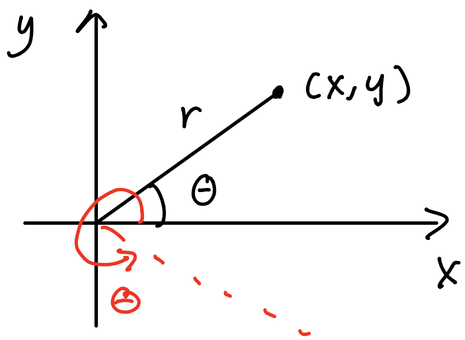
1.2.1 Intro



form: (r, θ)

$x = r\cos\theta$

$y = r\sin\theta$

$r = \sqrt{x^2+y^2}$

$\tan\theta = \frac{y}{x}$

① θ is defined only up to adding mod of 2π

② Sometime we allow $r < 0$.

<u>Curves in Polar coordinates</u>

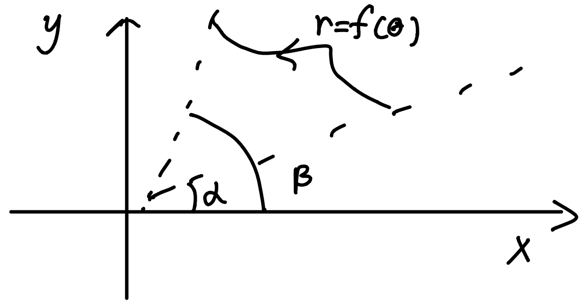
$r = f(\theta)$, $\alpha \leq \theta \leq \beta$

$x = r\cos\theta$
$y = r\sin\theta$

parametrized curve in x, y coords

Parametrized curve in x, y coordinates

$$\boxed{\begin{aligned} x &= f(\theta)\cos\theta \\ y &= f(\theta)\sin\theta \\ \alpha &\leq \theta \leq \beta \end{aligned}}$$



How to sketch $r=f(\theta)$, $\alpha \leq \theta \leq \beta$.

① Plot points

e.g. $r = 2\sin\theta$, $0 \leq \theta \leq \pi$

θ	r
0	0
$\frac{\pi}{4}$	$\sqrt{2}$
$\frac{\pi}{2}$	2
$3\pi/4$	$\sqrt{2}$
π	0

connect dots, arrow goes in dir. of increasing θ.

② Convert to Cartesian (x, y) coordinates

$x = r\cos\theta$

$y = r\sin\theta$

$r^2 = 2r\sin\theta$

$x^2 + y^2 = 2y$

$x^2 + (y-1)^2 = 1$ circle of radius 1 with center $(0,1)$.

1.2.2. Slope and area in polar coordinates

Slope of a polar curve $r = f(\theta)$

$x = f(\theta)\cos\theta$

$y = f(\theta)\sin\theta$

$$\text{slope} = \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\frac{d}{d\theta}(f(\theta)\sin\theta)}{\frac{d}{d\theta}(f(\theta)\cos\theta)}$$

e.g. slope of $r = 1-2\cos\theta$ at $\theta = \frac{\pi}{2}$

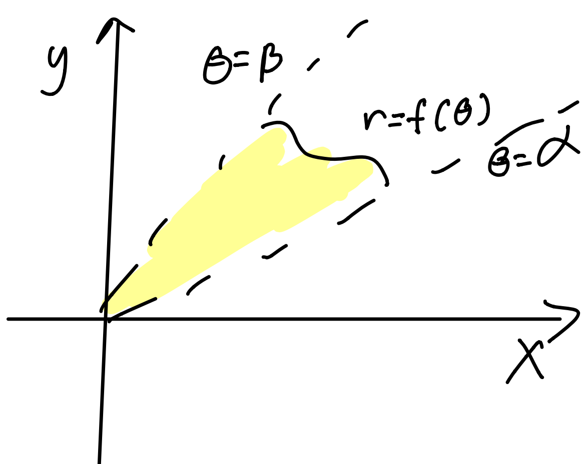
$x = (1-2\cos\theta)\cos\theta$

$y = (1-2\cos\theta)\sin\theta$

$$\text{slope} = \frac{dy/d\theta}{dx/d\theta} = \frac{\frac{d}{d\theta}[(1-2\cos\theta)\sin\theta]}{\frac{d}{d\theta}[(1-2\cos\theta)\cos\theta]} = \frac{(2\sin\theta)\sin\theta + (1-2\cos\theta)\cos\theta}{(2\sin\theta)(\cos\theta) + (1-2\cos\theta)(-\sin\theta)}$$

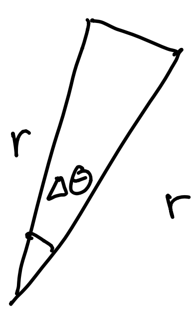
$\cdots\cdots = -2$

• area : $r = f(\theta)$, $\alpha \le \theta \le \beta$, assume $f \ge 0$



Area =

$$\lim_{\Delta\theta\to 0} \sum \text{Area(Pie Slice)}$$
$$= \lim_{\Delta\theta\to 0} \sum \frac{r^2\Delta\theta}{2}$$
$$= \int_\alpha^\beta \frac{r^2}{2}\,d\theta = \int_\alpha^\beta \frac{f(\theta)^2}{2}\,d\theta$$

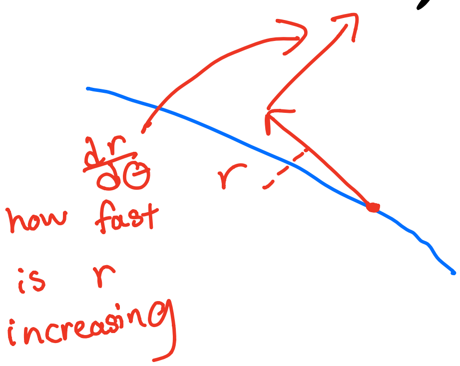


Area : $\frac{r^2\Delta\theta}{2}$

1.2.4 Length of a polar coordinate

$r = f(\theta)$, $\alpha \leq \theta \leq \beta$

$$L = \int_{\alpha}^{\beta} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$



- Cartesian Explanation

$x = r(\theta)\cos\theta$, $y = r(\theta)\sin\theta$

$$L = \int_{\alpha}^{\beta} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\, d\theta$$

$$\frac{dx}{d\theta} = \frac{dr}{d\theta}\cos\theta - r\sin\theta$$

$$\frac{dy}{d\theta} = \frac{dr}{d\theta}\sin\theta + r\cos\theta$$

... gives u: