

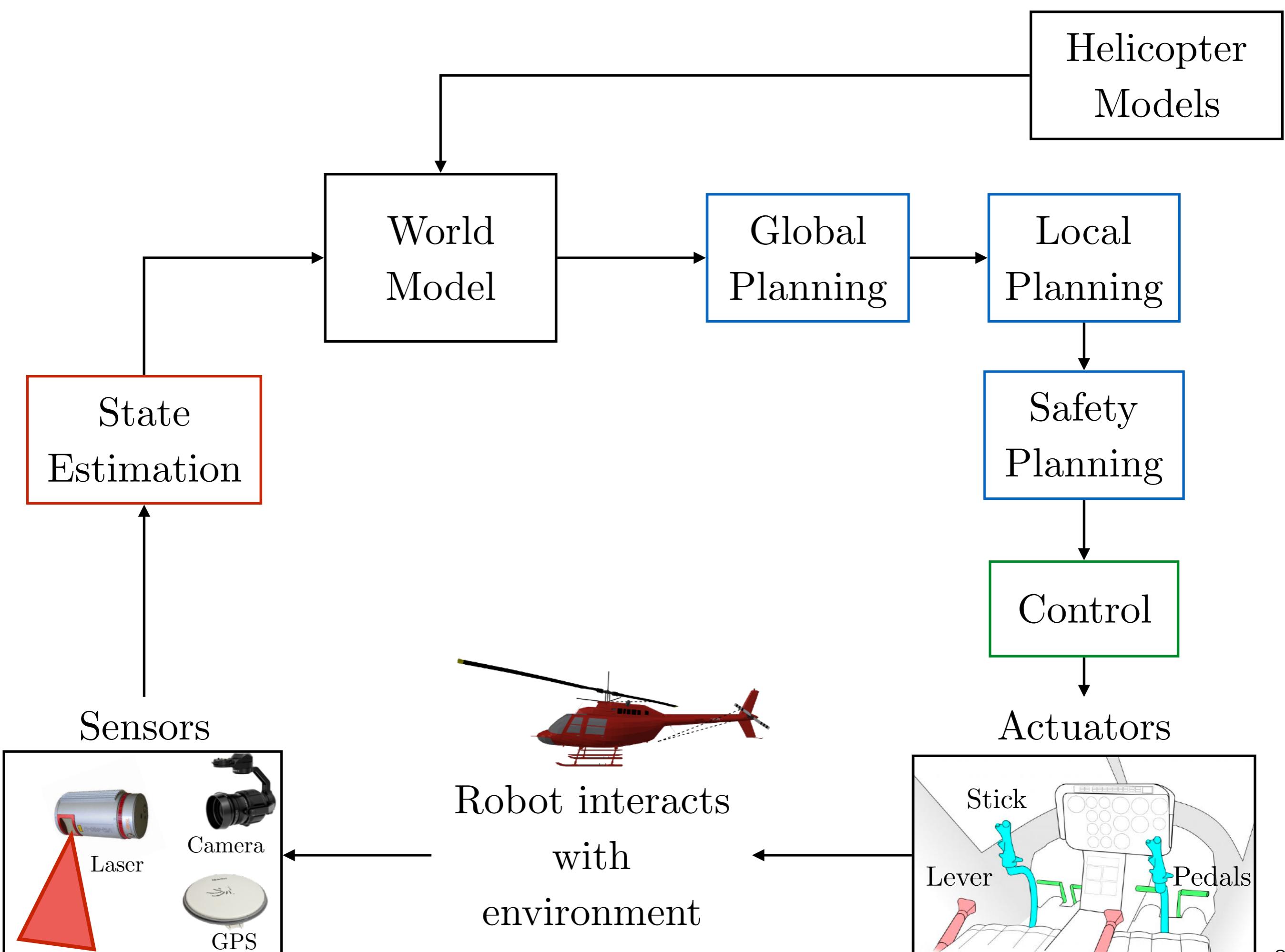
Introduction to State Estimation

Sanjiban Choudhury

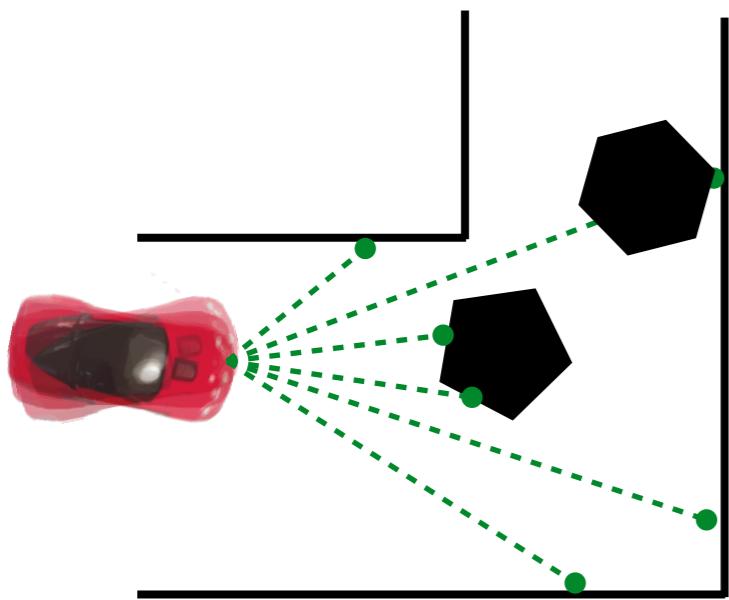
TAs: Matthew Rockett, Gilwoo Lee, Matt Schmittle

