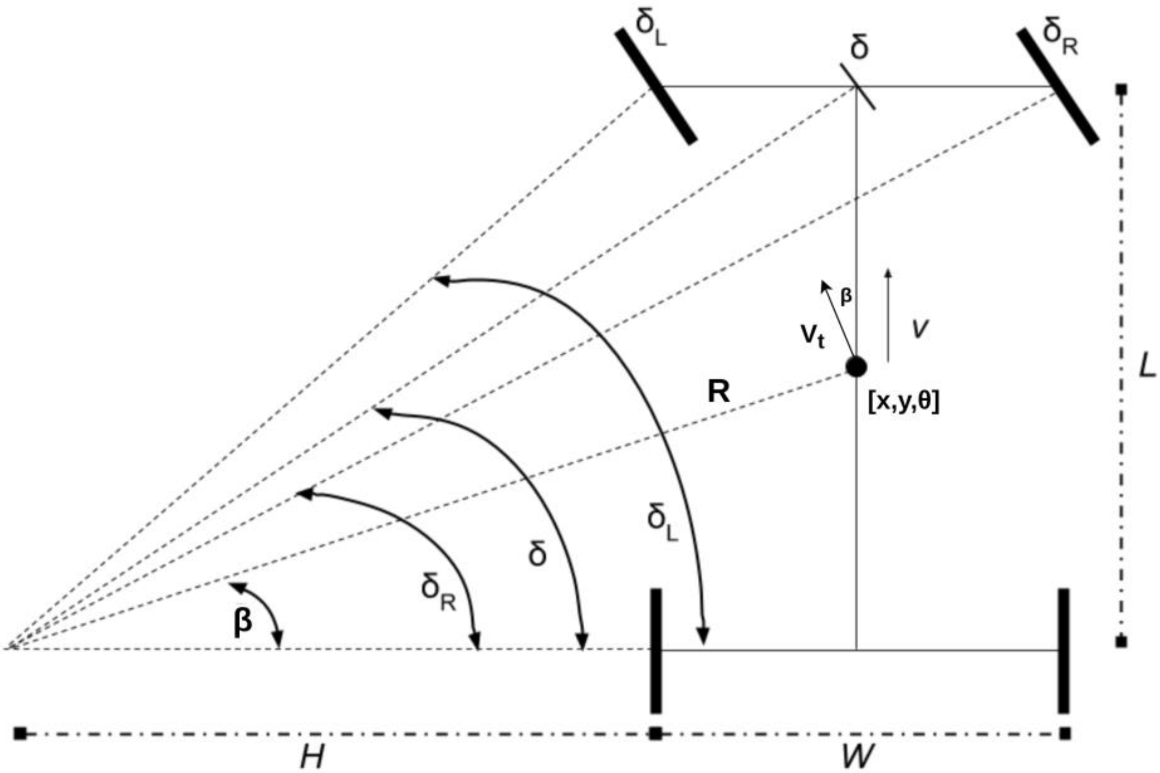
1. Derive the car model for the center of mass



$$\dot{x} = vcos(\theta)$$

$$\dot{y} = vsin(\theta)$$

$$\mathrm{v}_t = vcos(\beta)$$

$$R^2 = \left(\left(H + \frac{w}{2}\right)\right)^2 + \frac{L^2}{4}$$

$$H + \frac{w}{2} = \frac{L}{\tan(\delta)}$$

$$R^2 = \frac{L^2}{\tan^2(\delta)} + \frac{L^2}{4} = \frac{L^2(\tan^2(\delta) + 4)}{4\tan^2(\delta)}$$

$$R = \frac{L\sqrt{\tan^2(\delta) + 4}}{2\tan(\delta)}$$

$$\dot{\theta} = \omega = \frac{v}{R} = \frac{2v\tan(\delta)}{L\sqrt{\tan^2(\delta) + 4}}$$

$$\frac{\partial x}{\partial t} = vcos(\theta)$$

$$\frac{\partial y}{\partial t} = vsin(\theta)$$

$$\frac{\partial \theta}{\partial t} = \frac{2v\tan(\delta)}{L\sqrt{\tan^2(\delta) + 4}}$$

