq t \leq \beta$

$$L=\int_{\alpha}^{\beta} \sqrt{x'(t)^2+y'(t)^2}\,dt$$

Definition : $\text{length} = \lim_{N\to\infty} \sum_{i=1}^{N} \text{length}($ line segment from $(x(t_{i-1}), y(t_{i-1}))$ to $(x(t_i), y(t_i))$ $)$

## 1.1.8. Area of surface of revo of a parametrized curve

$$\text{Area} = \lim_{\Delta s \to 0} \sum \text{Area(Ribbon)} \quad \text{where } \text{Area(Ribbon)} \approx 2\pi y \,\Delta s$$

$$= \int_{\alpha}^{\beta} 2\pi y \, ds = \int_{\alpha}^{\beta} 2\pi y(t) \underbrace{\sqrt{x'(t)^2+y'(t)^2}\,dt}_{\text{length}}$$