Logistics

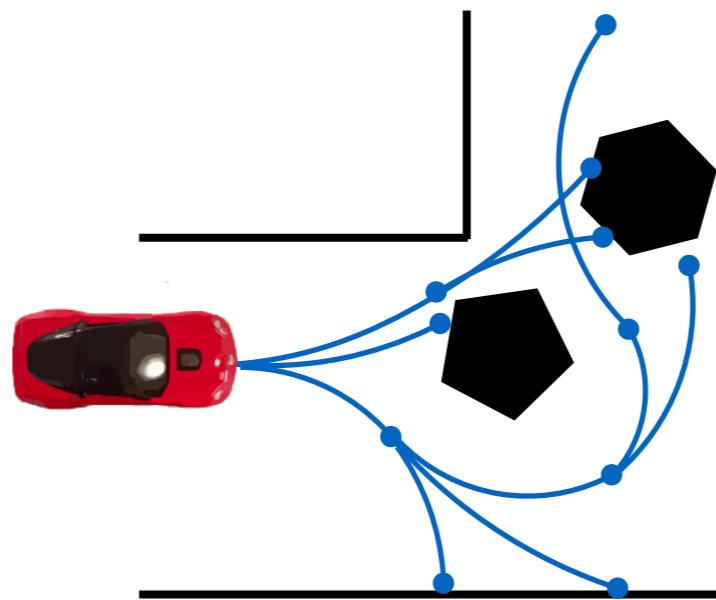
- Begin working on Assignment 0!
- Card key access - mail cardkey@cs.washington.edu with your student ID
- Post questions, discuss any issues you are having
- If you did not attend recitation yesterday, come talk to me after class.



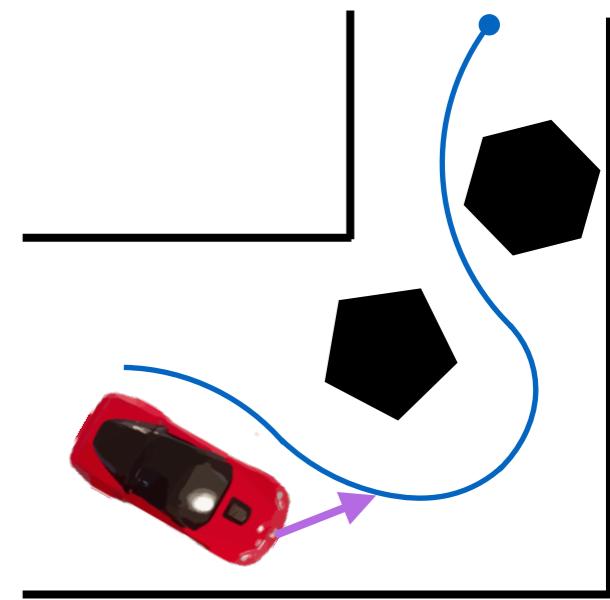
Estimate
state



Plan a
sequence of
motions



Control
robot to
follow plan



Today's objective

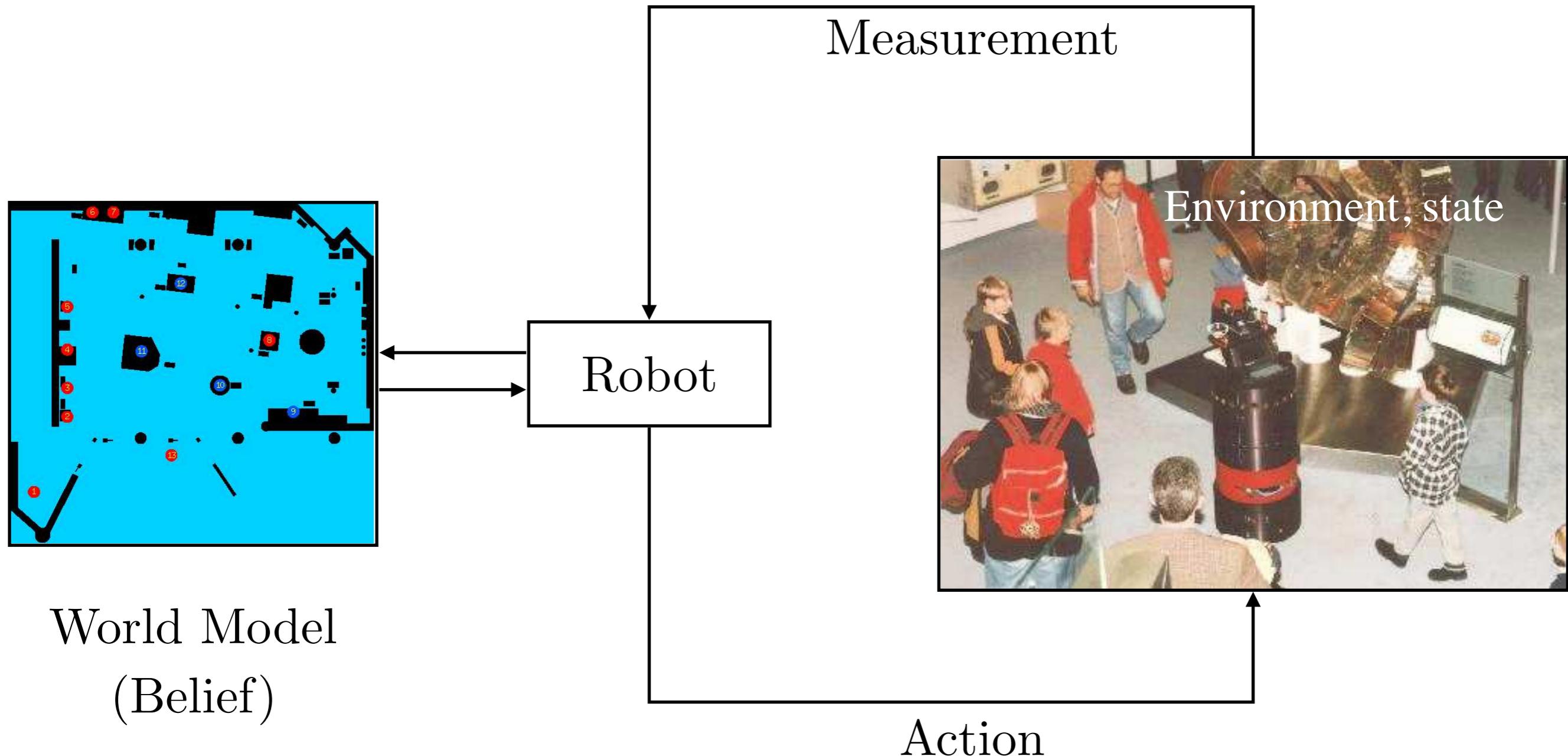
1. Formulate state estimation as a Bayes filtering problem
2. Discuss various components of a Bayes filter
3. Intuition behind Bayes filtering

A robot interacting with the world



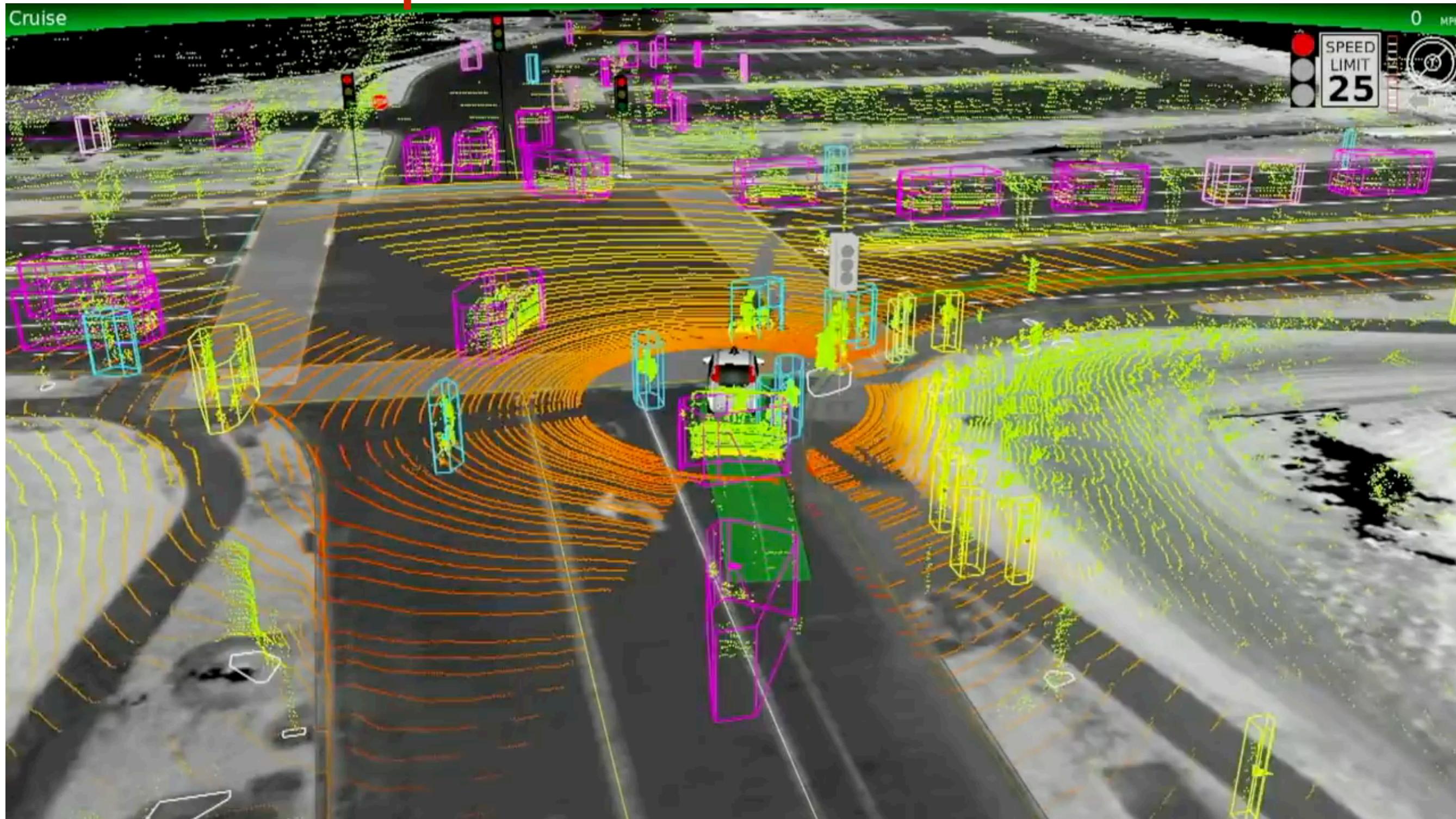
The **Minerva**
Experience

A robot interacting with the world



Bayes filtering in action!

Example 1: Rich information



Measurement:

LiDAR, camera, GPS

World Model (Belief):

Location, cars, people..

Action:

Steering, speed

Example 2: Medium information

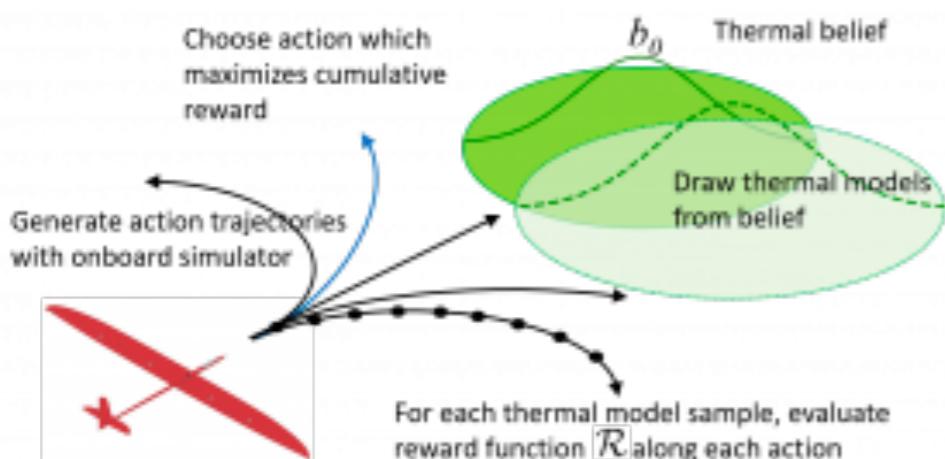


Measurement:
Planar LiDAR

World Model (Belief):
State of robot

Action:
Pitch, Roll

Example 3: Very low information

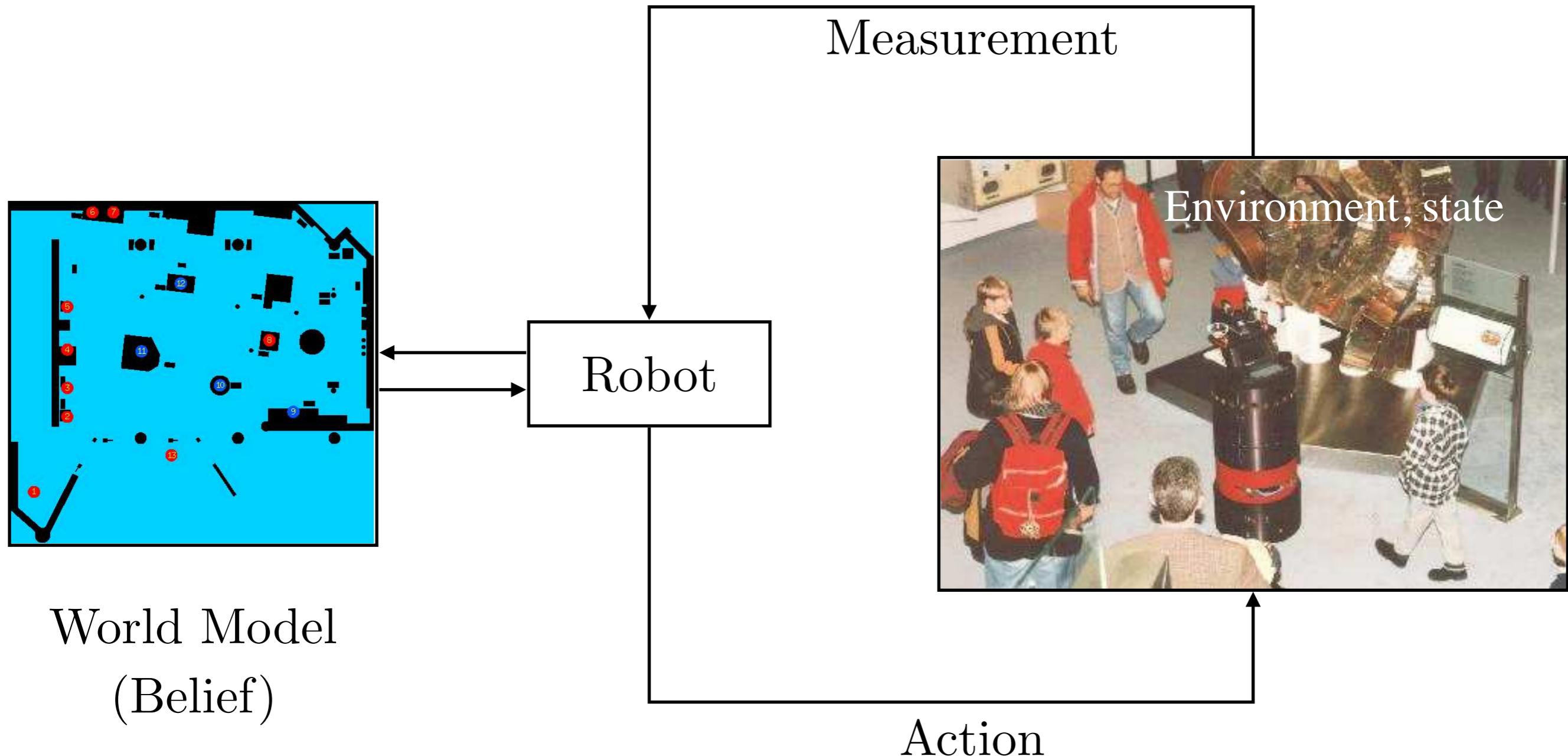


Measurement: Pitot tube (airspeed), GPS

World Model (Belief): Location of wind thermal sensor

Action: Pitch, Roll

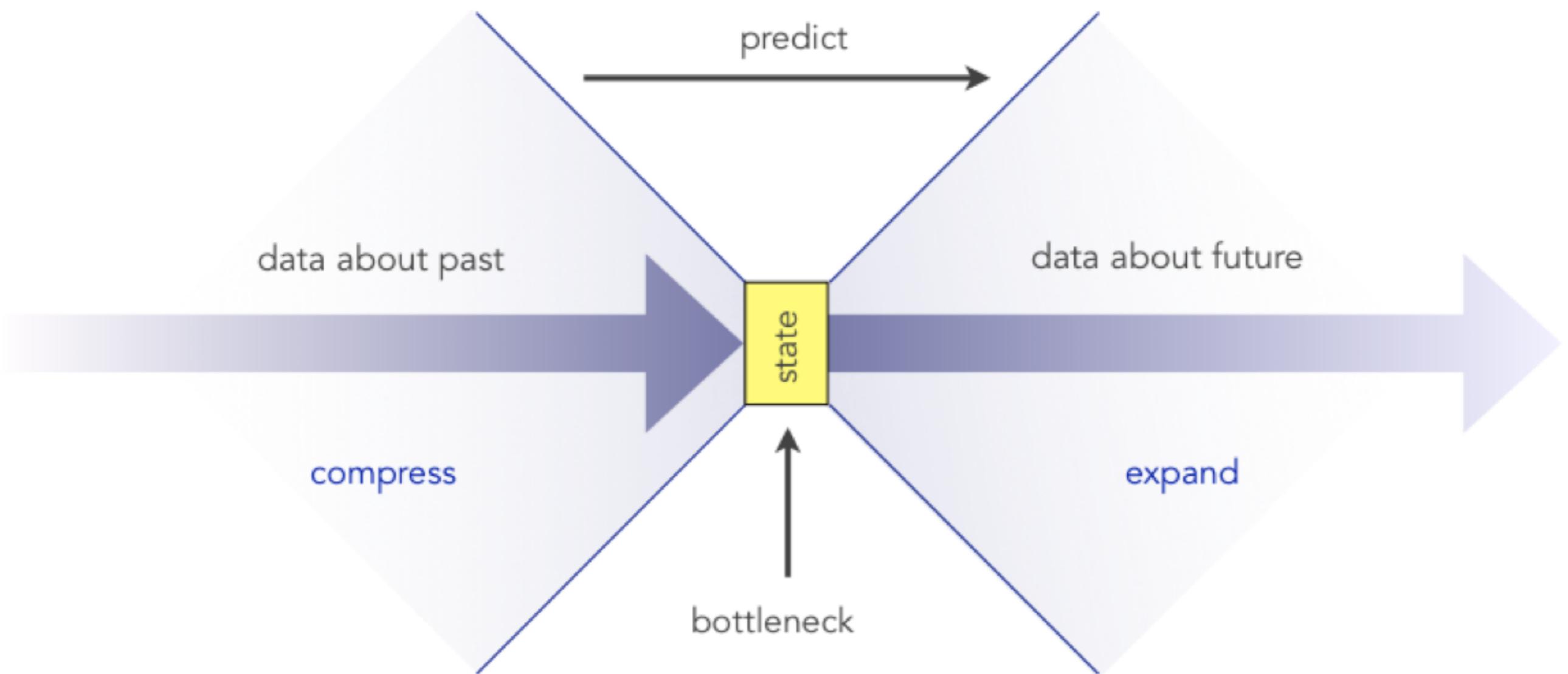
A robot interacting with the world





How do we formally define this problem?

State: A very abstract definition



State: statistic of history sufficient to predict the future

State (x_t)

Collection of variables sufficient to predict the ~~future~~
(future that we care about)

What are some examples of state?

1. Pose of a robot - Usually 6 dof (3 position, 3 for orientation)
- 3 dof for planar mobile robot (x,y,heading)
2. Configuration of a manipulator - Collection of joint angles
3. Location of objects in environment

State can be static/dynamic, discrete/continuous/hybrid

Measurement (z_t)

Measurements are sensor values that provide
information about the current state.

(Measurement does not always tell you state directly!!)

What are some examples of measurements?

1. GPS - absolute information about robot pose
2. Laser scan - relative geometric information between pose and environment
3. Camera image - information about color / texture
(harder to model)

Action (u_t)

Actions affect how a state changes from one time to another

What are some examples of actions?

1. Active forces applied by the robot - (measure motor currents, force torque sensors, odometers)
2. Passive actions that change environment - weather (can detect with sensors)
3. NOP actions - doing nothing is also an action. State does not change.

Fundamental problem: State is hidden

All the robot sees is a stream of actions and measurements

$$u_1, z_1, u_2, z_2, u_3, z_3, \dots$$

But robot never sees the state

$$x_1, x_2, x_3, \dots$$

Fundamental problem: State is hidden

But all decision making depends on knowing state

Solution: Estimate belief over state

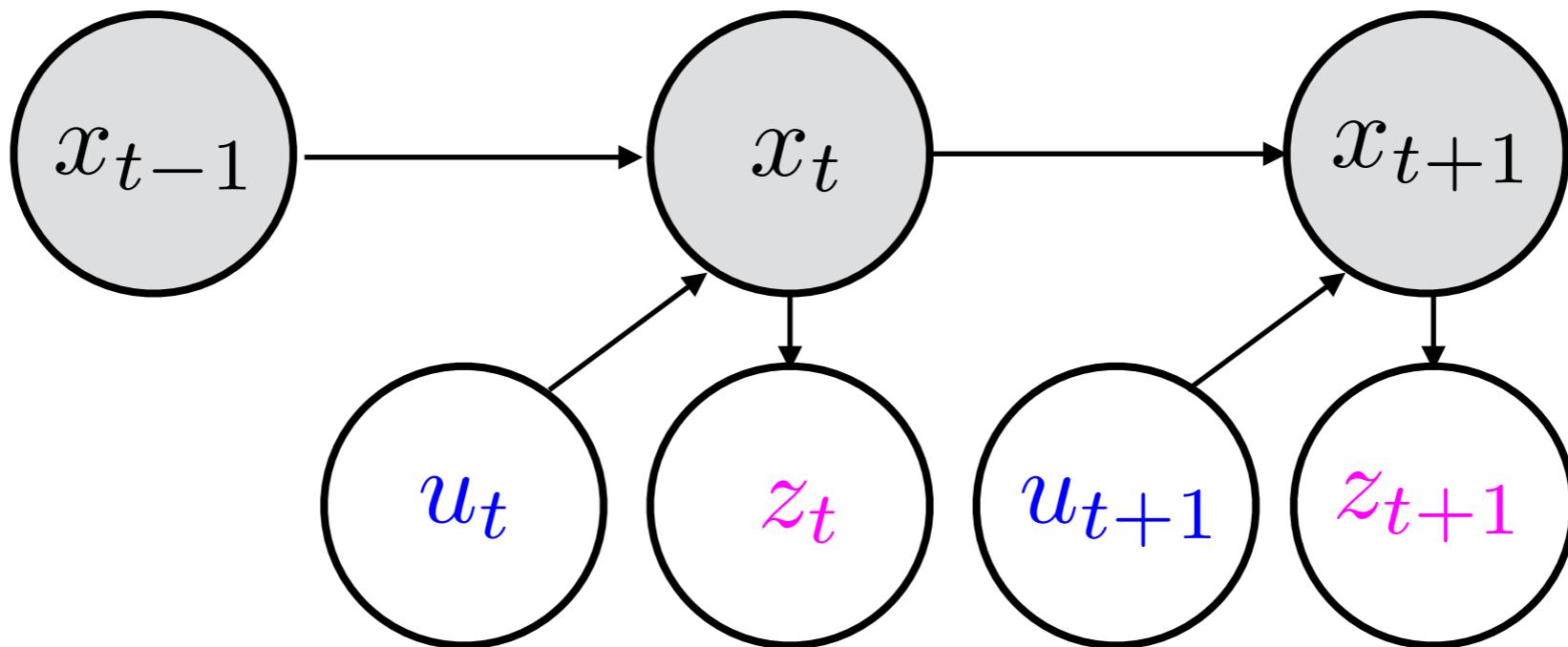
$$bel(x_t) = P(x_t | z_{1:t}, u_{1:t})$$

