

Slides adapted from Cyrill Stachniss, Michael Kaess, S.Scherer

Map Representations

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TAs: Matthew Rockett, Gilwoo Lee, Matt Schmittle

Announcements

Deadline for lab1 extended to **Wednesday 4/30 at 11:59 p.m**

This is the due date for the writeup

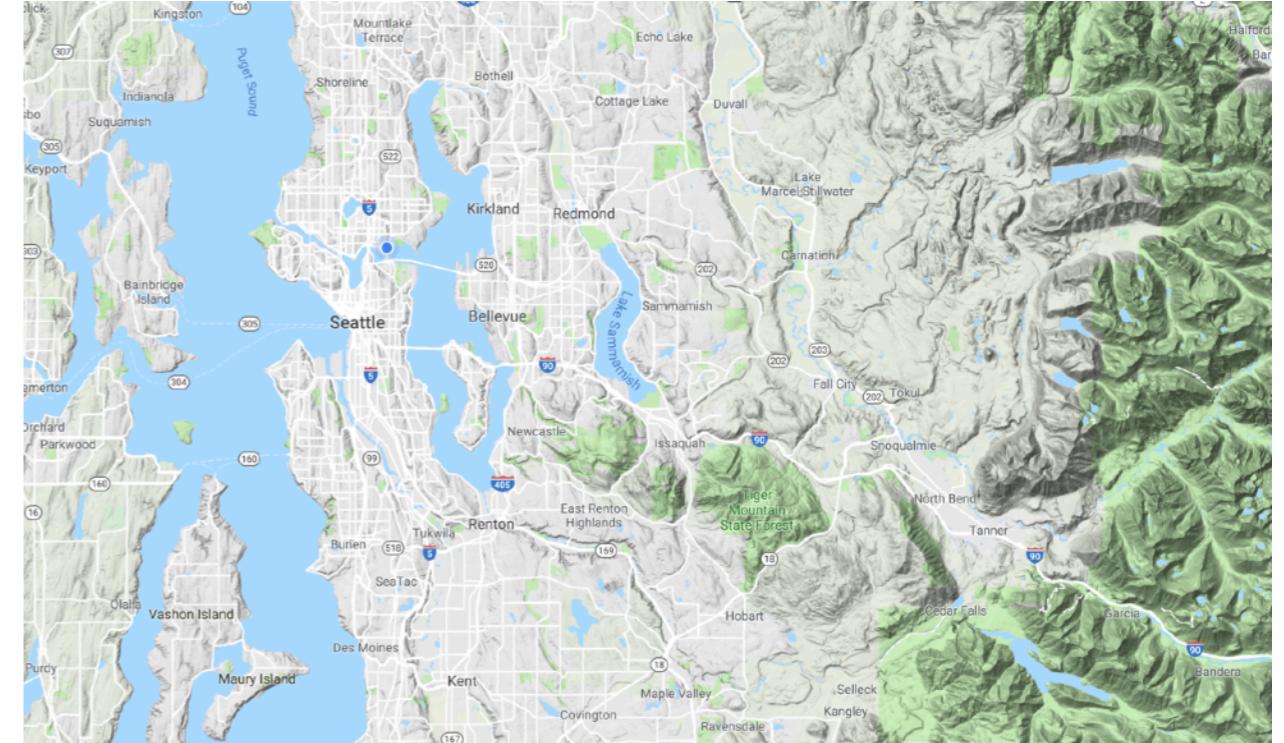
The lab evaluation is still on **Thursday 4/25**
from **9.00 a.m - 12:00 p.m**

Please continue to update blogs by Friday of **each week**

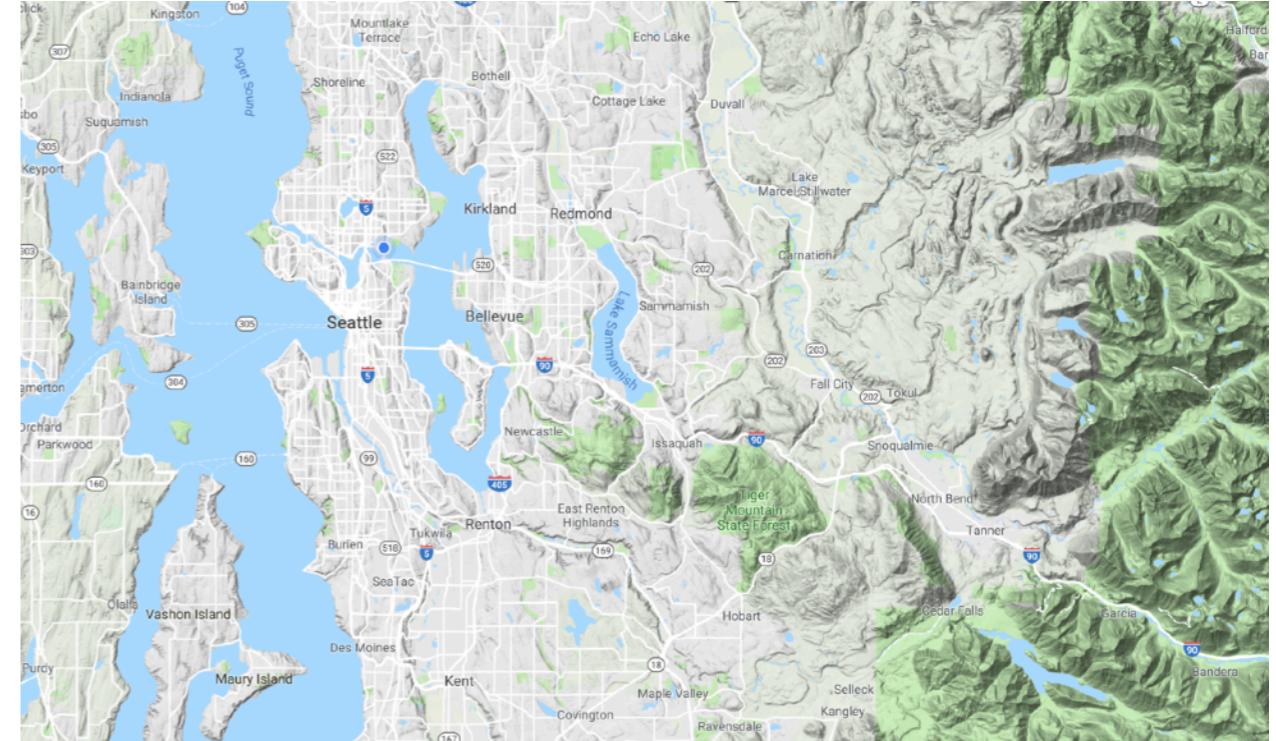
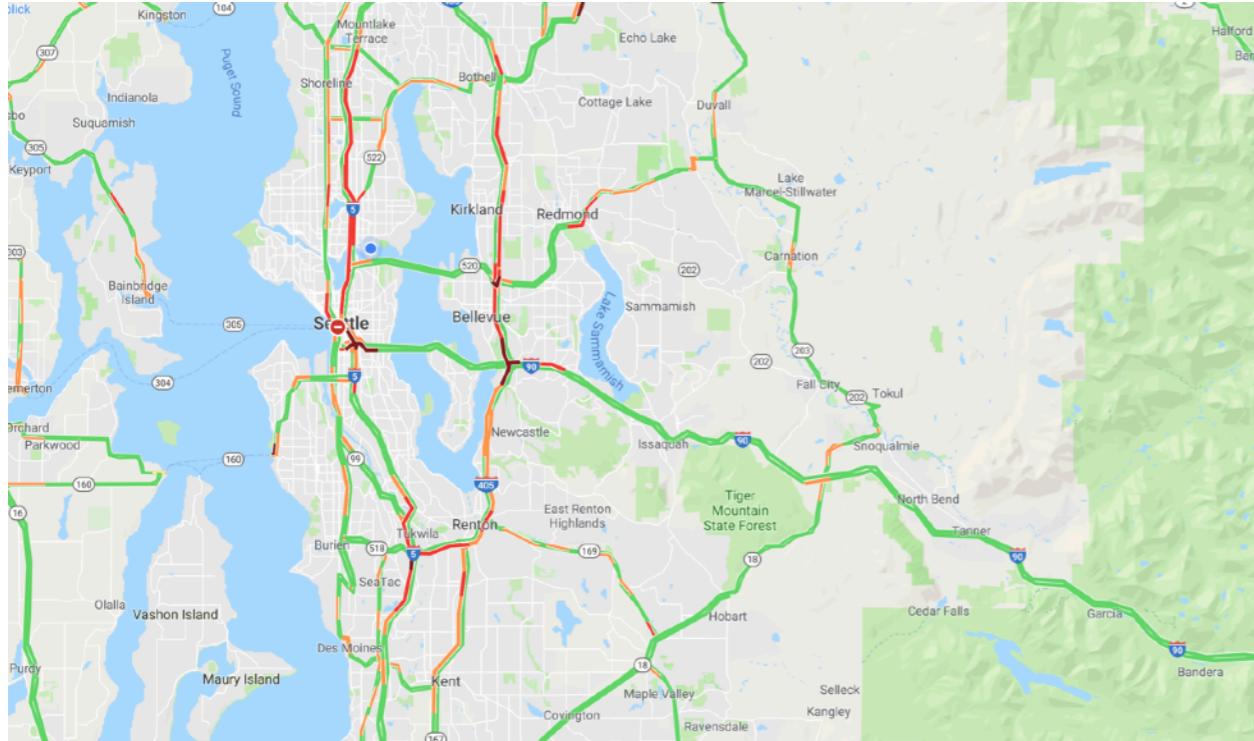
What is a map?

Do all maps convey the same information?

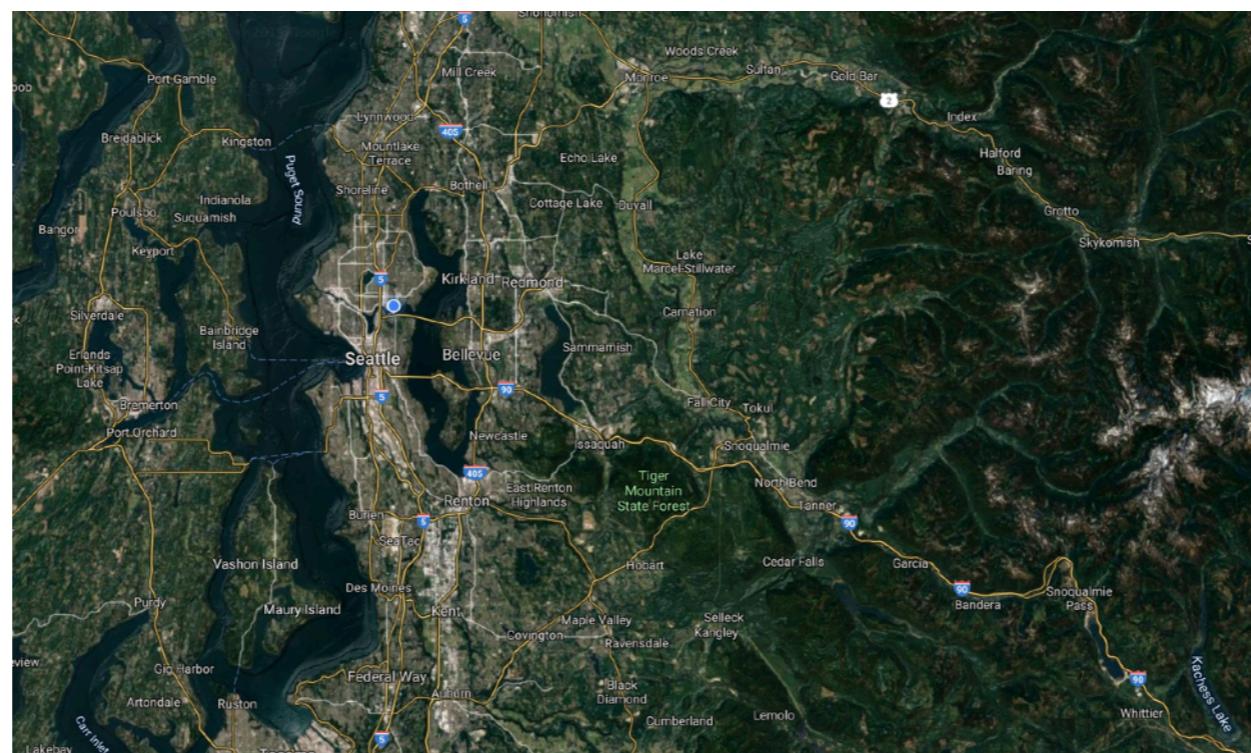
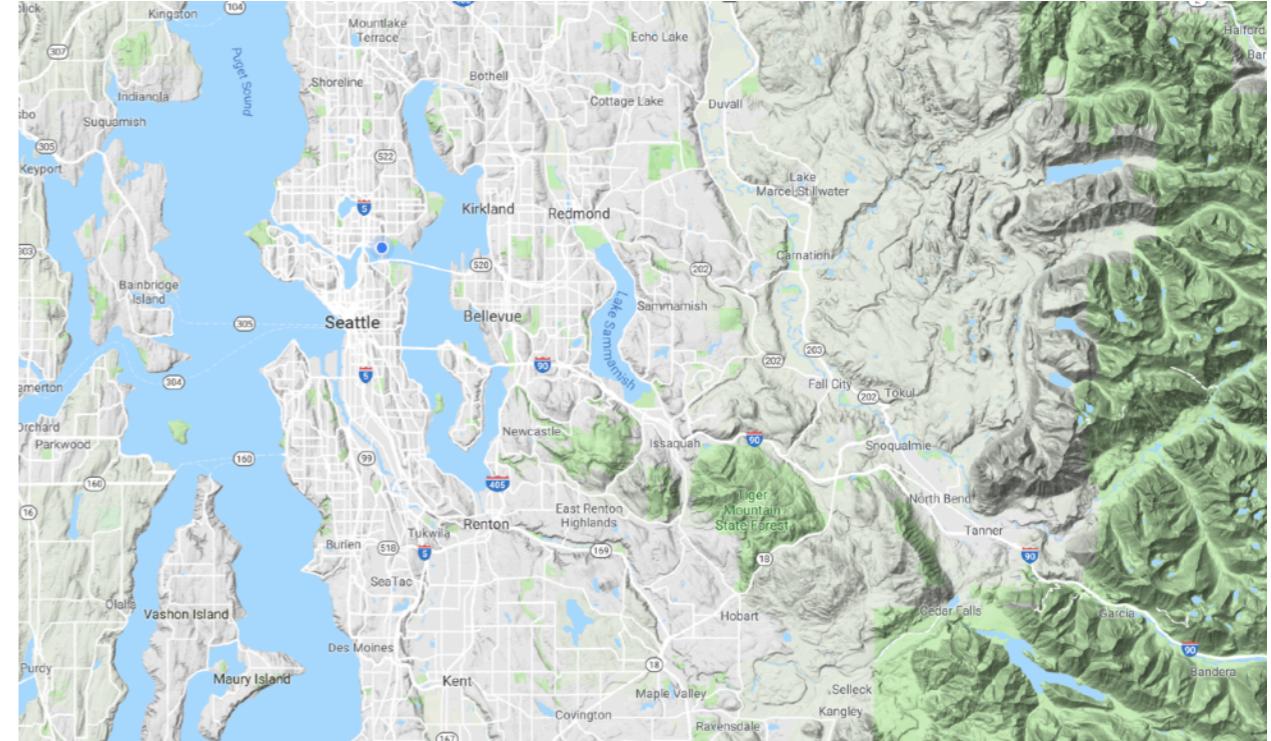
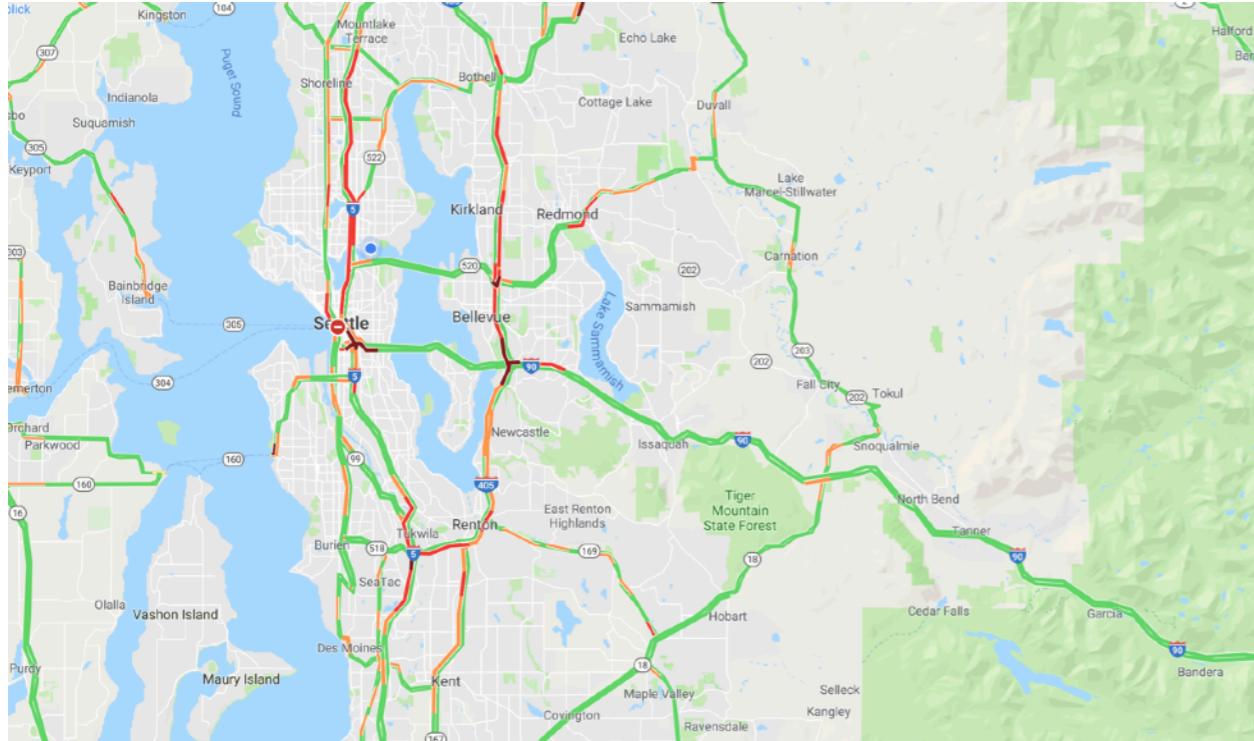
Do all maps convey the same information?



