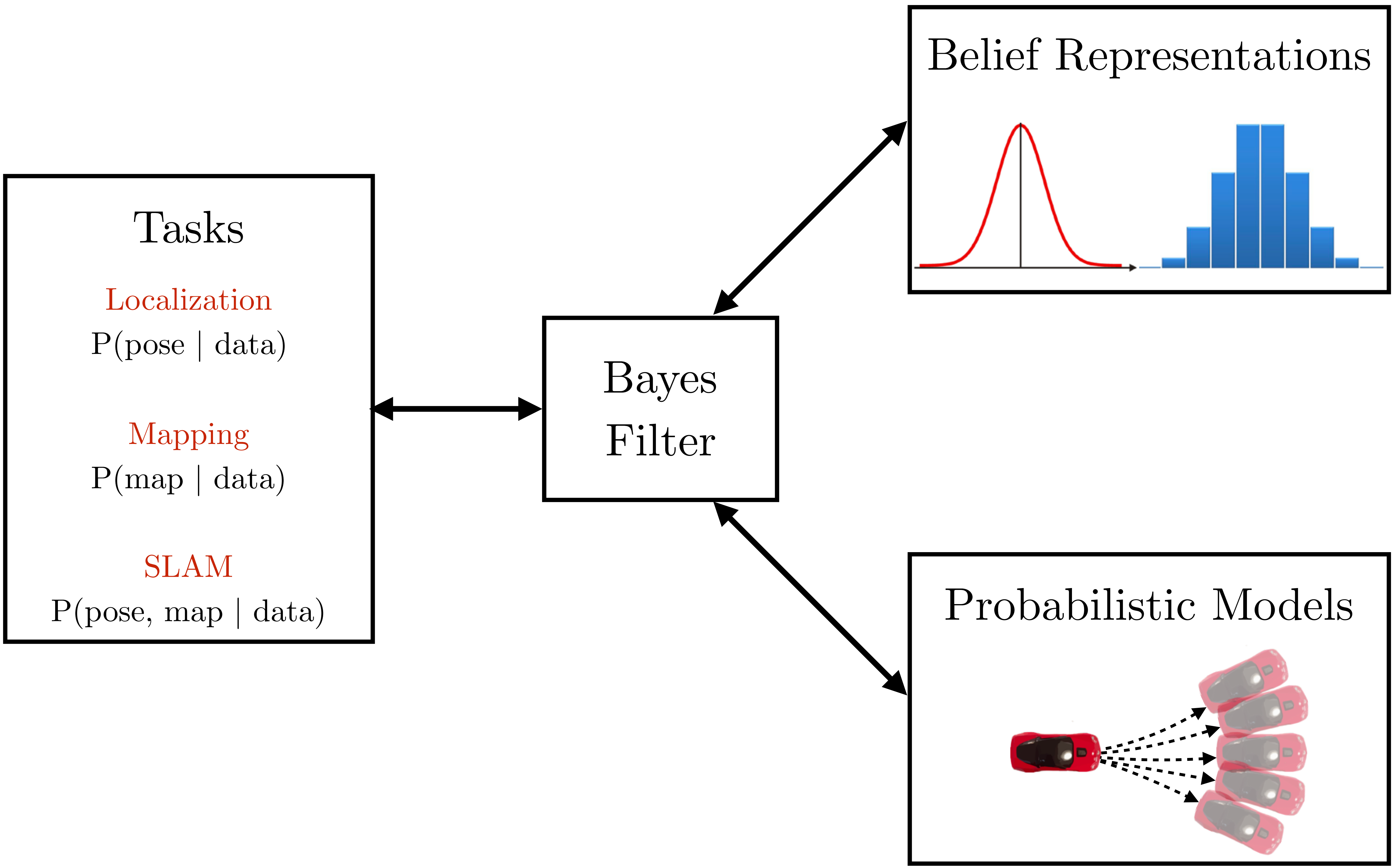
(Slides adapted from Drew Bagnell, Dieter Fox)

