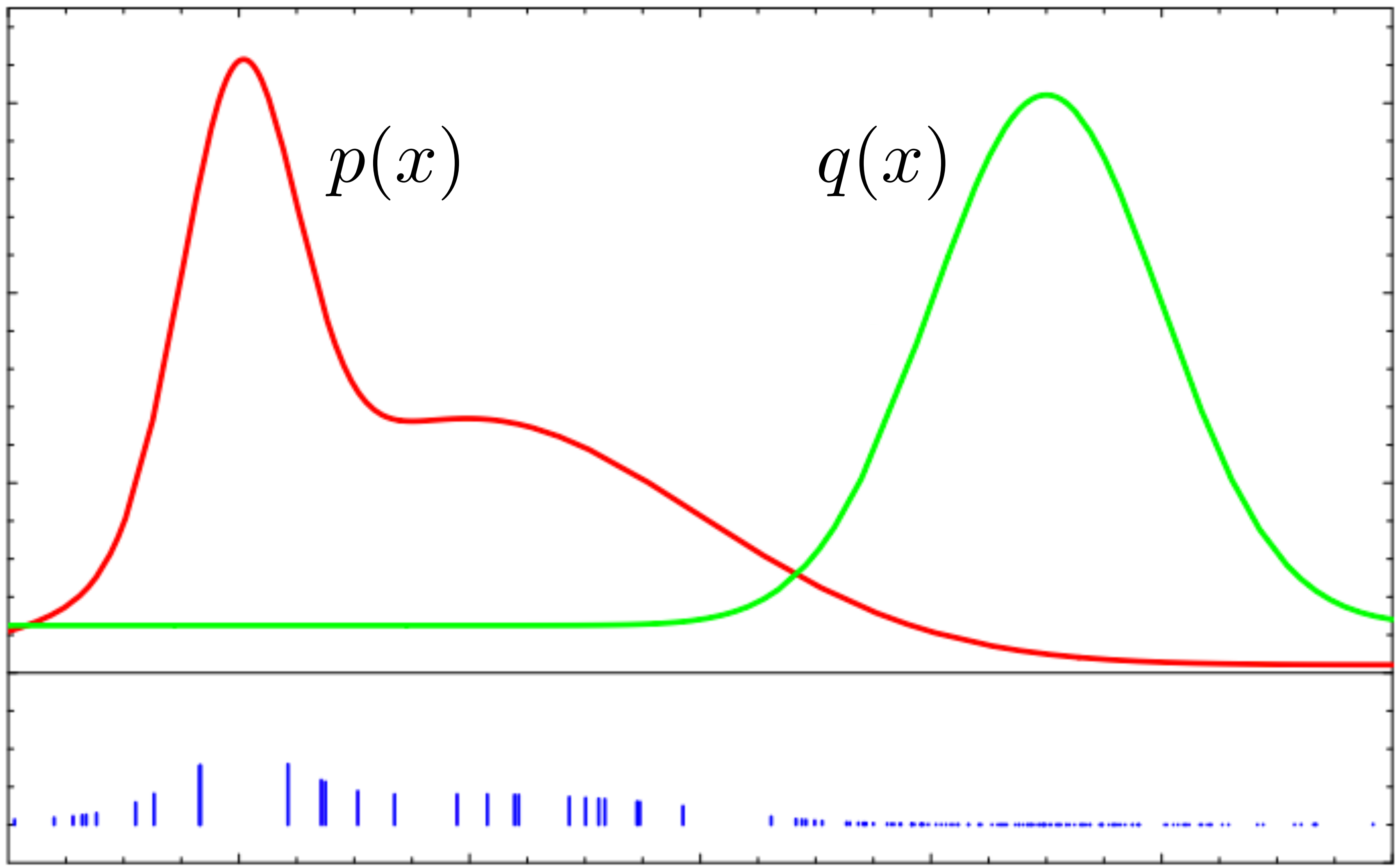
(Slides adapted from Drew Bagnell, Dieter Fox)

Particle Filtering

Sanjiban Choudhury

TAs: Matthew Rockett, Gilwoo Lee, Matt Schmittle

Question: What makes a good proposal distribution?



Applying importance sampling to Bayes filtering

Target distribution : Posterior

$$bel(x_t) = \eta P(z_t|x_t) \int p(x_t|u_t, x_{t-1}) bel(x_{t-1}) dx_{t-1}$$

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Proposal distribution : After applying motion model

$$\overline{bel}(x_t) = \int p(x_t|u_t, x_{t-1}) bel(x_{t-1}) dx_{t-1}$$

Why is this easy to sample from?

Applying importance sampling to Bayes filtering

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Why is this easy to sample from?

Importance Ratio:

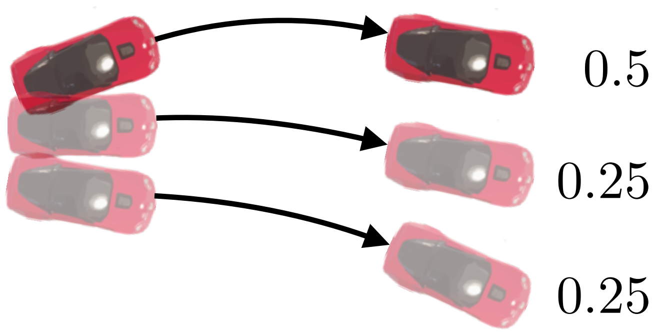
$$r = \frac{bel(x_t)}{\overline{bel}(x_t)} = \eta P(z_t|x_t)$$

Question: What are our options for non-parametric belief representations?

1. Histogram filter
2. Normalized importance sampling
3. Particle filter

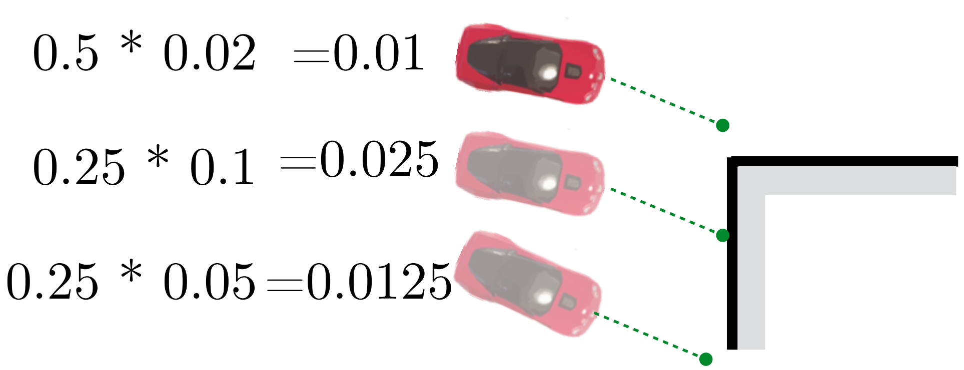
Approach 2: Normalized Importance Sampling

$$bel(x_{t-1}) = \left\{ \begin{matrix} x_{t-1}^1 \\ w_{t-1}^1 \end{matrix}, \begin{matrix} x_{t-1}^2 \\ w_{t-1}^2 \end{matrix}, \cdots, \begin{matrix} x_{t-1}^M \\ w_{t-1}^M \end{matrix} \right\}$$



for $i = 1$ to M

sample $\overline{x}_t^i \sim P(x_t | {\color{blue}u_t}, x_t^i)$



for $i = 1$ to M

$$w_t^i = P(z_t | \bar{x}_t^i) w_{t-1}^i$$

for $i = 1$ to M

$$w_t^i = \frac{w_t^i}{\sum_i w_t^i} \qquad bel(x_t) = \left\{ \begin{matrix} \bar{x}_t^1 \\ w_t^1 \end{matrix}, \cdots, \begin{matrix} \bar{x}_t^M \\ w_t^M \end{matrix} \right\}$$

Problem: What happens after enough iterations?

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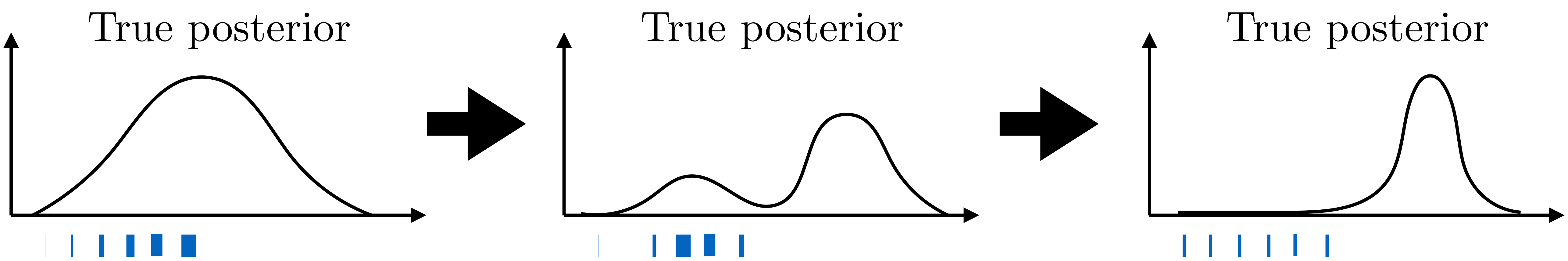
Particles don't move - can get stuck in regions of low probability



This is the same complaint we had about histogram filters!

Key Idea: Resample!