Do all maps convey the same information?



Do all maps convey the same information?



Maps are a **summary of information** about the world

Maps are a **summary of information** about the world

What sort of information? Depends on the **task**

Maps are a **summary of information** about the world

What sort of information? Depends on the **task**

Task also determines how we
query, update, store maps

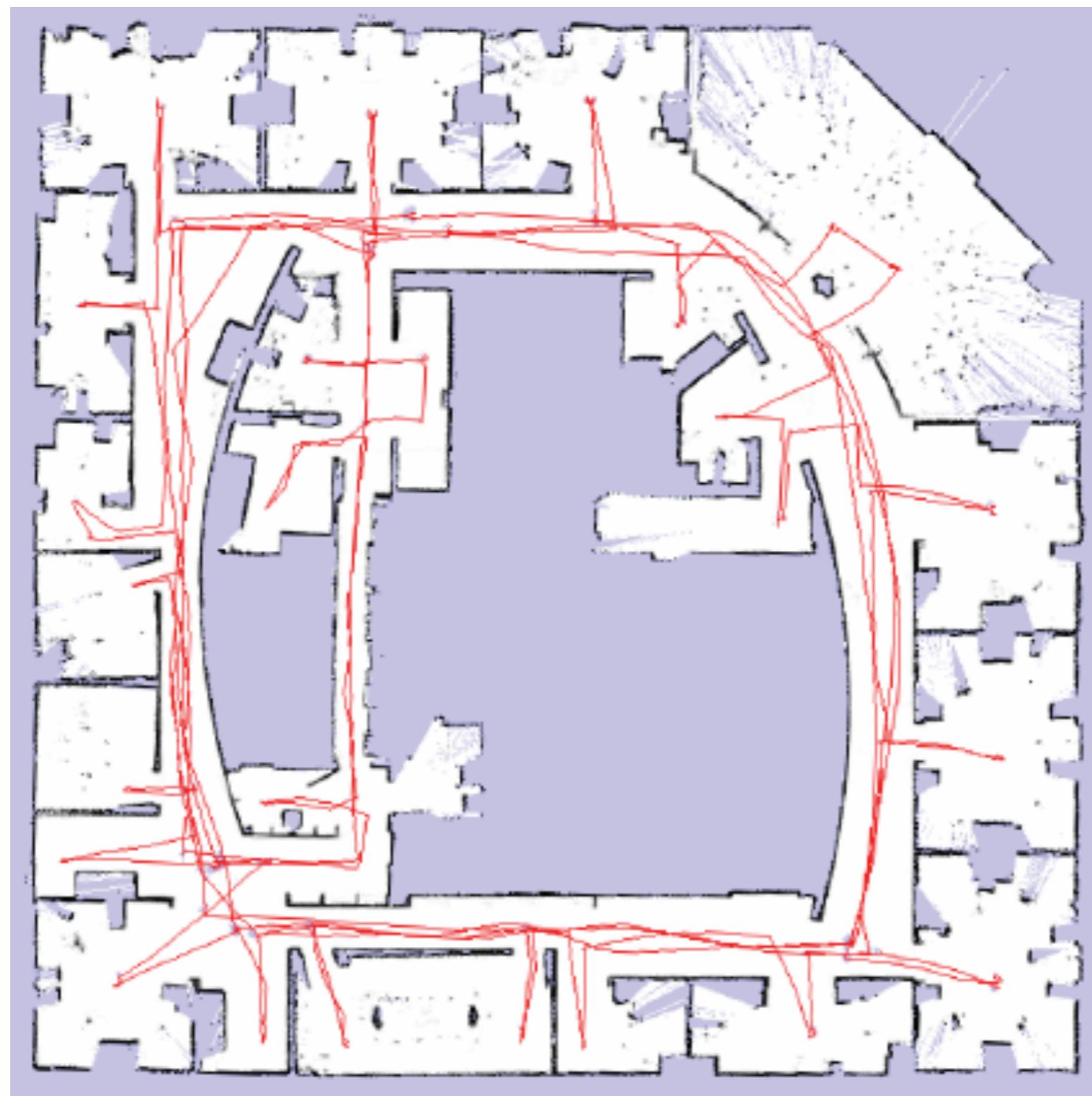
Today's objective

1. Framework / taxonomy to think about maps
2. Look at various maps and the underlying tasks they serve
3. Distance map

What do we want from maps?

1. **Information** - What task does it help me solve?
(Help me localize, help me navigate, help humans navigate / plan their lives etc)
2. **Query** - Can we query it online? How often?
3. **Updatable** - Can we update it online? Can it deal with noisy measurements?
4. **Memory** - How much storage does it need? Is it transportable? How does it scale with time? Scale with amount of stuff we see ?

Example 1: Occupancy grids



Example 1: Occupancy grids

Category

Details

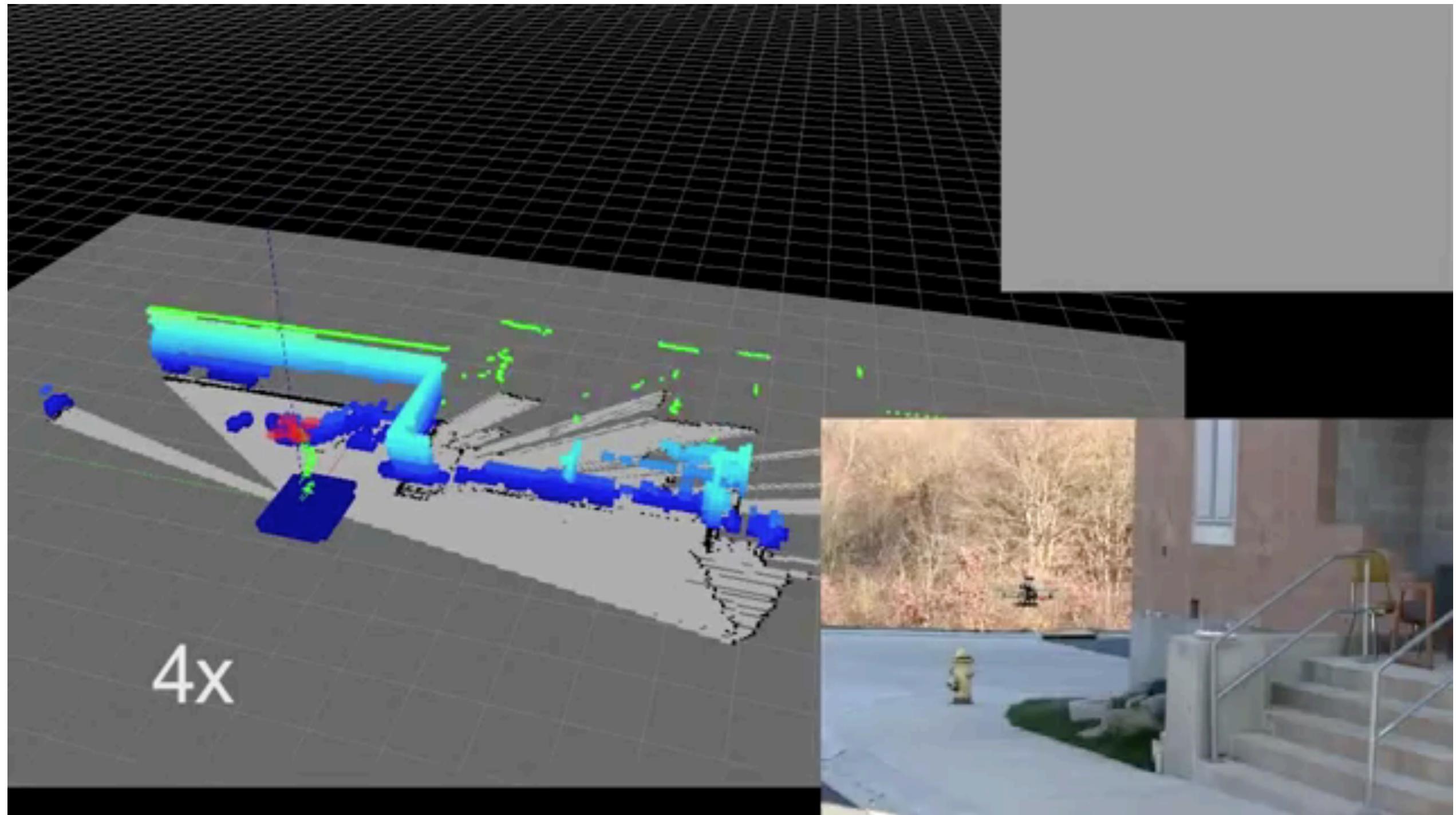
Information

Query

Update

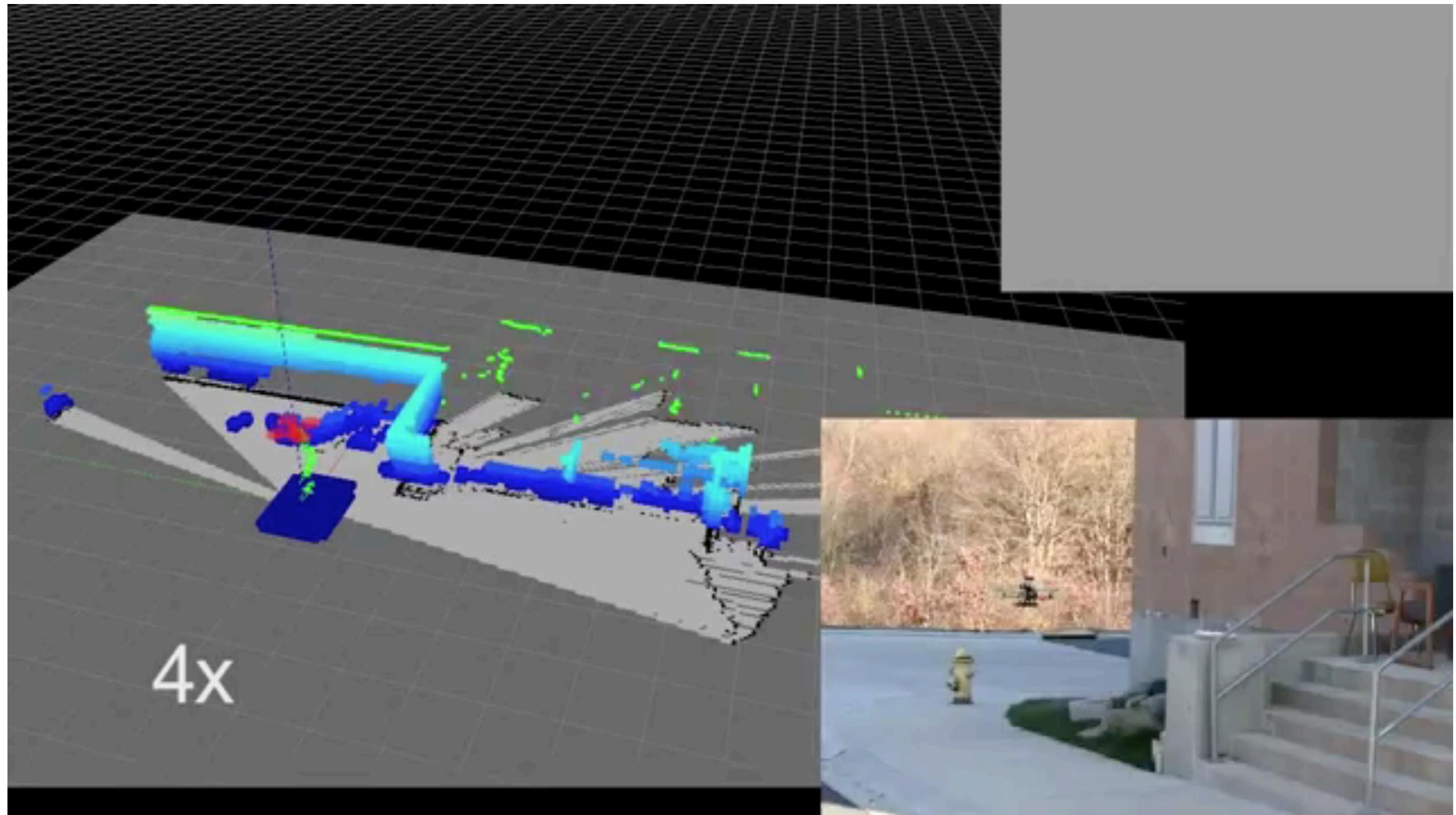
Memory

Occupancy grids in action



“Autonomous Multi-Floor Indoor Navigation with a Computationally Constrained MAV”, S. Shen, N. Michael, V.Kumar, 2010

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Example 1: Occupancy grids

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Example 1: Occupancy grids

Category	Details
Information	Discretized likelihood of occupancy (free/occ/unknown) Useful for exploration (go to unknown areas) Useful for safe navigation (keep robot in known free space)
Query	
Update	
Memory	

Example 1: Occupancy grids

Category	Details
Information	Discretized likelihood of occupancy (free/occ/unknown) Useful for exploration (go to unknown areas) Useful for safe navigation (keep robot in known free space)
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Update	
Memory	

Example 1: Occupancy grids

Category	Details
Information	Discretized likelihood of occupancy (free/occ/unknown) Useful for exploration (go to unknown areas) Useful for safe navigation (keep robot in known free space)
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Update	Can deal with noisy sensors (log likelihood update) Updates equal ray-casting ($O(l)$ where l is length of ray)
Memory	

Example 1: Occupancy grids

Category	Details
Information	<p>Discretized likelihood of occupancy (free/occ/unknown)</p> <p>Useful for exploration (go to unknown areas)</p> <p>Useful for safe navigation (keep robot in known free space)</p>
Query	Cheap: $O(1)$
Update	<p>Can deal with noisy sensors (log likelihood update)</p> <p>Updates equal ray-casting ($O(l)$ where l is length of ray)</p>
Memory	Bounded Can still be large if we want really fine resolution Need to allocate all the memory upfront

Problems with occupancy grids

Problems with occupancy grids

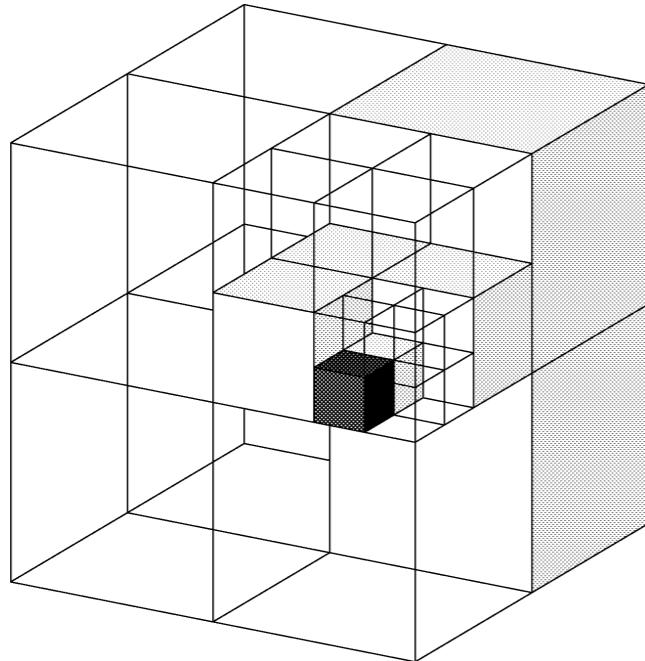
1. Memory scales with distance travelled in any one direction

Problems with occupancy grids

1. Memory scales with distance travelled in any one direction
2. Do I need high resolution information everywhere?

Example 2: Occupancy Trees (OctoMap)

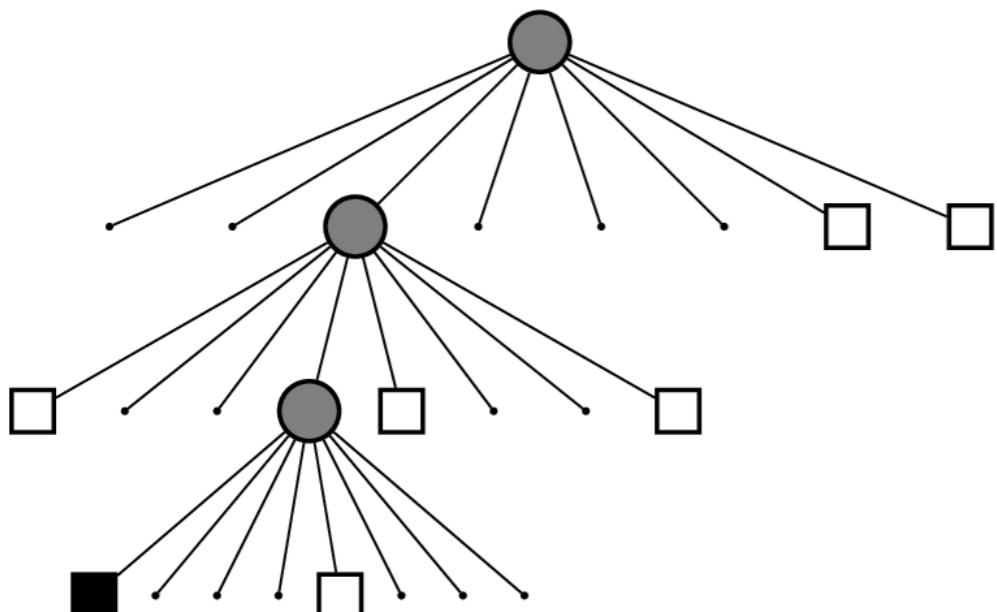
Hornung et al. 2013



Tree-based data structure

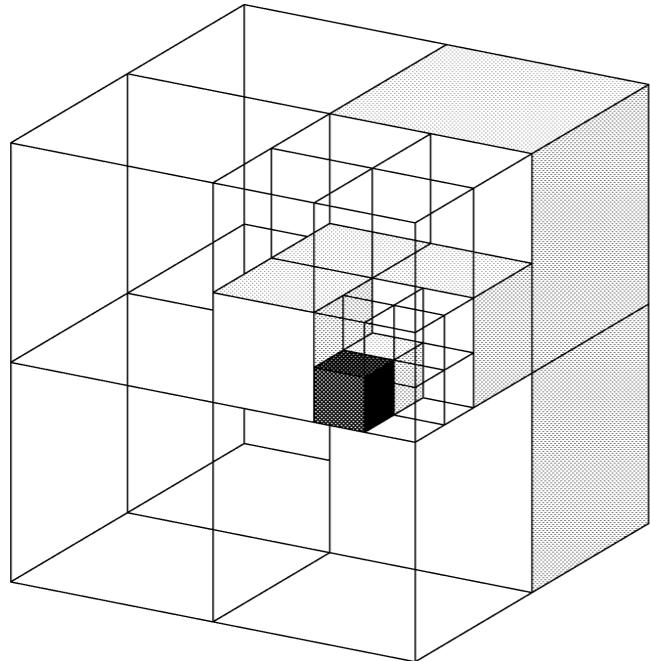
Recursive sub-division of space

Query maps at multiple-resolutions!



Example 2: Occupancy Trees (OctoMap)

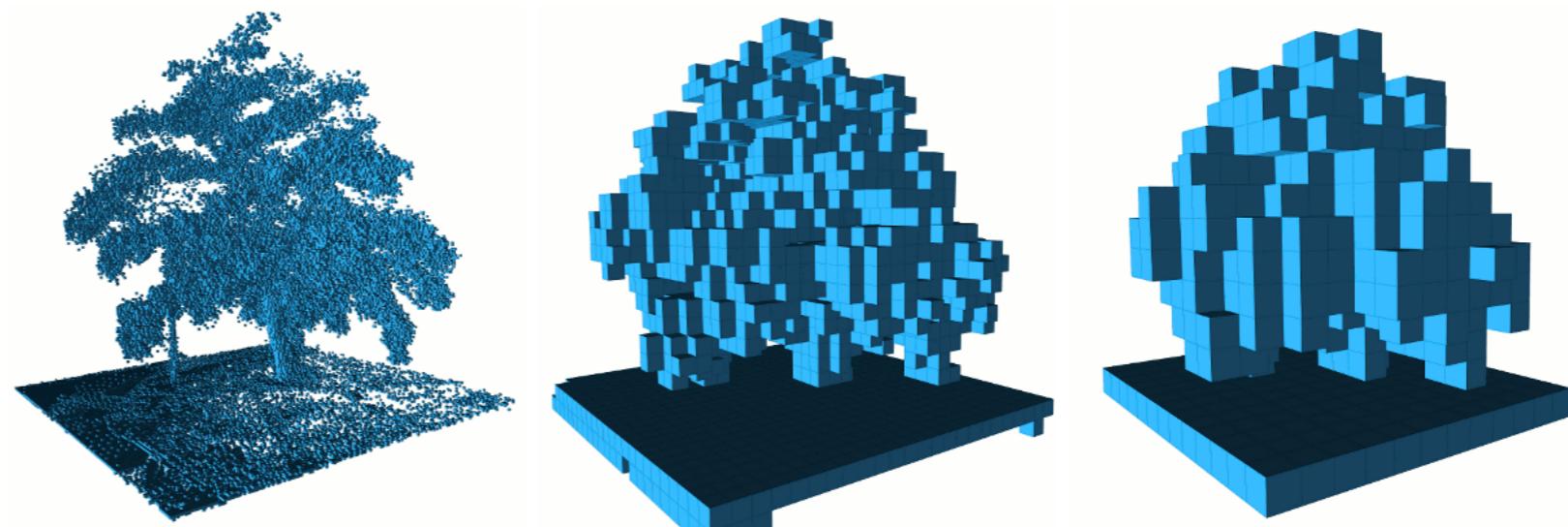
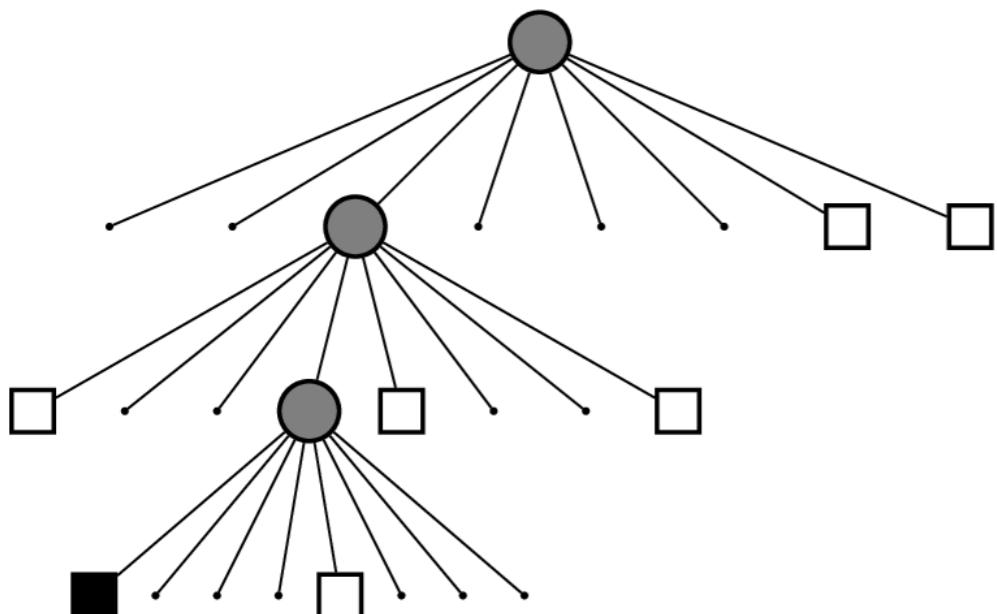
Hornung et al. 2013



Tree-based data structure

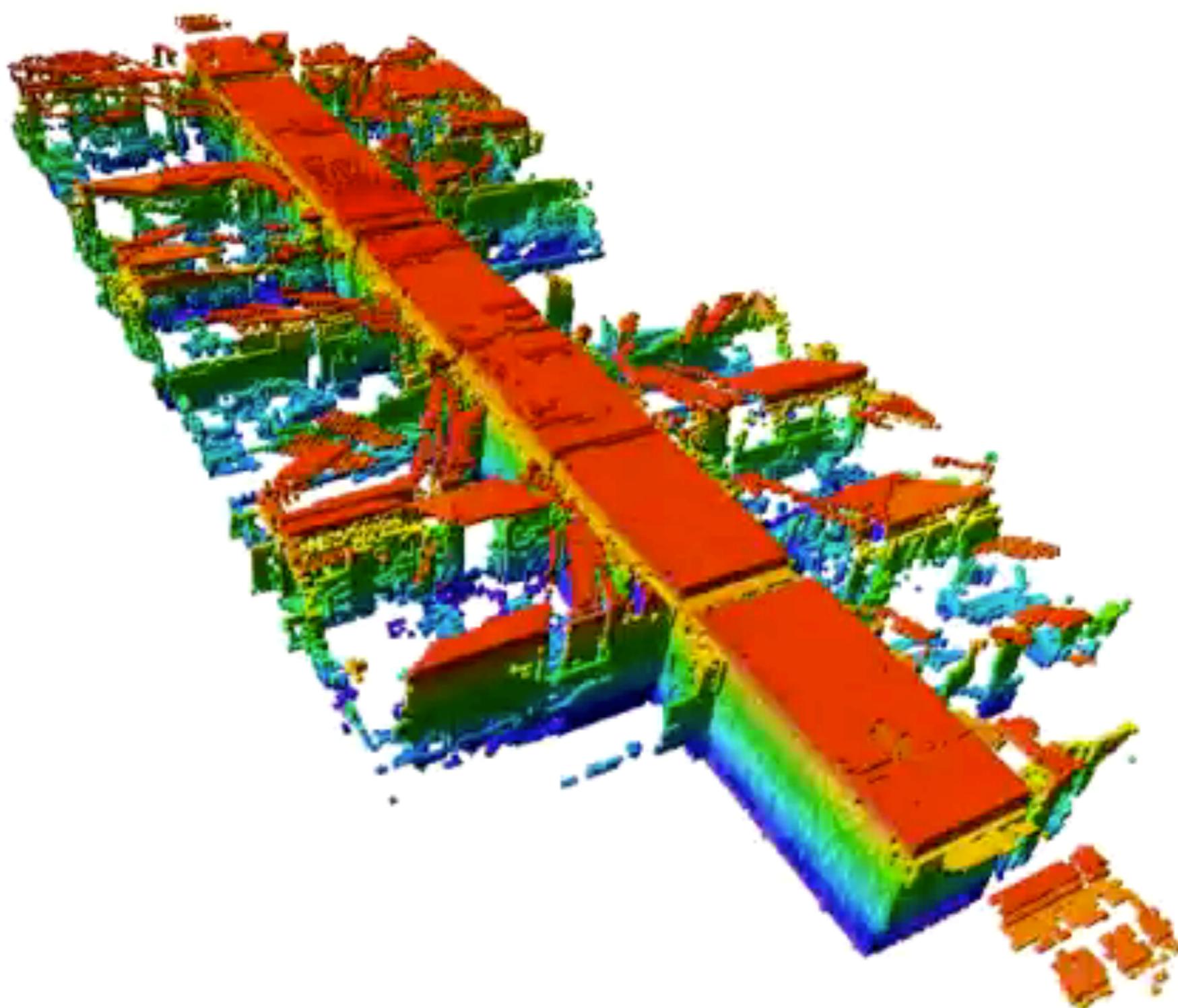
Recursive sub-division of space

Query maps at multiple-resolutions!

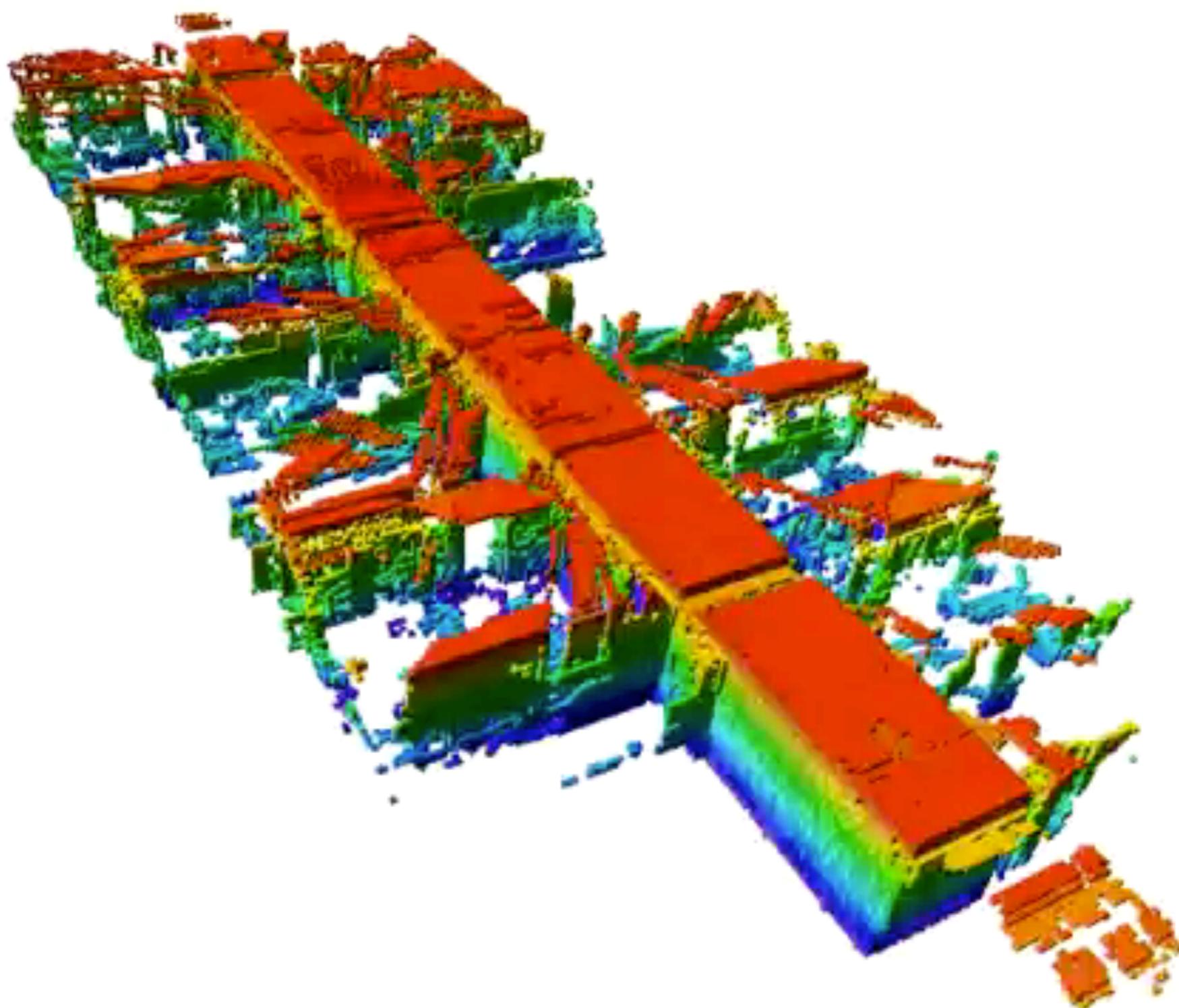


<https://octomap.github.io/>

Example 2: OctoMap



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Example 2: OctoMap

Category

Details

Information

Query

Update

Memory

Example 2: OctoMap

Category	Details
Information	Same as occupancy grids Stores information at multiple resolutions . Useful for large scale exploration, multi-res planning.
Query	
Update	
Memory	

Example 2: OctoMap

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Information	Same as occupancy grids Stores information at multiple resolutions . Useful for large scale exploration, multi-res planning.
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Update	
Memory	

Example 2: OctoMap

Category	Details
Information	Same as occupancy grids Stores information at multiple resolutions . Useful for large scale exploration, multi-res planning.
Query	Little expensive : $O(\log n)$, where n is the number of nodes in tree
Update	Similar to occupancy grids, extra $O(\log n)$ complexity
Memory	

Example 2: OctoMap

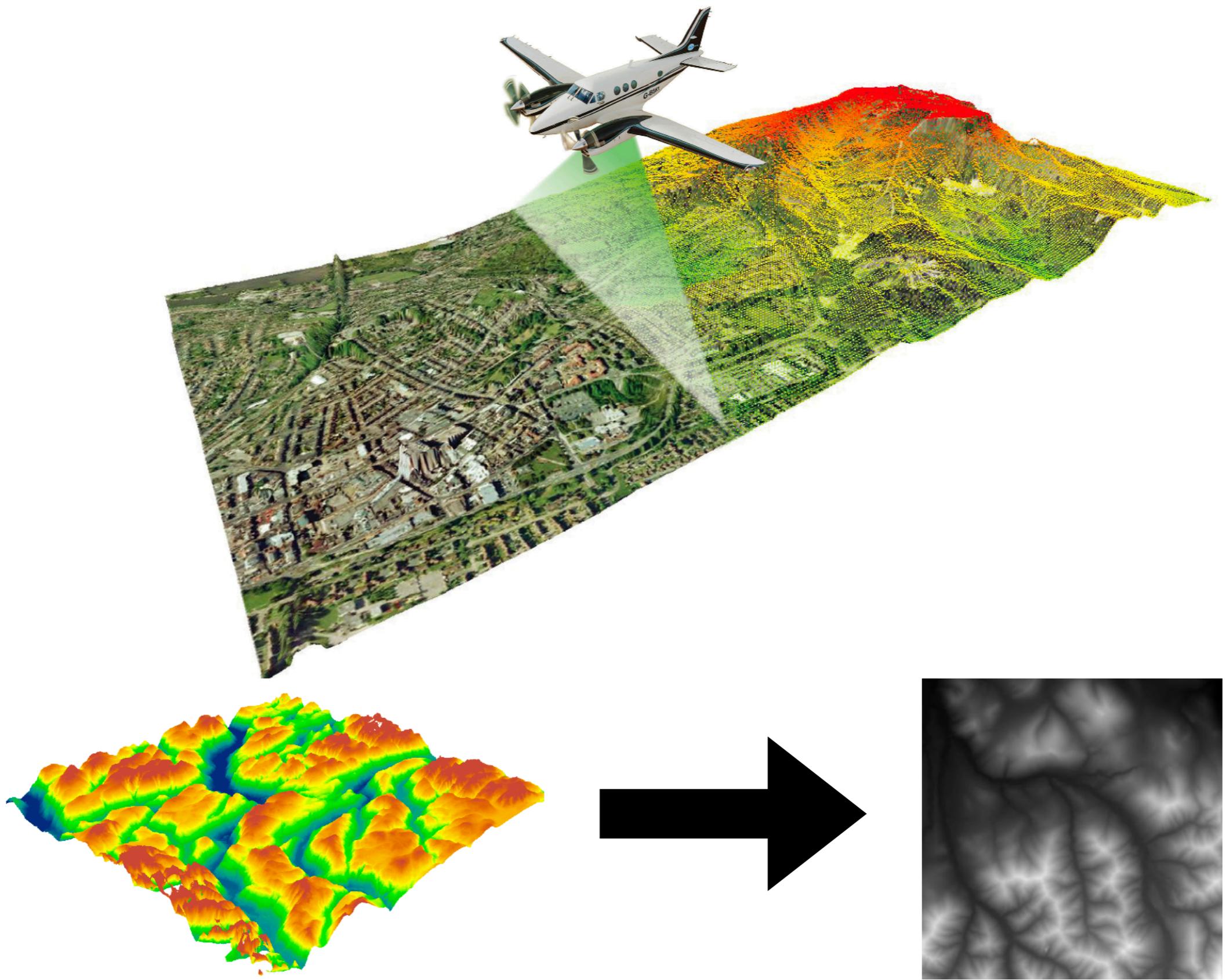
Category	Details
Information	Same as occupancy grids Stores information at multiple resolutions . Useful for large scale exploration, multi-res planning.
Query	Little expensive : $O(\log n)$, where n is the number of nodes in tree
Update	Similar to occupancy grids, extra $O(\log n)$ complexity
Memory	Much smaller than occupancy grids (proportional to amount of stuff in the world)

Is the world always 3D?

~~Is the world always 3D?~~

Do we care about 3D?