$$\int_{\theta_t}^{\theta_{t+1}} d\theta = \int_t^{t+\Delta t} \frac{2v\tan(\delta)}{L\sqrt{\tan^2(\delta) + 4}} dt$$

$$\theta_{t+1} - \theta_t = \frac{2v\tan(\delta)}{L\sqrt{\tan^2(\delta) + 4}}(t + \Delta t - t) = \frac{2v\tan(\delta)}{L\sqrt{\tan^2(\delta) + 4}}\Delta t$$

$$\theta_{t+1} = \theta_t + \frac{2v\tan(\delta)}{L\sqrt{\tan^2(\delta) + 4}}\Delta t$$

$$\int_{x_t}^{x_{t+1}} dx = \int_t^{t+\Delta t} vcos(\theta)\, dt = \int_{\theta_t}^{\theta_{t+1}} vcos(\theta)\frac{L\sqrt{\tan^2(\delta) + 4}\, d\theta}{2v\tan(\delta)} = \frac{L\sqrt{\tan^2(\delta) + 4}}{2\tan(\delta)} \int_{\theta_t}^{\theta_{t+1}} cos(\theta) d\theta$$

$$x_{t+1} - x_t = \frac{L\sqrt{\tan^2(\delta) + 4}}{2\tan(\delta)}[\sin(\theta_{t+1}) - \sin(\theta_t)]$$

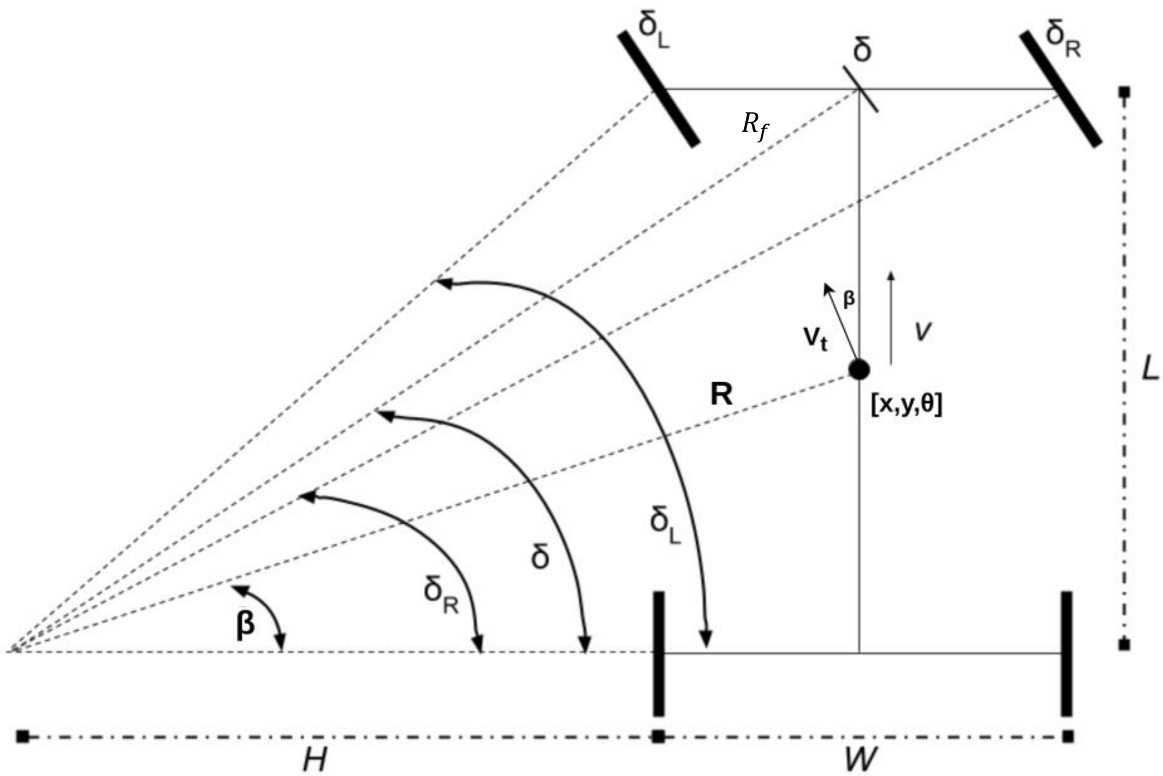
$$x_{t+1} = x_t + \frac{L\sqrt{\tan^2(\delta) + 4}}{2\tan(\delta)}[\sin(\theta_{t+1}) - \sin(\theta_t)]$$