Why? Get rid of bad particles

Approach 3: Particle Filtering

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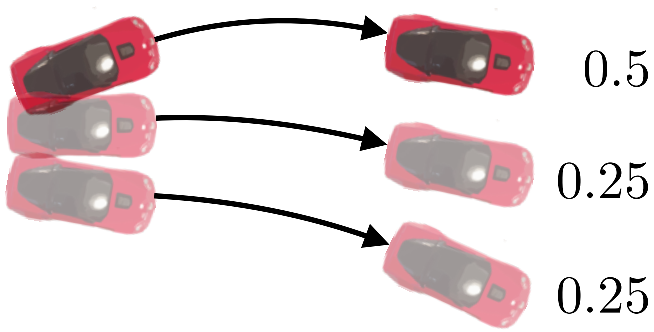

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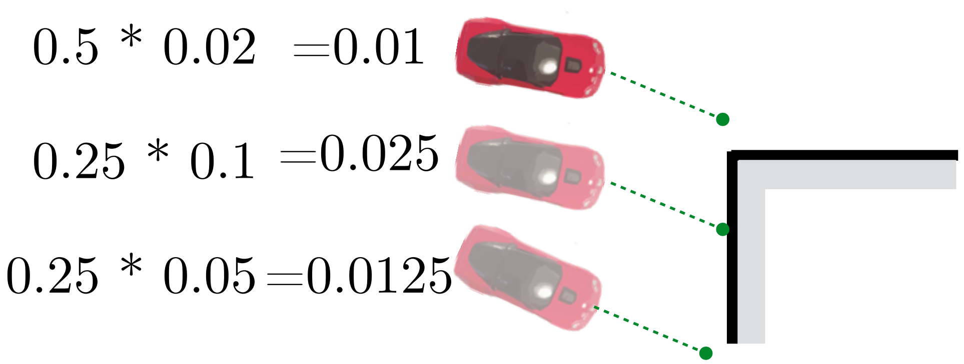


for $i = 1$ to M

$$w_t^i = P(z_t | \bar{x}_t^i) w_{t-1}^i$$

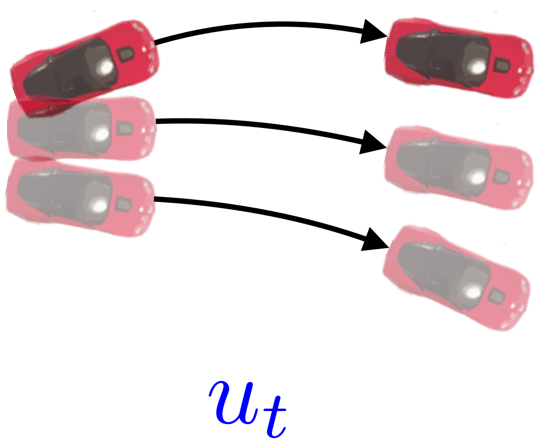
all weights = 1/M

for $i = 1$ to M

sample $x_t^i \sim w_t^i$

$$bel(x_t) = \left\{ \begin{matrix} x_t^1 \\ 1 \end{matrix}, \cdots, \begin{matrix} x_t^M \\ 1 \end{matrix} \right\}$$

Virtues of resampling



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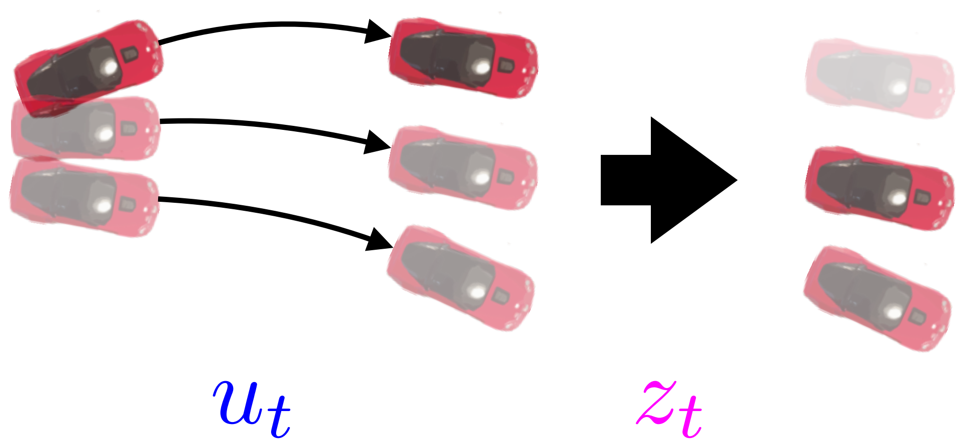

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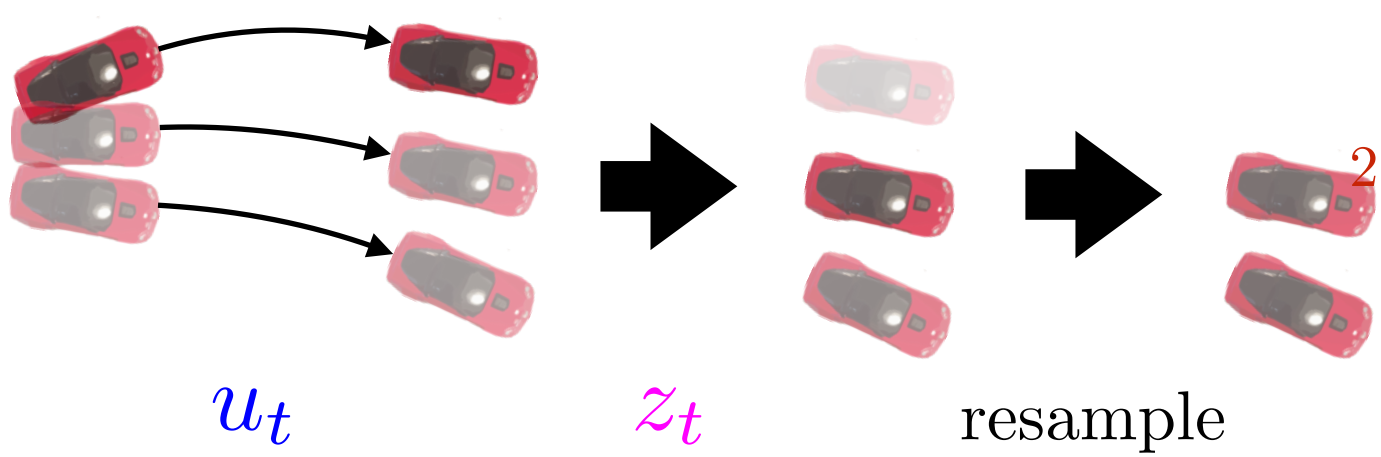

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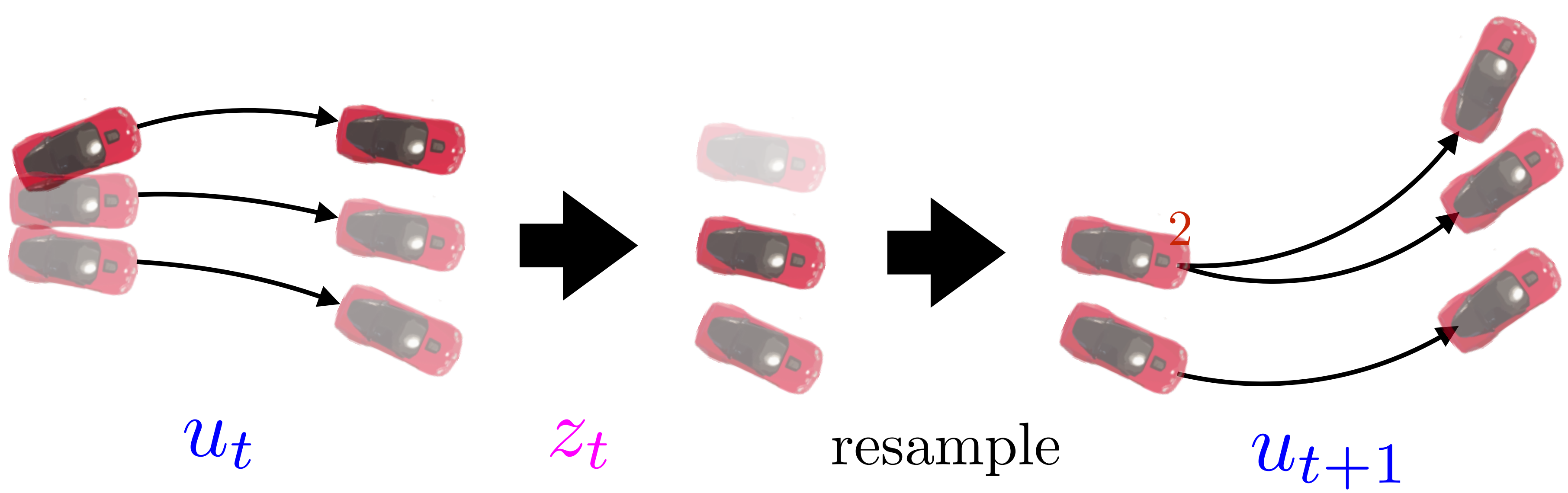