Example 3: 2.5D height map



Example 3: 2.5D height map

Category	Details
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Information	
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Query	
-------	--

Update	
--------	--

Memory	
--------	--

Example 3: 2.5D height map

Category	Details
Information	<p>Image where each pixel denotes height. Useful for mapping terrain where for overhead flight. <i>Don't use when flying underneath objects</i></p>
Query	
Update	
Memory	

Example 3: 2.5D height map

Category	Details
Information	<p>Image where each pixel denotes height. Useful for mapping terrain where for overhead flight. <i>Don't use when flying underneath objects</i></p>
Query	$O(1)$
Update	
Memory	

Example 3: 2.5D height map

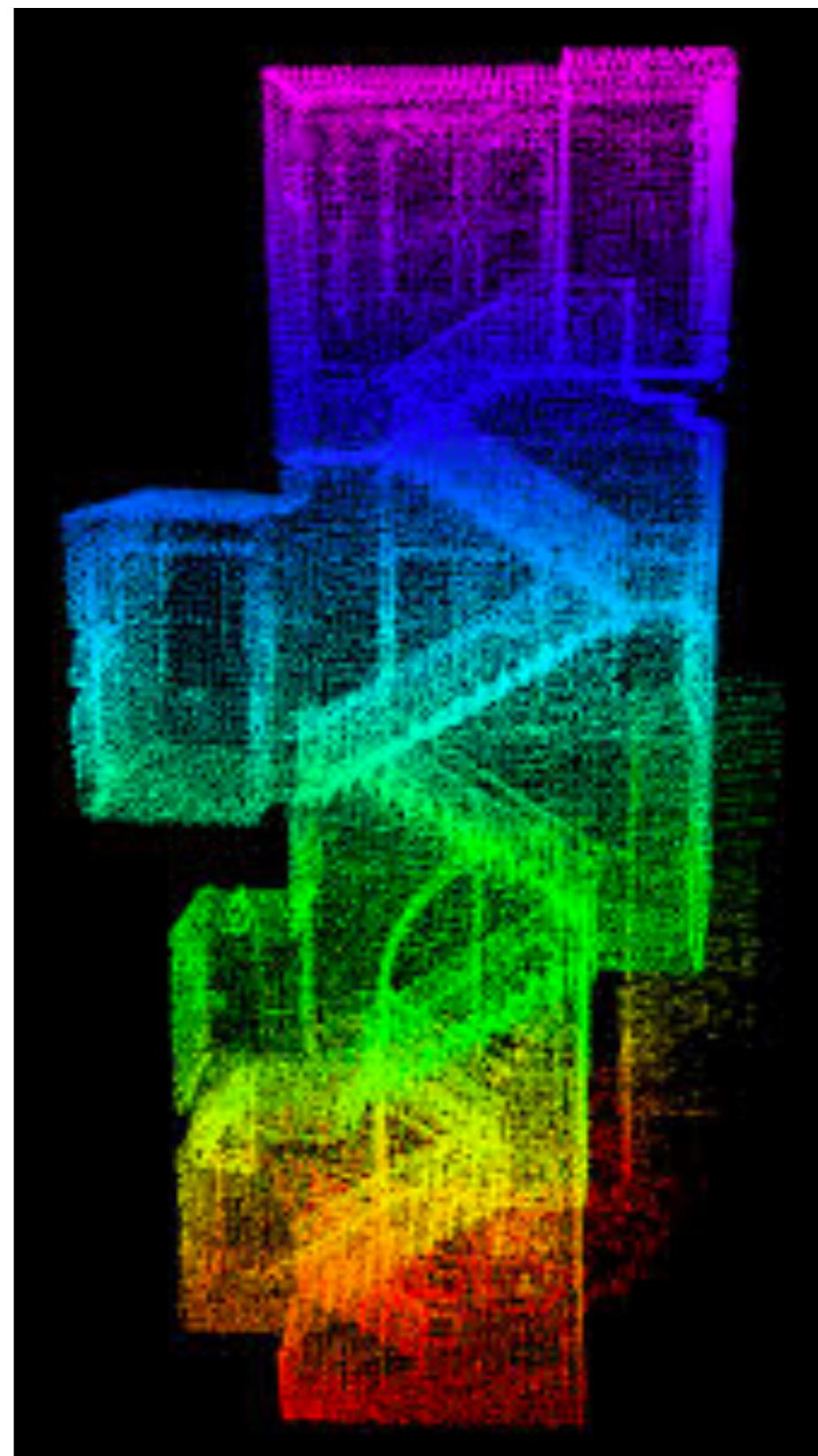
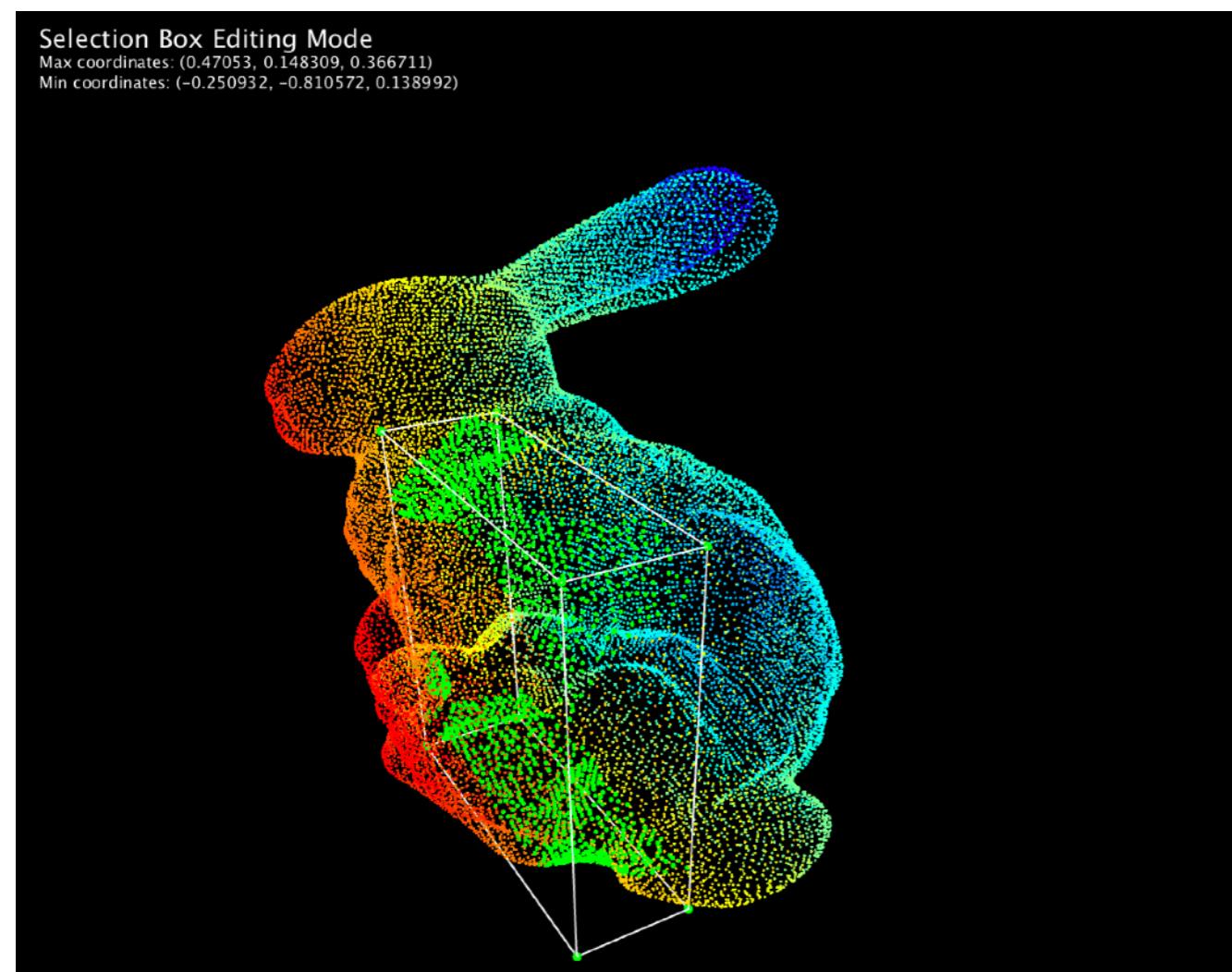
Category	Details
Information	<p>Image where each pixel denotes height. Useful for mapping terrain where for overhead flight. <i>Don't use when flying underneath objects</i></p>
Query	$O(1)$
Update	<p>Can handle noisy measurements by defining a Bayes filter for height of each cell.</p>
Memory	

Example 3: 2.5D height map

Category	Details
Information	<p>Image where each pixel denotes height. Useful for mapping terrain where for overhead flight. Don't use when flying underneath objects</p>
Query	$O(1)$
Update	<p>Can handle noisy measurements by defining a Bayes filter for height of each cell.</p>
Memory	Very cheap! (2D grid)

What are my options if I don't want
to discretize?

Example 4: Point cloud



courtesy Ji Zhang

Example 4: Point cloud



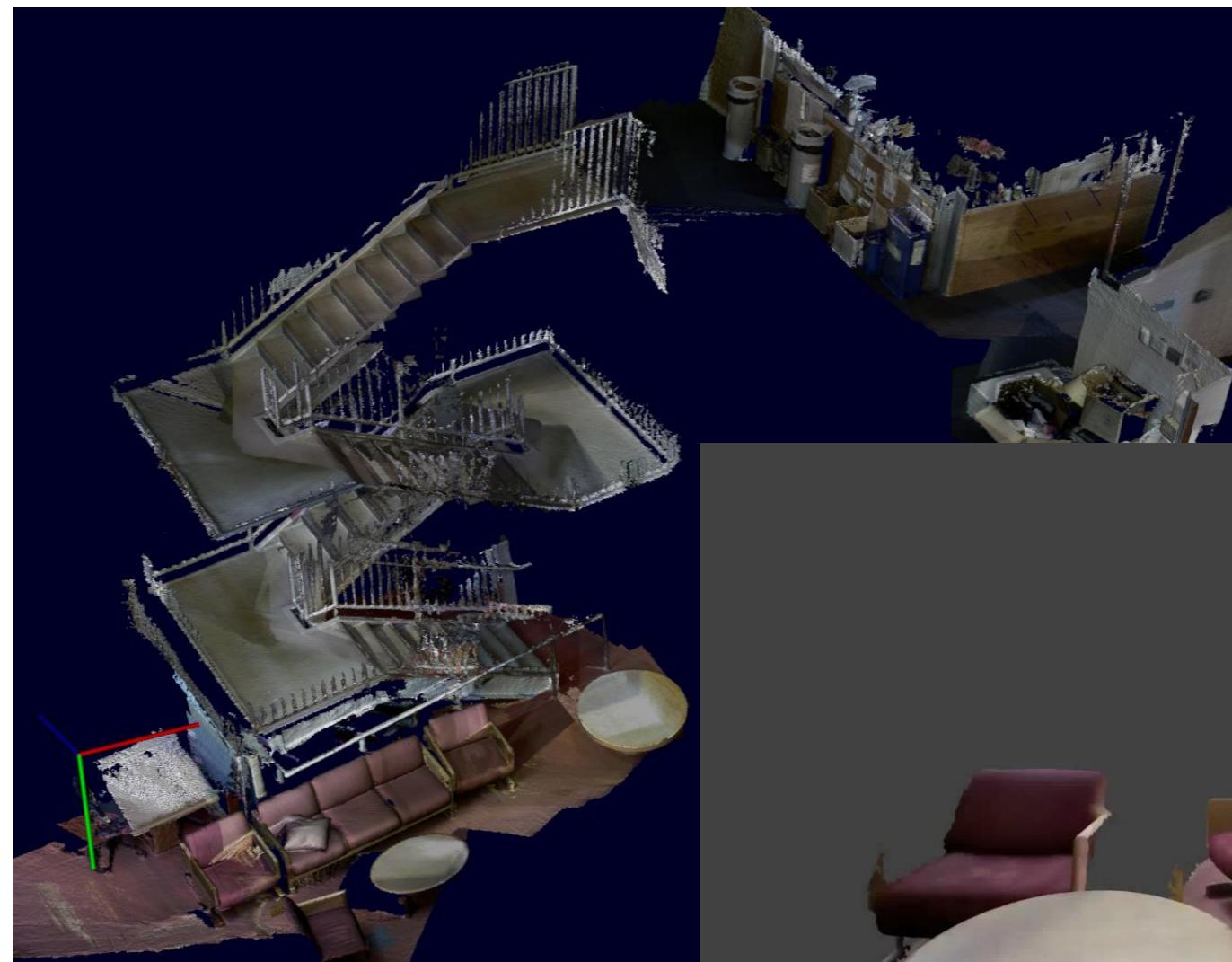
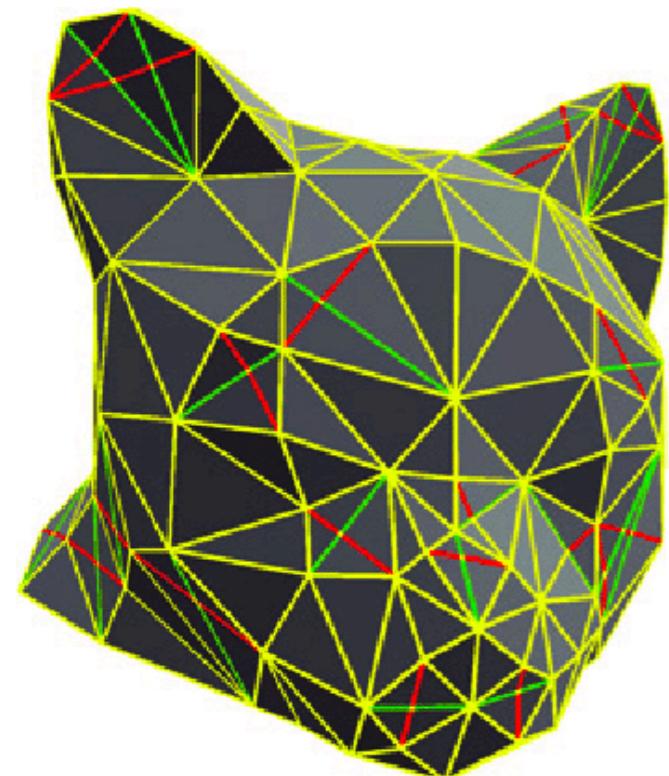
Example 4: Point cloud



Example 4: Point clouds

Category	Details
Information	<p>Surface of obstacles (no discretization)</p> <p>Useful for 3D reconstruction</p> <p>Very accurate laser based odometry.</p>
Query	<p>Typical query - give me the closest point / set of points</p> <p>Naive query is $O(N)$ (remember N is huge!!!)</p>
Update	<p>Easy to update (just dump points)</p> <p>Cannot deal with noisy measurements</p>
Memory	Unbounded - can always keep adding points on top of each other indefinitely.

Example 5: Surface representations



- Handheld RGB-D sensor (\$180)
- Real-time with GPU processing



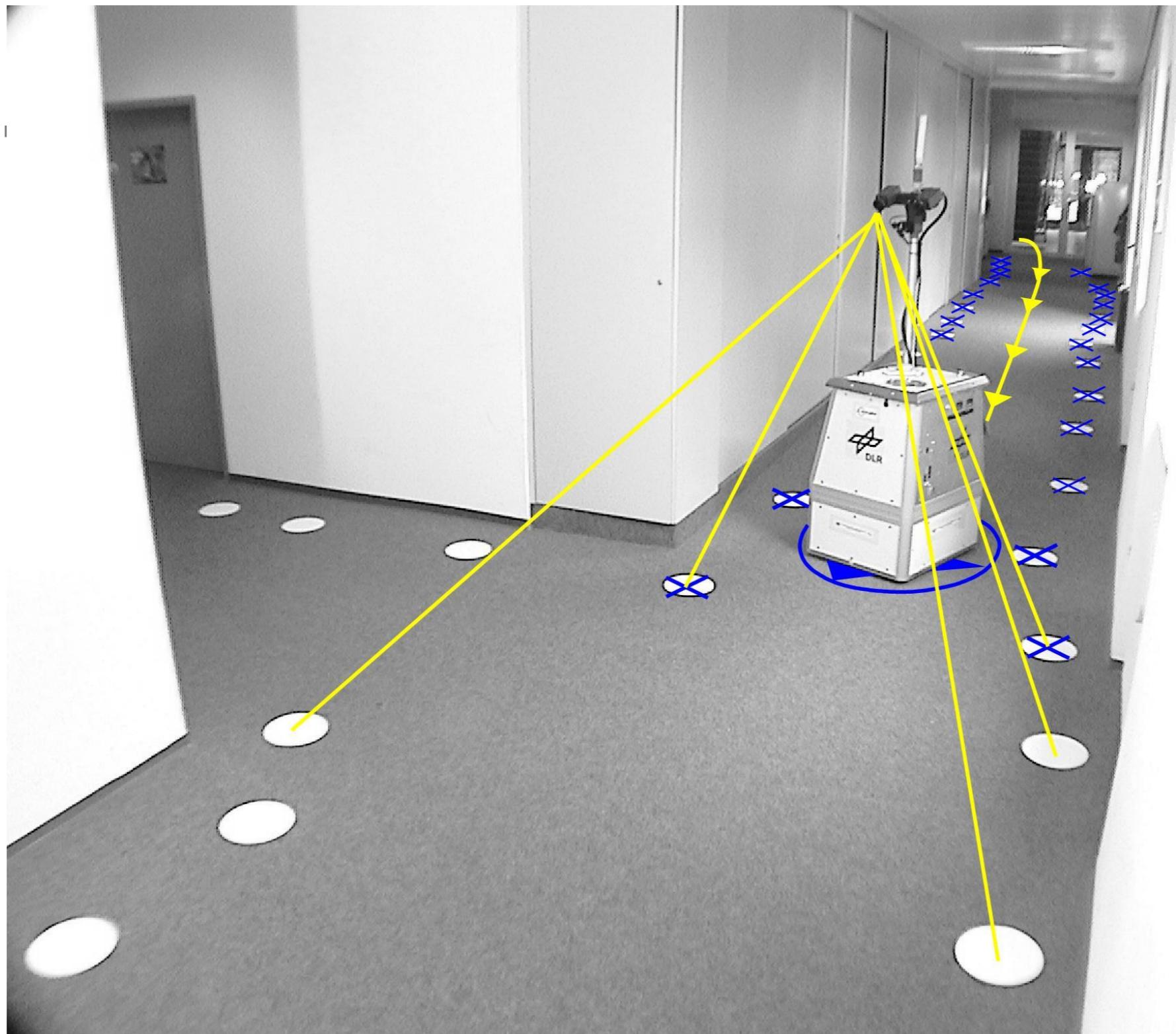
courtesy M.Kaess

Example 5: Surface representations

Category	Details
Information	<p>List of triangles representing surface No discretization, arbitrary surfaces Used for computing object object interactions</p>
Query	<p>Find the closest surface. Very naively $O(N)$ but can get massive speedups</p>
Update	<p>Can be updated online (albeit non-trivial) Very susceptible to noisy sensors</p>
Memory	<p>Proportional to amount of surface</p>

