Slides adapted from Eric Westman, Cyrill Stachniss

Occupancy Grid Mapping

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What is an occupancy map?

What is an occupancy map?

Probabilistic representation of world

from

noisy, uncertain sensor measurement data,

with the assumption

that the robot pose is known.

What is an occupancy grid map?

Discretize world into cells

Assign a probability [0,1] to each cell

Courtesy: C. Stachniss

When do need to map online?

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1. Cant rely on floor plans

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2. Mapping disaster regions

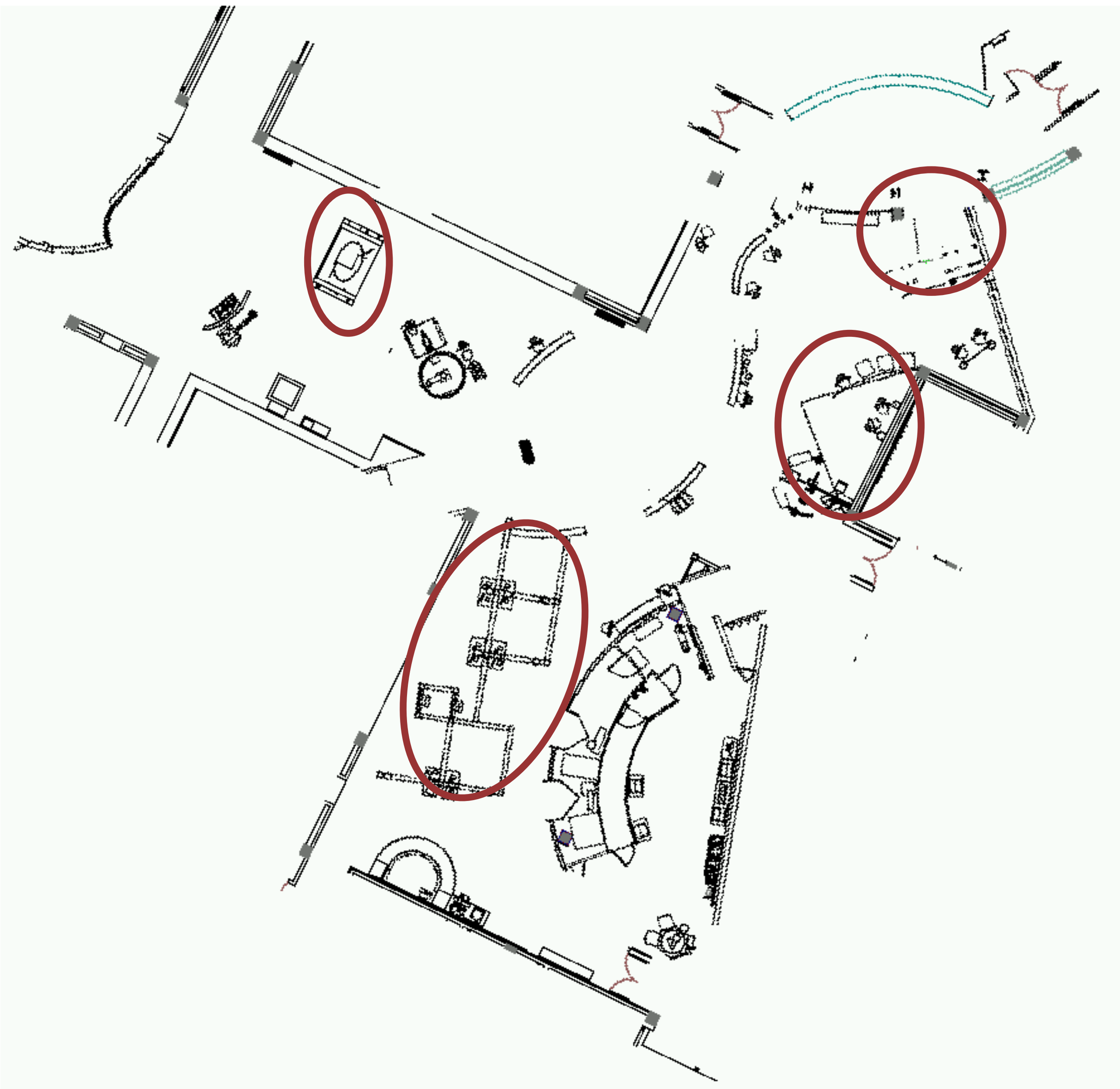
When do need to map online?

1. Cant rely on floor plans

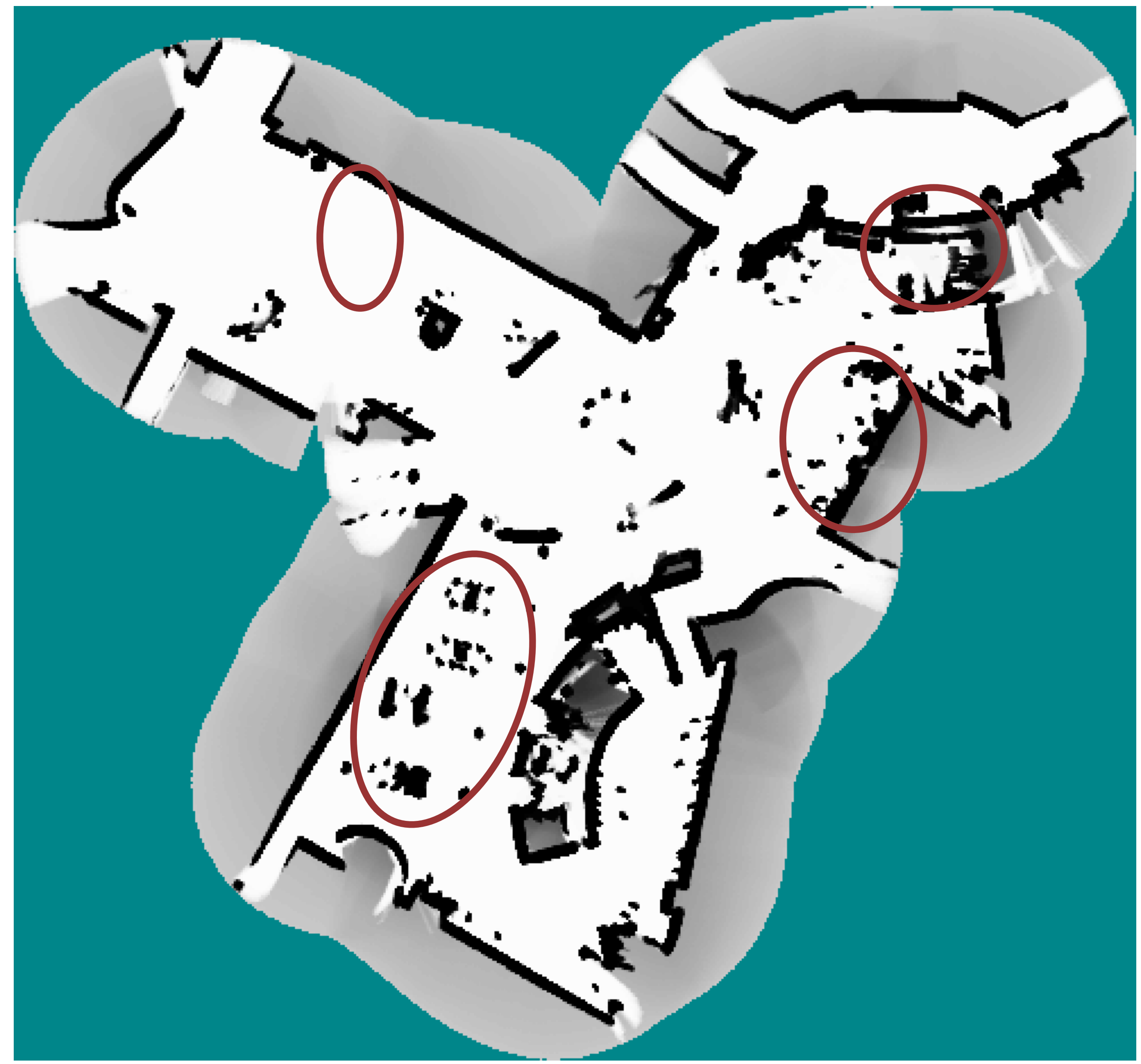
2. Mapping disaster regions

3. Mapping unstructured environments

Even floor plans can be wrong...