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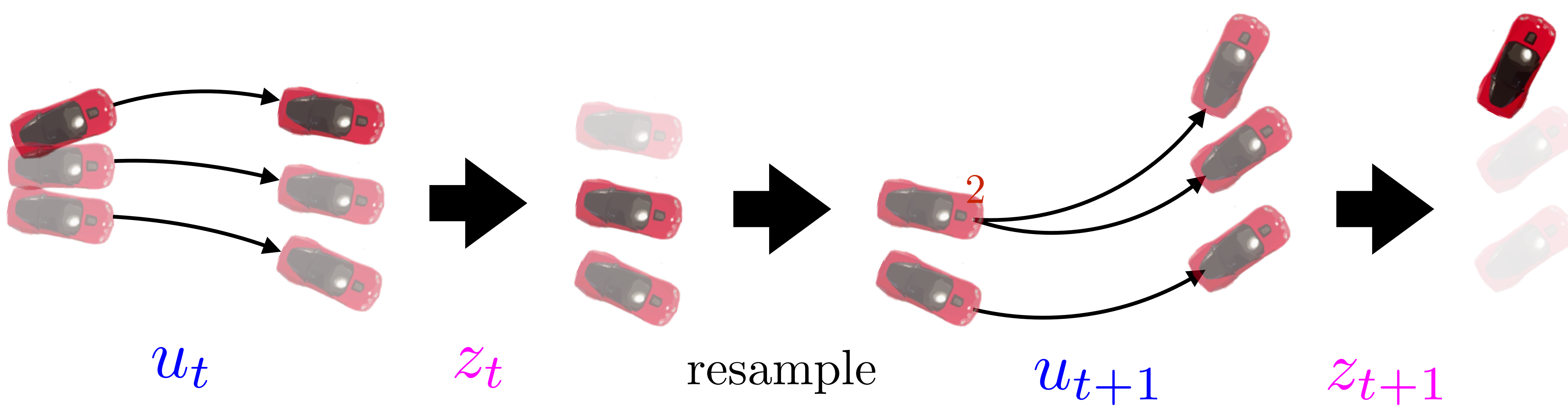

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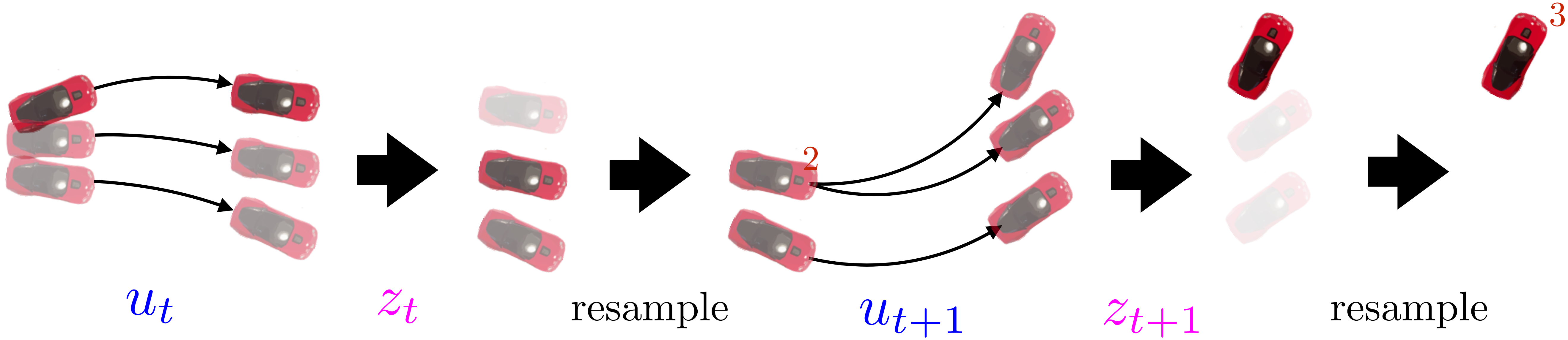
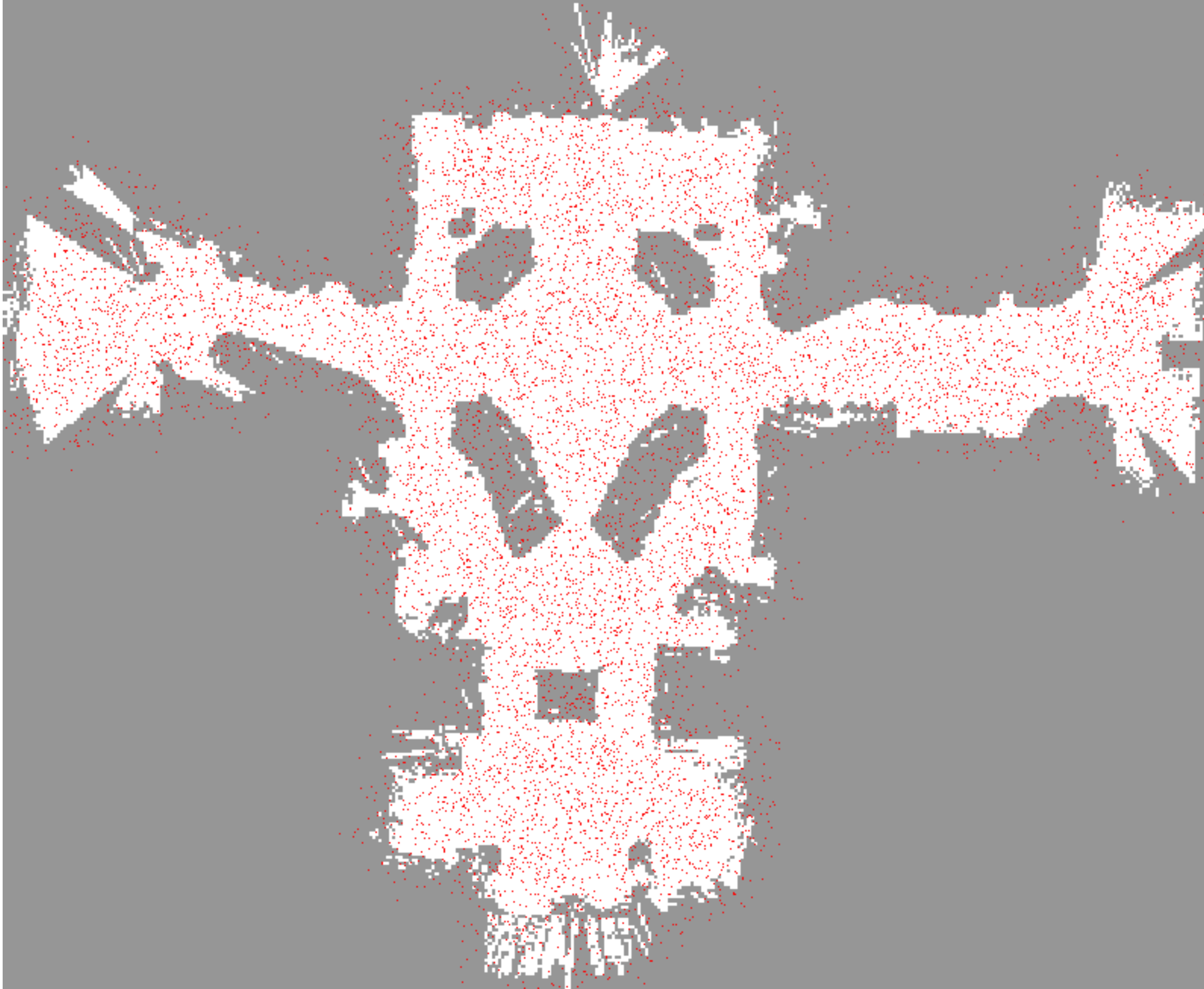
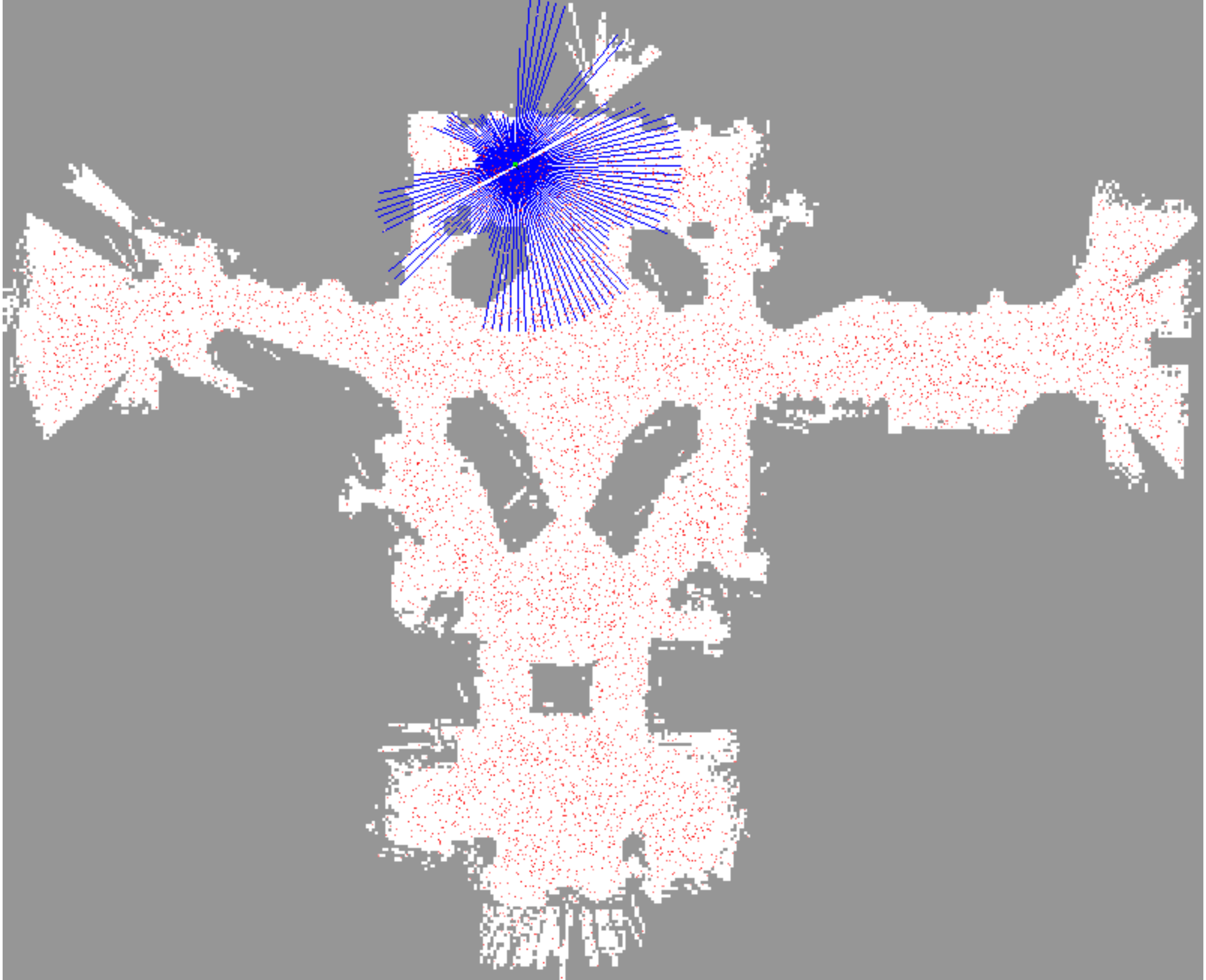
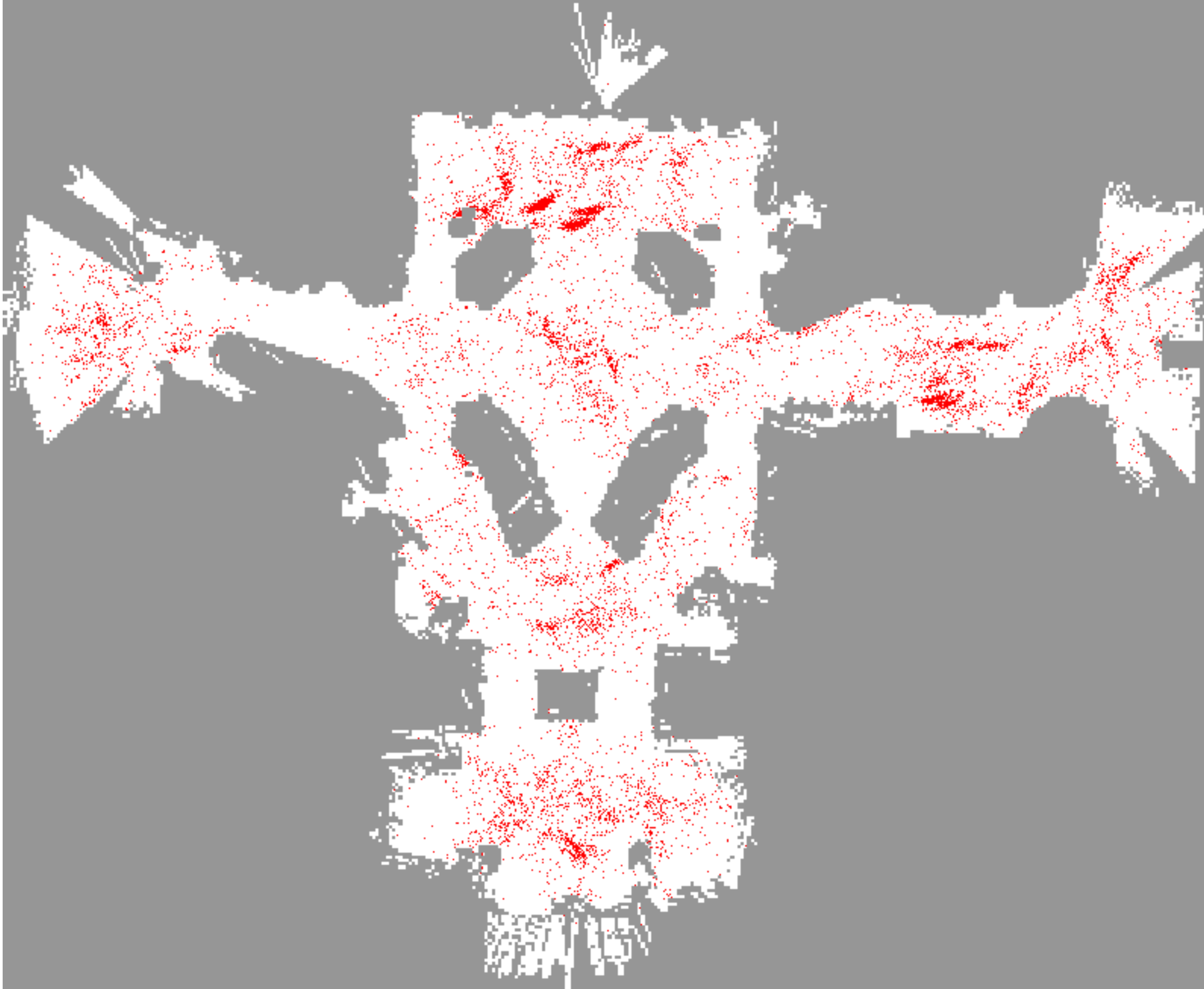