$$\int_{y_t}^{y_{t+1}} dy = \int_t^{t+\Delta t} vsin(\theta)\, dt = \int_{\theta_t}^{\theta_{t+1}} vsin(\theta)\frac{L\sqrt{\tan^2(\delta) + 4}\, d\theta}{2v\tan(\delta)} = \frac{L\sqrt{\tan^2(\delta) + 4}}{2\tan(\delta)} \int_{\theta_t}^{\theta_{t+1}} \sin(\theta)\, d\theta$$

$$y_{t+1} - y_t = \frac{L\sqrt{\tan^2(\delta) + 4}}{2\tan(\delta)}[-\cos(\theta_{t+1}) + \cos(\theta_t)]$$

$$y_{t+1} = y_t + \frac{L\sqrt{\tan^2(\delta) + 4}}{2\tan(\delta)}[-\cos(\theta_{t+1}) + \cos(\theta_t)]$$

2. Derive the car model for the front axle.



$$\dot{x} = vcos(\theta)$$

$$\dot{y} = vsin(\theta)$$

$$\mathrm{v}_t = vcos(\beta)$$

$$R_f = \frac{L}{\cos(\delta)}$$

$$\dot{\theta} = \omega = \frac{v}{R_f} = \frac{v}{\frac{L}{\cos(\delta)}} = \frac{vcos(\delta)}{L}$$

$$\frac{\partial x}{\partial t} = vcos(\theta)$$

$$\frac{\partial y}{\partial t} = vsin(\theta)$$

$$\frac{\partial \theta}{\partial t} = \frac{vcos(\delta)}{L}$$

$$\int_{\theta_t}^{\theta_{t+1}} d\theta = \int_t^{t+\Delta t} \frac{vcos(\delta)}{L} dt$$

$$\theta_{t+1} - \theta_t = \frac{vcos(\delta)}{L}(t + \Delta t - t) = \frac{vcos(\delta)}{L}\Delta t$$

$$\theta_{t+1} = \theta_t + \frac{vcos(\delta)}{L}\Delta t$$

$$\int_{x_t}^{x_{t+1}} dx = \int_t^{t+\Delta t} vcos(\theta)\, dt = \int_{\theta_t}^{\theta_{t+1}} vcos(\theta)\frac{Ld\theta}{vcos(\delta)} = \frac{L}{vcos(\delta)}\int_{\theta_t}^{\theta_{t+1}} cos(\theta)d\theta$$

$$x_{t+1} - x_t = \frac{L}{vcos(\delta)}[\sin(\theta_{t+1}) - \sin(\theta_t)]$$

$$x_{t+1} = x_t + \frac{L}{vcos(\delta)}[\sin(\theta_{t+1}) - \sin(\theta_t)]$$

$$\int_{y_t}^{y_{t+1}} dy = \int_t^{t+\Delta t} vsin(\theta)\, dt = \int_{\theta_t}^{\theta_{t+1}} vsin(\theta)\frac{Ld\theta}{vcos(\delta)} = \frac{L}{vcos(\delta)}\int_{\theta_t}^{\theta_{t+1}} \sin(\theta)\, d\theta$$

$$y_{t+1} - y_t = \frac{L}{vcos(\delta)}[-\cos(\theta_{t+1}) + \cos(\theta_t)]$$

$$y_{t+1} = y_t + \frac{L}{vcos(\delta)}[-\cos(\theta_{t+1}) + \cos(\theta_t)]$$