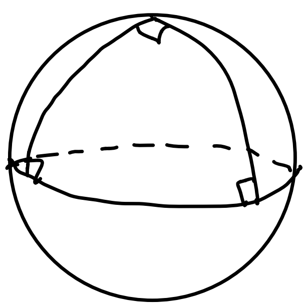
1.3.1 Distance in Euclidean space

In 2d: $\text{dist}((x_1, y_1), (x_2, y_2)) = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

In 3d: $\text{dist}((x_1, y_1, z_1), (x_2, y_2, z_2))$
$= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

Non-Euclidean Geometry



sum of angles: $\frac{3\pi}{2}$

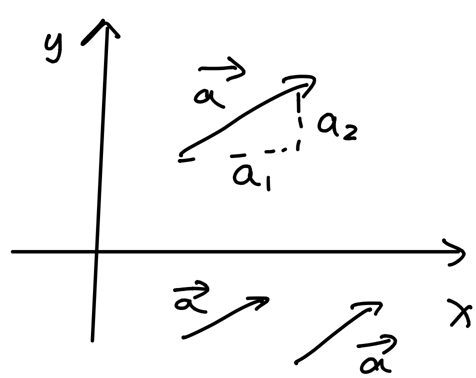
in this course only Euclidean space.

1.3.2 Vectors in 2 and 3 dimensions

2d $\quad \vec{a} = \langle a_1, a_2 \rangle$

vector

components (real num)



Addition of vectors $\vec{b} = \langle b_1, b_2 \rangle$

Define $\vec{a} + \vec{b} = \langle a_1 + b_1, a_2 + b_2 \rangle$

Multiplication of vector $\vec{a} = \langle a_1, a_2 \rangle$ by a scalar c (c real)

Define $c\vec{a} = \langle ca_1, ca_2 \rangle$

Length of $\vec{a} = \langle a_1, a_2 \rangle$ (norm, magnitude)

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2}$$

Fact $|c\vec{a}| = \underbrace{|c|}_{\text{abs. val.}} |\vec{a}|$

Vectors in 3d $\vec{a} = \langle a_1, a_2, a_3 \rangle$

operations similar

Dot Product

How to multiply two vectors $\vec{a} = \langle a_1, a_2, a_3 \rangle$ and $\vec{b} = \langle b_1, b_2, b_3 \rangle$?

$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + \cdots + a_n b_n$

geometric interpretation: $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$

<u>Define</u> " $\vec{a} \perp \vec{b}$ " means $\vec{a} \cdot \vec{b} = 0$

is perpendicular to

Dot product is commutative and distributive over addition

① $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$

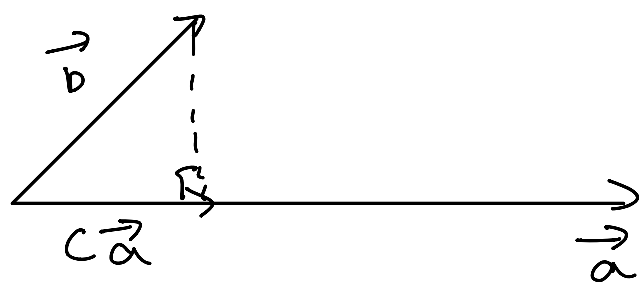
② $\vec{a} \cdot (\vec{b} + \vec{c}) = (\vec{a} \cdot \vec{b}) + (\vec{a} \cdot \vec{c})$

* Not associative

<u>Pythagorean Theorem</u>

If $\vec{a} \perp \vec{b}$, then $|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2$

* Given $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$



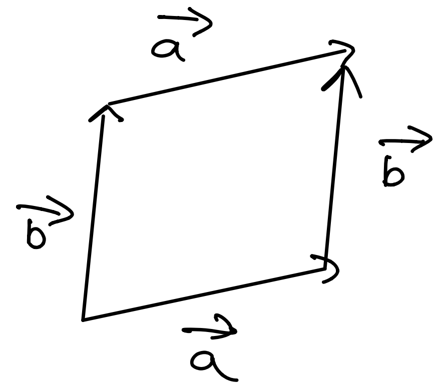
$c\vec{a}$ = <u>orthogonal projection</u> of $\vec{b}$ onto $\vec{a}$

1.3.5. Determinants

$$\det\begin{pmatrix} a_1 & a_2 \\ b_1 & b_2 \end{pmatrix} = \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1$$

Geometric meaning

$\vec{a} = \langle a_1, a_2 \rangle$
$\vec{b} = \langle b_1, b_2 \rangle$



$$\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} = \pm \text{ Area (Parellelogram)}$$

$\therefore$ + when $\vec{a}$ points to the right of b.
– when $\vec{a}$ points to the left of b.

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

Geometric Interpretation

$\vec{a} = \langle a_1, a_2, a_3 \rangle$
$\vec{b} = \langle b_1, b_2, b_3 \rangle$
$\vec{c} = \langle c_1, c_2, c_3 \rangle$

det = $\pm$ Volume (Parallelopiped)