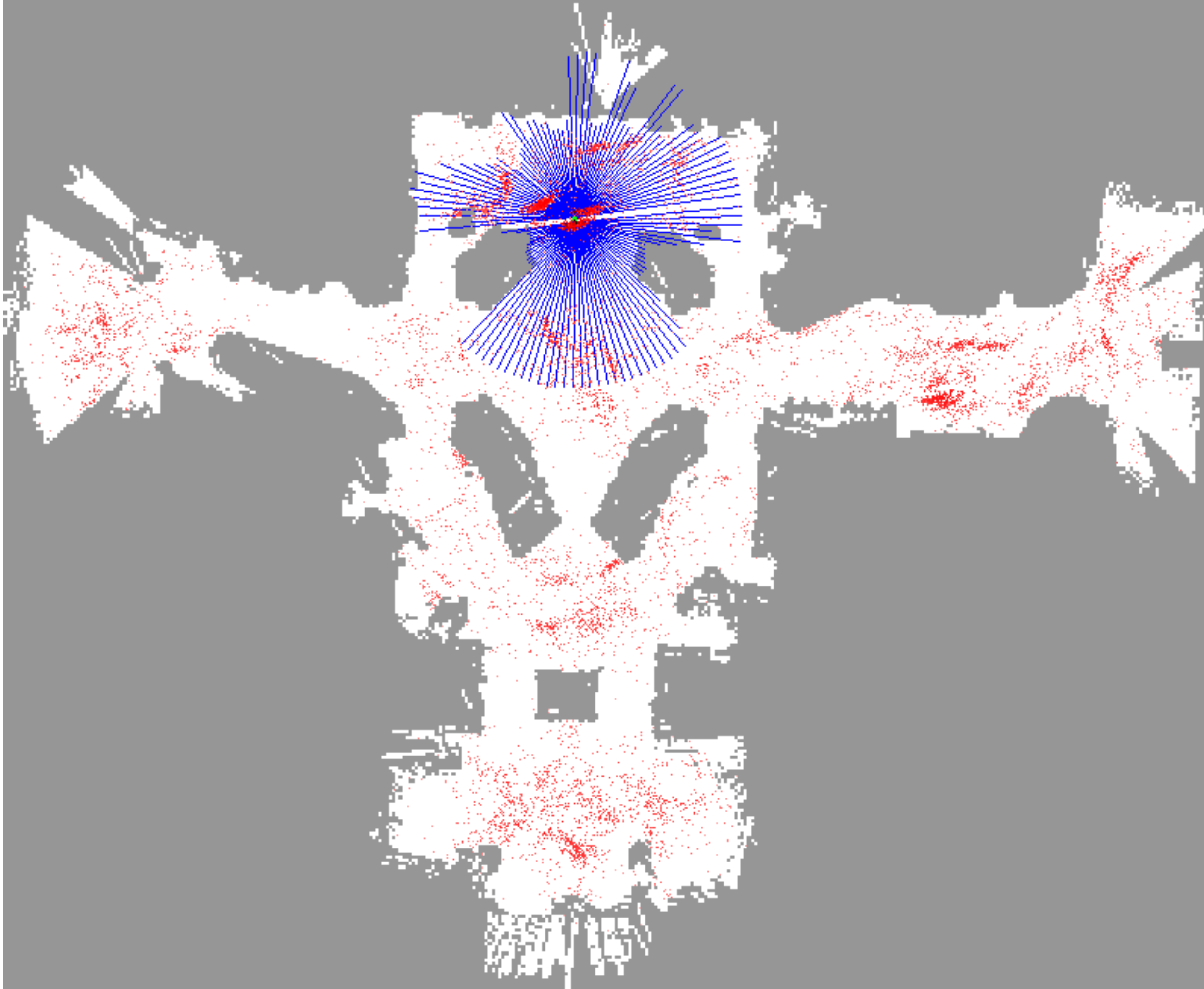
Why use particle filters?

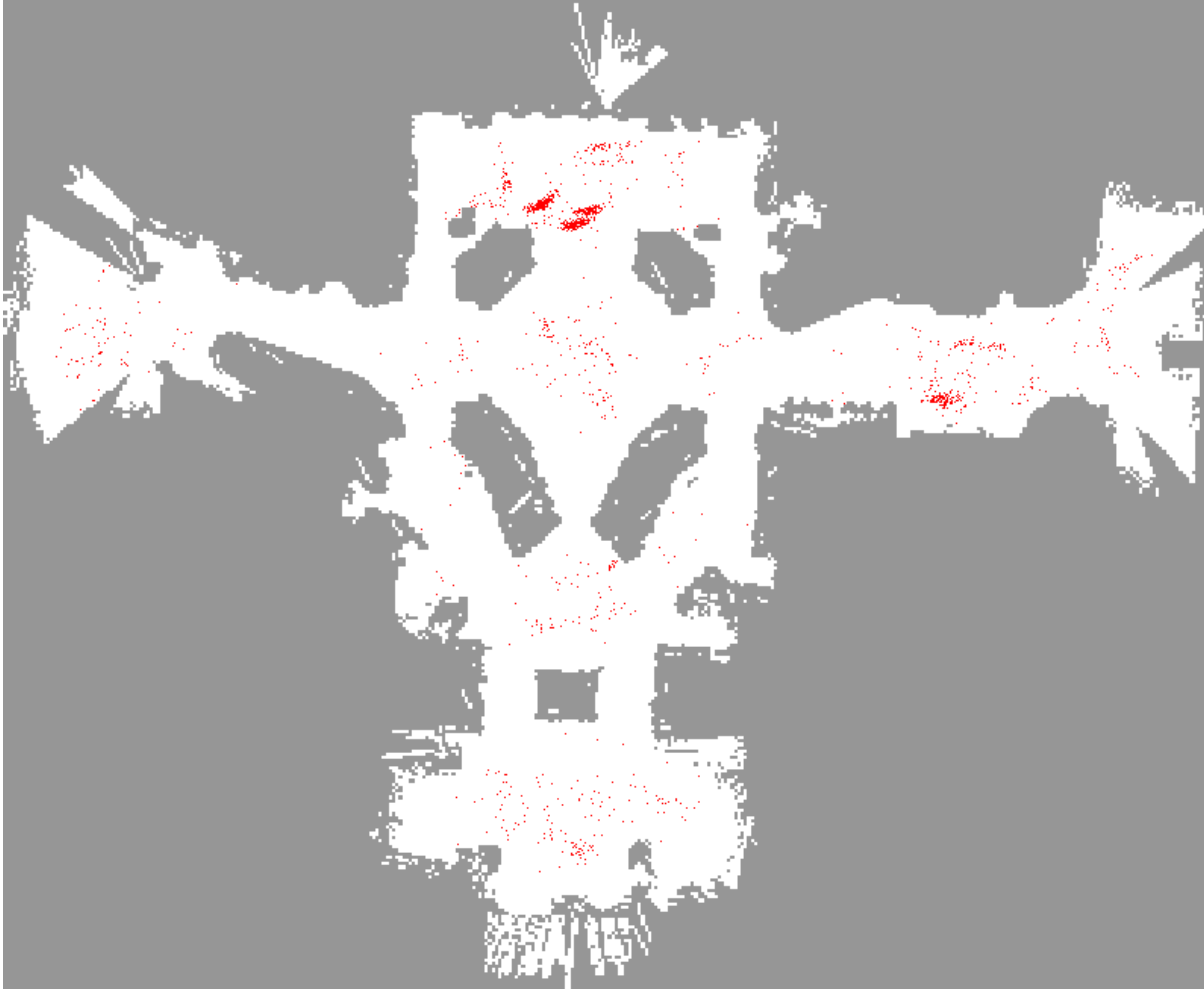
1. Can answer any query
2. Will work for any distribution, including multi-modal (unlike Kalman filter)
3. Scale well in computational resources (embarassingly parallel)
4. Easy to implement!

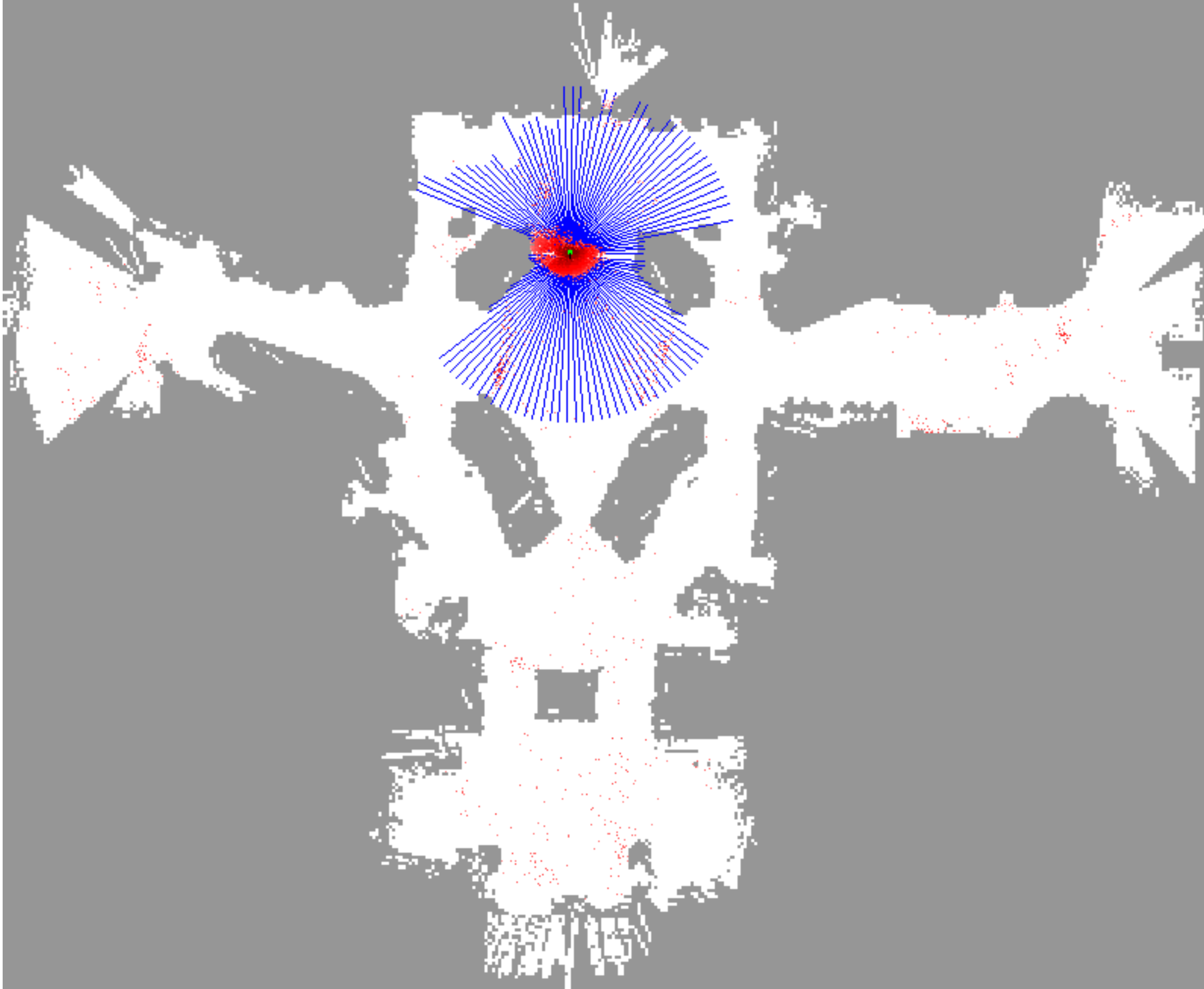


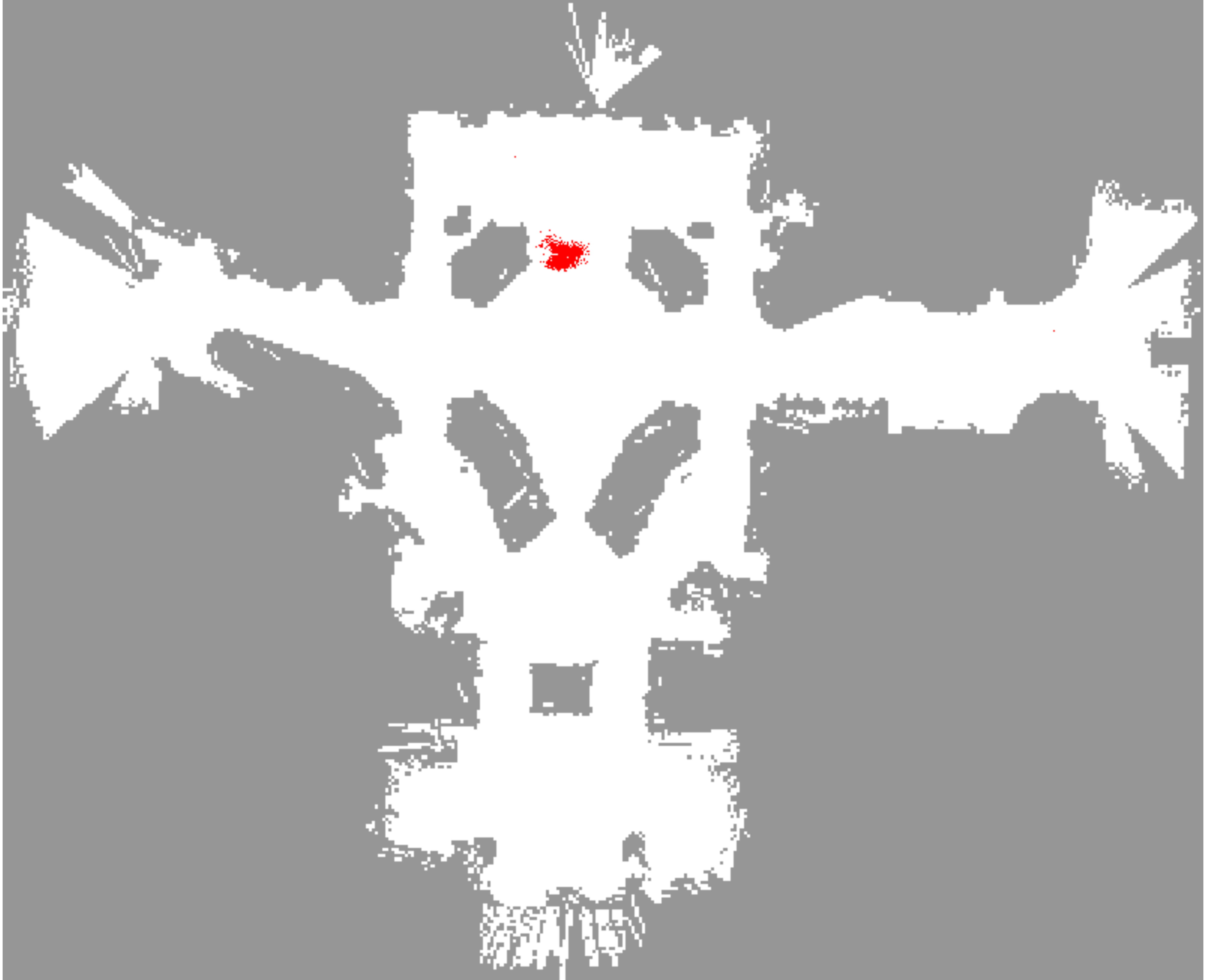












Non-parametric Filters

Grid up state space

Histogram Filter

Use a fixed set of samples

Normalized Importance Sampling

Resample

Particle Filter

Same fundamental Bayes rule again and again ...

Are we done?

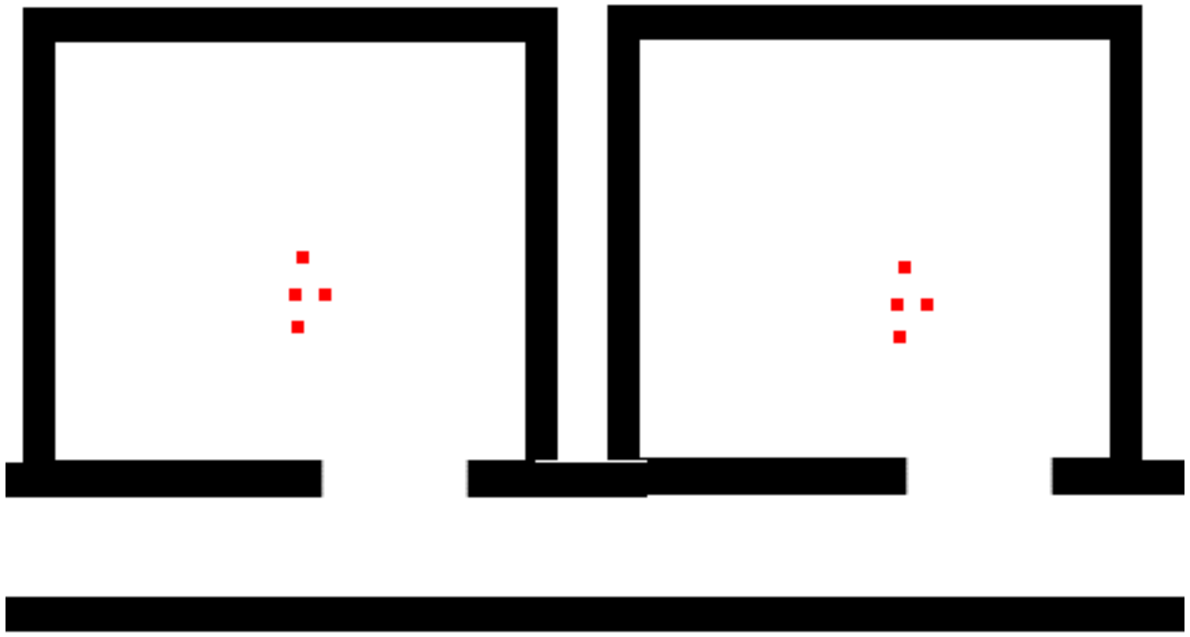
No!

Lots of practical problems to deal with

Problem 1: Two room challenge

Given: Particles equally distributed, no motion, no observation

What happens?



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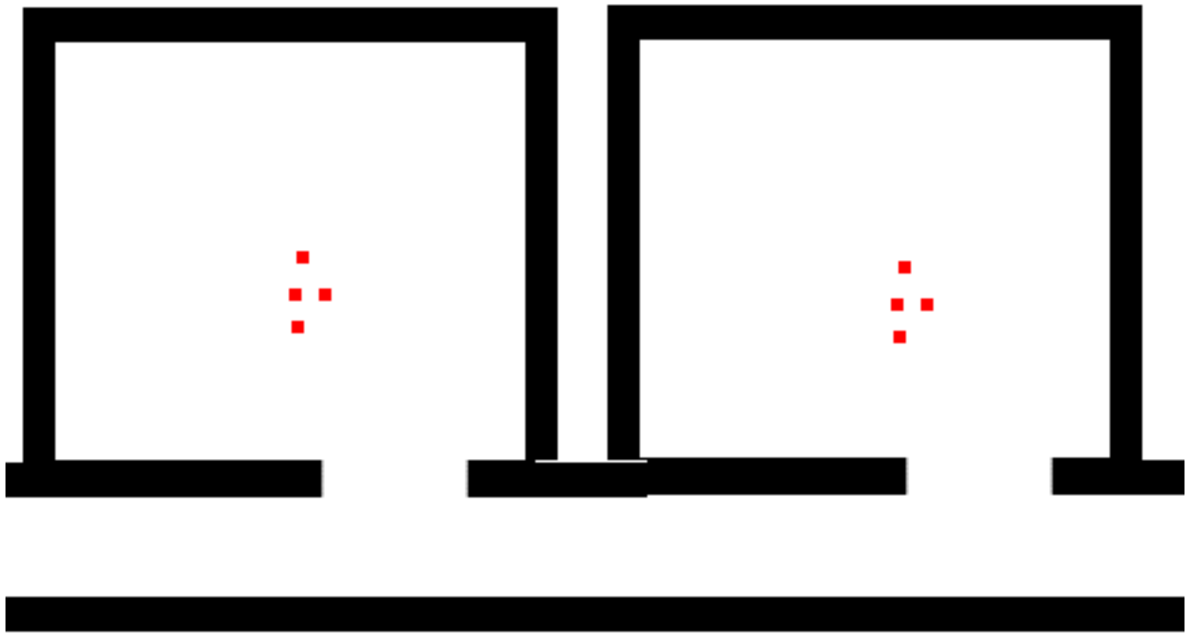
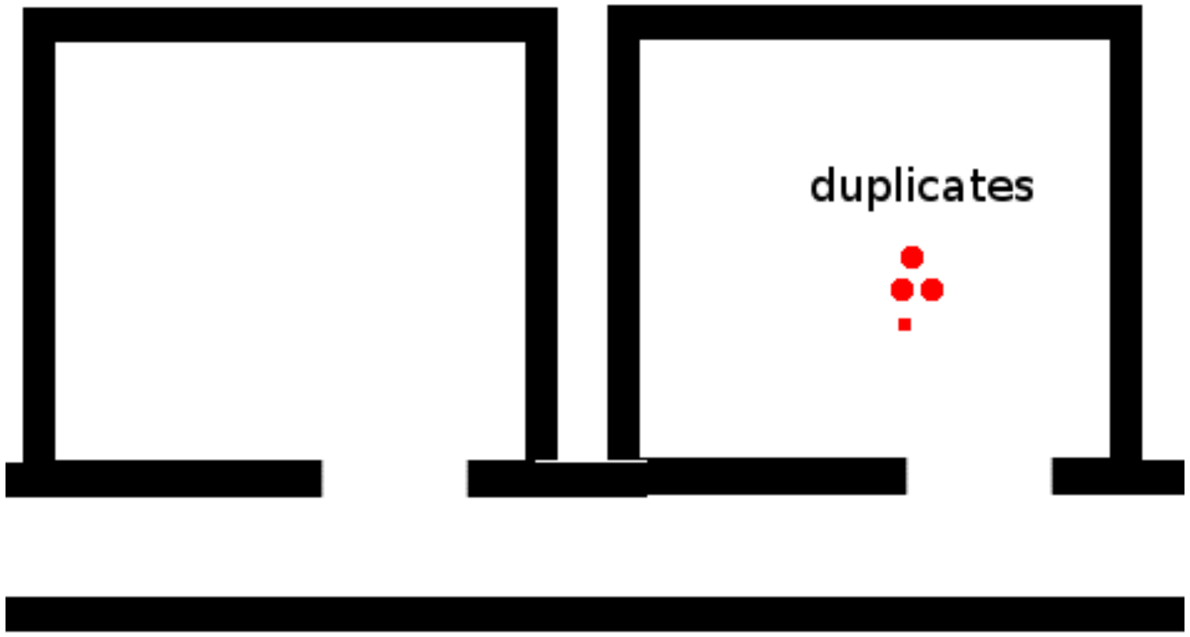
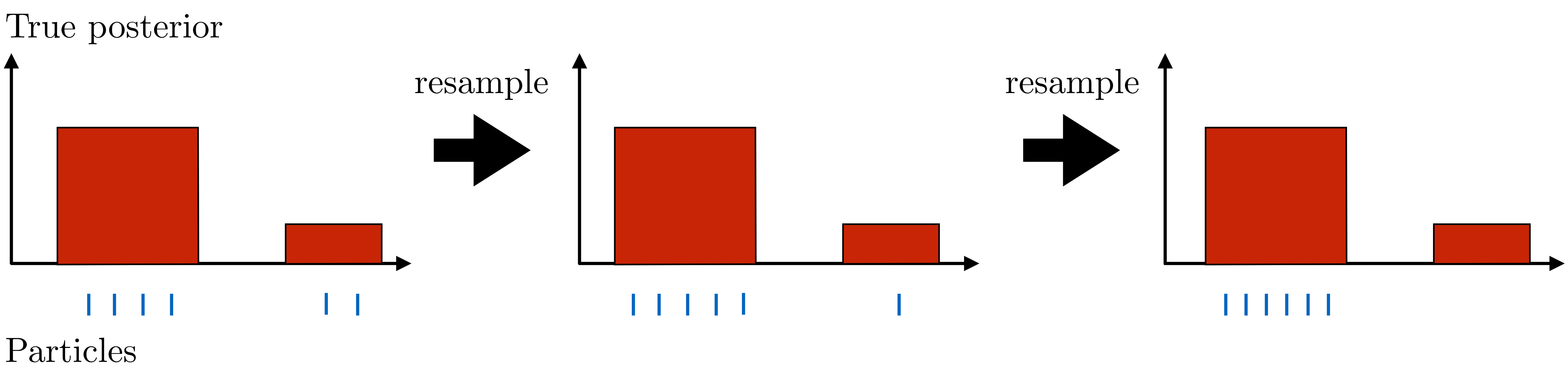




All particles migrate to the other room!!

Reason: Resampling increases variance



Resampling collapses particles, reduces diversity, increases variance w.r.t true posterior

Fix 1: Choose when to resample

Key idea: If variance of weights low, don't resample

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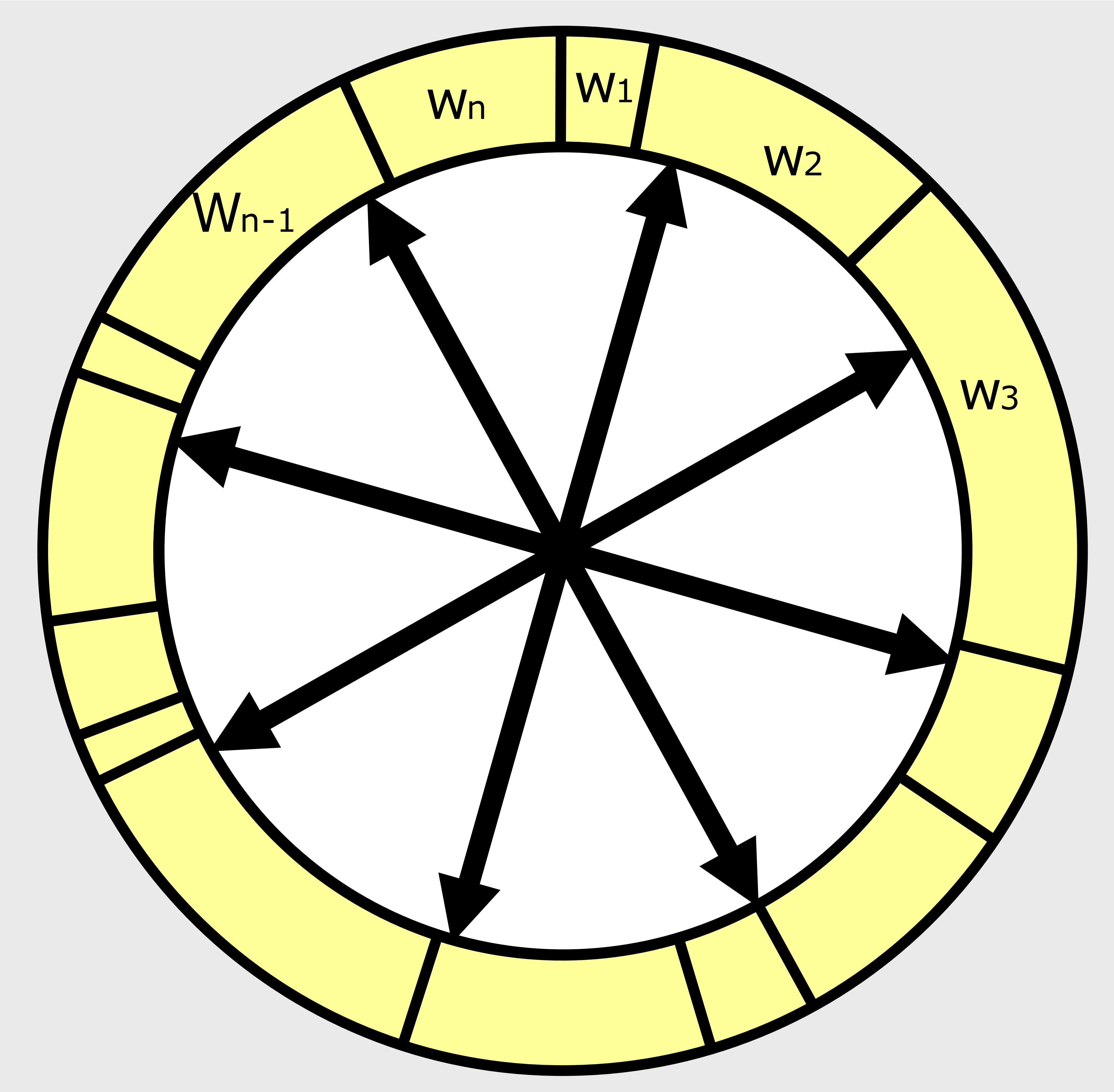
We can implement this condition in various ways

1. All weights are equal - don't resample

2. Entropy of weights high - don't resample

3. Ratio of max to min weights low - don't resample

Fix 2: Low variance sampling



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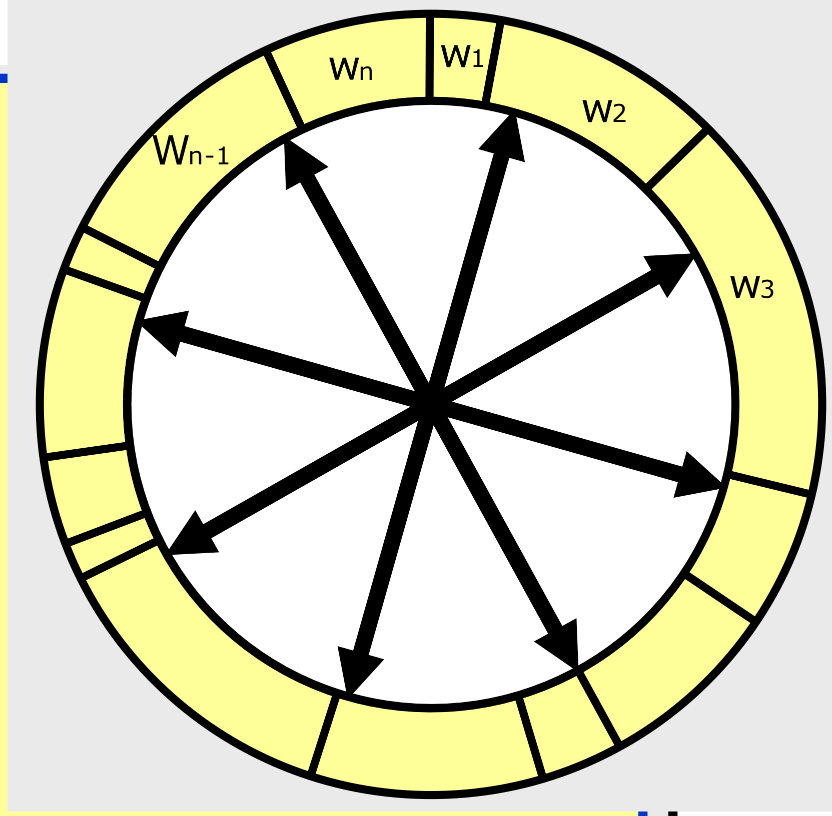
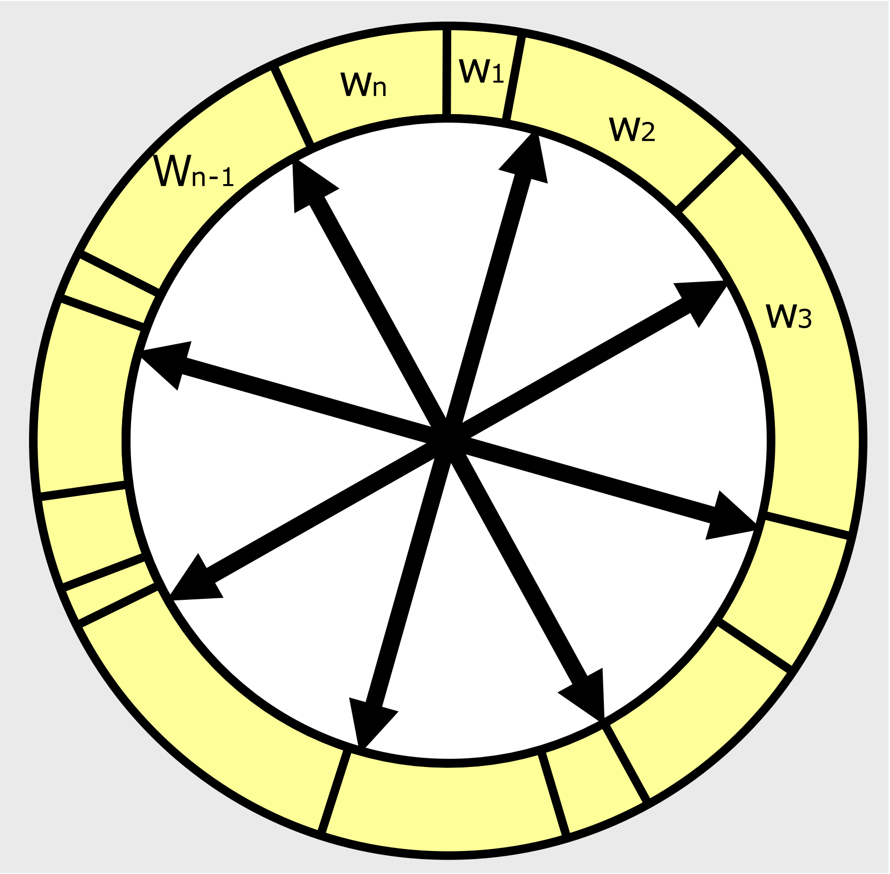



```
Algorithm systematic_resampling(S,n):

S' = ∅, c_1 = w^1                      Assumption: weights sum to 1
For i = 2…n                            Generate cdf
    c_i = c_{i-1} + w^i
u_1 ~ U[0, n^{-1}], i = 1              Initialize threshold

For j = 1…n                            Draw samples …
    While ( u_j > c_i )                Skip until next threshold reached
        i = i + 1
    S' = S' ∪ {< x^i, n^{-1} >}        Insert
    u_j = u_j + n^{-1}                 Increment threshold

Return S'
```

Also called **stochastic universal sampling**

Why does this work?