Belief is a probability of each possible state given history

Also called Posterior / Information state / State of knowledge

Represent belief? Parametric (Gaussian), Non-parametric (Histogram)

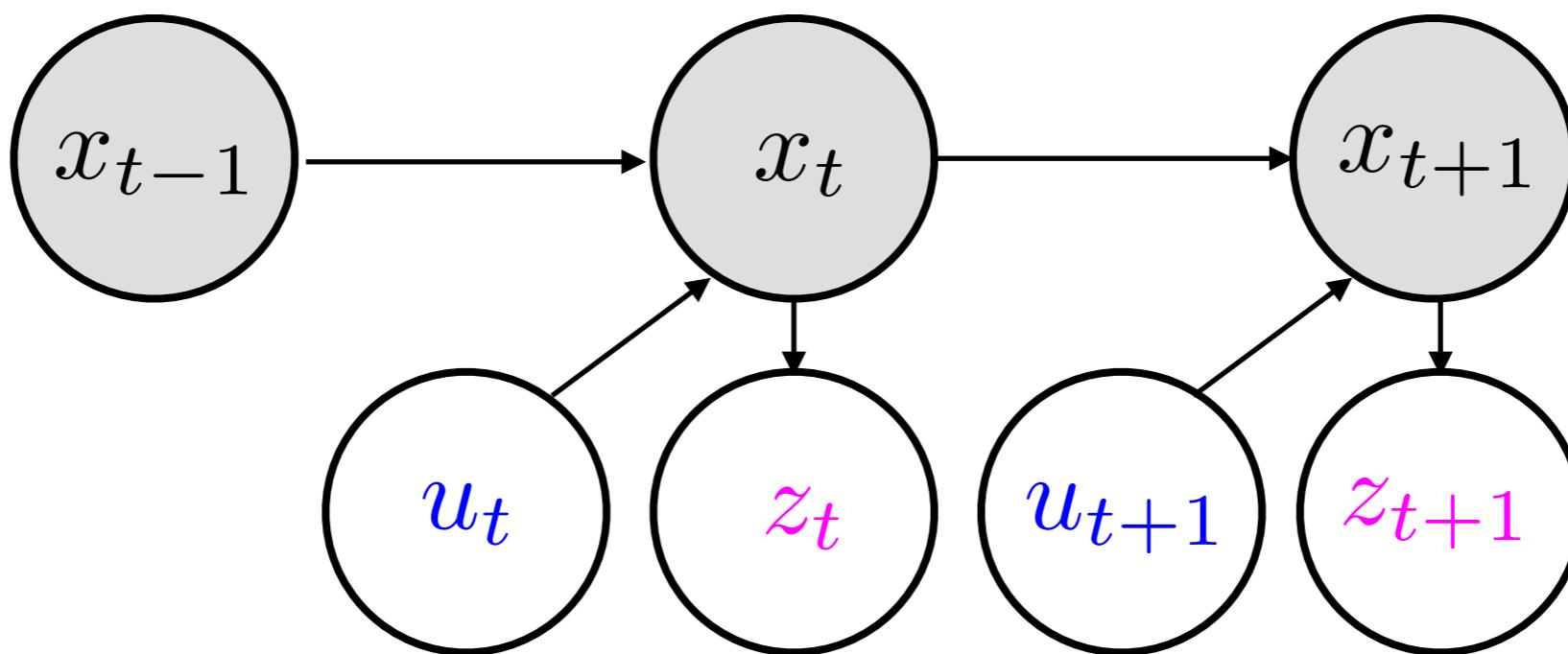
Let's think about causality of events



Assumptions:

1. Robot receives a stream of measurements / actions.
2. One measurement / action per time-step.

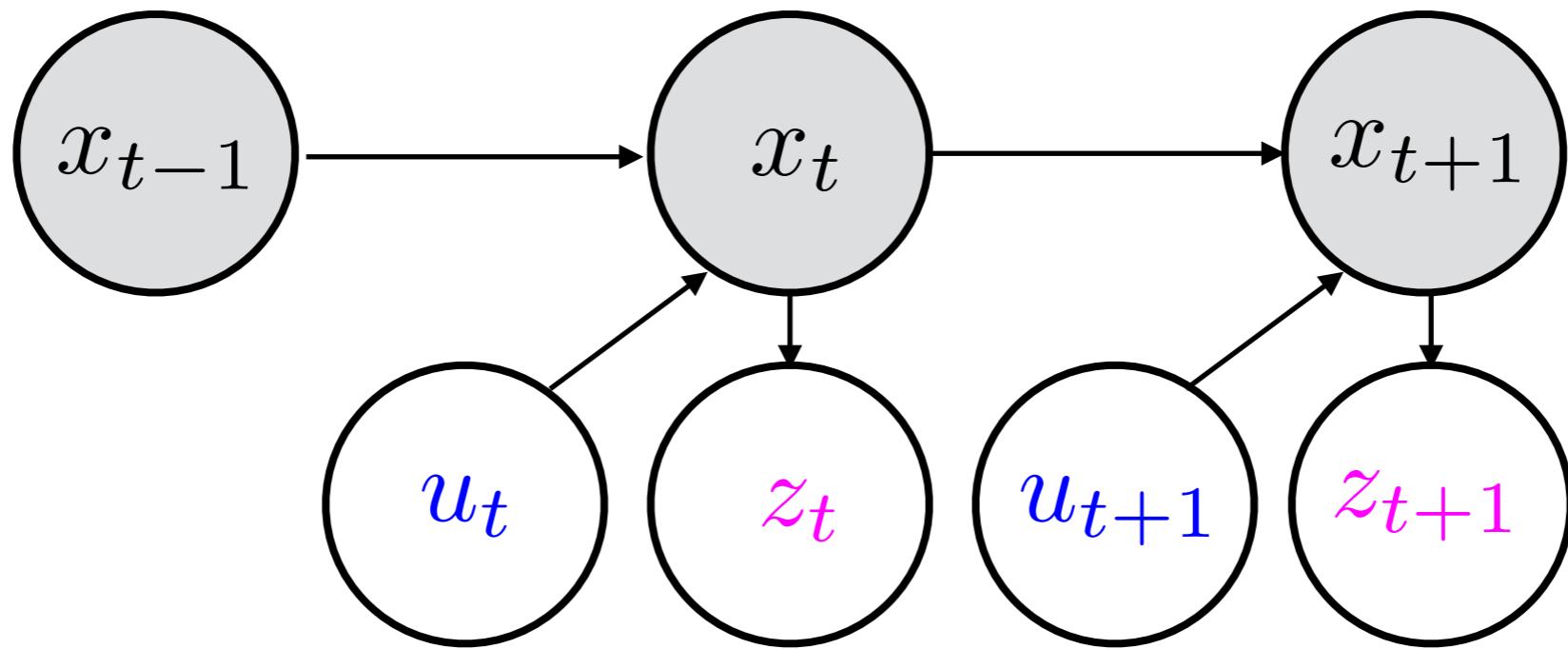
Problem: How do we estimate belief tractably?



$P(\text{ current state} \mid \text{all past information})$

$$P(x_t \mid z_t, u_t, x_{t-1}, \dots)$$

Solution: Markov assumption



Markov assumption : Conditional independence given state

$$P(x_t | \textcolor{blue}{u_t}, x_{t-1}, z_{t-1}, u_{t-1}, \dots) = P(x_t | \textcolor{blue}{u_t}, x_{t-1})$$

$$P(\textcolor{magenta}{z_t} | x_t, u_t, x_{t-1}, z_{t-1}, u_{t-1}, \dots) = P(\textcolor{magenta}{z_t} | x_t)$$

Probabilistic models

State transition probability / dynamics / motion model

$$P(x_t | x_{t-1}, u_t)$$

Measurement probability / Observation model

$$P(z_t | x_t)$$

When does Markov not hold?

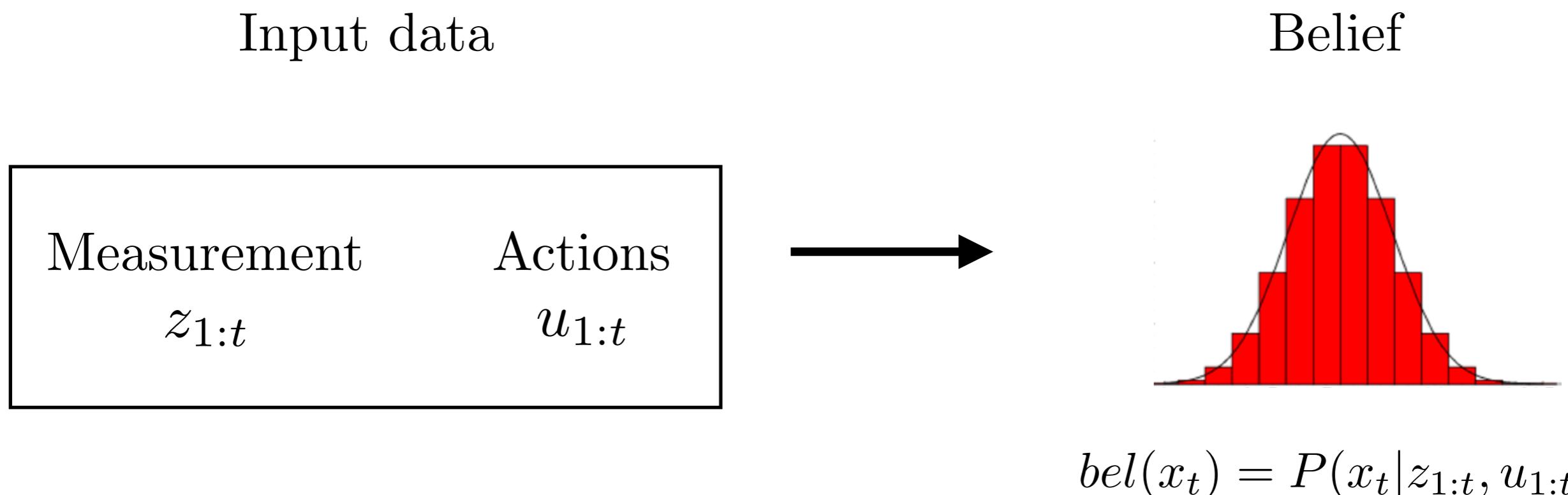
$$P(x_t | x_{t-1}, u_t) \quad P(\textcolor{magenta}{z}_t | x_t)$$

whenever state doesn't capture all requisite information

- Camera images at different times of the day
- Unmodelled pedestrians in front of laser
- Steady gusts of wind

Central Question:

How do we tractably calculate belief?



Ans: Bayes filter!

Bayes filter in a nutshell

Key Idea: Apply Markov to get a **recursive** update!