Maps that help robots localize

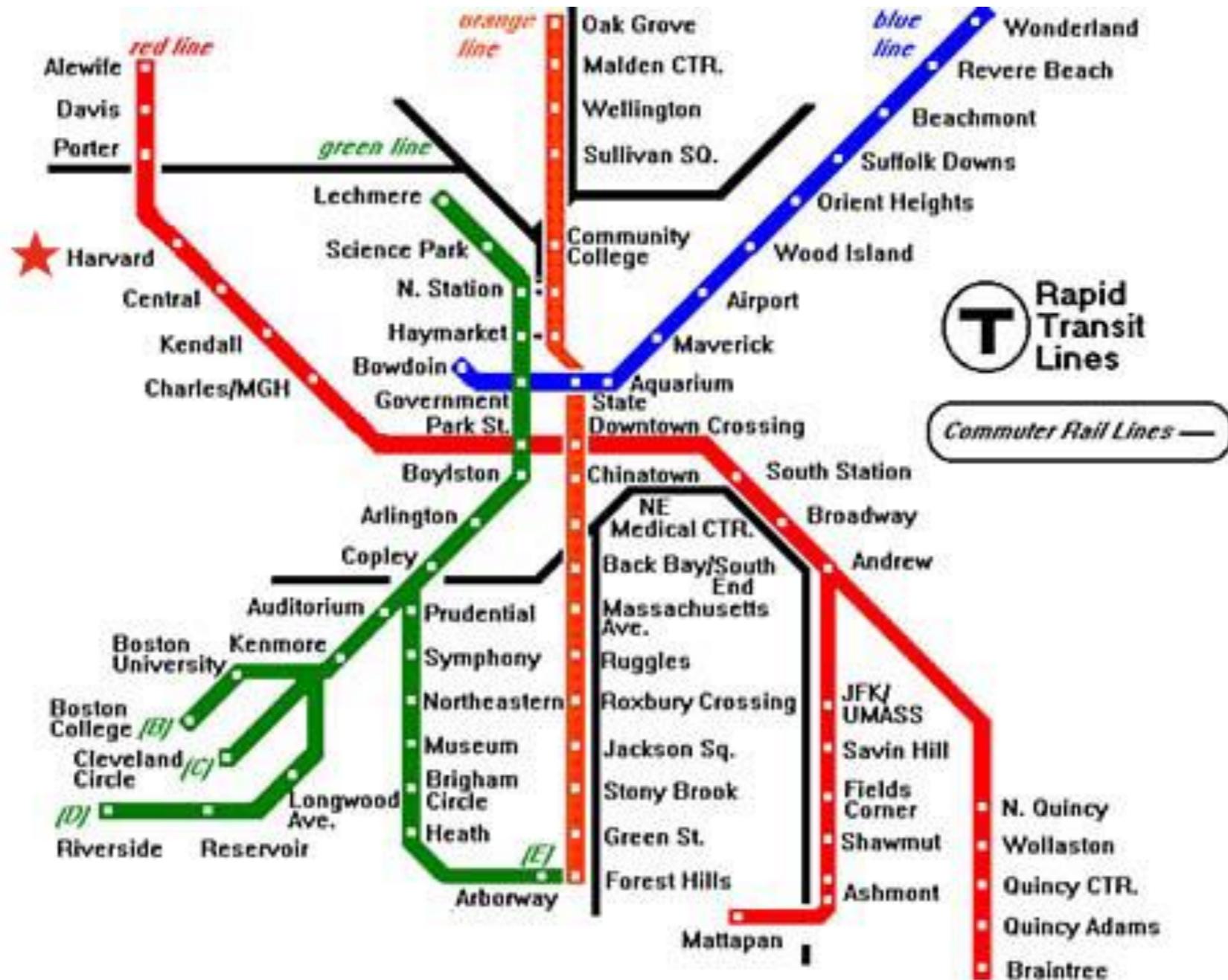
Example 6: Landmark maps



Example 6: Landmark maps

Category	Details
Information	Localization (correspondence between images at different timesteps)
Query	Typical query - give me the closest landmark Naive query is $O(N)$
Update	Easy to update (just dump landmarks) Need outlier rejection
Memory	Unbounded (but usually small as landmarks are sparse)

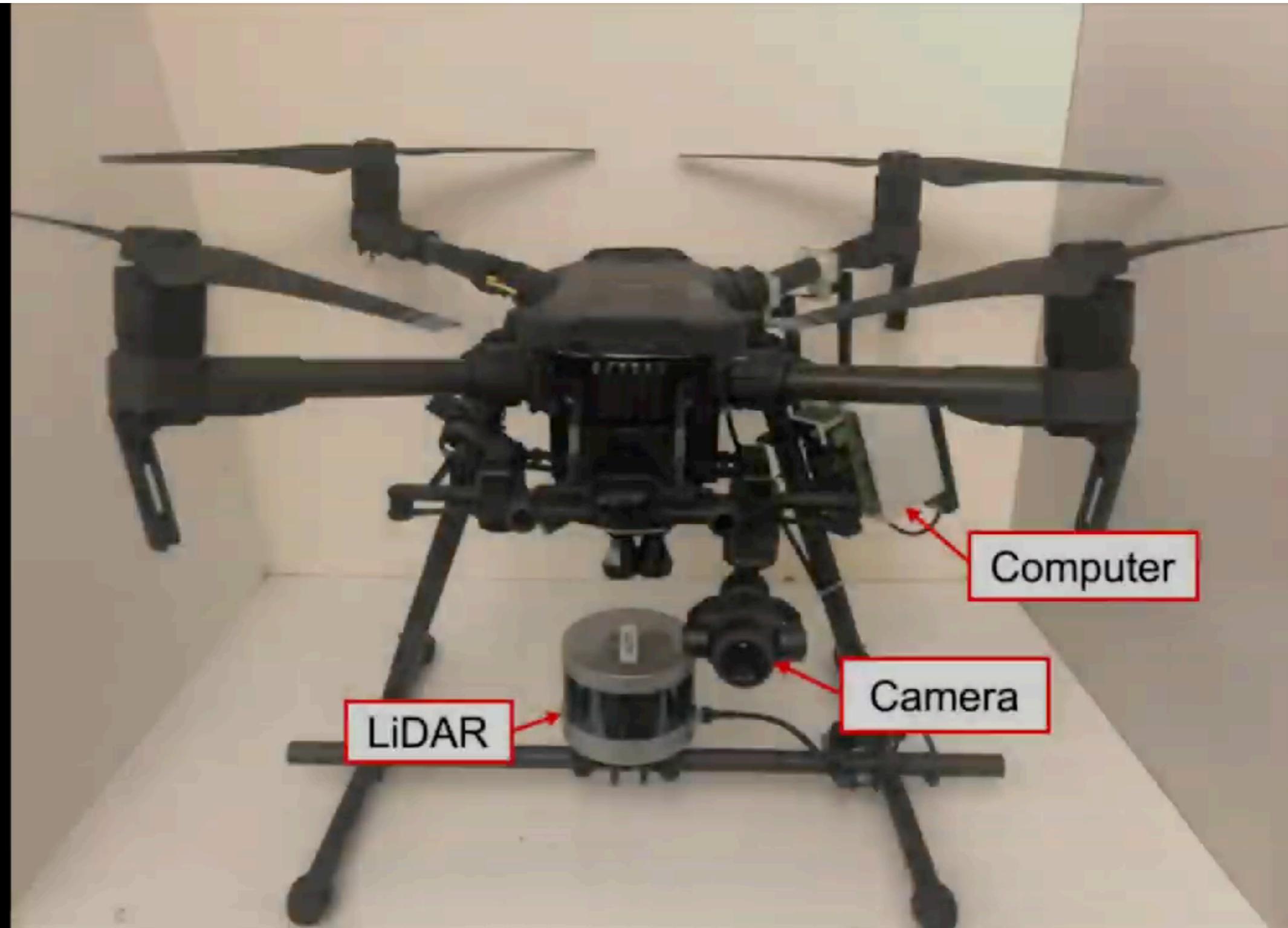
Example 7: Topological representations



Example 7: Topological representations

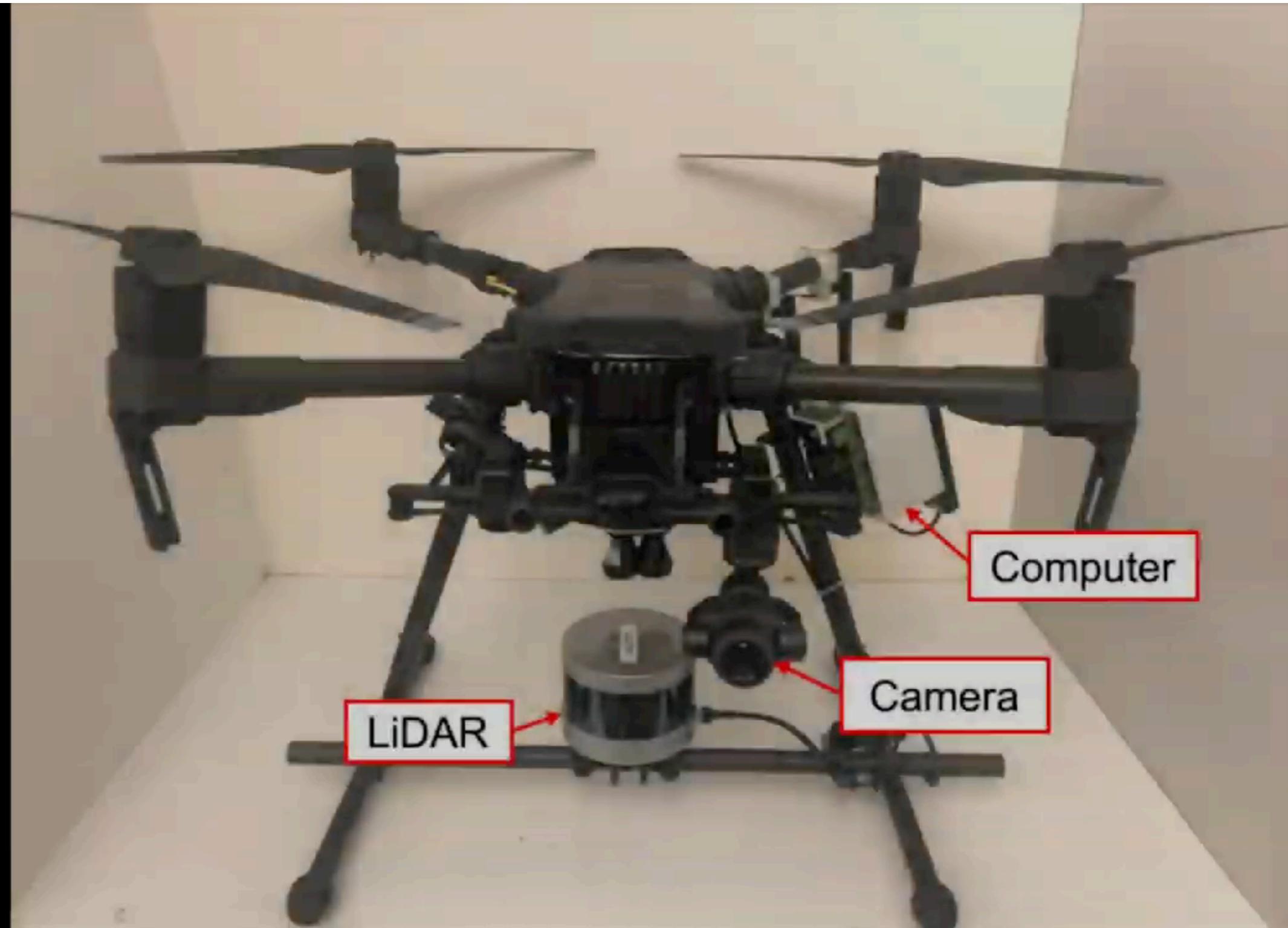
Category	Details
	Graph where vertices are landmarks (e.g. rooms in a building), and edges represent relationships (connections)
Information	High level navigation tasks which are specified on the topomap.
	Localize robot on the map by finding correspondence with vertices.
Query	Cheap graph query
Update	Non-trivial / mostly done offline
Memory	Low

Applications with multiple map representations



Bonnatti et al. 2019

Applications with multiple map representations



Bonnatti et al. 2019

Maps are not just ways of storing
sensor data

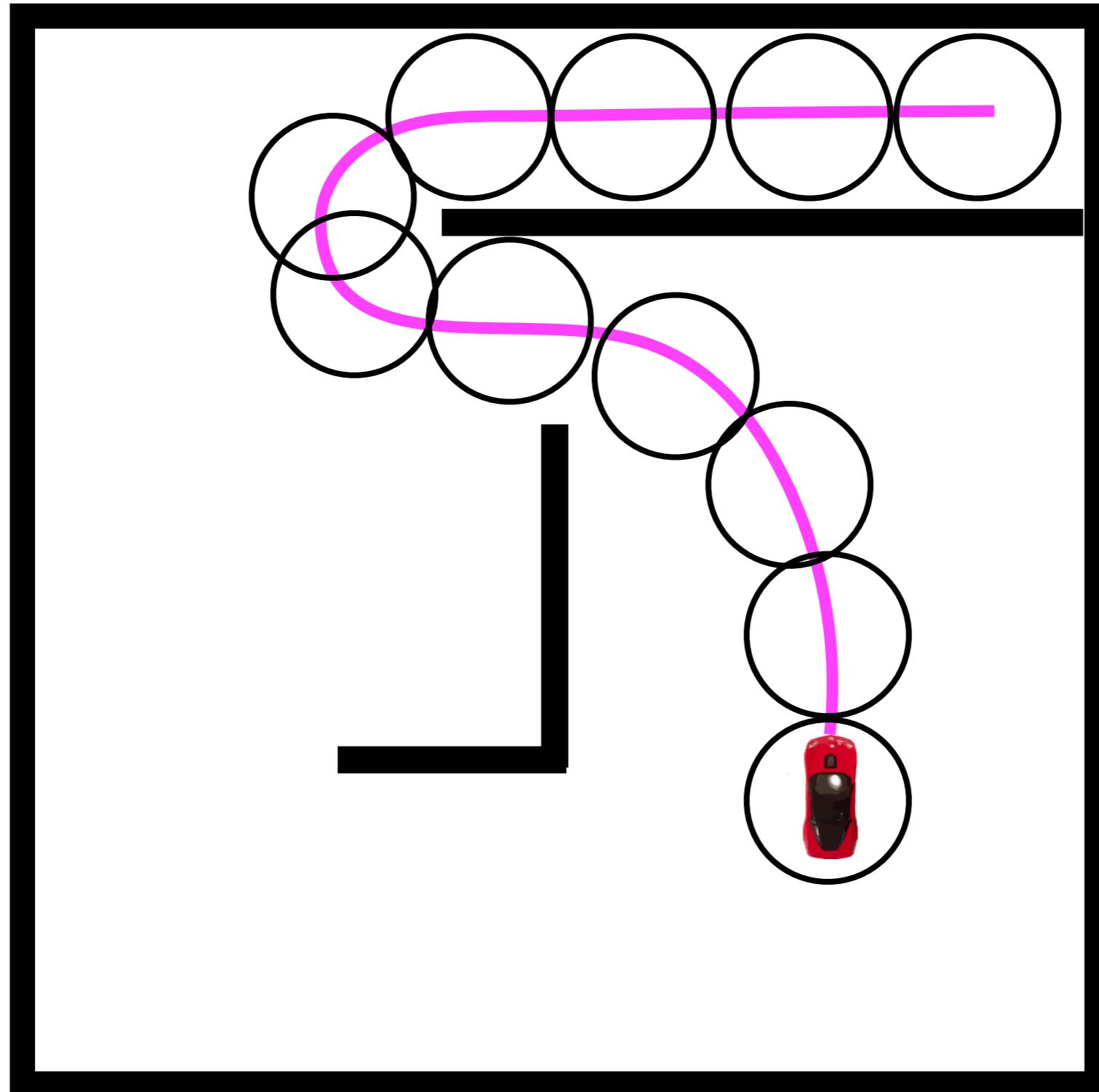
Some maps are computational
operations on other maps

Distance map

Why do we need distance?

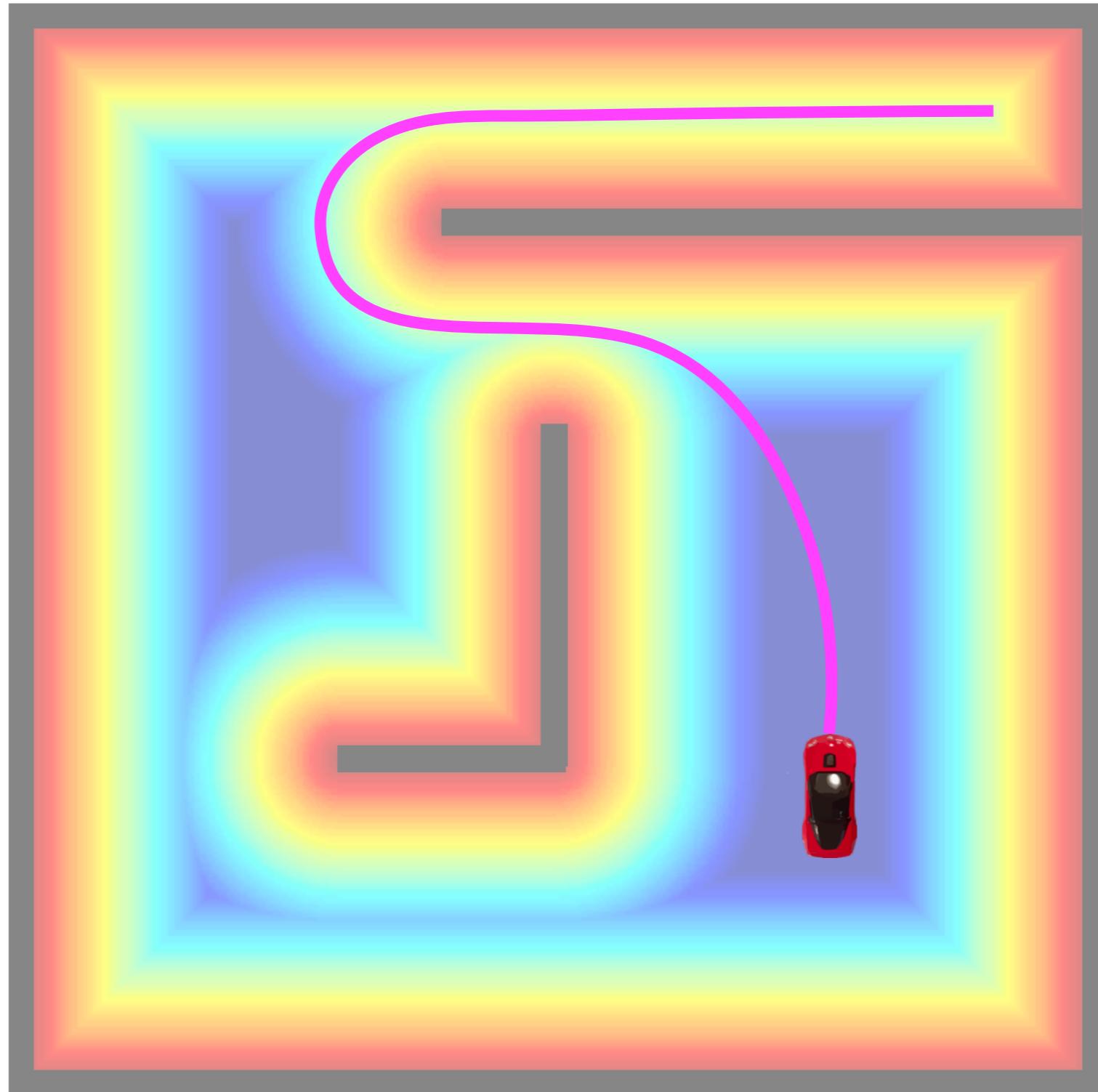
Plan a path
that penalizes
proximity to
obstacles

Why do we
need a map?



Desiderata: Map storing (truncated) distance

Input:
Binary map
of the world



Output:
Map of
same size
storing
truncated
distance

Example 8: Distance map

Category	Details
Information	Truncated distance to obstacles
Query	$O(1)$
Update	We want to incrementally update this map Ideally $O(k)$ where k is the number of cells which changed distance value
Memory	Same as the underlying occupancy grid

How do we efficiently calculate
distance map?

Dynamic programming to the rescue!

Initialize distance $d(i)$ for free cells to Inf

Insert all boundary pixels to queue Q

While Q not empty

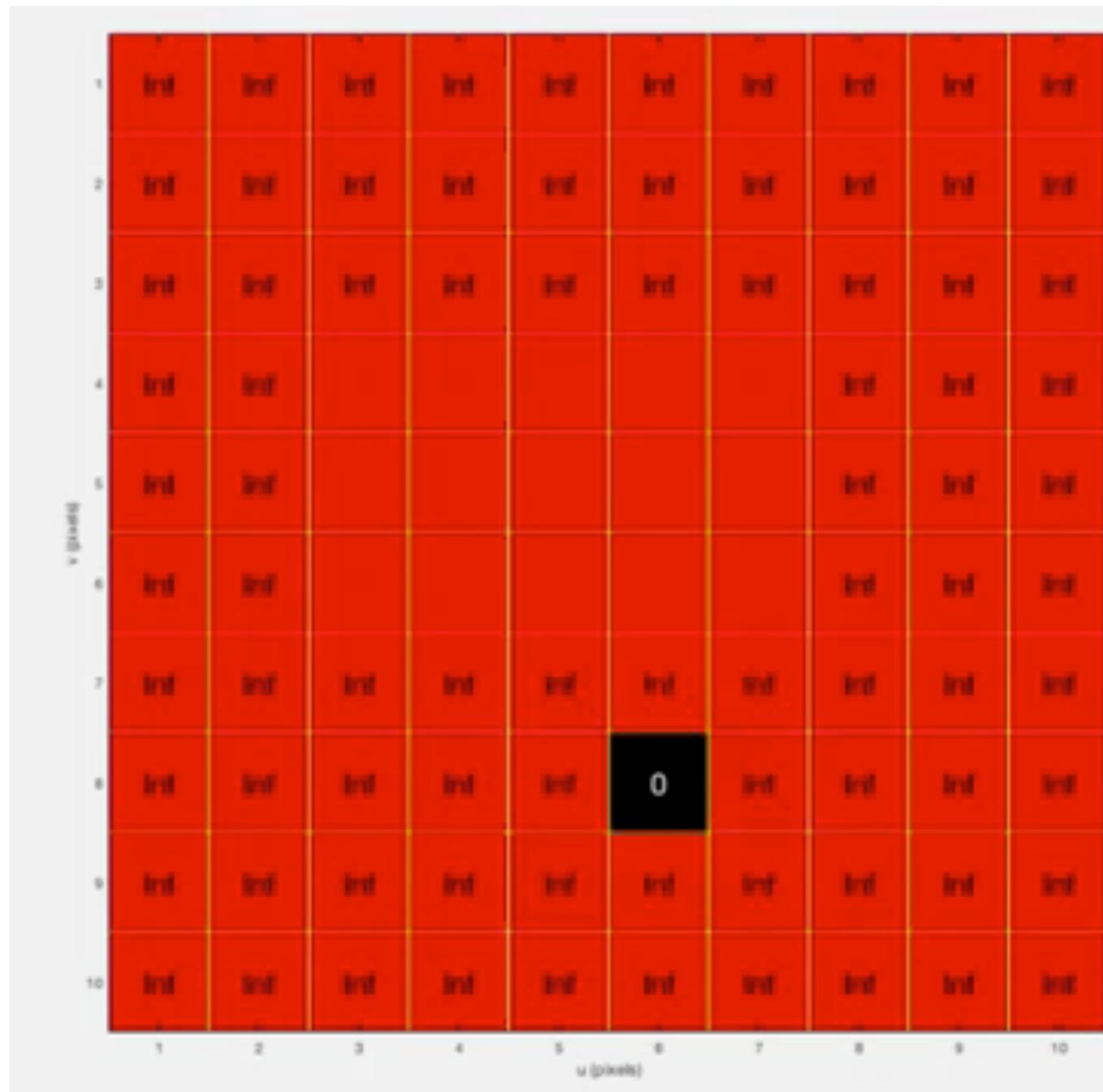
$x = Q.pop()$

for each n **in** Neigbour(x)

$d(n) = \min(d(n), v(x) + \text{dis}(x,n))$

if $d(n) \leq d_{\max}$

$Q.insert(n)$



Dynamic programming to the rescue!

Initialize distance $d(i)$ for free cells to Inf

Insert all boundary pixels to queue Q

While Q not empty

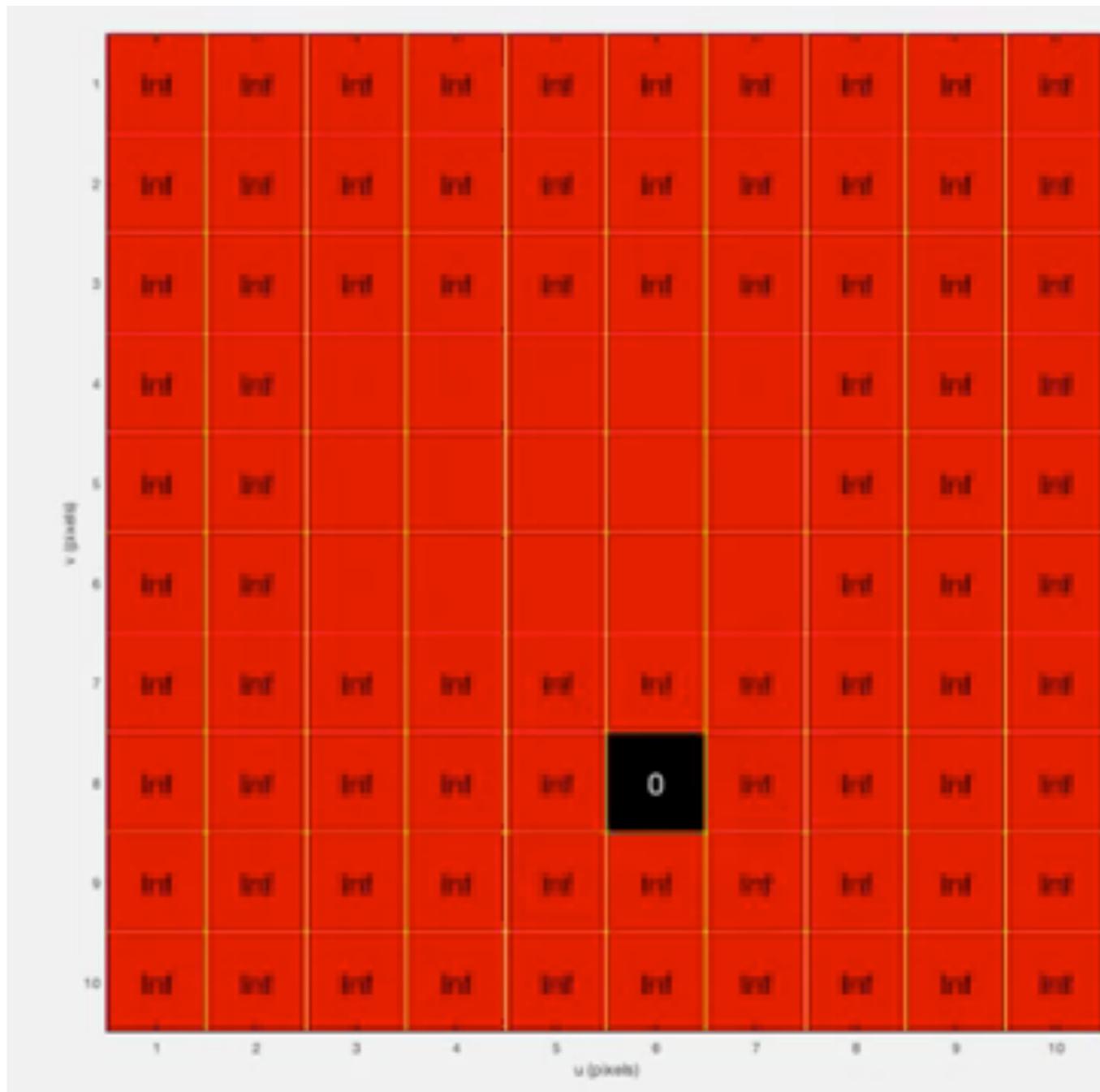
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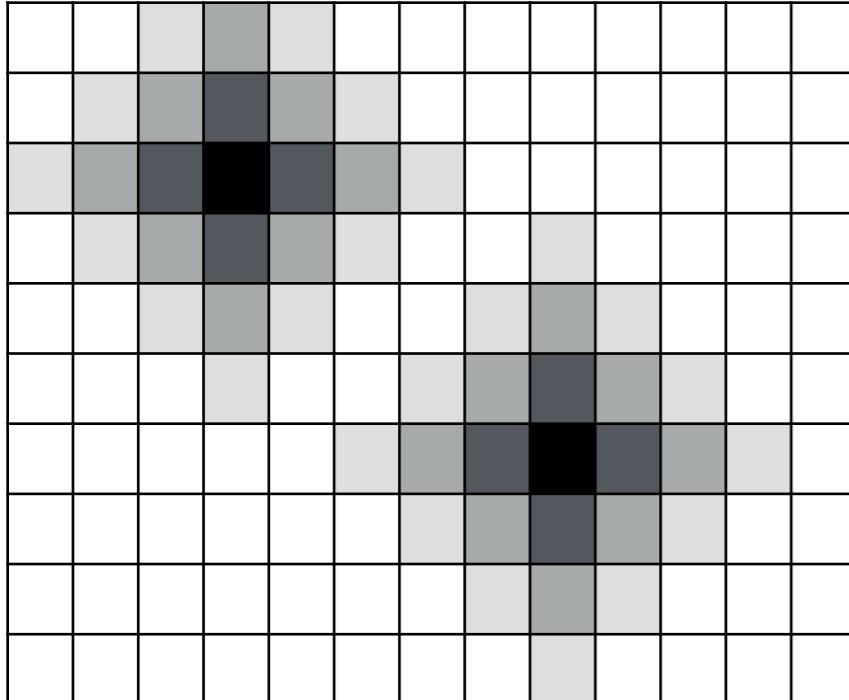
How can we incrementally update
this map?

Tale of two wavefronts

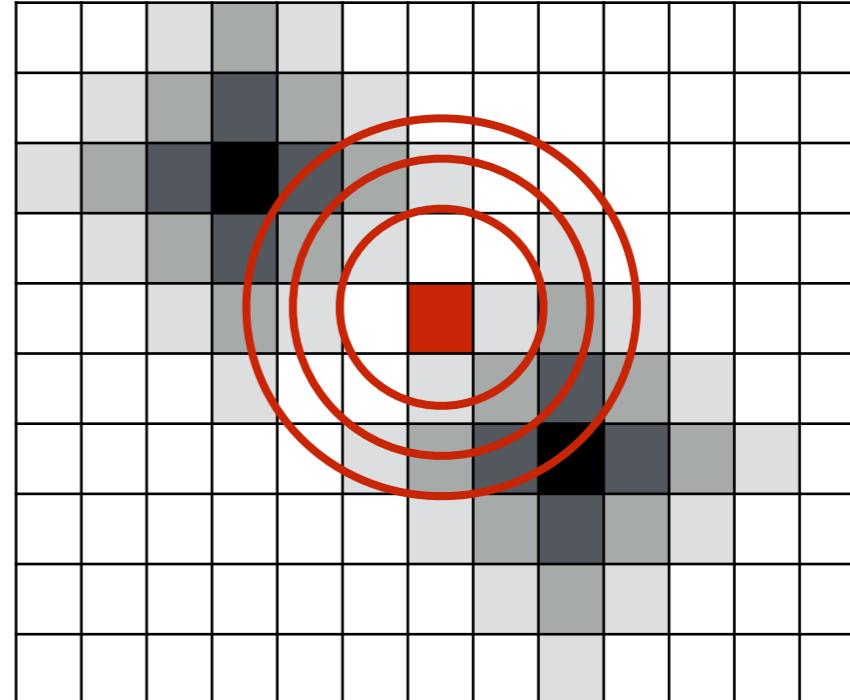
LOWER (when you add obstacle)

RAISE (when you delete obstacle)

