Probabilistic Models

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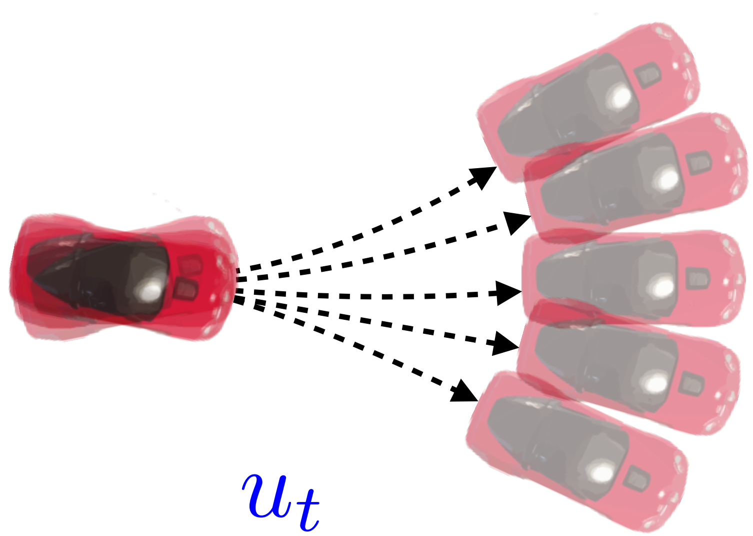
Probabilistic models in localization