Bayes filter in a nutshell

Step 0. Start with the belief at time step t

$$bel(x_{t-1})$$

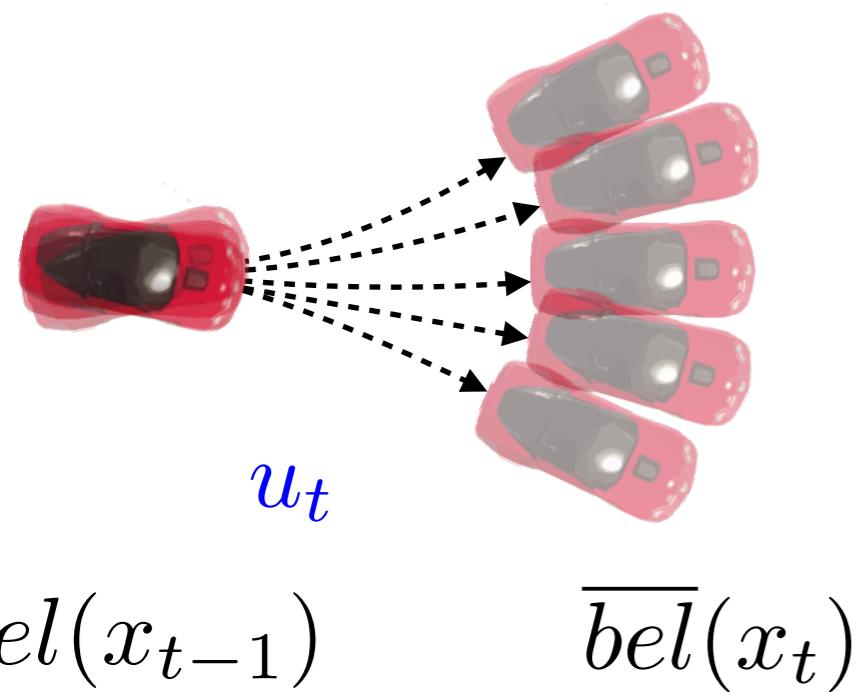


$$bel(x_{t-1})$$

Bayes filter in a nutshell

Step 1: Prediction - push belief through dynamics given action

$$\overline{bel}(x_t) = \int P(x_t | \textcolor{blue}{u}_t, x_{t-1}) bel(x_{t-1}) dx_{t-1}$$

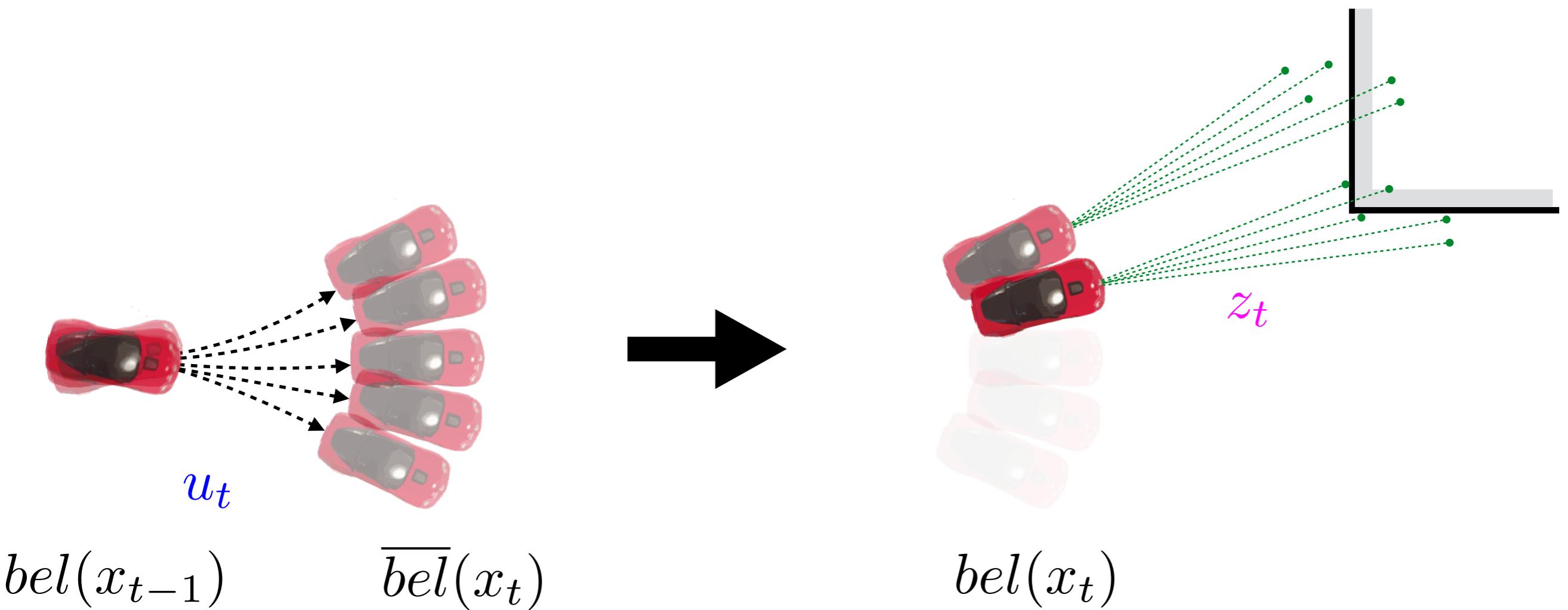


$$bel(x_{t-1}) \quad \overline{bel}(x_t)$$

Bayes filter in a nutshell

Step 2: Correction - apply Bayes rule given measurement

$$bel(x_t) = \eta P(z_t|x_t) \overline{bel}(x_t)$$



Bayes filter in a nutshell

Key Idea: Apply Markov to get a recursive update!

Step 0. Start with the belief at time step t

$$bel(x_{t-1})$$

Step 1: Prediction - push belief through dynamics given **action**

$$\overline{bel}(x_t) = \int P(x_t | \textcolor{blue}{u}_t, x_{t-1}) bel(x_{t-1}) dx_{t-1}$$

Step 2: Correction - apply Bayes rule given **measurement**

$$bel(x_t) = \eta P(\textcolor{violet}{z}_t | x_t) \overline{bel}(x_t)$$

Bayes filter is a powerful tool



Localization



Mapping



SLAM



POMDP