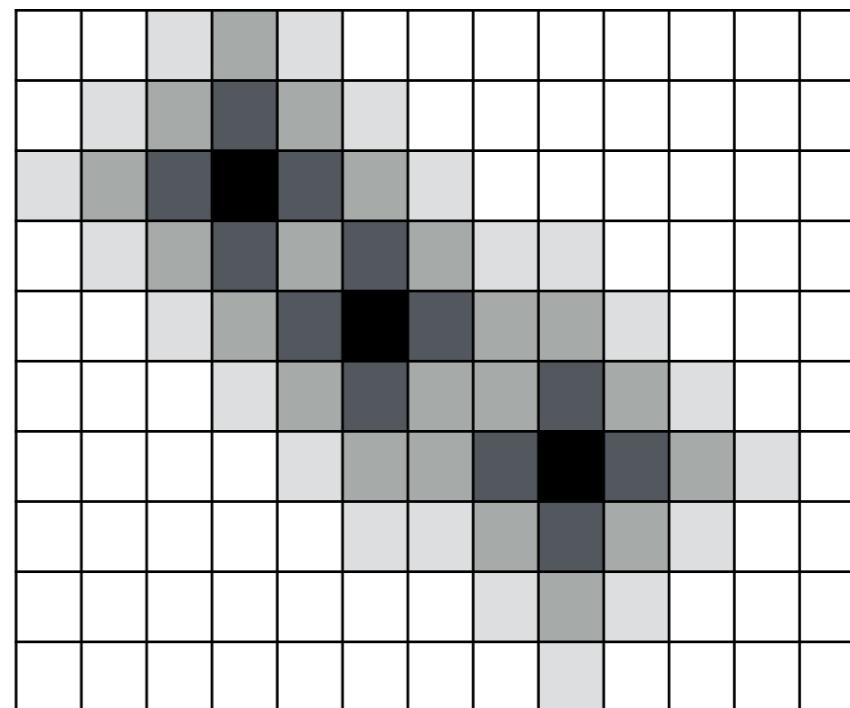
When obstacle is added



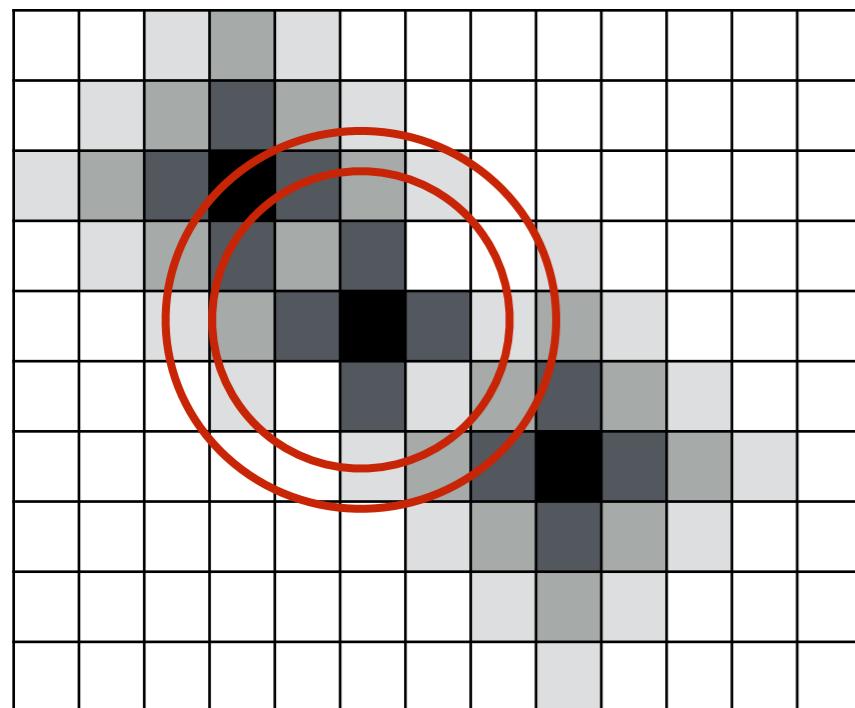
Existing
distance
map



New obstacle
added.
LOWER
wavefront
started



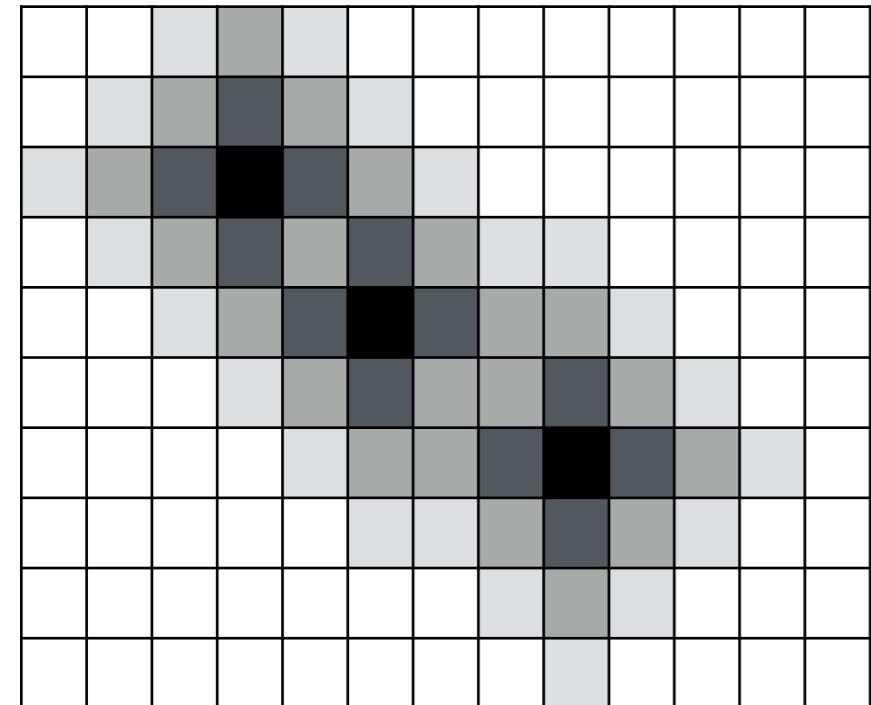
Stop
wavefront
whenever
you meet
higher
distance



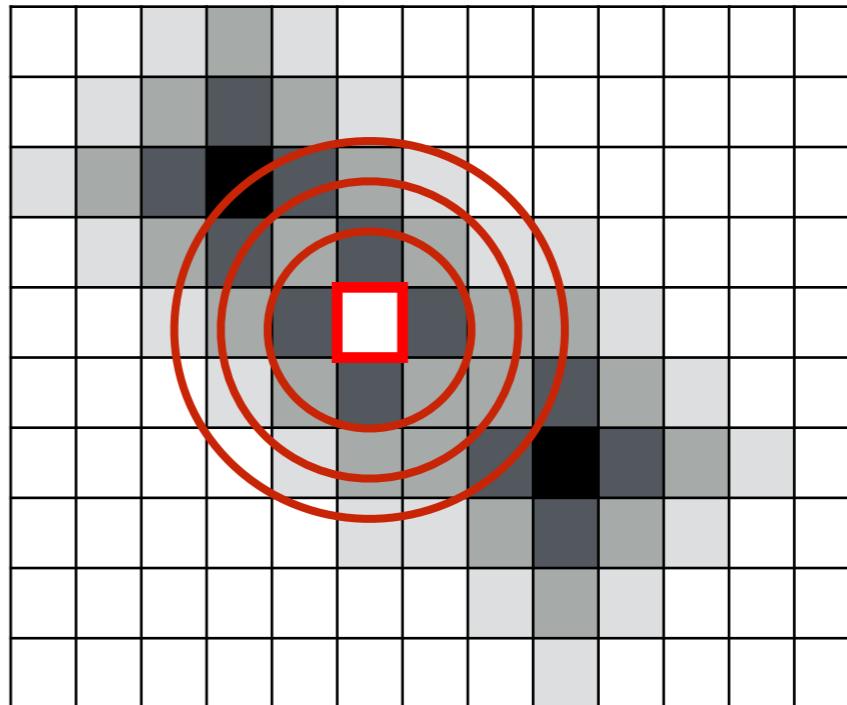
Overwrite
distances
if smaller
value.

Remember
closest
obstacle

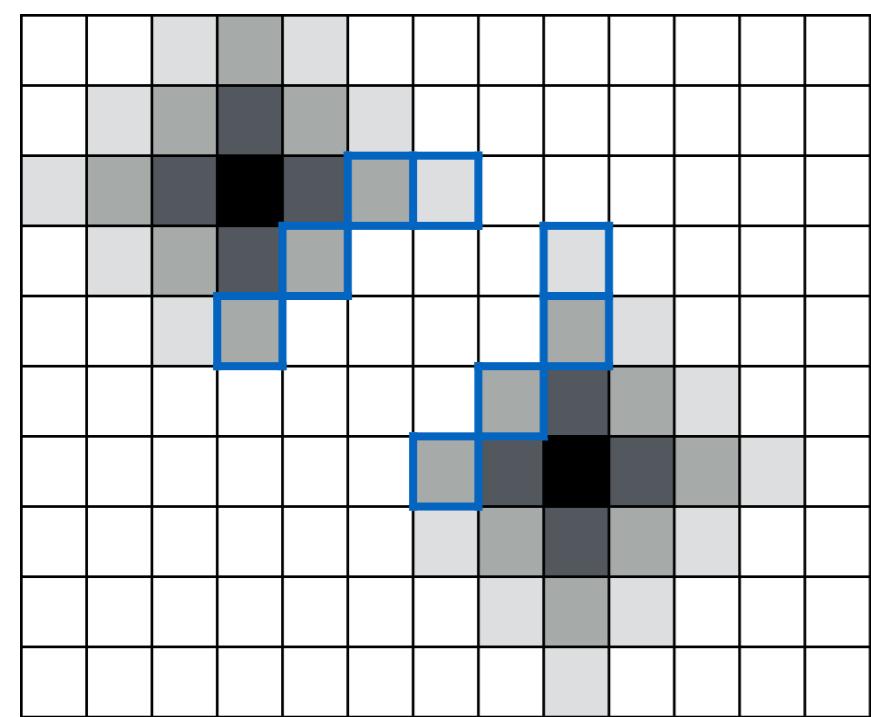
When obstacle is deleted



Existing
distance
map

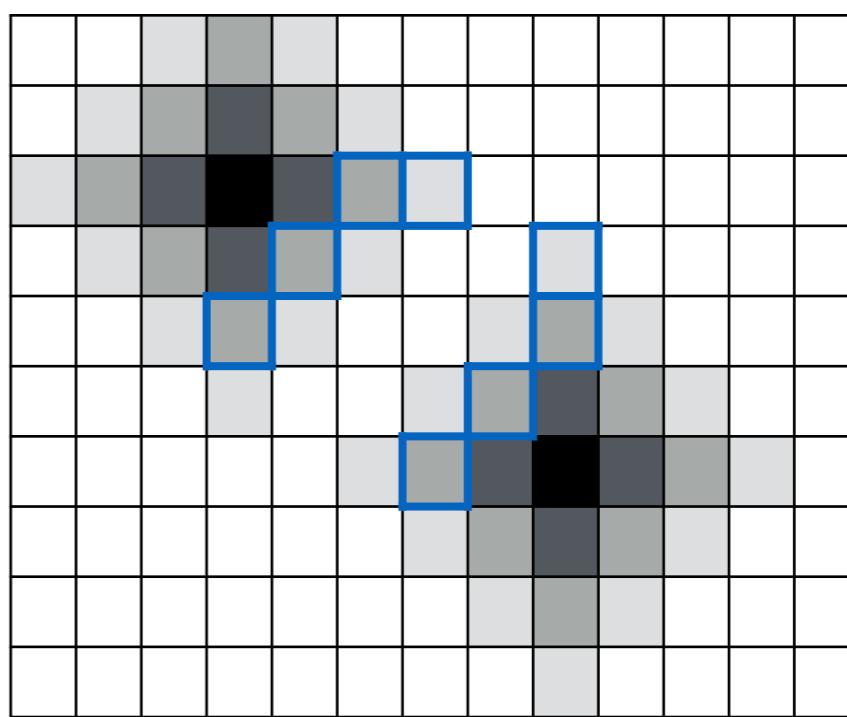


Obstacle
deleted
RAISE
wavefront
started



Set $d = d_{\max}$
if closest
obstacle was
deleted.

Stop
wavefront
otherwise.



Boundary
cells
trigger
LOWER
wavefront

Template for incremental dynamic programming

Input: Cells which changed status (obstacles added / removed)

Insert all changed cells into a queue Q

While Q not empty

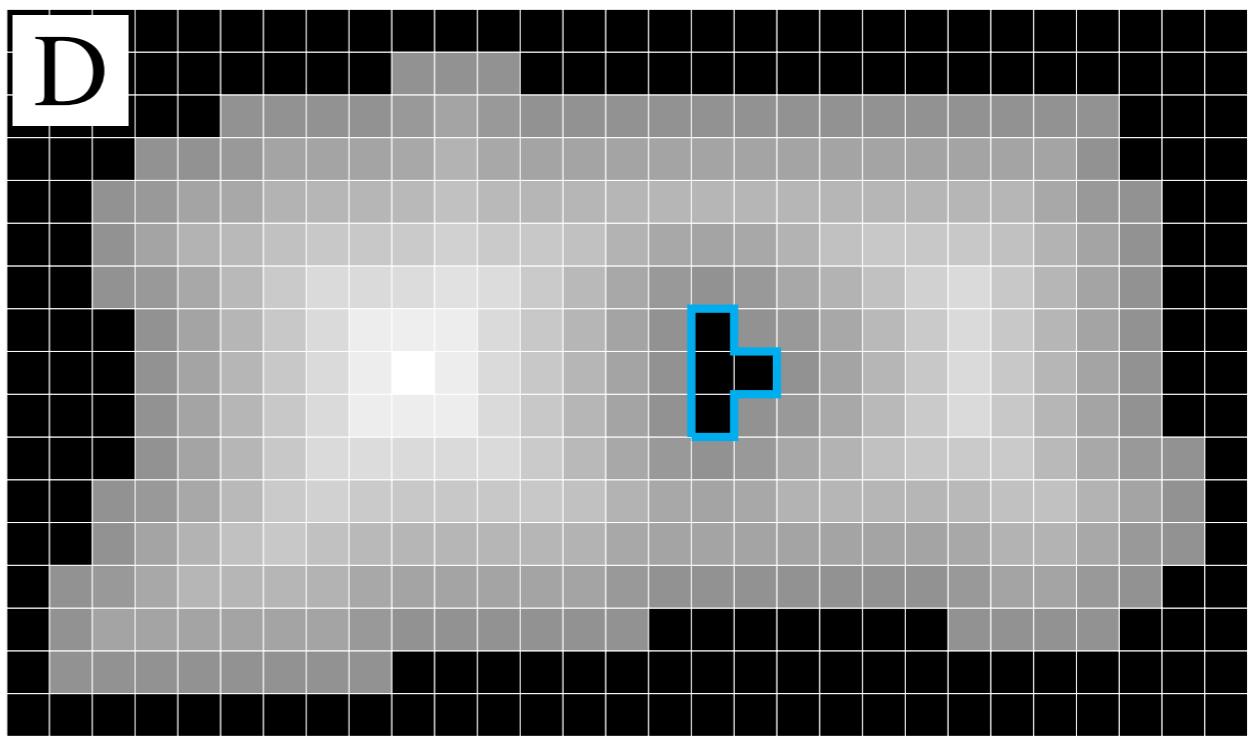
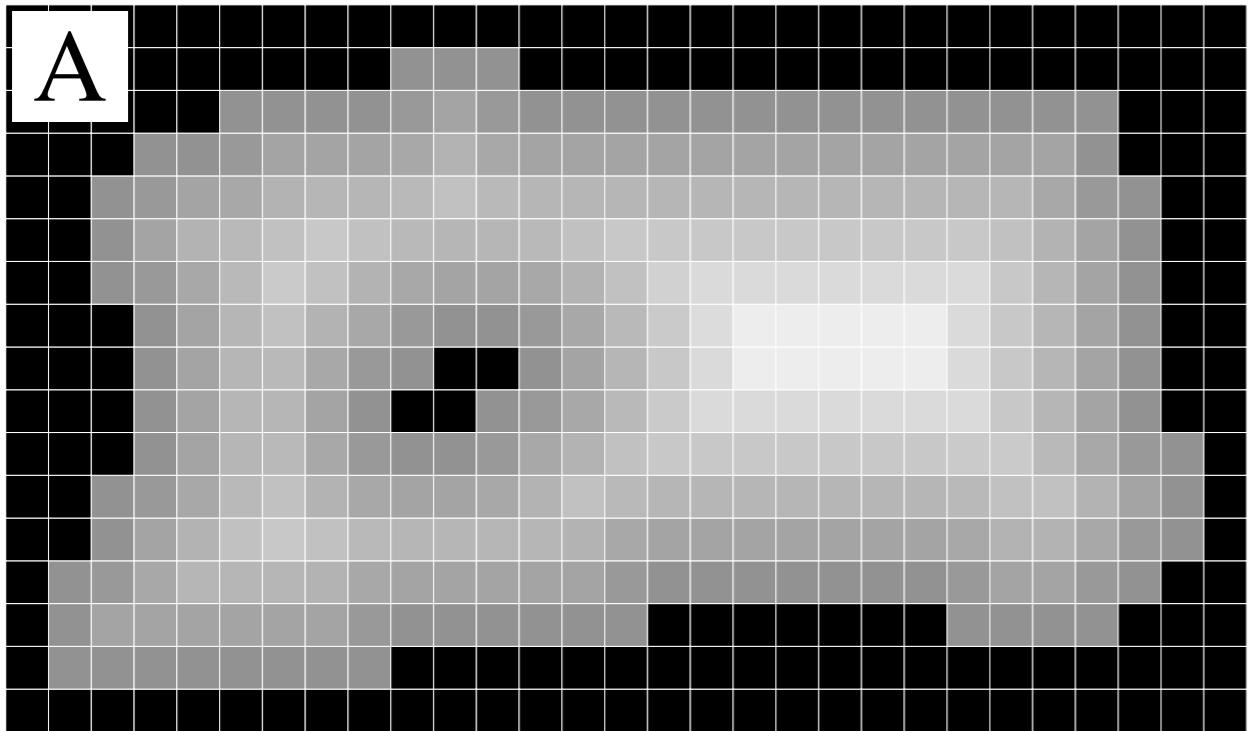
Node n = Q.pop()