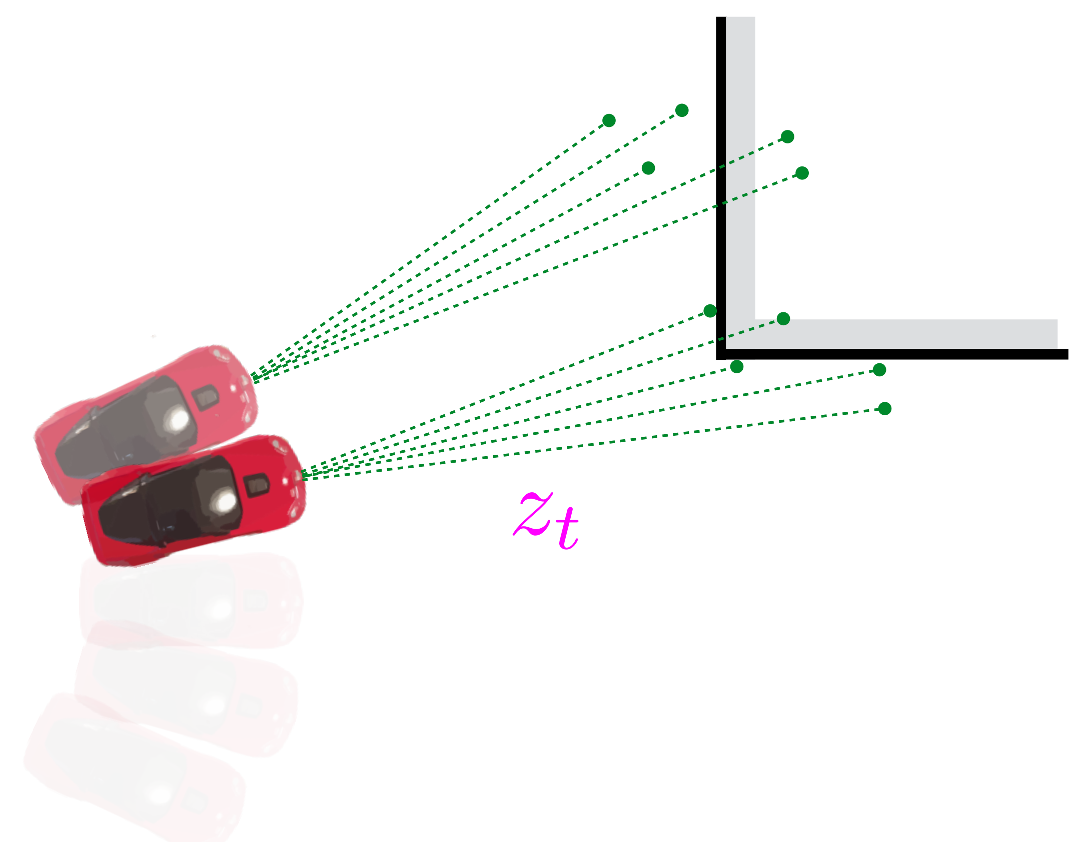
Motion model

$$P(x_t | u_t, x_{t-1})$$

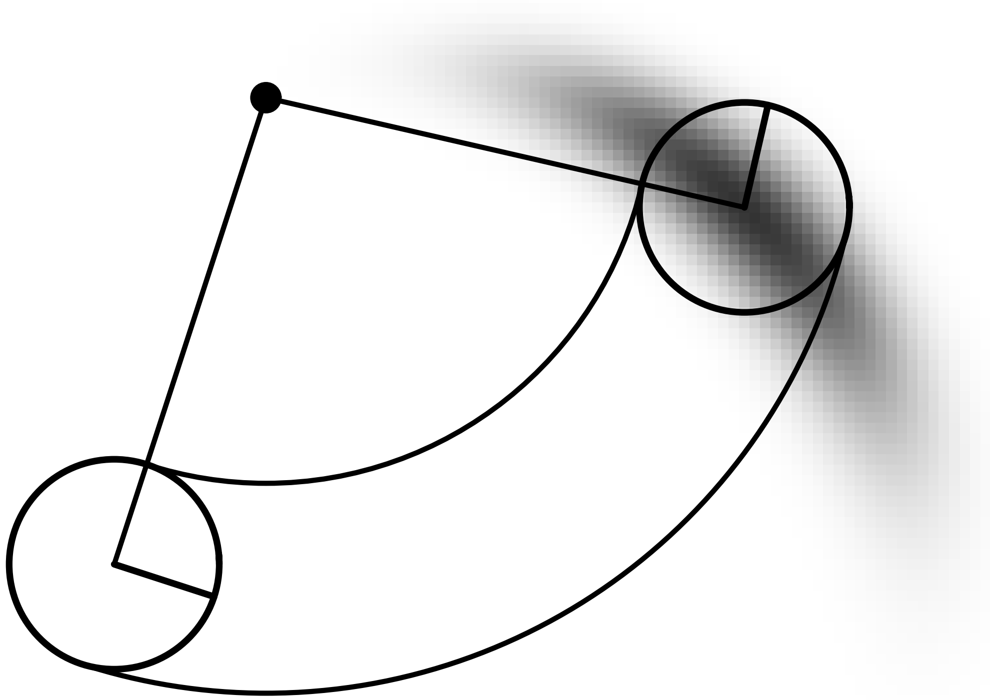


Measurement model

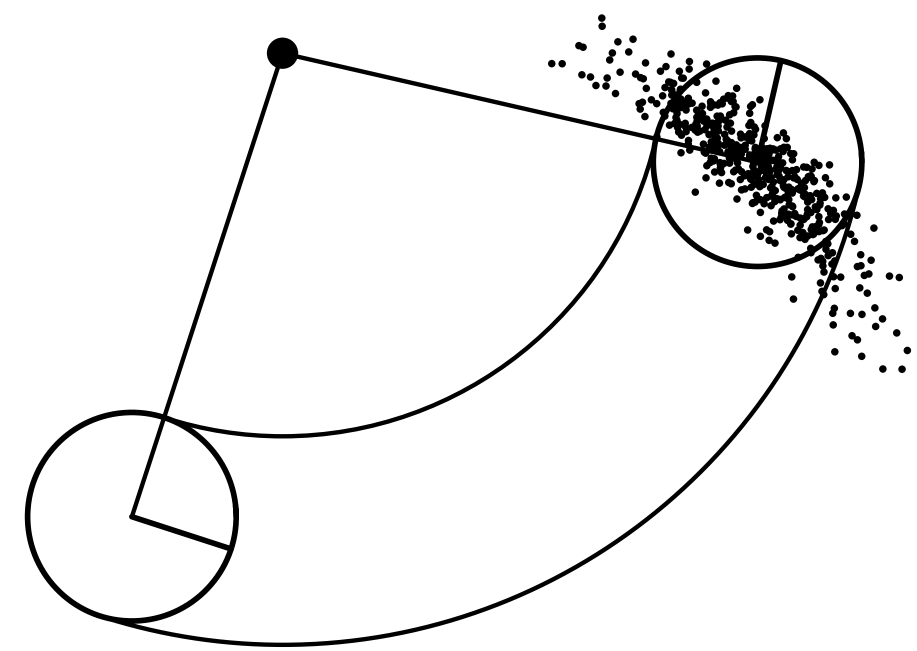
$$P(z_t | x_t)$$



Example of a motion model



Probability density function



Samples from the pdf

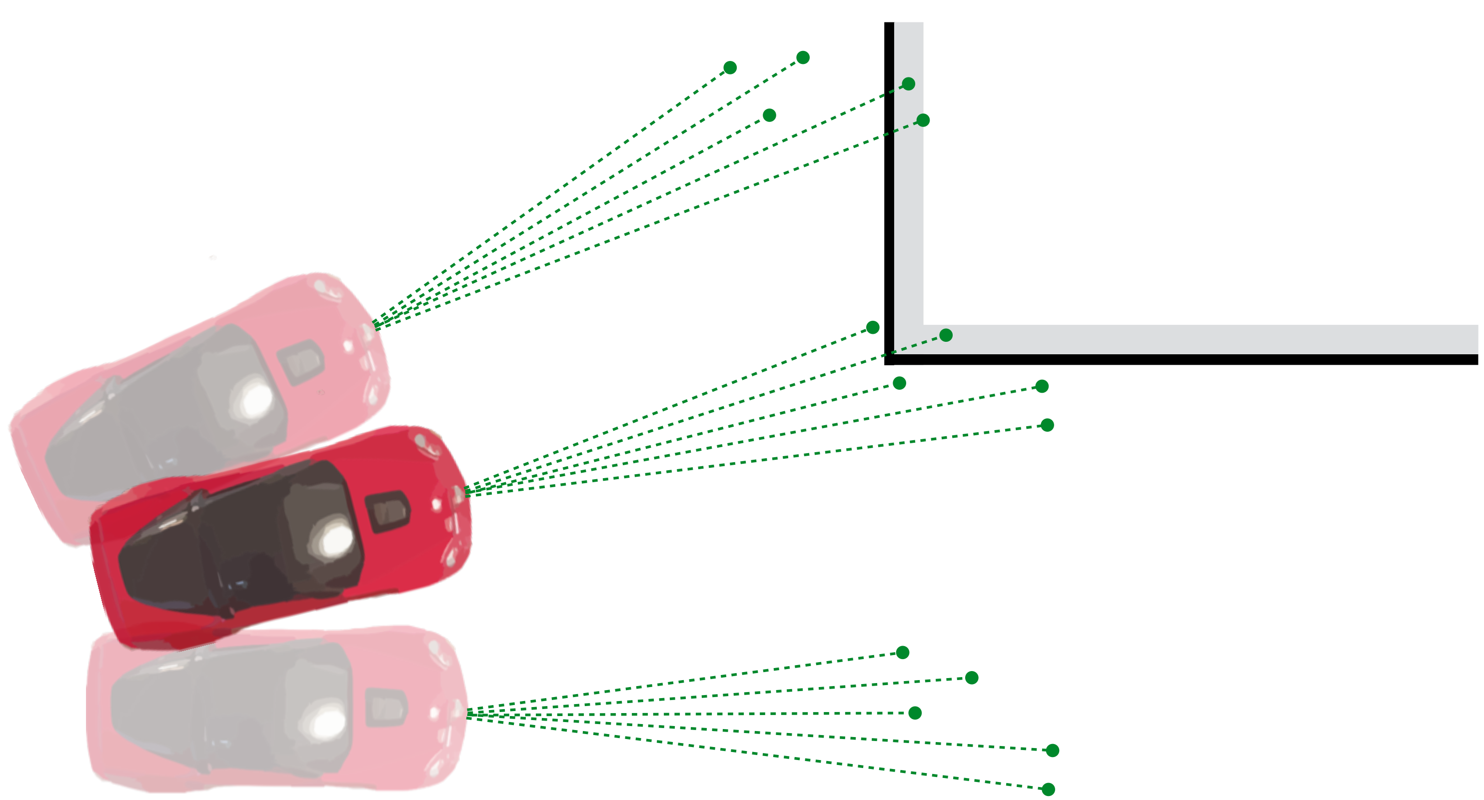
Measurement Model

$$P(z_t | x_t, m)$$

sensor reading

state

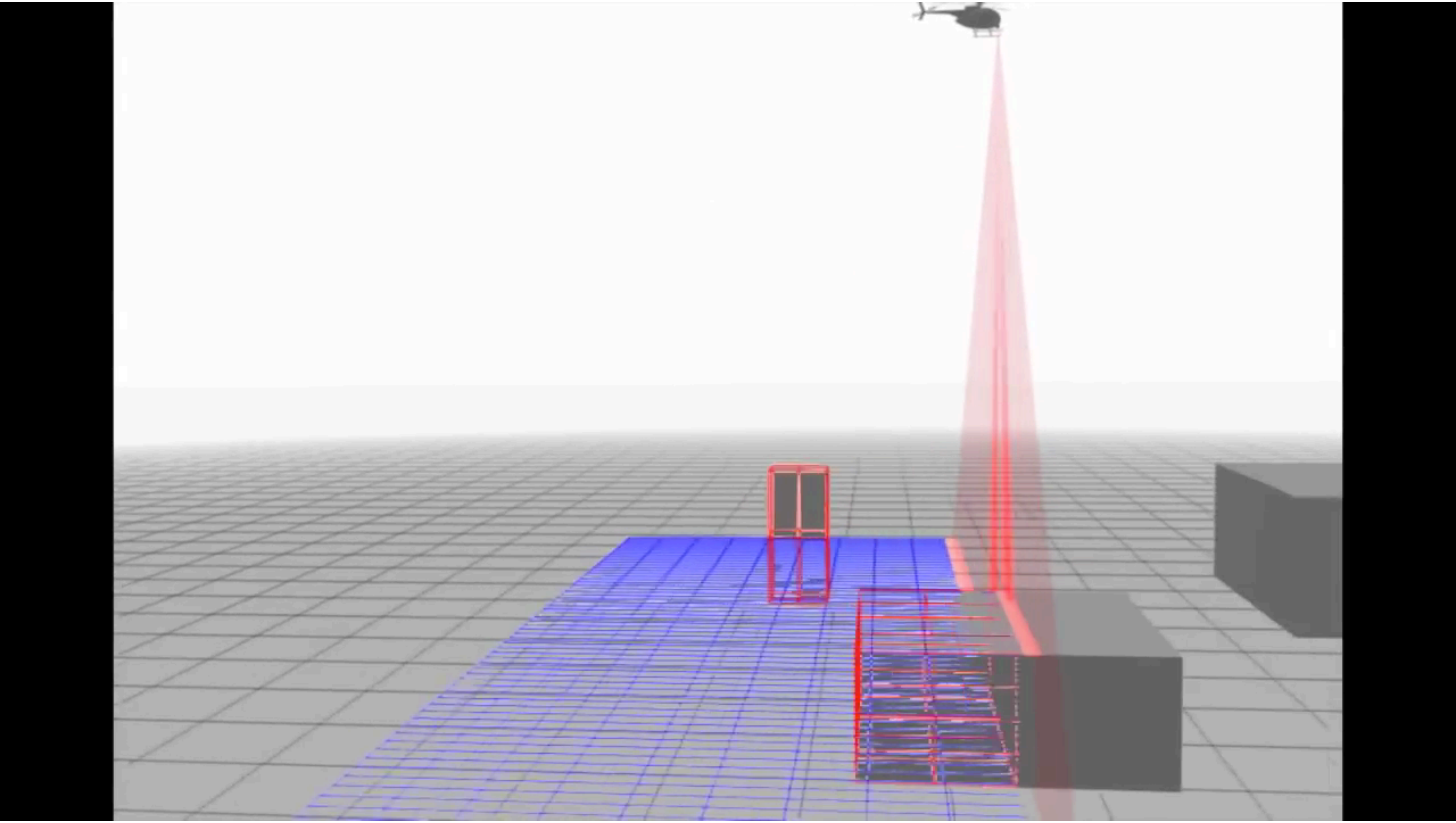
map



How does a LiDAR work?

https://www.youtube.com/watch?v=NZKvf1cXe8s&t=67s

Working with lasers in the real world



(courtesy Lyle Chamberlain)

Three questions you should ask

1. Why is the model probabilistic?

2. What defines a good model?