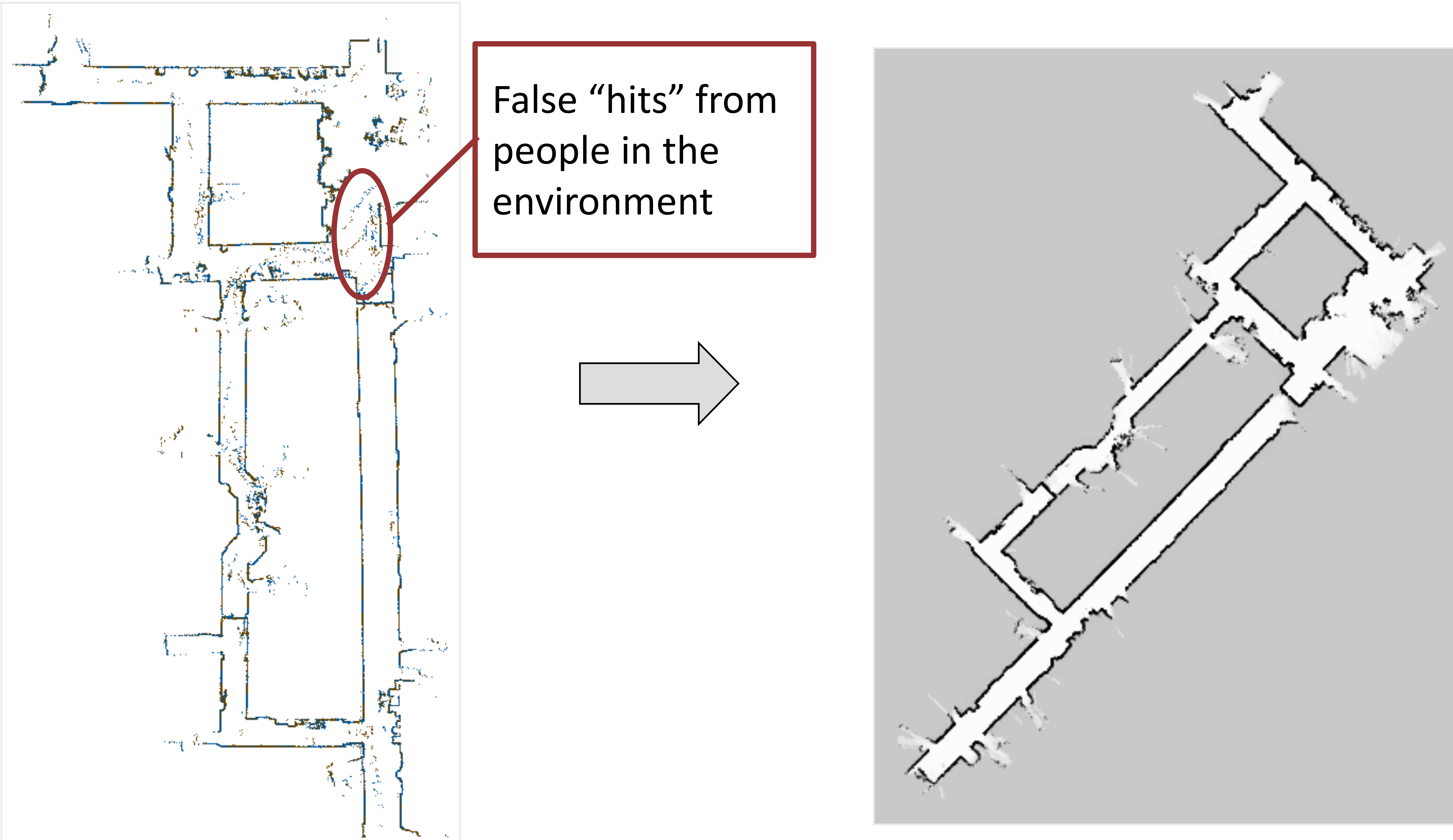
CAD map



occupancy grid map

Tech Museum, San Jose

Why do we need estimation?

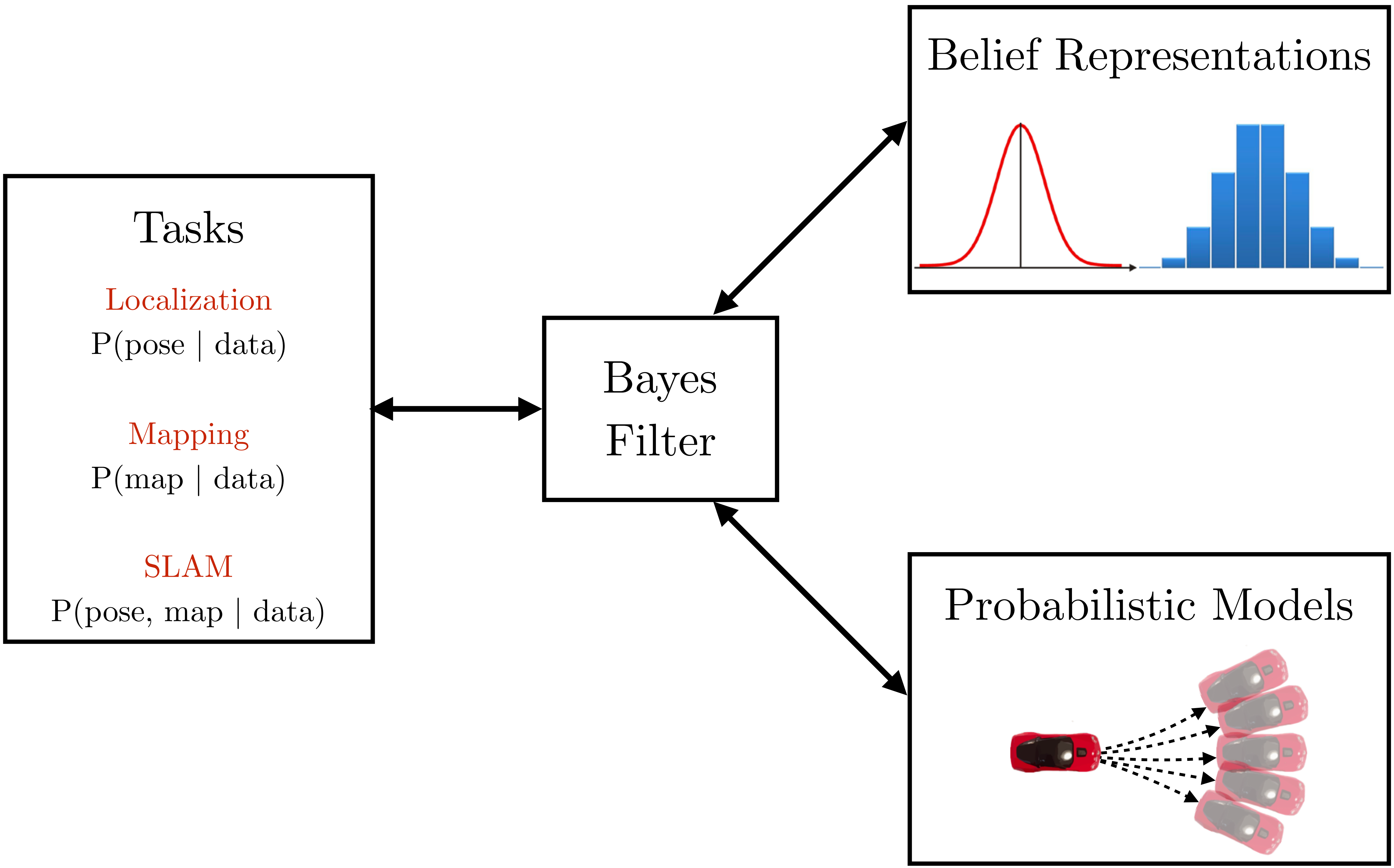


Scans are noisy - if you just added them they will contradict each other and create a mess!

Bayes filter is a powerful tool

Localization   Mapping   SLAM   POMDP

Assembling Bayes filter



Different tasks as Bayes filtering

Tasks	**Belief Representation**	**Probabilistic Models**
Localization P(pose | data)	Gaussian / Particles	Motion model Measurement model
Mapping P(map | data)	Discrete (binary)	Inverse measurement model
SLAM P(pose, map | data)	Gaussian (pose, landmarks)	Motion model, measurement model, correspondence model

Today's objective

1. Understand occupancy grid mapping intuitively

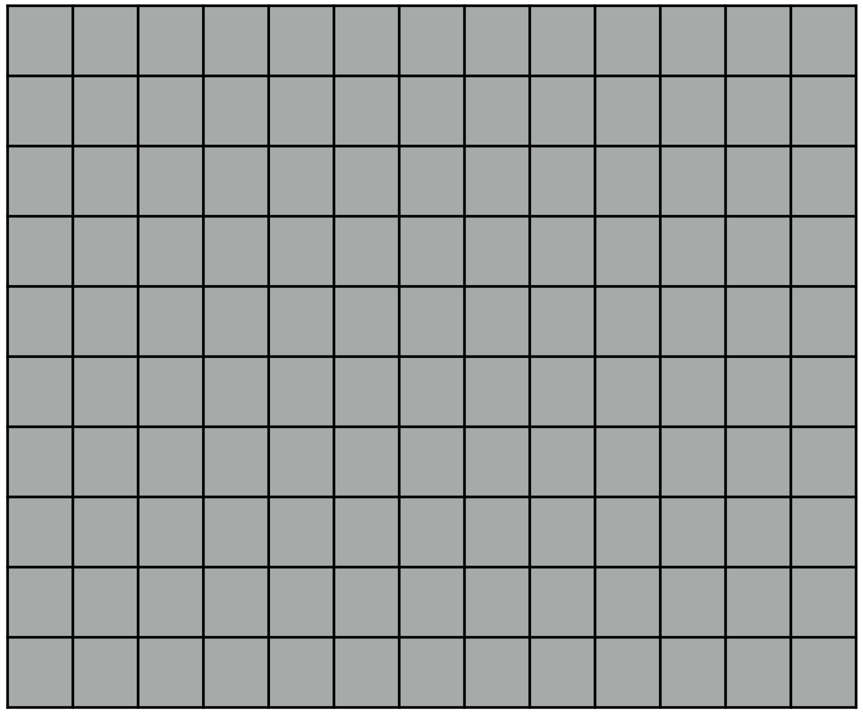
2. Work through Bayes filter derivation

3. Examine when assumptions get violated

Spilling the beans on mapping

Spilling the beans on mapping

Step 1:
Start with an empty map.
0.5 prob of being free.



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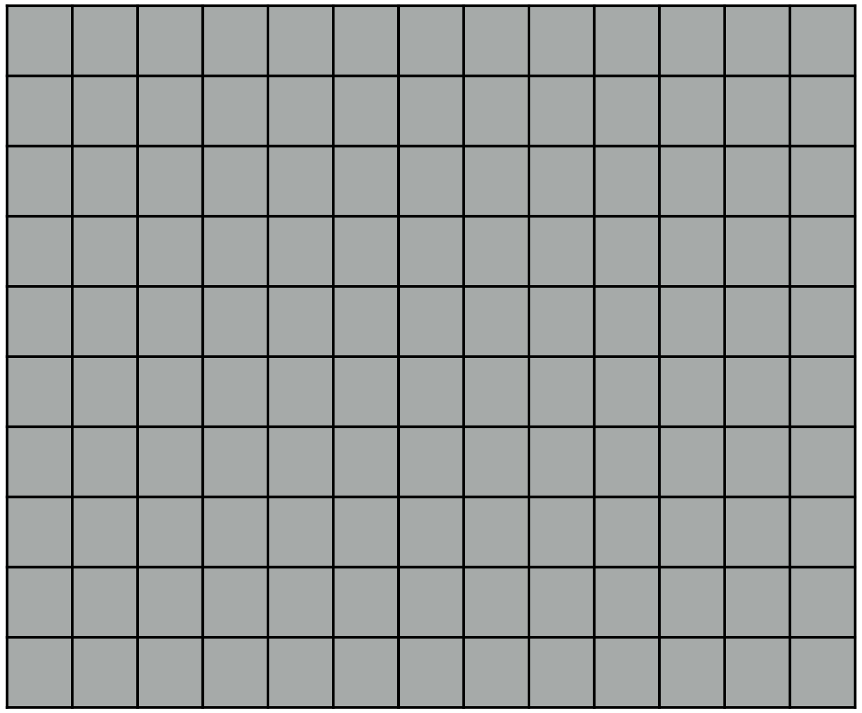
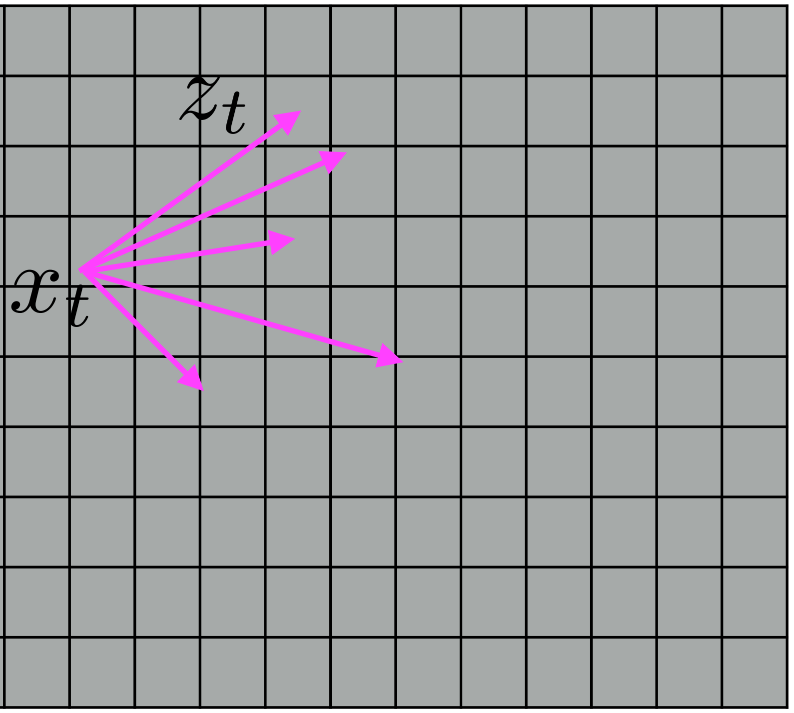


Step 2:
Accept the latest measurement and pose



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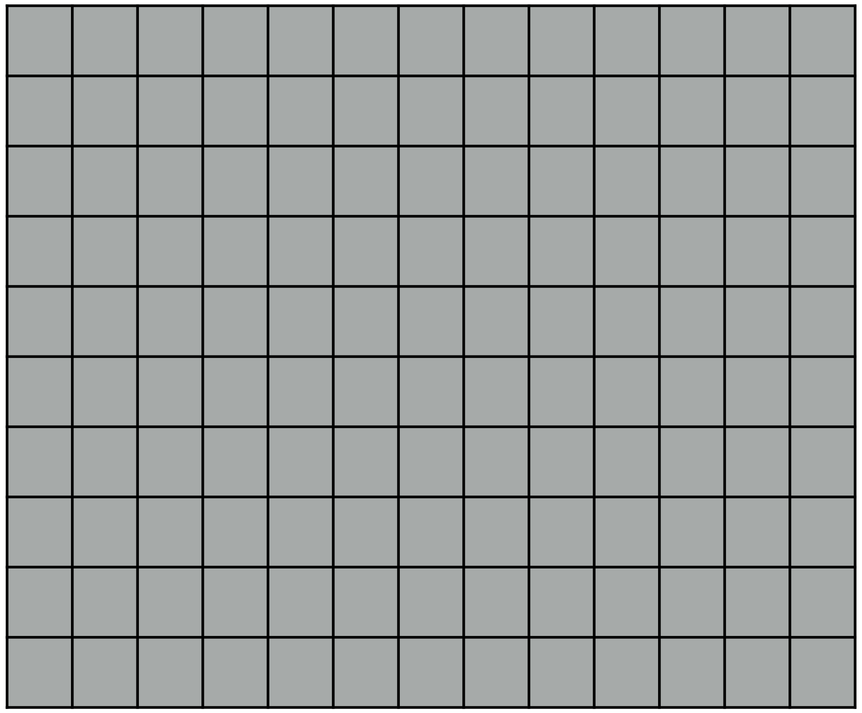

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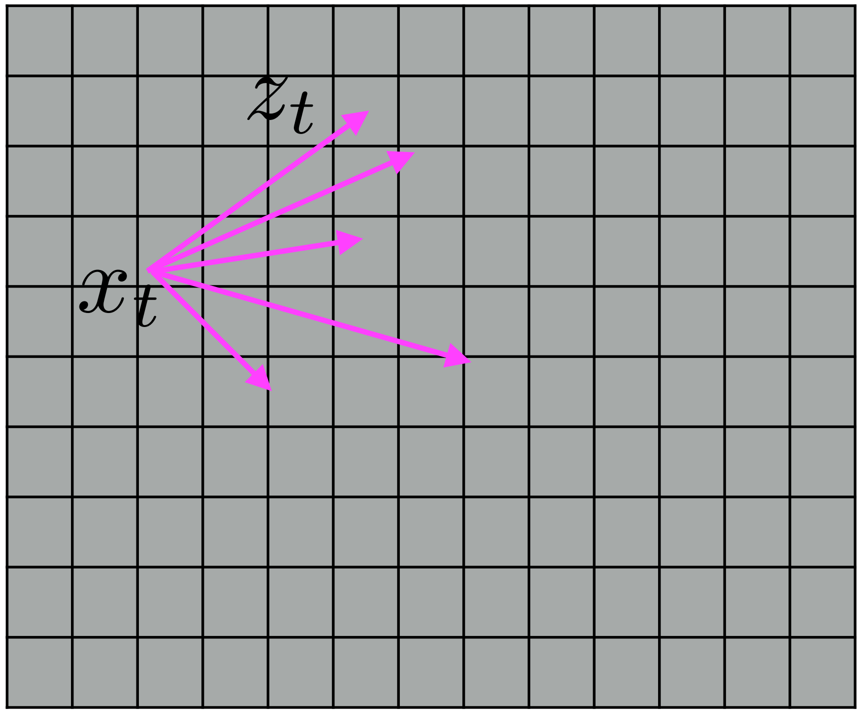
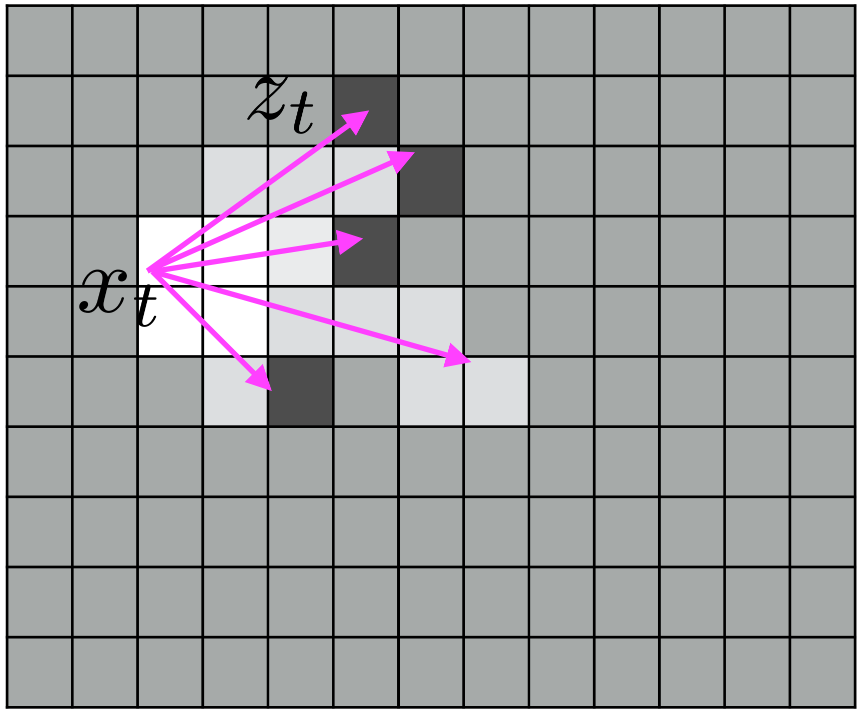


Step 3:
Raycast every beam. Group cells as HIT and MISS.

HIT: Bump down probability.
MISS: Bump up.



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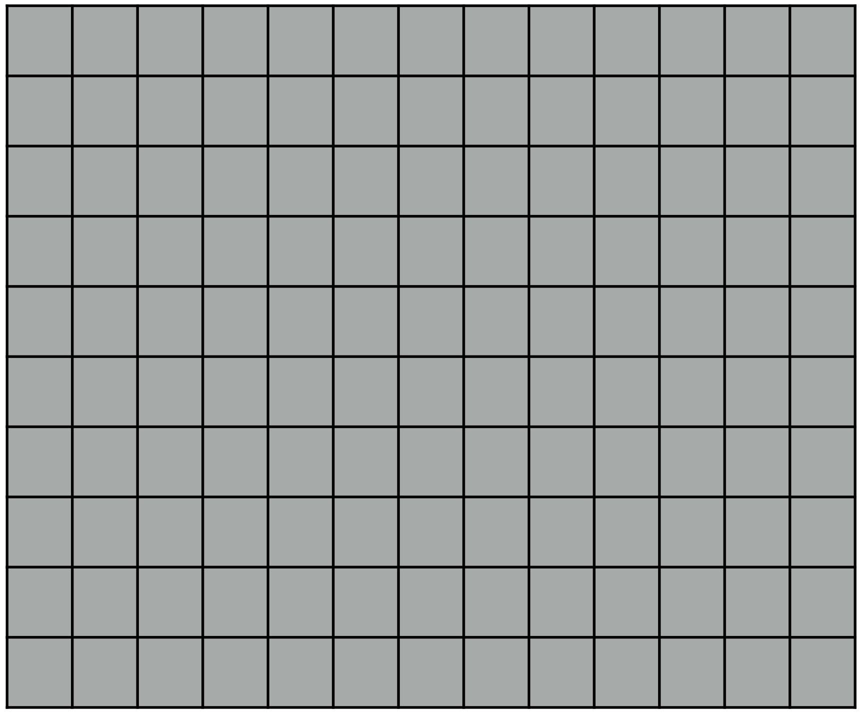

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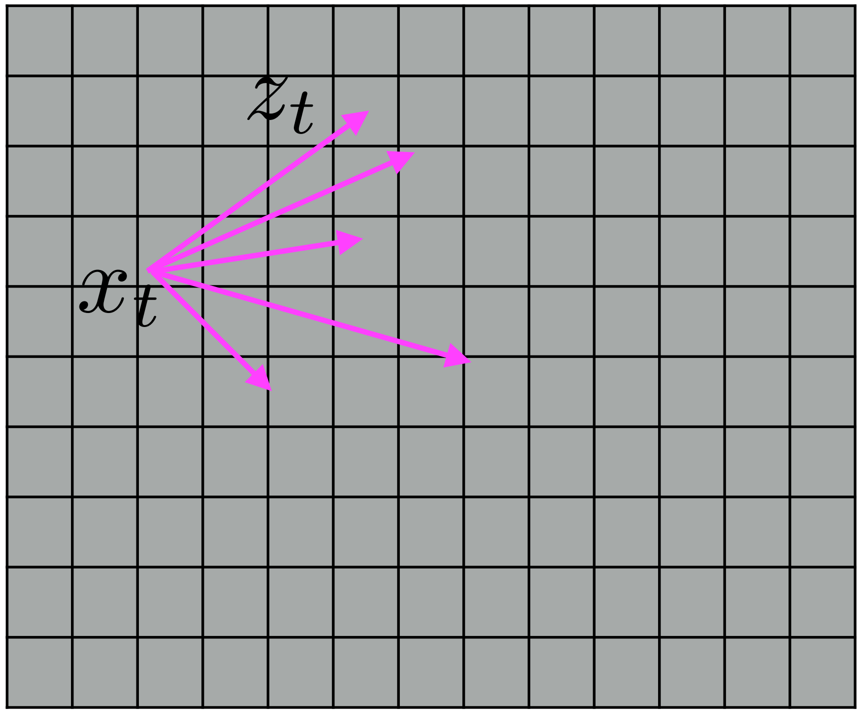

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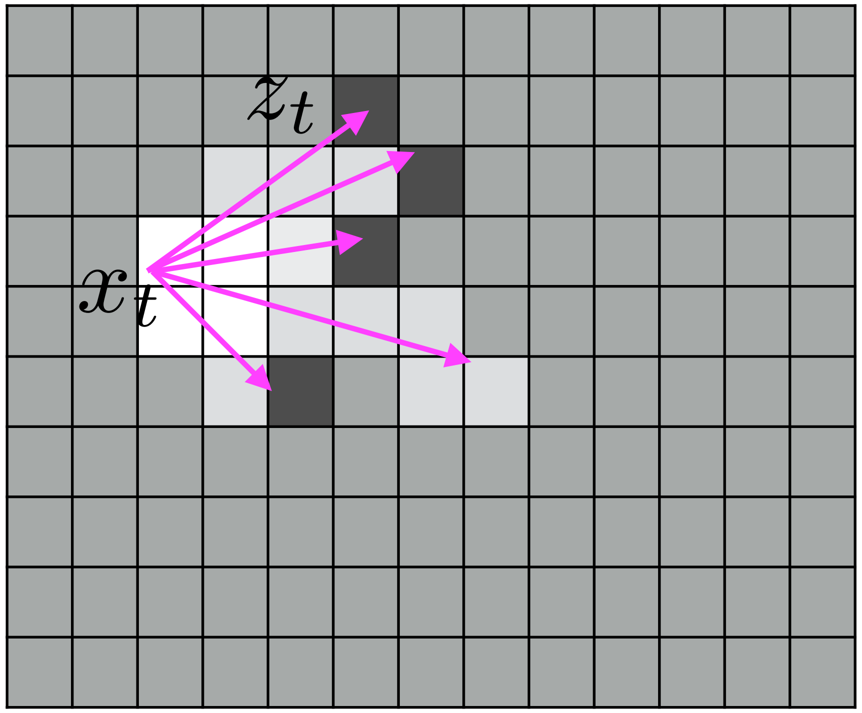
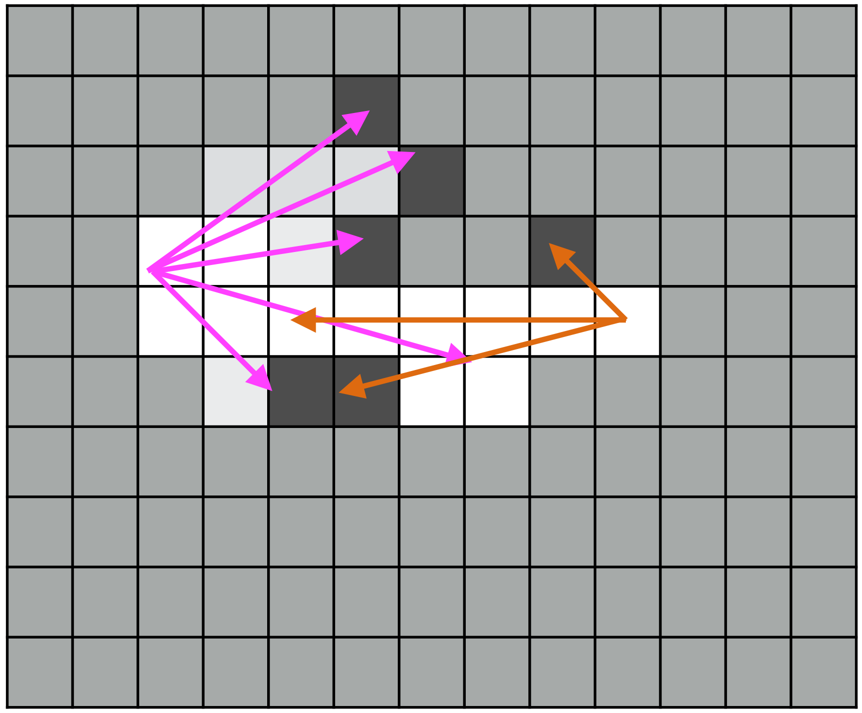


Step 4:
Get new measurement.

Update the map.



Mapping as just another Bayes filtering problem

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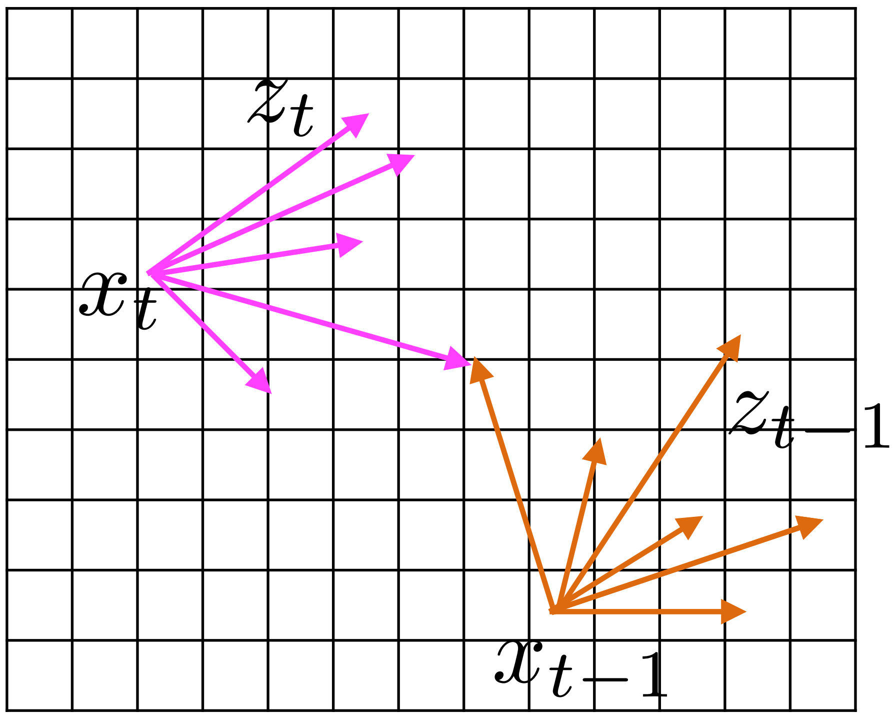
Task: $$\begin{gathered}\text{Mapping}\\ \mathrm{P}(\text{map} \mid \text{data})\end{gathered}$$

Mapping as just another Bayes filtering problem

Task: Mapping

P(map | data)

What is the data?



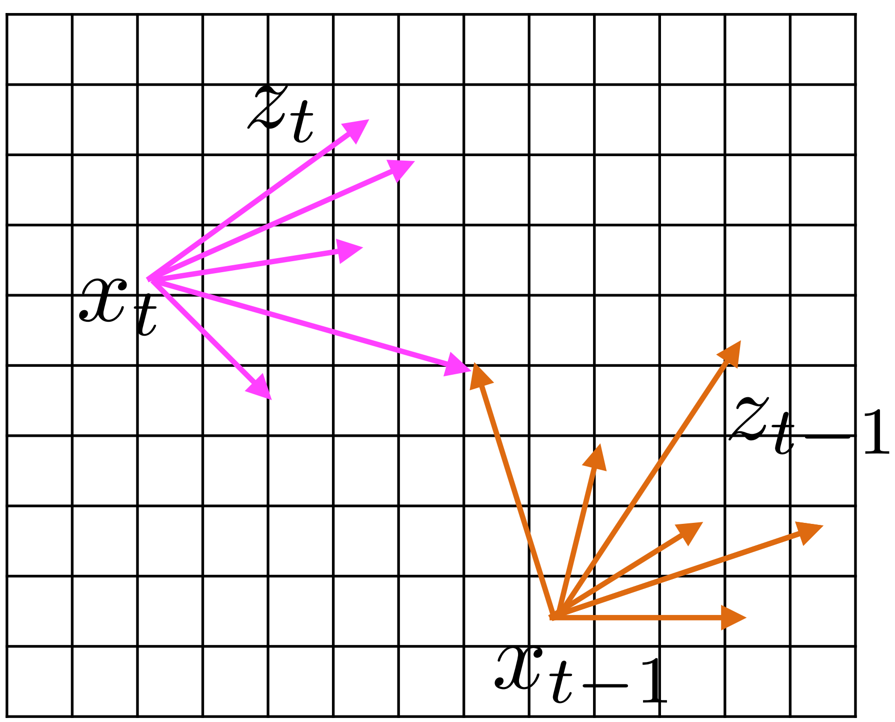
Stream of pose and laser scans $x_{1:t}, z_{1:t}$

Mapping as just another Bayes filtering problem

Task: Mapping

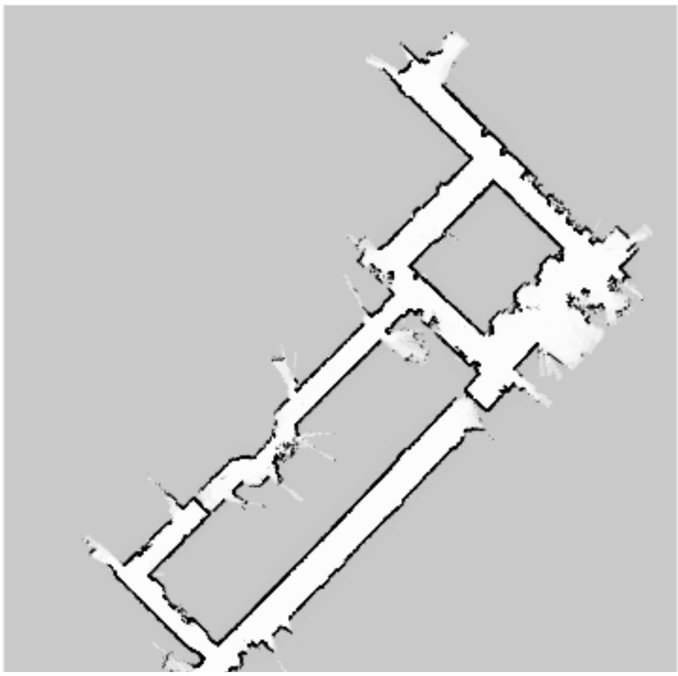
P(map | data)

What is the data?