If n is over consistent ($d_{\text{old}} > d_{\text{new}}$), lower value

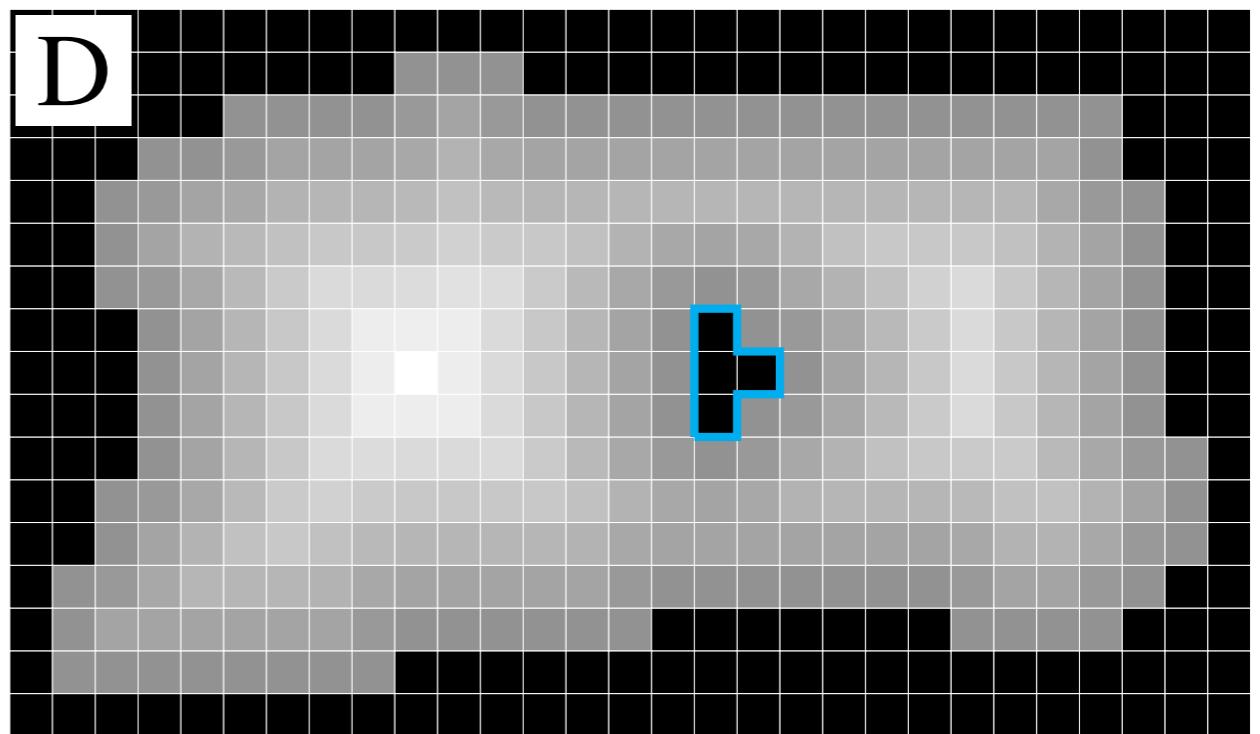
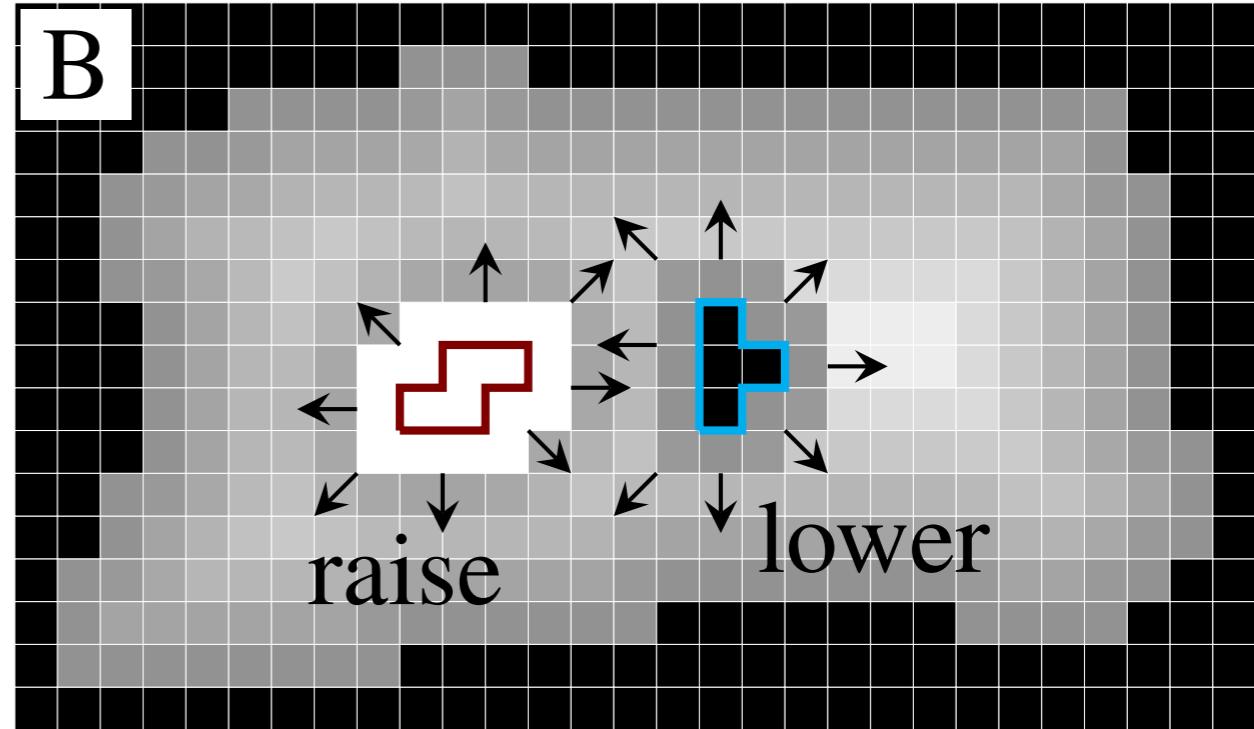
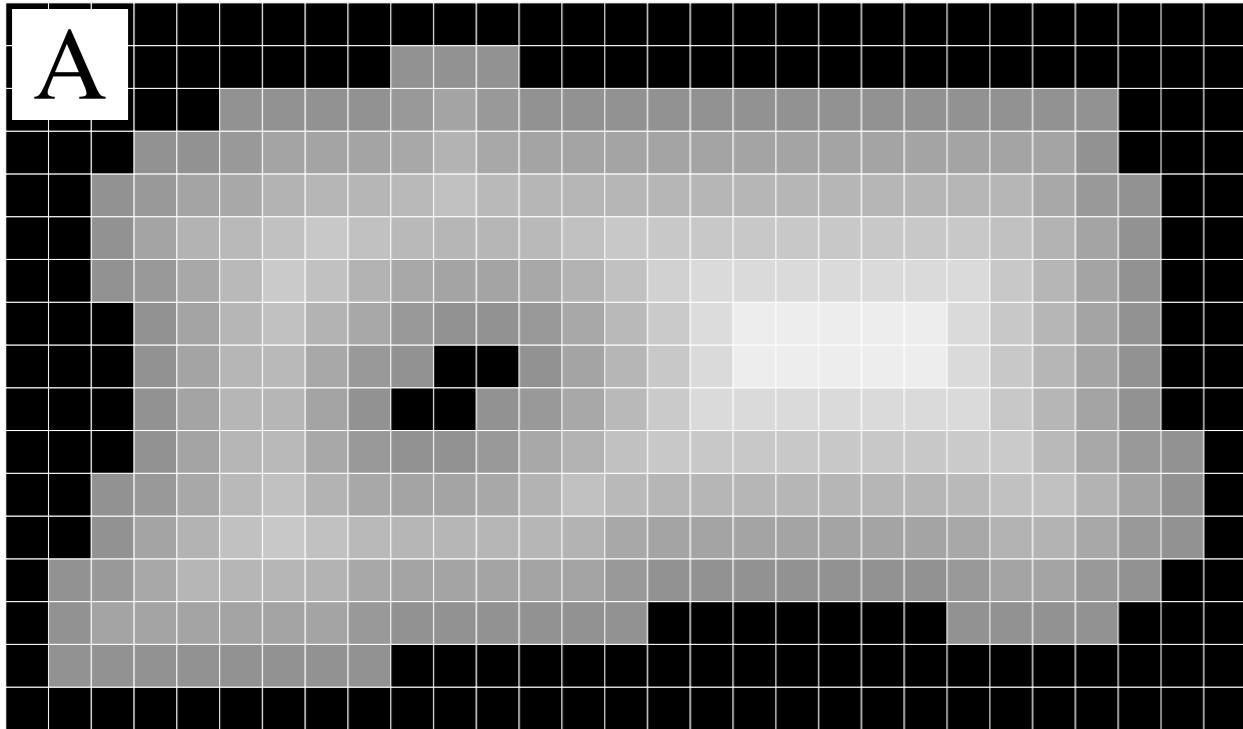
If n is under consistent ($d_{\text{old}} < d_{\text{new}}$), raise value

Add neighbors whose values need to be changed.

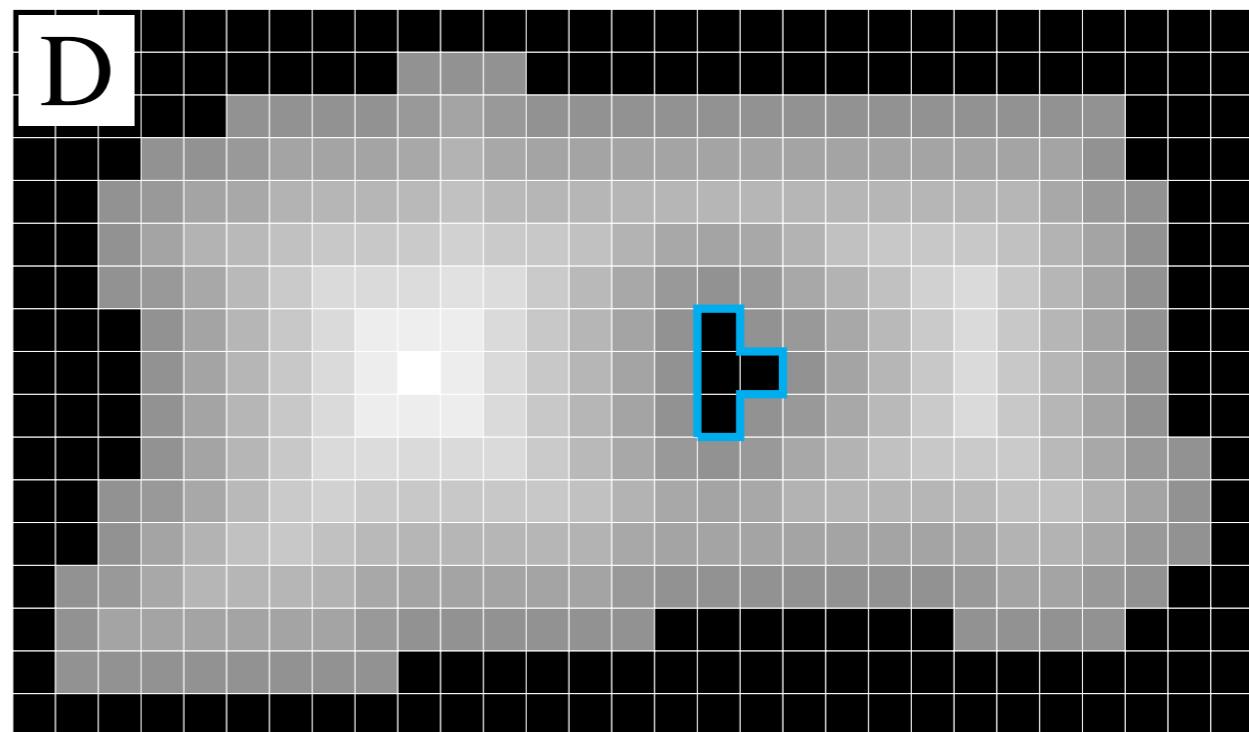
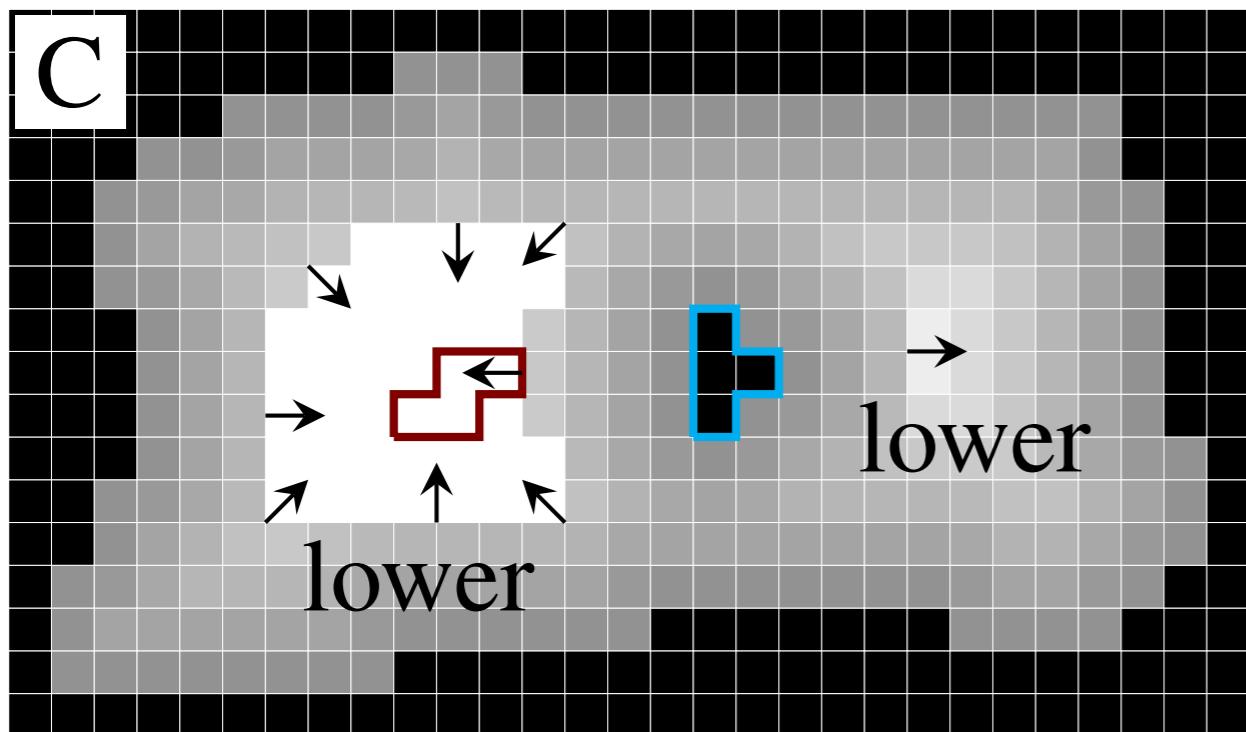
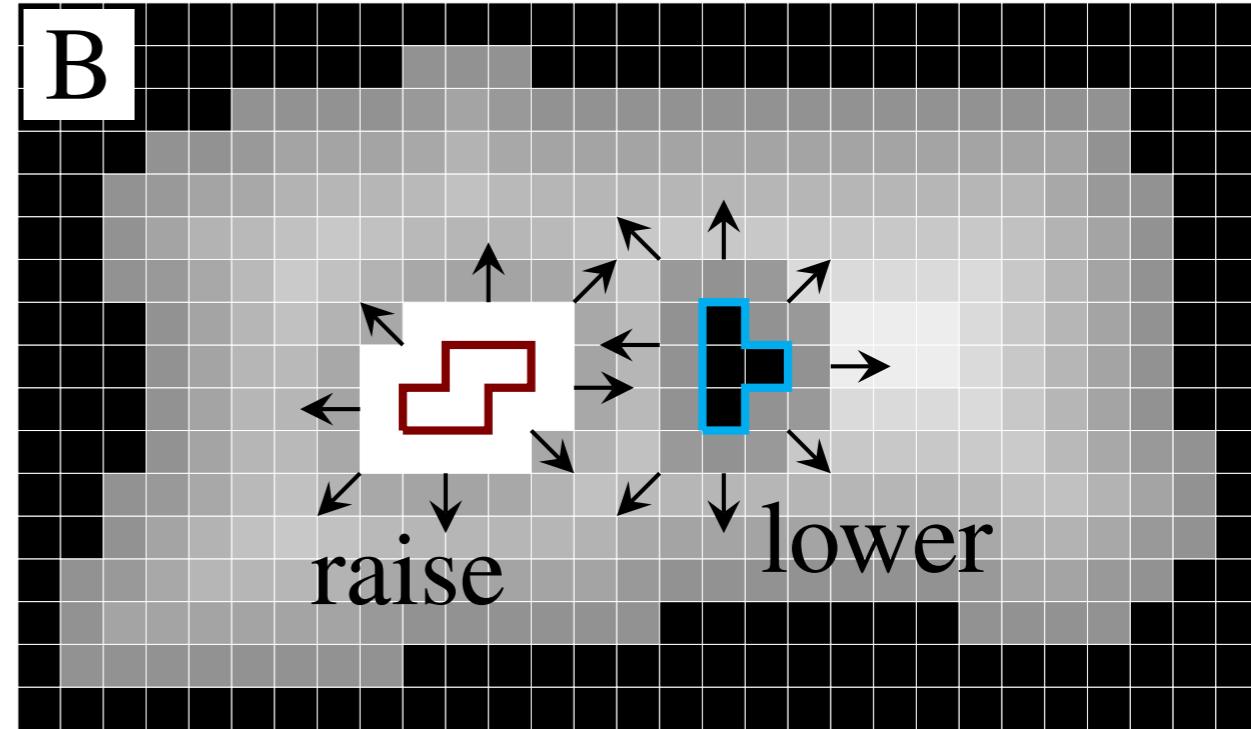
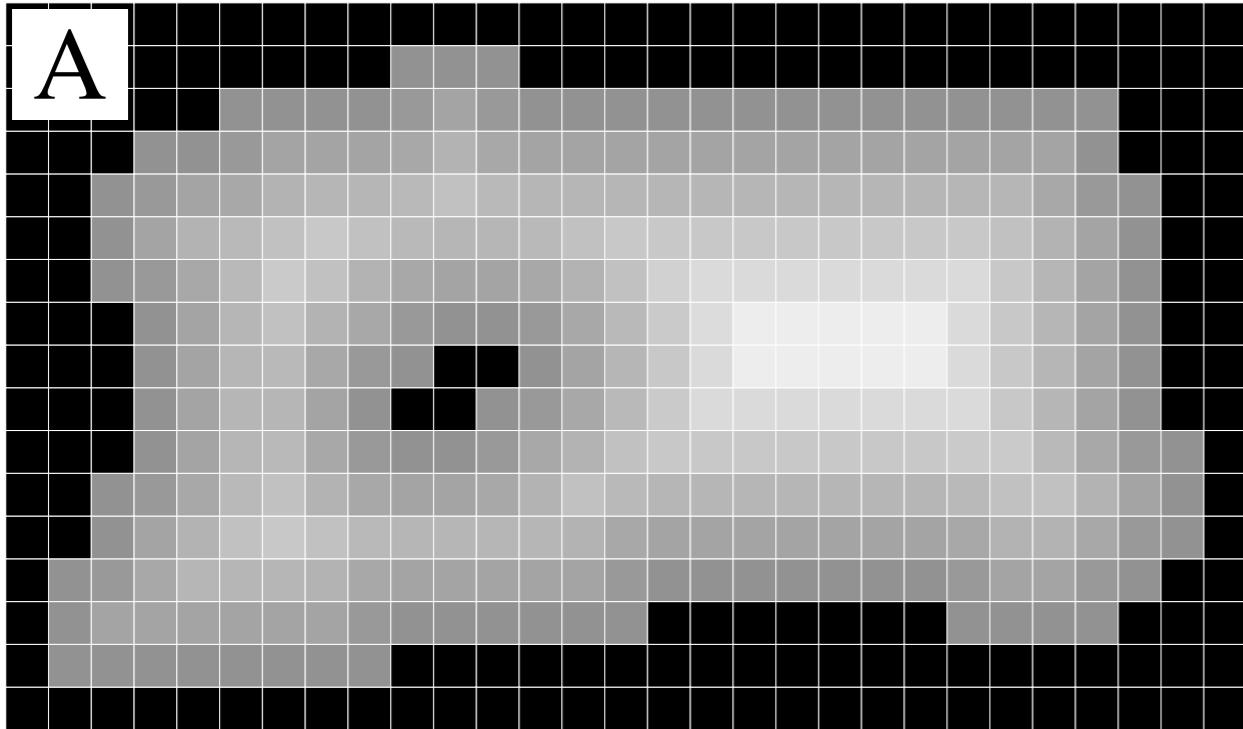
Incremental Euclidean Distance Mapping



Incremental Euclidean Distance Mapping



Incremental Euclidean Distance Mapping



Truncated signed distance map

We can easily modify this algorithm to tell us distance
inside an object.

Signed distance - negative inside object, positive outside

Signed distance important in motion planning!