3. What model should I use for my robot?

Why is the measurement model probabilistic?

Several sources of stochasticity

Three questions you should ask

1. Why is the model probabilistic?

2. What defines a good model?

3. What model should I use for my robot?

What defines a good model?

Good news: LiDAR is very precise!
A handful of measurements is enough to localize robot

However, has distinct modes of failures

Problem: Overconfidence in measurement can be catastrophic

Solution: Anticipate specific types of failures and
add stochasticity accordingly.

Three questions you should ask

1. Why is the model probabilistic?

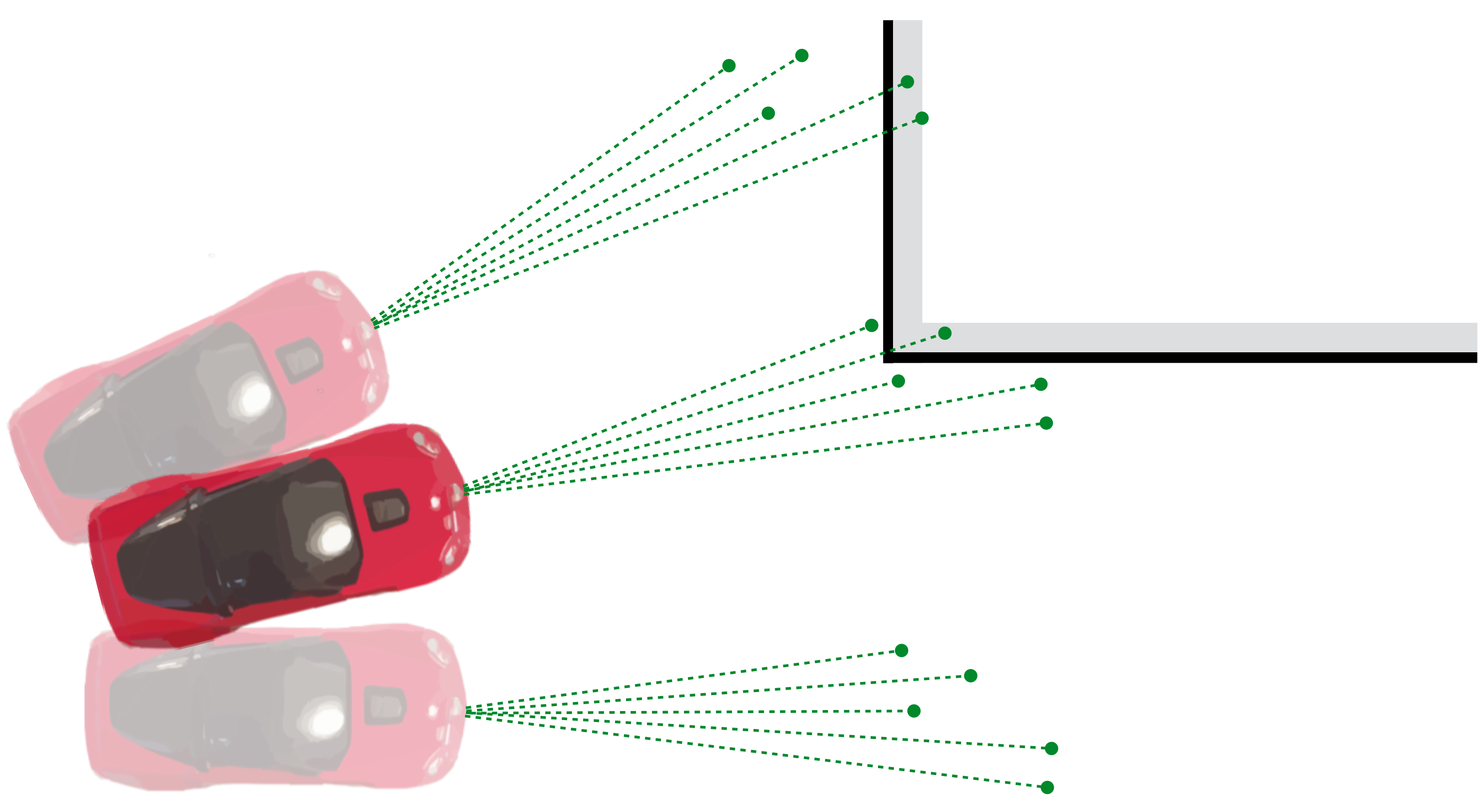
2. What defines a good model?