Stream of pose and laser scans $x_{1:t}, z_{1:t}$

What is the belief representation?



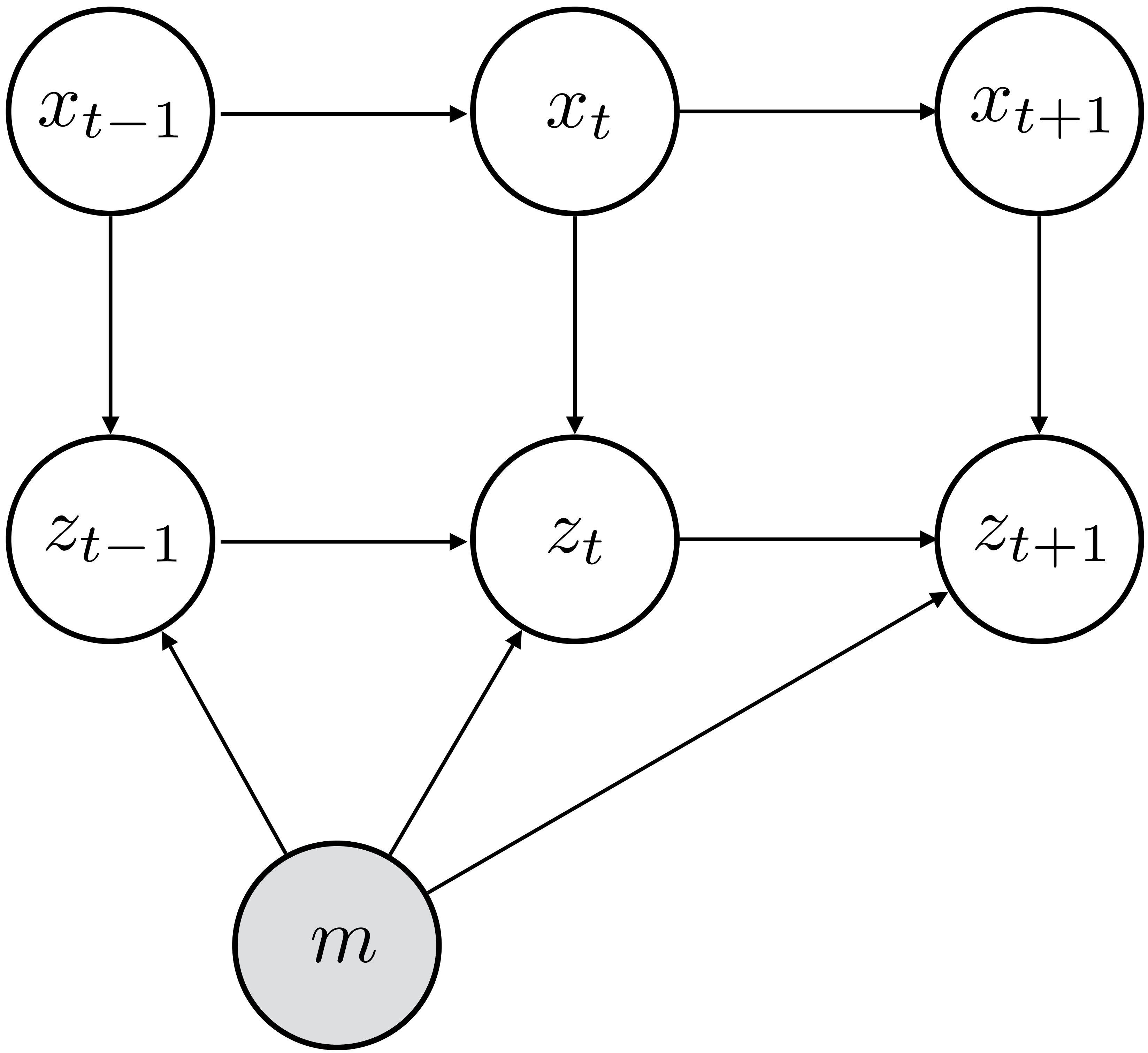
0 is occupied, 1 is free, 0.5 unknown

Represent world as a collection of cells

Each belief is [0,1]

$$P(m) = P(m_1, m_2, \ldots, m_n)$$

Graphical model of mapping



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The state is a matrix of binary values

The belief assigns a probability to all states

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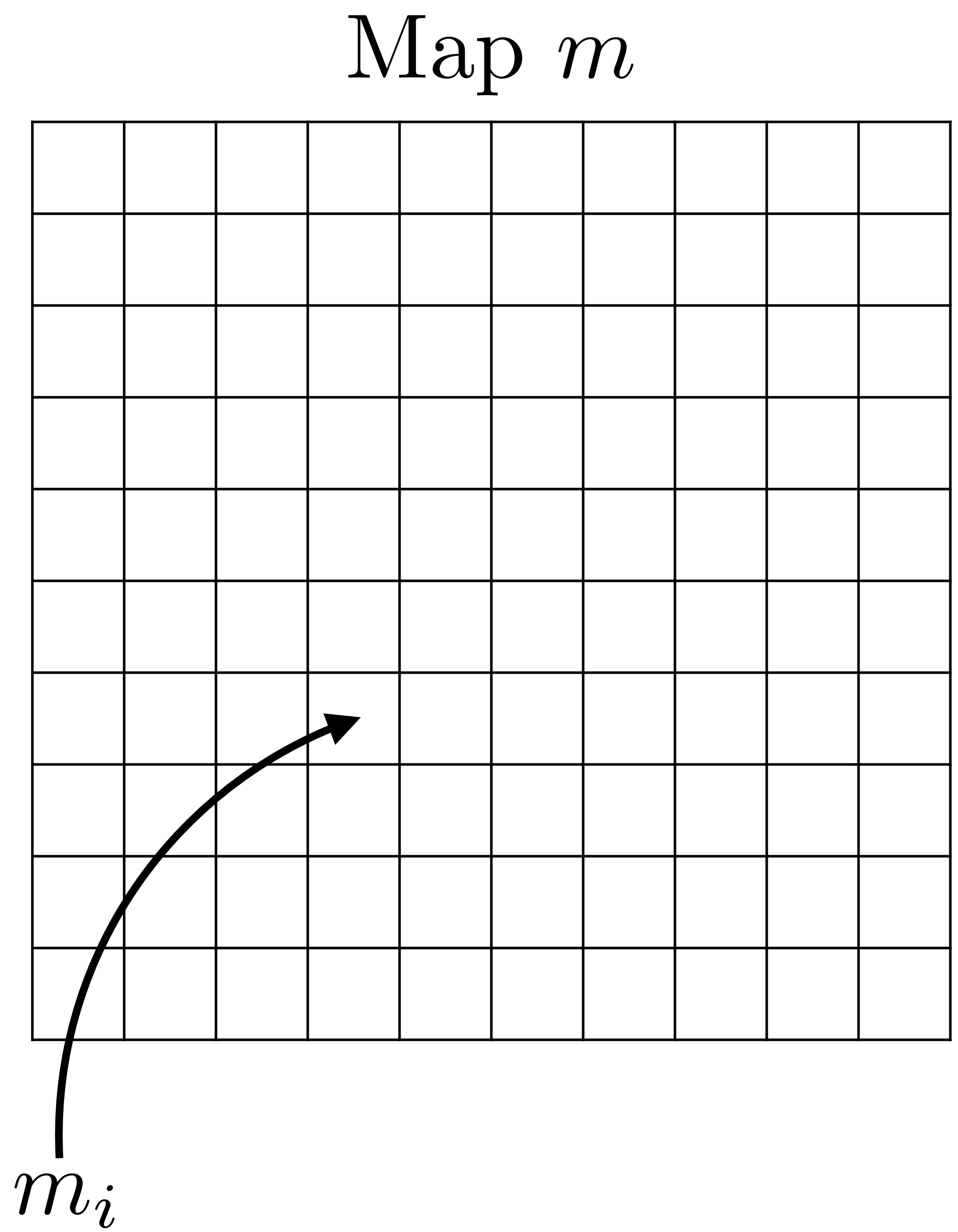

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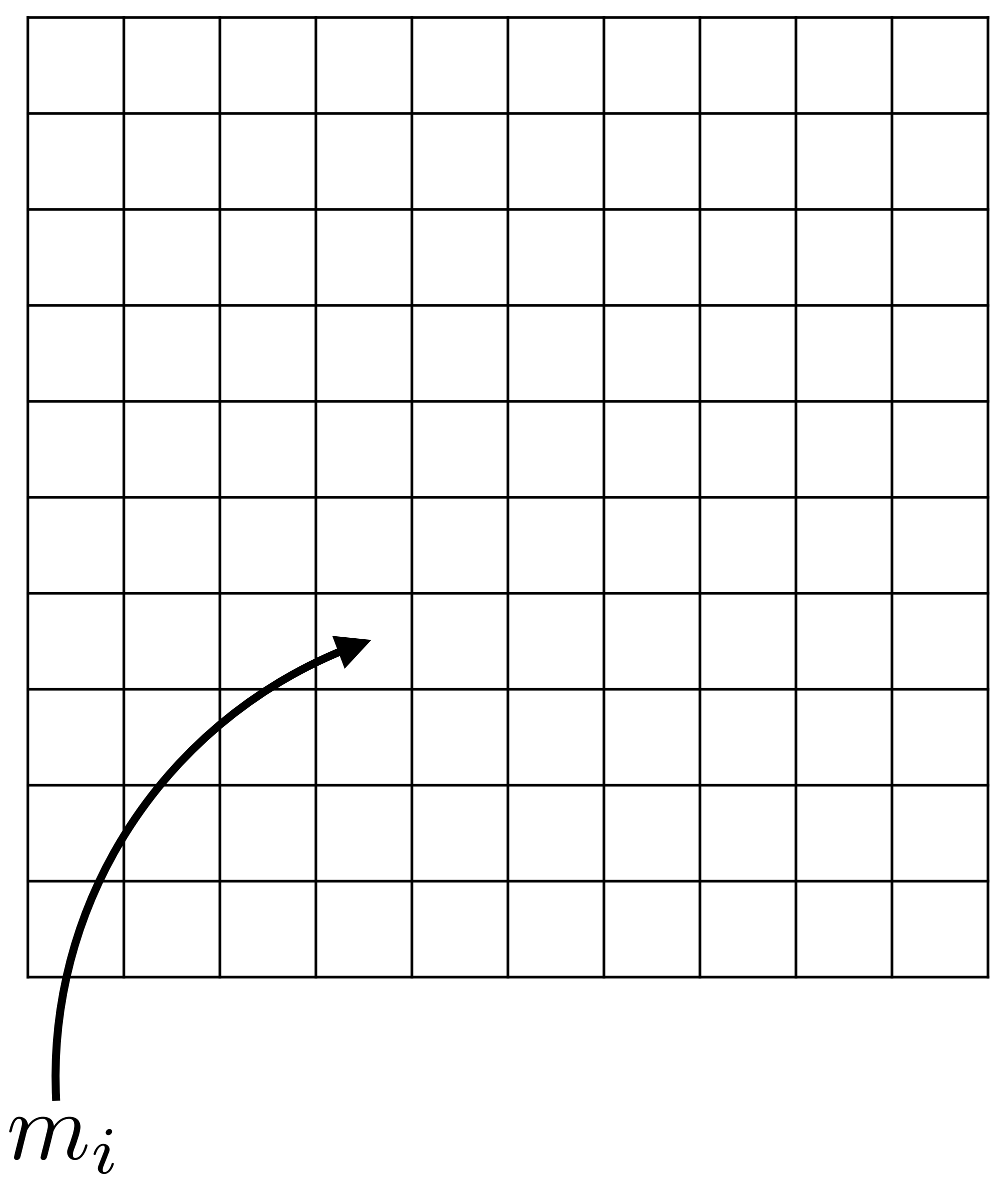
Map m



