1.4.1 Planes in 3d space

- Parametrized curves in 3d
- point P $\longleftrightarrow$ arrow from origin to P
  (point in 3d) (3-component vector)

$$\underbrace{\vec{r}(t) = \vec{r_0}}\ + \underbrace{t}_{t \text{ can be all real number}}\vec{v}$$

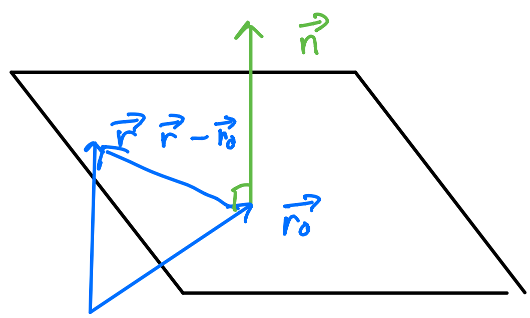
↓ vector form

$$\boxed{\begin{aligned} &\langle x(t), y(t), z(t) \rangle = \langle x_0, y_0, z_0 \rangle + t\langle a, b, c \rangle \\ &x(t) = x_0 + ta \\ &y(t) = y_0 + tb \\ &z(t) = z_0 + tc \end{aligned}}$$

1.4.2 Planes

$\vec{r_0}$ = a point on the plane

$\vec{n}$ = a normal (perpendicular) vector to the plane



$\vec{r}$ is on the plane when $\vec{r}-\vec{r_0} \perp \vec{n}$

i.e. $(\vec{r}-\vec{r_0}) \cdot \vec{n} = 0$

Cross product and determinant

Given two nonzero vector $a = \langle a_1, a_2, a_3 \rangle$ and $b = \langle b_1, b_2, b_3 \rangle$

We want to find a nonzero vector c s.t. it's perpendicular to both a and b.

This means that $c \perp a$ & $c \perp b$, OR:

$a \cdot c = 0$ and $b \cdot c = 0$ by how we define "perpendicular to" using dot product.

Turns out that $c = \langle c_1, c_2, c_3 \rangle$

$$= \langle a_2b_3 - a_3b_2,\ a_3b_1 - a_1b_3,\ a_1b_2 - a_2b_1 \rangle$$

$$\Downarrow$$

$$a \times b = \langle a_2b_3 - a_3b_2,\ a_3b_1 - a_1b_3,\ a_1b_2 - a_2b_1 \rangle$$

this is a vector not a scalar

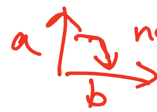
* defined only when a & b are 3d

- Another way to write out cross product:

$$a \times b = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \swarrow \text{det}$$

Area of parallelogram by $a \times b$:

positive



negative

area given by $\det\left(\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}\right)$

or area of parallelogram

- length of cross product: $|a \times b| = |a||b| \sin\theta$

* $a \times b$ is orthogonal to both a and b.

Moreover $a \times b$ is a vector pointing to the direction satisfied by the right-hand rule.

- The length of the cross product $a \times b$ is equal to the area of the parallelogram determined by a and b.