3. What model should I use for my robot?

Measurement model for LiDAR

$$P(z_t | x_t, m)$$

laser scan   state   map



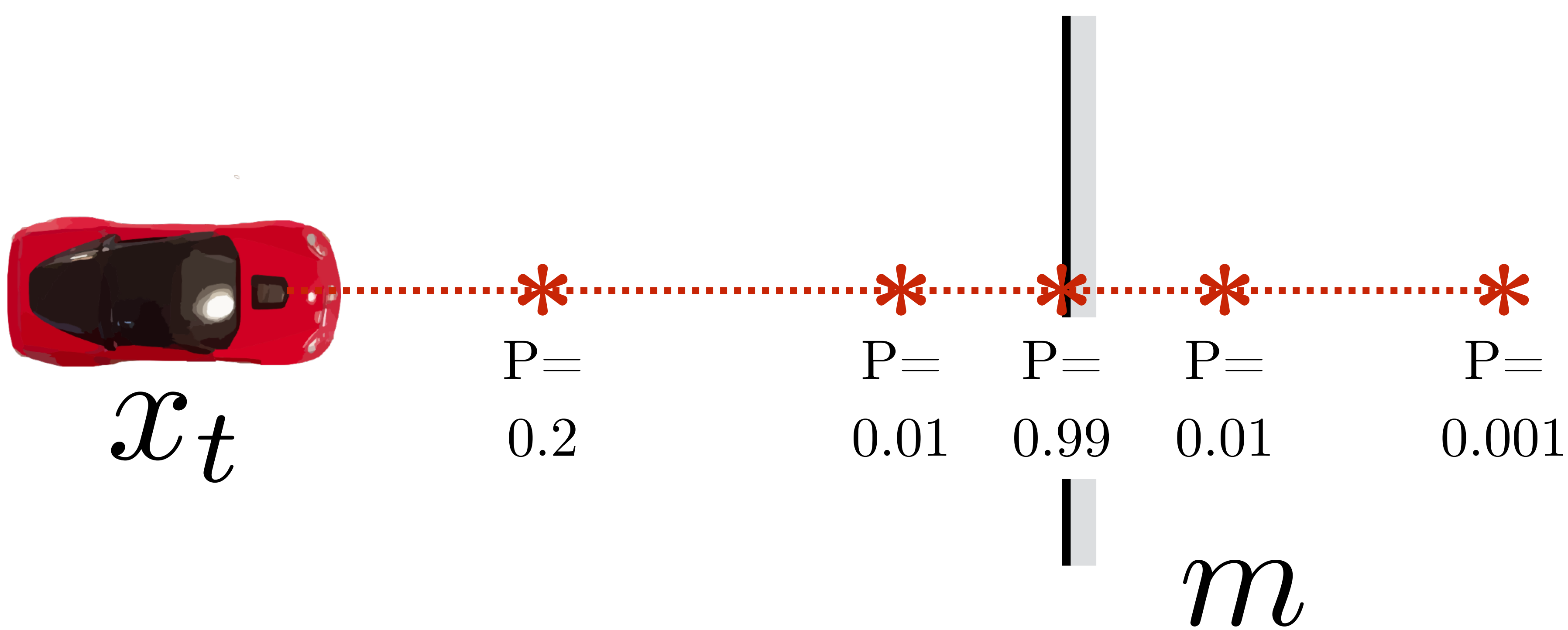
Measurement model for LiDAR

Assume individual beams are conditionally independent given map

Measurement model for single beam

$$P(z_t^k | x_t, m)$$

distance value   state   map



Pseudo-algorithm for sensor model

Input: State of the robot x, Map m, True laser scan z

Output: Probability p

1. Use x to figure out the pose p of the sensor

2. Ray-cast (shoot out rays) from p on the map m

3. Get back a simulated laser-scan z^*

4. Go over every ray in z^* and compare with z. Compute a likelihood based on how much they match / mismatch.

5. Multiply all probabilities to get p

What kind of stochasticity should we consider?

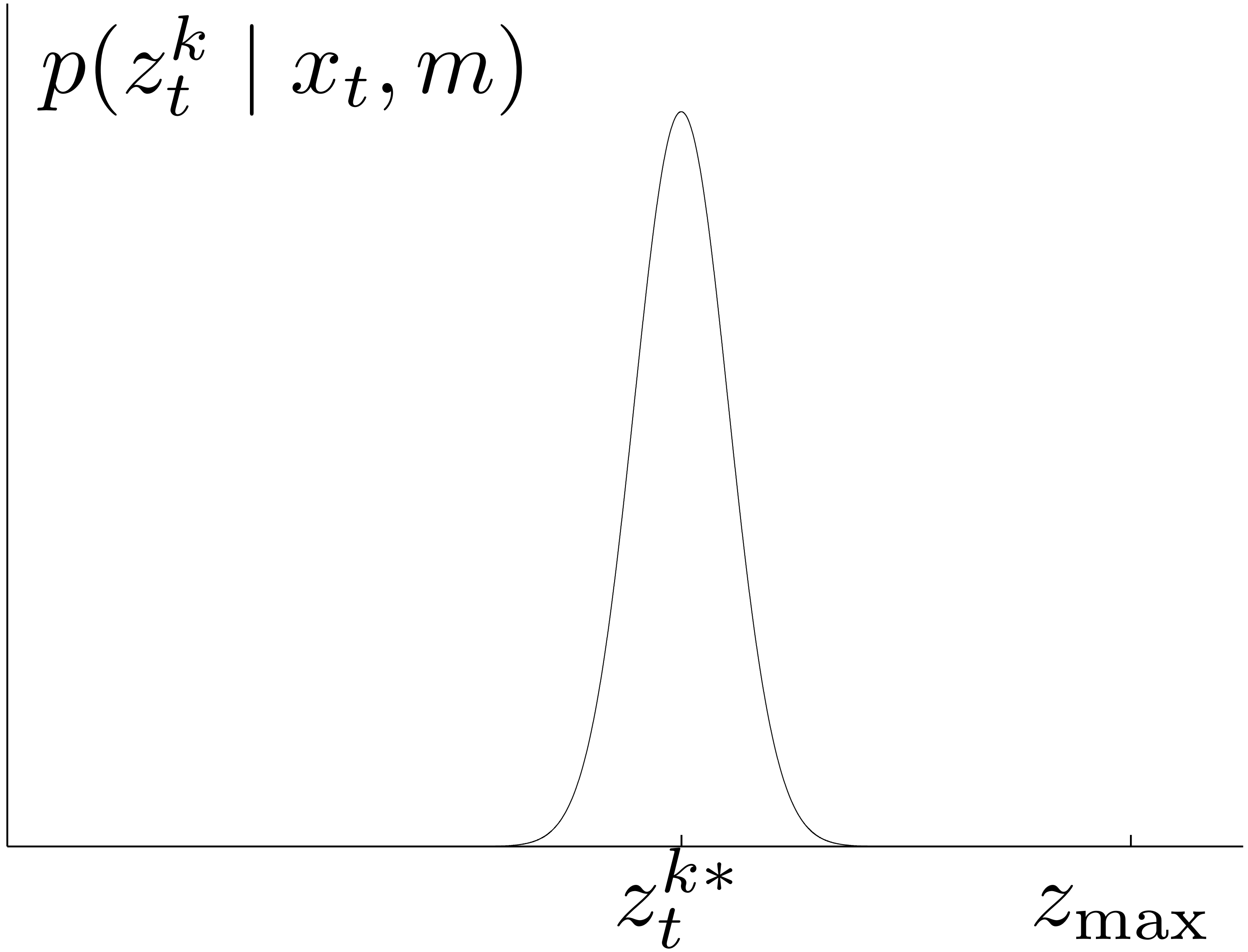
1. Simple measurement noise in distance value

2. Presence of unexpected objects

3. Laser returns max range when no objects

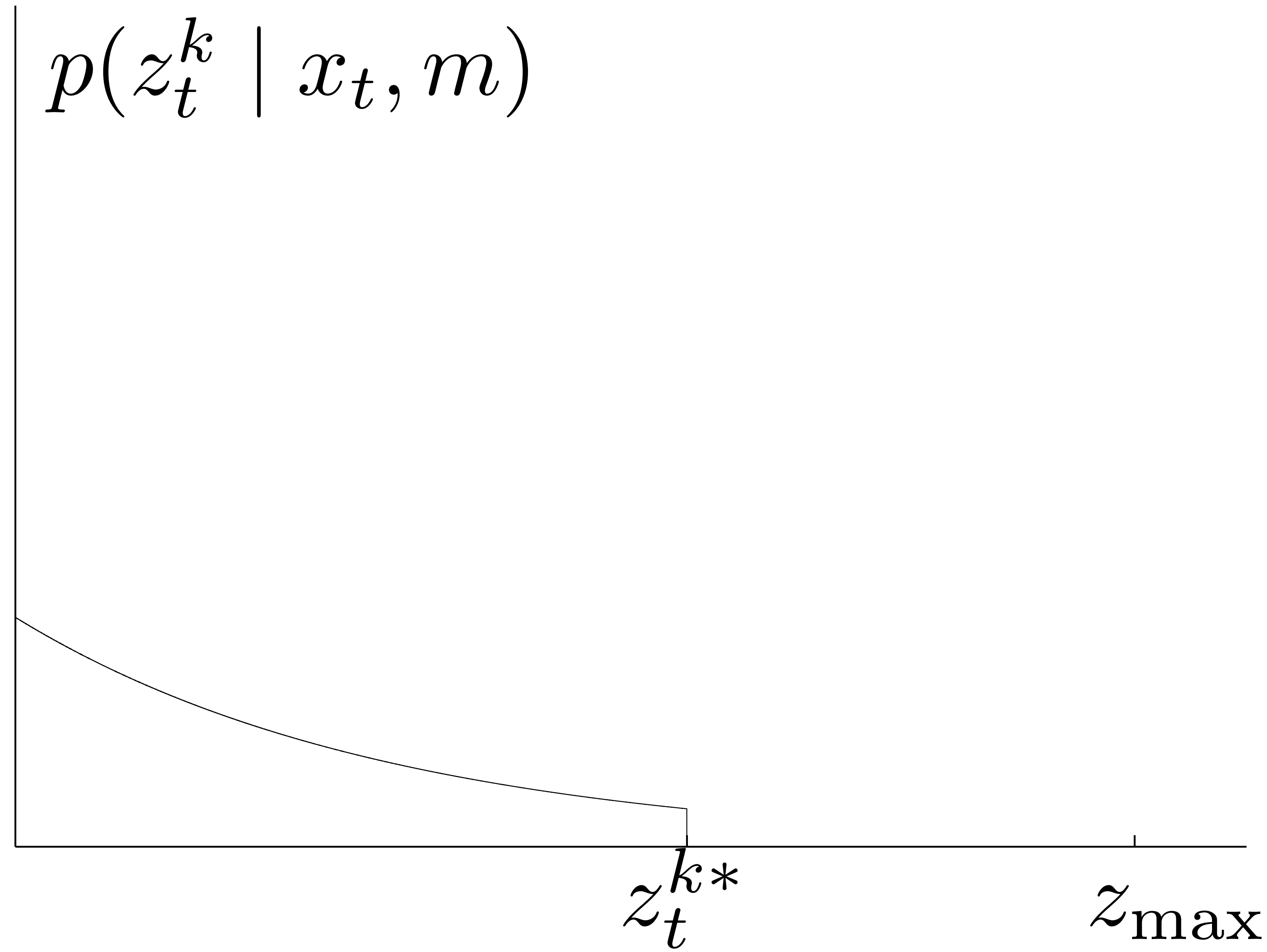
4. Failures in sensing

Factor 1: Simple measurement noise



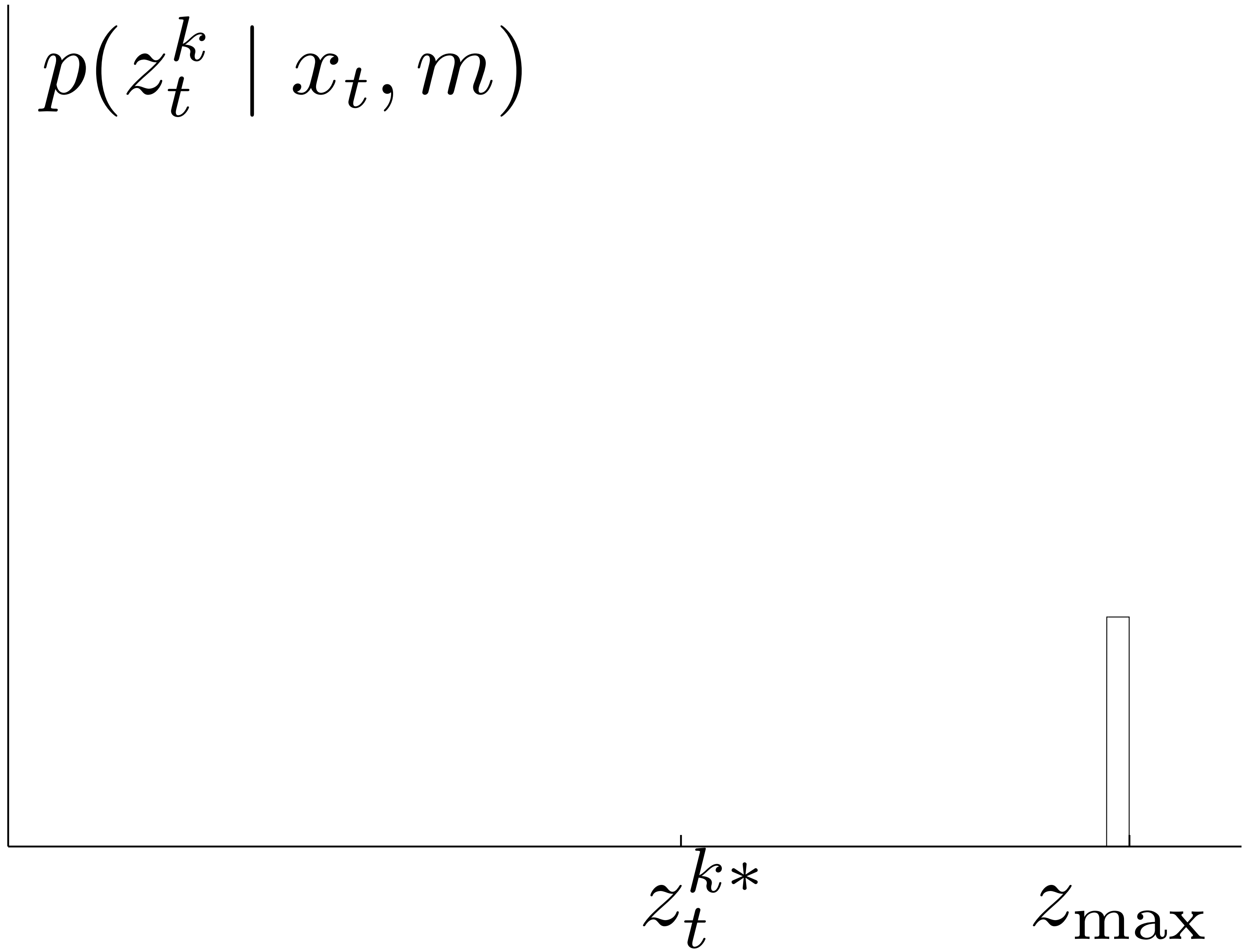
$$p_{\text{hit}}(z_t^k \mid x_t, m) = \begin{cases} \eta\, \mathcal{N}(z_t^k; z_t^{k*}, \sigma_{\text{hit}}^2) & \text{if } 0 \le z_t^k \le z_{\max} \\ 0 & \text{otherwise} \end{cases}$$

Factor 2: Unexpected objects



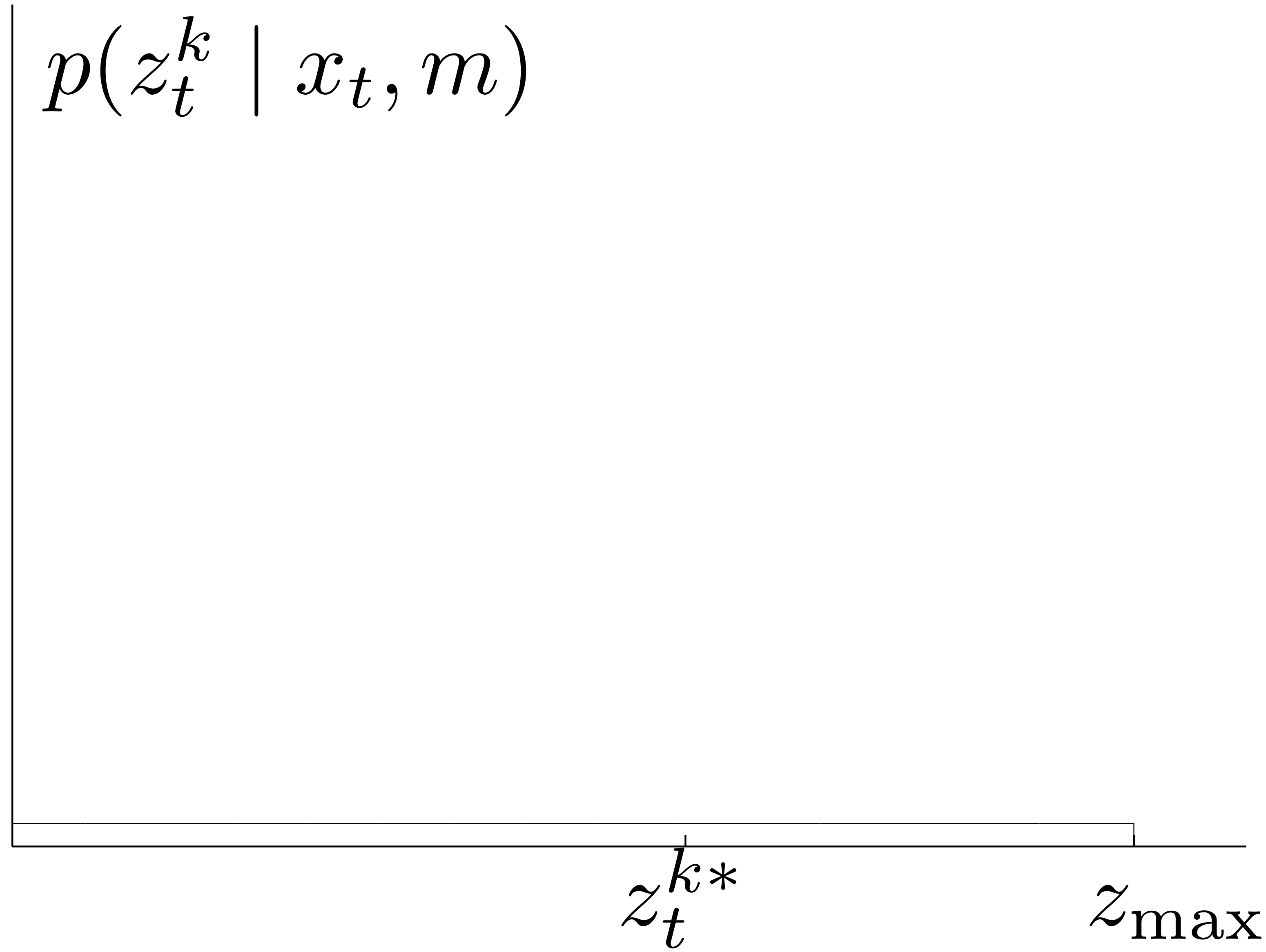
$$p_{\text{short}}(z_t^k \mid x_t, m) = \begin{cases} \eta\, \lambda_{\text{short}}\, e^{-\lambda_{\text{short}} z_t^k} & \text{if } 0 \le z_t^k \le z_t^{k*} \\ 0 & \text{otherwise} \end{cases}$$