Example: We are mapping 25m x 25m area at 25 cm resolution.

100 x 100 grid = 10,000 cells

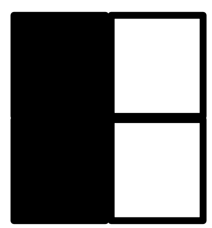
How many possible maps can there be?

2^10000 !!!

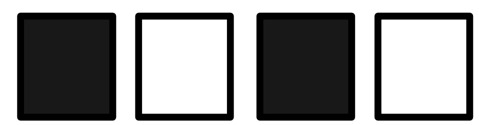
Solution: Approximate by independent cells

Joint probability is approximated by product of individual probabilities

$$P(m) = \prod_i P(m_i)$$



Structured
map (4-dim state)



4 individual
cells

$$P(m|x_{1:t}, z_{1:t}) = \prod_i P(m_i|x_{1:t}, z_{1:t})$$

Binary r.v.

Let's crank through Bayes filter

Let's crank through Bayes filter

$$P(m_i|z_{1:t}, x_{1:t}) \quad = \quad P(m_i|z_{1:t-1}, x_{1:t-1}, {\color{magenta}z_t}, {\color{blue}x_t})$$

$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{[old data]} \qquad\qquad \text{[new data]}$$

Let's crank through Bayes filter

$$P(m_i|z_{1:t}, x_{1:t}) \quad = \quad P(m_i|z_{1:t-1}, x_{1:t-1}, z_t, x_t)$$

[old data] [new data]

$$\text{(Bayes)} \qquad = \quad \eta P(z_t|m_i, z_{1:t-1}, x_{1:t-1}, x_t) P(m_i|z_{1:t-1}, x_{1:t-1}, x_t)$$

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(Cond Ind.) $= \eta P(z_t|m_i, z_{1:t-1}, x_{1:t-1}, x_t) P(m_i|z_{1:t-1}, x_{1:t-1})$

[old data] [old filter value]

Problem 1: Can we apply conditional indep?

$$P(z_t | m_i, z_{1:t-1}, x_{1:t-1}, x_t) = P(z_t | m_i, x_t)$$

[map value] [old data] [sensor pose]

Let's crank through Bayes filter

$$P(m_i|z_{1:t},x_{1:t}) \quad = \quad P(m_i|z_{1:t-1},x_{1:t-1},z_t,x_t)$$

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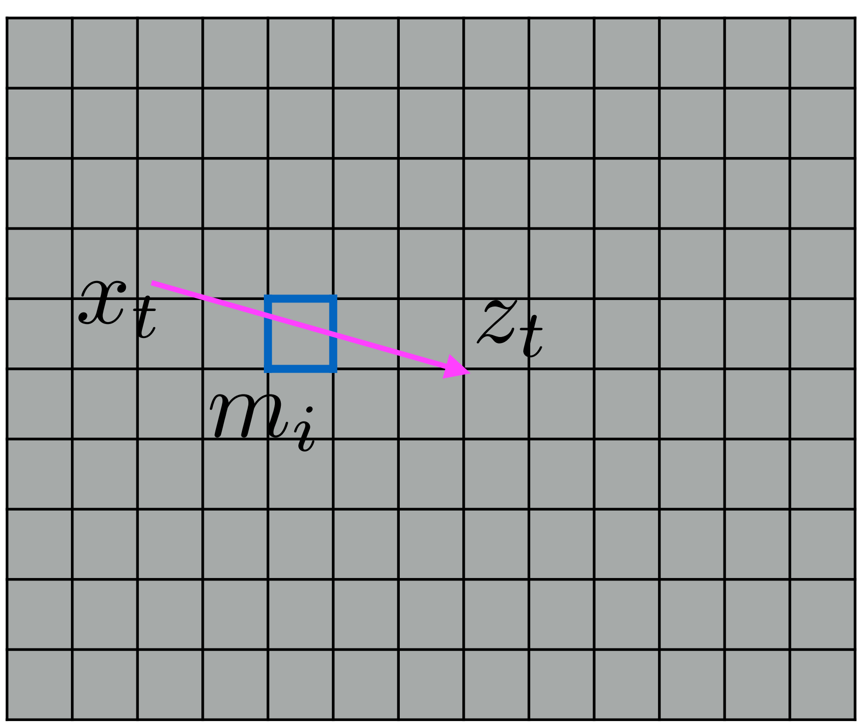
Problem 2: Sensor model is hard to define

Why is this hard to specify? $P(z_t | m_i, x_t)$

[ray] [cell]



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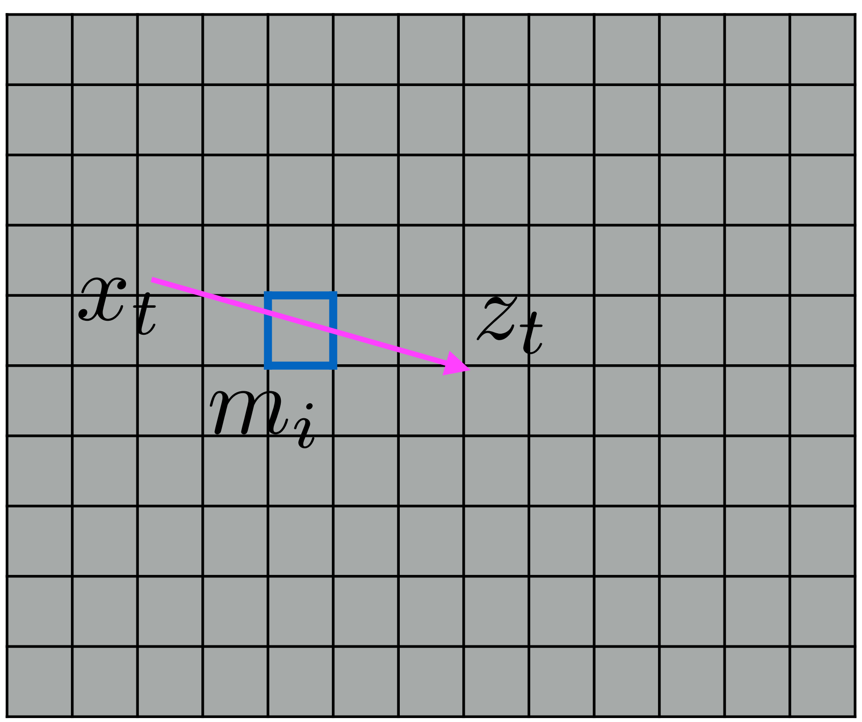

Is this easier to specify?

$$P(m_i | z_t, x_t)$$

[cell] [ray]

Problem 2: Sensor model is hard to define

$$P(z_t | m_i, x_t)$$

We can't specify this...

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$$P(z_t | m_i, x_t)$$

We can't specify this...

Solution: Apply Bayes to get an inverse sensor model

$$P(z_t | m_i, x_t) = \frac{\boxed{P(m_i | z_t, x_t)} P(z_t | x_t)}{P(m_i | x_t)}$$

Let's crank through Bayes filter

$$P(m_i|z_{1:t}, x_{1:t}) \quad = \quad P(m_i|z_{1:t-1}, x_{1:t-1}, z_t, x_t)$$

$$\text{(Bayes)} \quad = \quad \eta P(z_t|m_i, z_{1:t-1}, x_{1:t-1}, x_t) P(m_i|z_{1:t-1}, x_{1:t-1}, x_t)$$

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$$\text{(Cond Ind.)} \quad = \eta P(z_t|m_i,x_t)P(m_i|z_{1:t-1},x_{1:t-1})$$

$$\text{(Bayes.)} \qquad = \eta \frac{P(m_i|z_t,x_t)P(z_t|x_t)}{P(m_i|x_t)} P(m_i|z_{1:t-1},x_{1:t-1})$$

Let's crank through Bayes filter

$$
\begin{aligned}
P(m_i|z_{1:t}, x_{1:t}) &= P(m_i|z_{1:t-1}, x_{1:t-1}, z_t, x_t) \\
\text{(Bayes)} \quad &= \eta P(z_t|m_i, z_{1:t-1}, x_{1:t-1}, x_t) P(m_i|z_{1:t-1}, x_{1:t-1}, x_t) \\
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\text{(Cond Ind.)} \quad &= \eta \frac{P(m_i|z_t, x_t) P(z_t|x_t)}{P(m_i)} P(m_i|z_{1:t-1}, x_{1:t-1})
\end{aligned}
$$

Let's look at likelihood ratios

$$\underset{\text{[cell=1]}}{P(m_i | z_{1:t}, x_{1:t})} \;=\; \eta \frac{P(m_i | z_t, x_t) P(z_t | x_t)}{P(m_i)} P(m_i | z_{1:t-1}, x_{1:t-1})$$

There are terms we don't know and would not like to calculate!