1. What happens when all weights equal?

2. What happens if you have ONE large weight and many tiny weights?

$$\mathrm{w1} = 0.5, \;\; \mathrm{w2} = 0.5/1000, \; \mathrm{w3} = 0.5/1000, \; .... \; \mathrm{w1001} = 0.5/1000$$

Problem 2: Particle Starvation

No particles in the vicinity of the current state

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No particles in the vicinity of the current state

Why?

1. Unlucky set of samples

2. Committed to the wrong mode in a multi-modal scenario

3. Bad set of measurements

Fix: Add new particles

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Which distribution should be used to add new particles?

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Which distribution should be used to add new particles?

1. Uniform distribution

2. Biased around last good measurement

3. Directly from the sensor model

Fix: Add new particles

When should we add new samples?

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Key Idea: As soon as importance weights become too small, add more samples

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1. Threshold the total sum of weights

2. Fancy estimator that checks rate of change.

Problem 3: Observation model too good!

Observation model is so peaky, that all particles die!

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Observation model is so peaky, that all particles die!

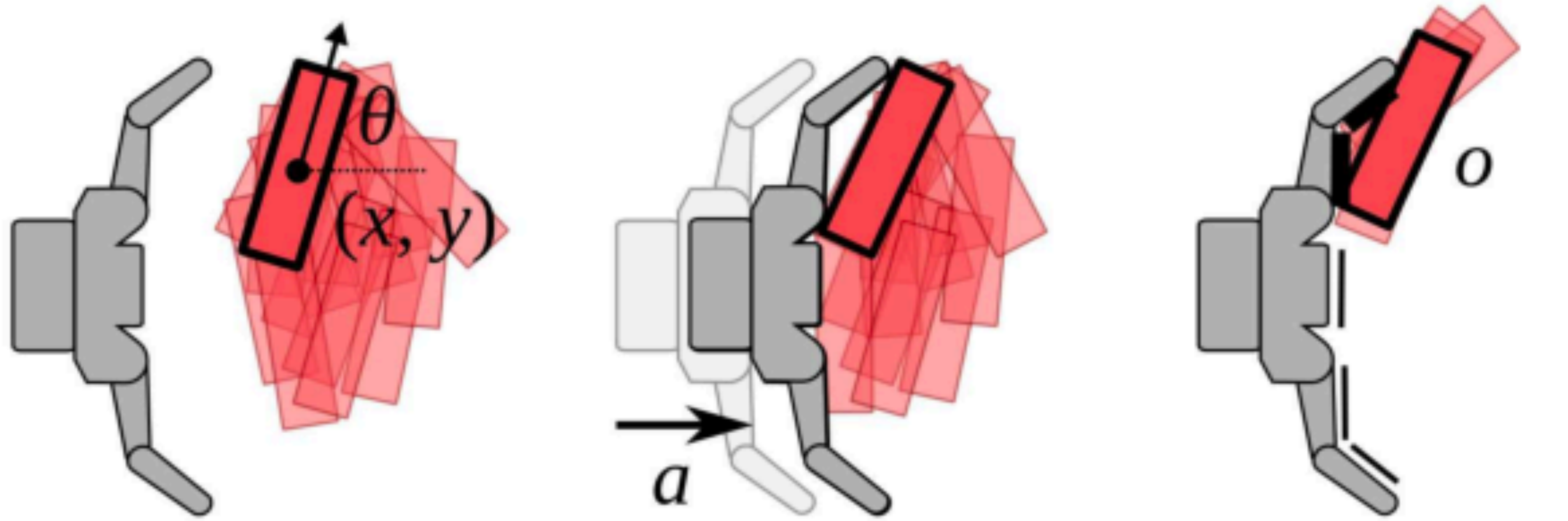
Fixes

1. Sample from a better proposal distribution than motion model!

2. Squash the observation model (apply a power of $1/m$ to all probabilities. m observations count as one)

3. Last resort: Smooth your observation model with a Gaussian (you are pretending your observation model is worse than it is)

Fix 1: Sample from a better proposal distribution



Koval et al. 2017

Contact observation may kill ALL particles!

Key Idea: Sample and weigh particles correctly

$$bel(x_t) = \eta P(z_t|x_t) \int P(x_t|x_{t-1}, u_t)\, bel(x_{t-1}) dx_{t-1}$$

(Sample) (Reweigh)

Problem 4: How many samples is enough?

Example: We typically need more particles at the beginning of run

Key idea: KLD Sampling (Fox et al. 2002)

1. Partition the state-space into bins

2. When sampling, keep track of the number of bins

3. Stop sampling when you reach a statistical threshold that depends on the number of bins

(If all samples fall in a small number of bins -> lower threshold)

KLD sampling

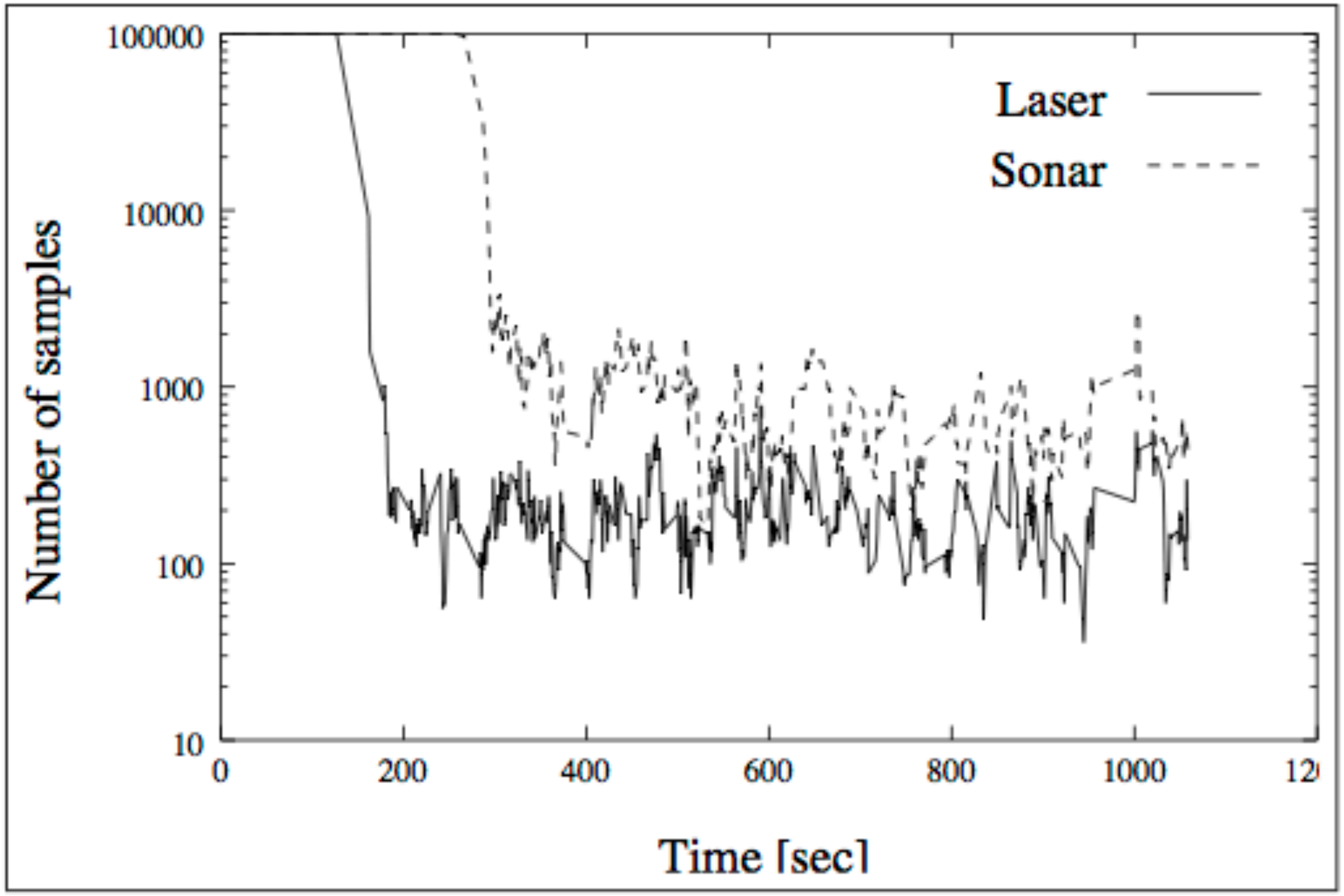


Figure 8.18 KLD-sampling: Typical evolution of number of samples for a global localization run, plotted against time (number of samples is shown on a log scale). The solid line shows the number of samples when using the robot's laser range-finder, the dashed graph is based on sonar sensor data.

Closing: Myth busting Particle filters

1. Particle Filter = Sample from motion model, weight by observation

2. Particle filters are for localization

3. Particle filters are to do with samples

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(sample from any good proposal distribution)

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BUSTED

(sample from any good proposal distribution)

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(any continuous space estimation problem)

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(sample from any good proposal distribution)

2. Particle filters are for localization

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(any continuous space estimation problem)

3. Particle filters are to do with samples

BUSTED

(normalized importance sampling also uses samples but no resampling)