Factor 3: Maximum range



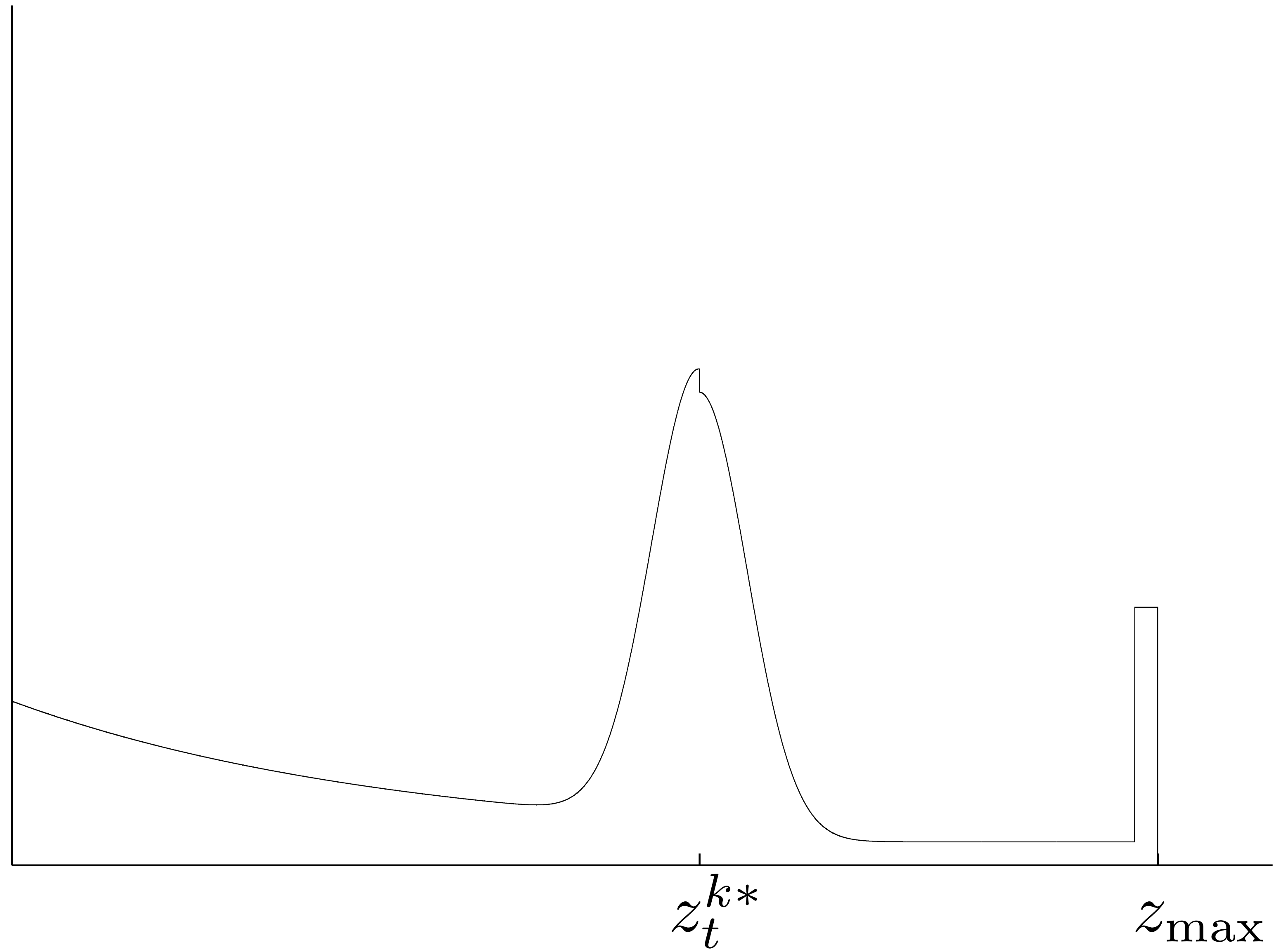
$$p_{\max}(z_t^k \mid x_t, m) \quad = \quad I(z = z_{\max}) \quad = \quad \begin{cases} 1 & \text{if } z = z_{\max} \\ 0 & \text{otherwise} \end{cases}$$

Factor 4: Failures in sensing



$$p_{\mathrm{rand}}(z_t^k \mid x_t, m) \quad = \quad \begin{cases} \frac{1}{z_{\max}} & \text{if } 0 \le z_t^k < z_{\max} \\ 0 & \text{otherwise} \end{cases}$$