Let's look at likelihood ratios

$$\underset{\text{[cell=1]}}{P(m_i|z_{1:t}, x_{1:t})} = \eta \frac{P(m_i|z_t, x_t)P(z_t|x_t)}{P(m_i)} P(m_i|z_{1:t-1}, x_{1:t-1})$$

There are terms we don't know and would not like to calculate!

Let's look at the opposite probability!

$$\underset{\text{[cell=0]}}{P(\neg m_i|z_{1:t}, x_{1:t})} = \eta \frac{P(\neg m_i|z_t, x_t)P(z_t|x_t)}{P(\neg m_i)} P(\neg m_i|z_{1:t-1}, x_{1:t-1})$$

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Let's look at the opposite probability!

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Let's look at the ratio

$$\frac{P(m_i|z_{1:t}, x_{1:t})}{P(\neg m_i|z_{1:t}, x_{1:t})} = \frac{P(m_i|z_t, x_t)}{P(\neg m_i|z_t, x_t)} \frac{P(\neg m_i)}{P(m_i)} \frac{P(m_i|z_{1:t-1}, x_{1:t-1})}{P(\neg m_i|z_{1:t-1}, x_{1:t-1})}$$

Log likelihood ratios

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(inverse sensor model) (prior) (new ratio)

Log likelihood ratios

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(inverse sensor model) (prior) (new ratio)

Taking logs of all terms

$$\log\left(\frac{P(m_i|z_{1:t}, z_{1:t})}{P(\neg m_i|z_{1:t}, z_{1:t})}\right) = \log\left(\frac{P(m_i|z_t, x_t)}{P(\neg m_i|z_t, x_t)}\right) - \log\left(\frac{P(m_i)}{P(\neg m_i)}\right) + \log\left(\frac{P(m_i|z_{1:t-1}, x_{1:t-1})}{P(\neg m_i|z_{1:t-1}, x_{1:t-1})}\right)$$

Log likelihood ratios

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$$l_t \;=\; l(m_i|z_t, x_t) - l_0 + l_{t-1}$$

(updated belief) (inverse model) (prior) (old belief)

Pseudo code of occupancy mapping

for every ray r in (x_t, z_t)

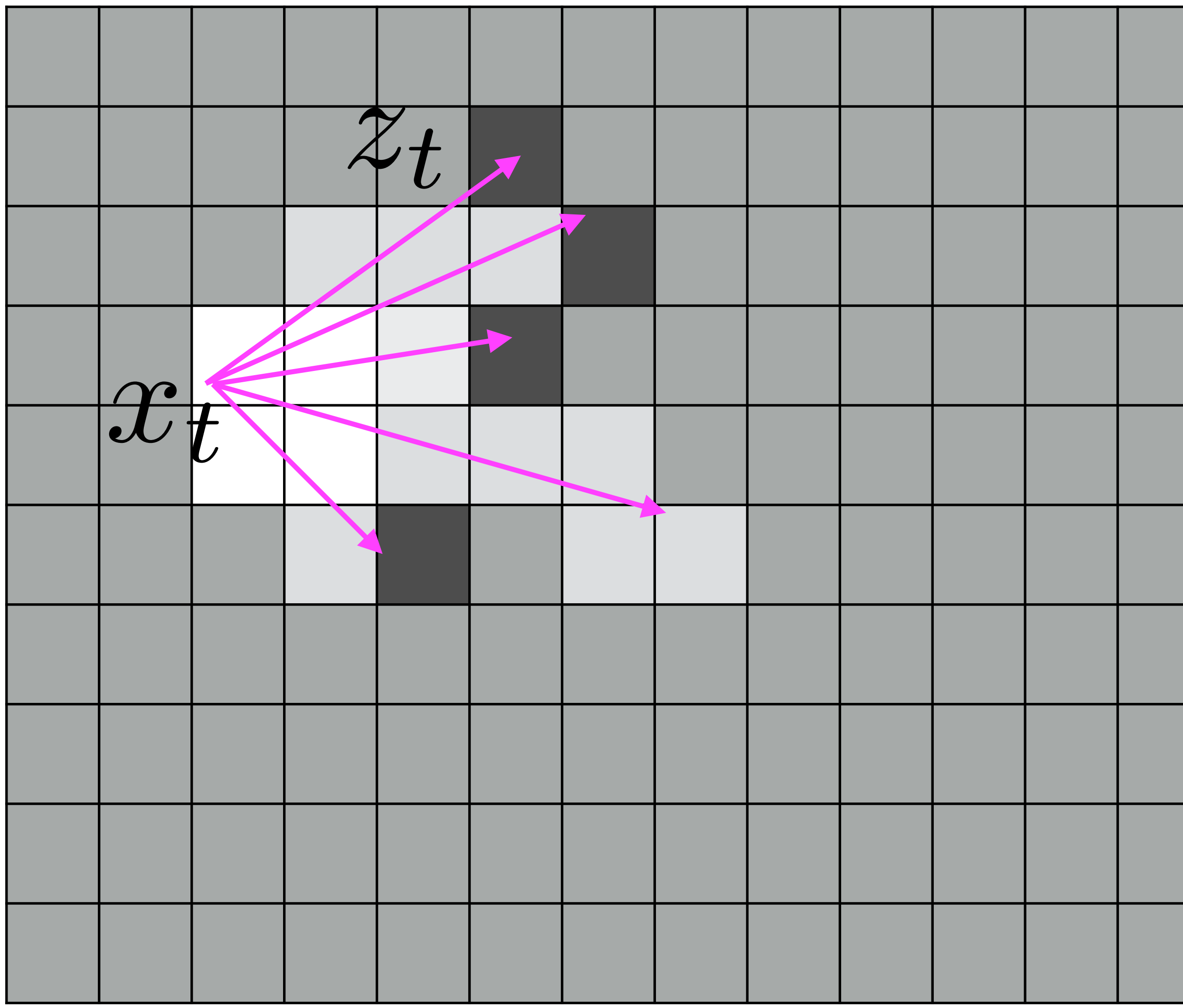
for every cell m_i in r

if m_i is MISS

$$l_i = l_i + l(\mathrm{MISS}) - \mathrm{l}_0$$

else

$$l_i = l_i + l(\mathrm{HIT}) - \mathrm{l}_0$$

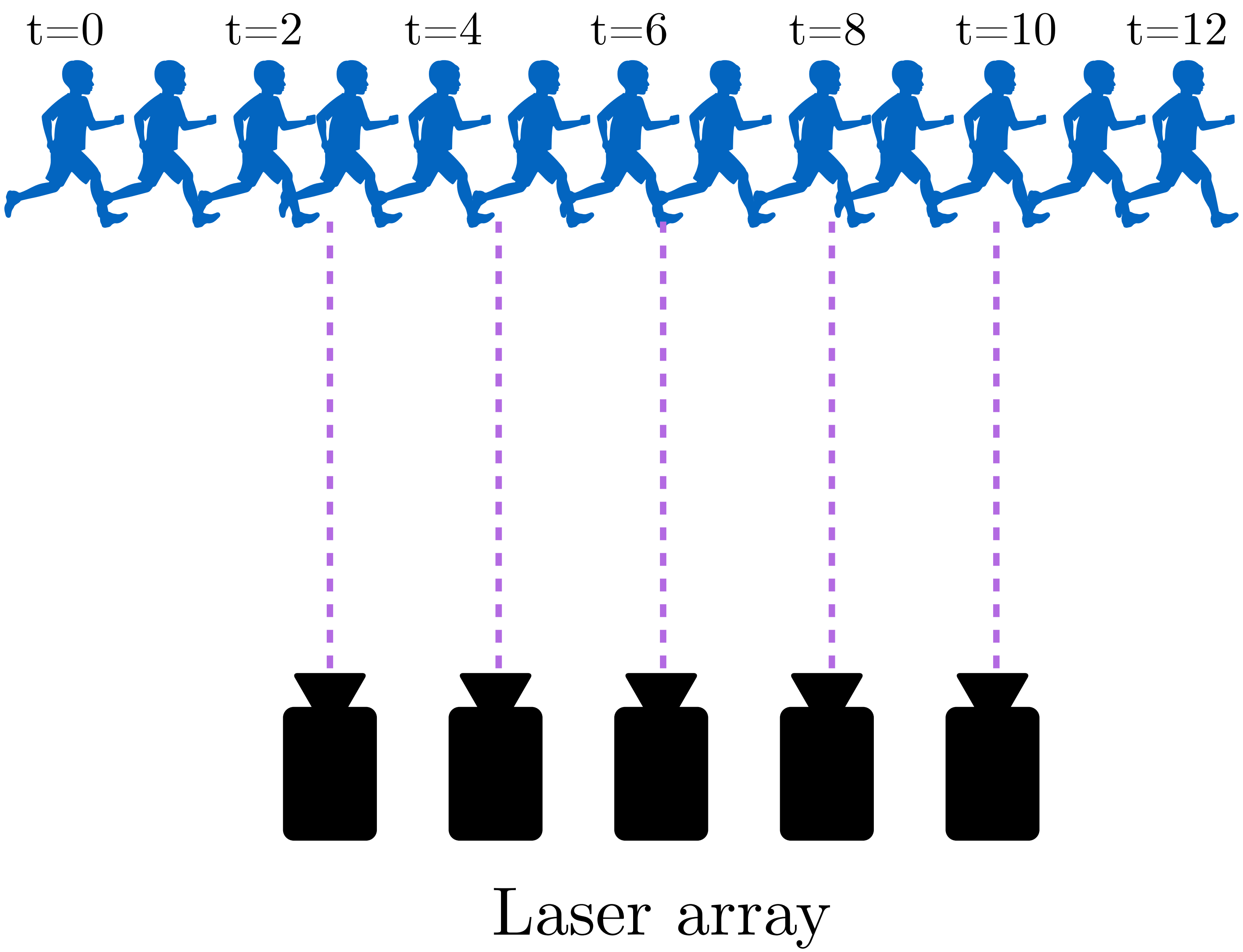


What is the inverse sensor model $P(m_i|z_t, x_t)$?

$$P(m_i|z_t, x_t) = \begin{cases} P(m_i) + \delta & \text{if ray passes through cell} \\ P(m_i) & \text{if ray does not intersect cell} \\ P(m_i) - \delta & \text{if ray stops in cell} \end{cases}$$