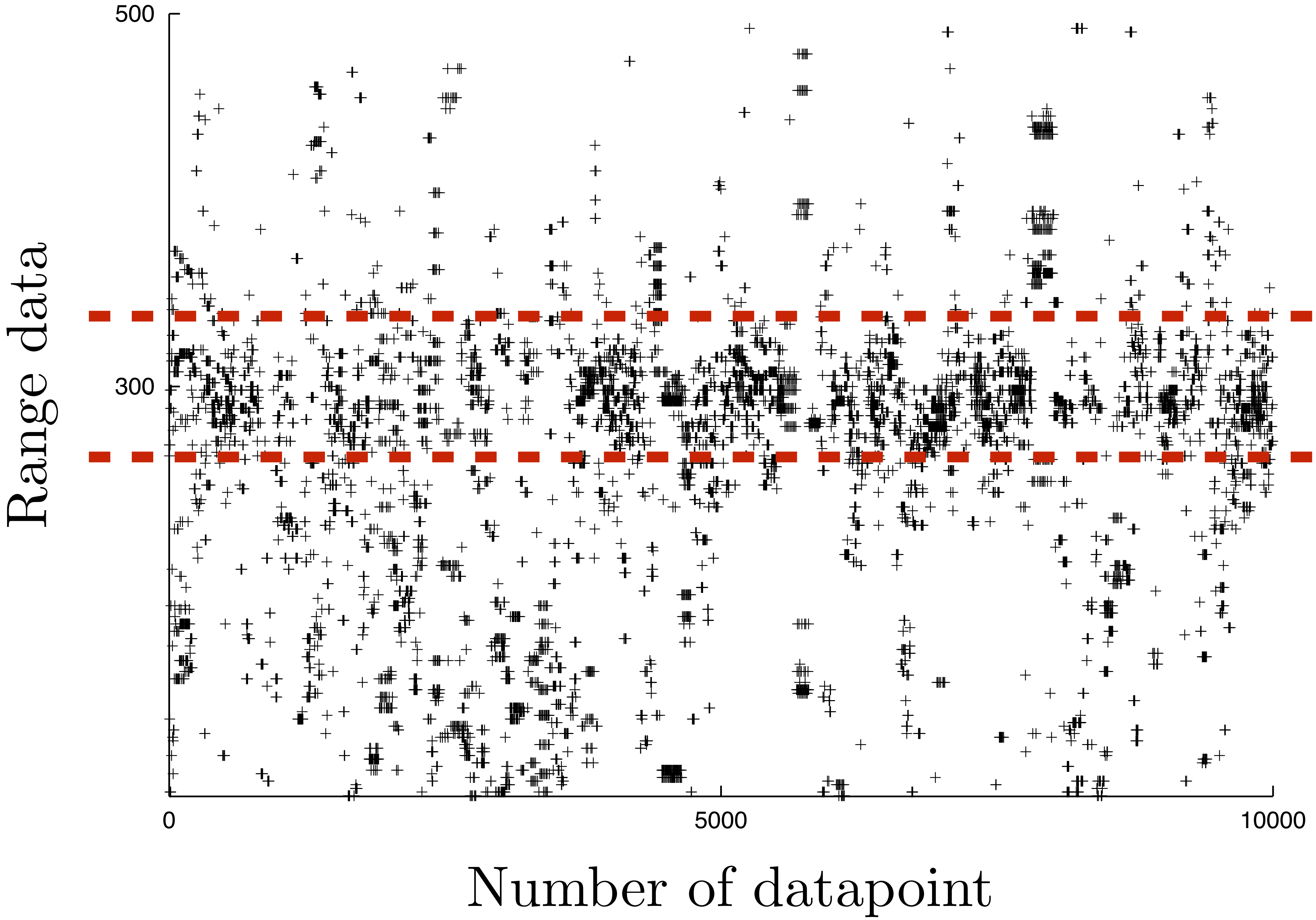
Combined probabilistic model



$$p(z_t^k \mid x_t, m) \quad = \quad \begin{pmatrix} z_{\text{hit}} \\ z_{\text{short}} \\ z_{\max} \\ z_{\text{rand}} \end{pmatrix}^T \cdot \begin{pmatrix} p_{\text{hit}}(z_t^k \mid x_t, m) \\ p_{\text{short}}(z_t^k \mid x_t, m) \\ p_{\max}(z_t^k \mid x_t, m) \\ p_{\text{rand}}(z_t^k \mid x_t, m) \end{pmatrix}$$

Question: How do we tune parameters?

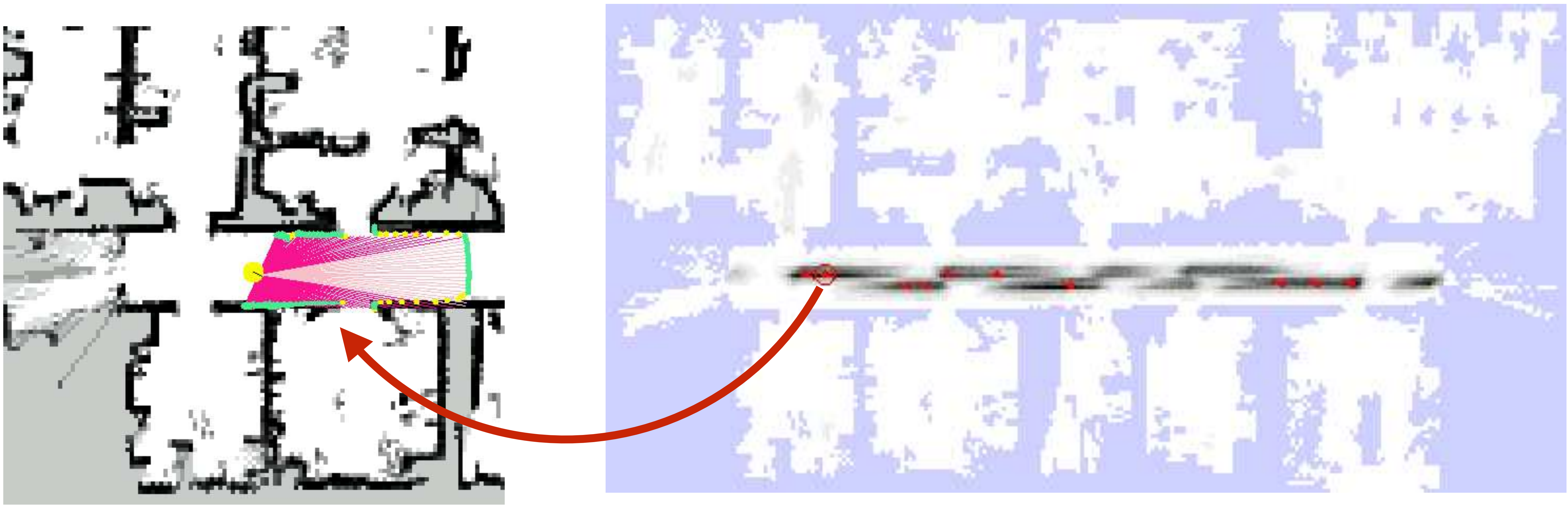
In theory: Collect lots of data and optimize parameters to maximize data likelihood



Example: Place a robot 300 cm from a wall and collect lots of data

Question: How do we tune parameters?

In practice: Simulate a scan and plot the likelihood from different positions



Actual scan

Likelihood at various locations

Problem: Overconfidence

$$P(z_t|x_t, m) = \prod_{i=1}^{K} P(z_t^k|x_t, m)$$

Independence assumption may result in repetition of mistakes