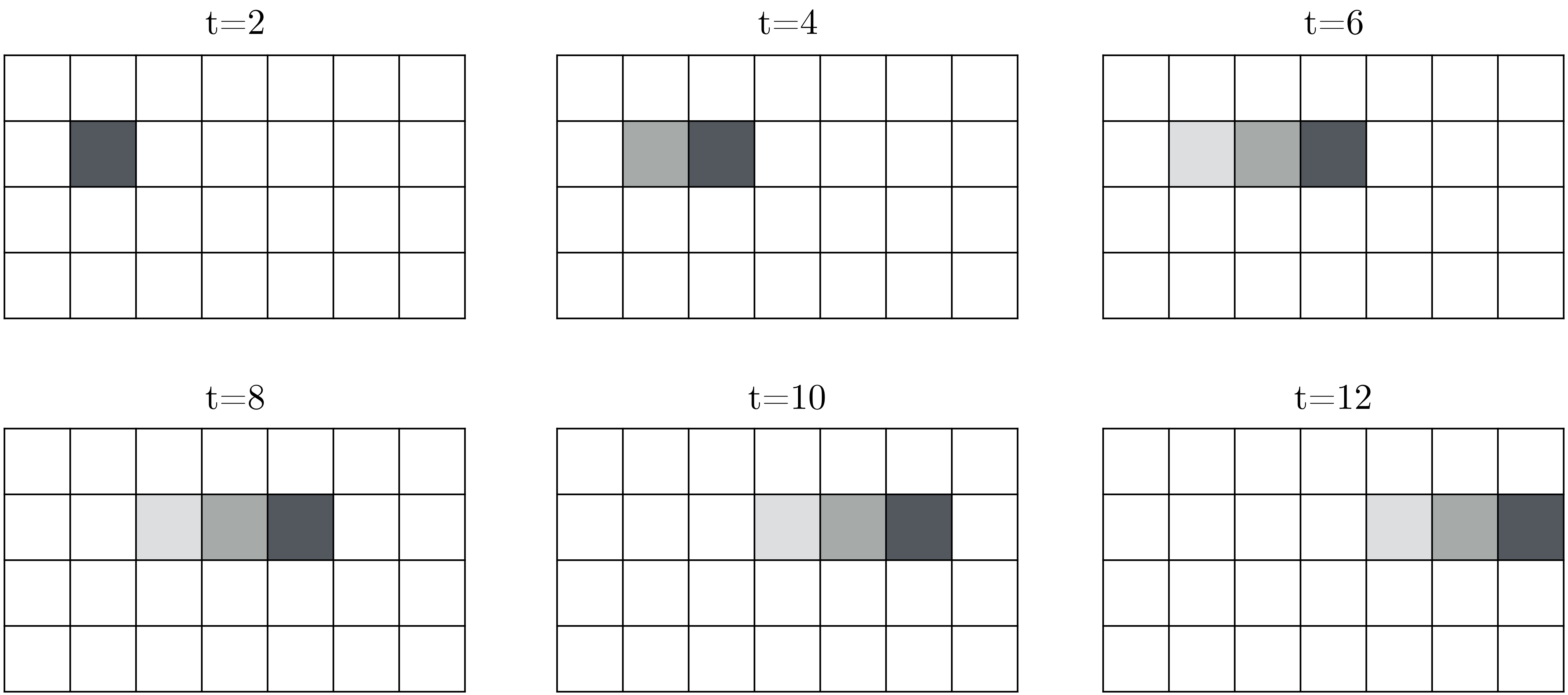
[cell] [ray]

Problem: Dynamic obstacles



What will the occupancy map look like?

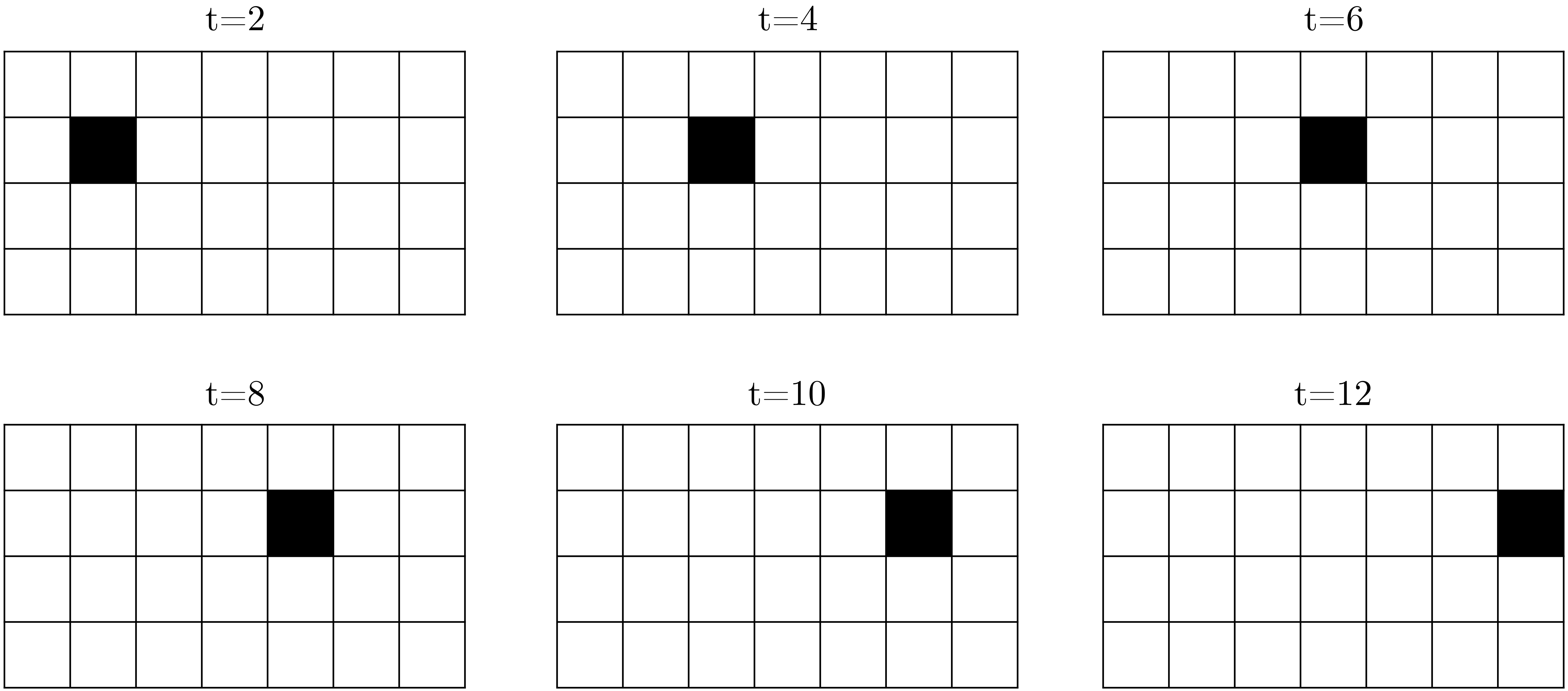
Problem: Dynamic obstacles



But is this what we want??

Solution: Don't use independence

If we directly guessed the best explanation, we can come up with this!



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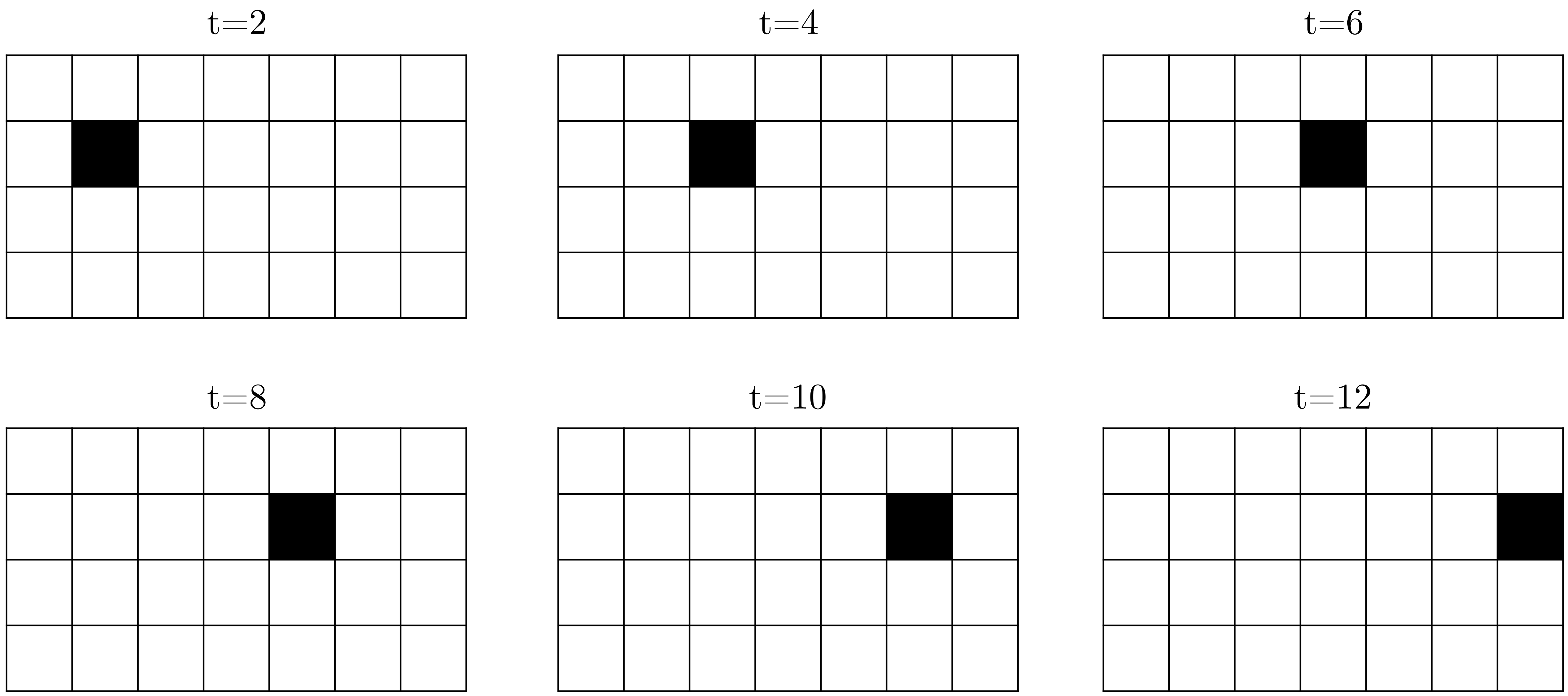

What is wrong with independence?

If I know there is ONE obstacle and if a cell gets a MISS

and neighbor gets a HIT,

then cell must be FREE (P=1).

Dynamic obstacle mapping in general

"Map building with mobile robots in dynamic environments" D. Hähnel, R. Triebel, W. Burgard, and S. Thrun. 2003

"Occupancy Grid Models for Robot Mapping in Changing Environments" D. Meyer-Delius, M. Beinhofer and W. Burgard 2003