Particle Filtering

Sanjiban Choudhury

TAs: Matthew Rockett, Gilwoo Lee, Matt Schmittle

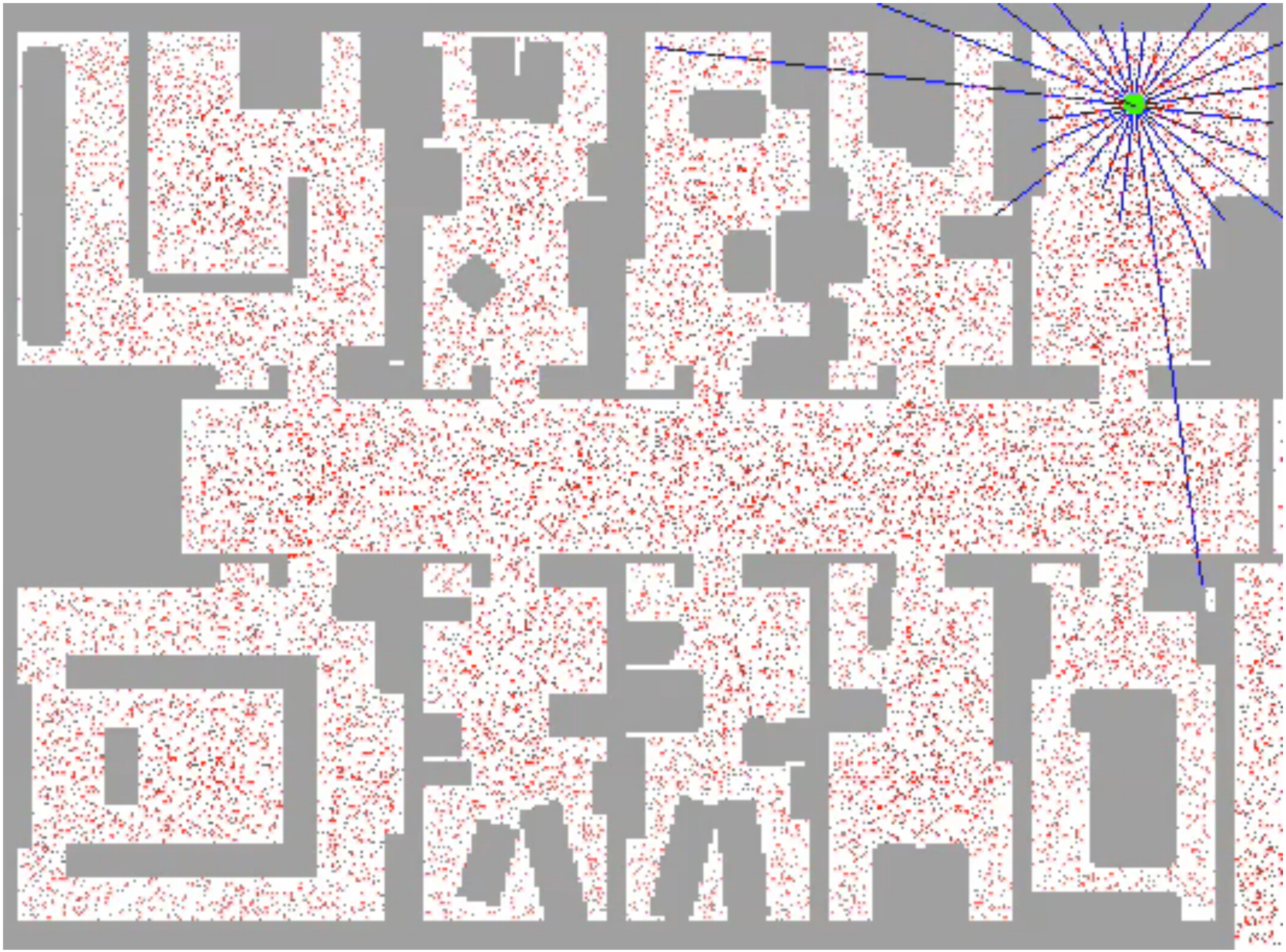
Assembling Bayes filter



Tasks that we will cover

Tasks	Belief Representation	Probabilistic Models
Localization P(pose | data) (Week 3)	Gaussian / Particles	Motion model Measurement model
Mapping P(map | data) (Week 4)	Discrete (binary)	Inverse measurement model
SLAM P(pose, map | data) (Week 4)	Particles+Gaussian (pose, landmarks)	Motion model, measurement model, correspondence model

Example: Indoor localization



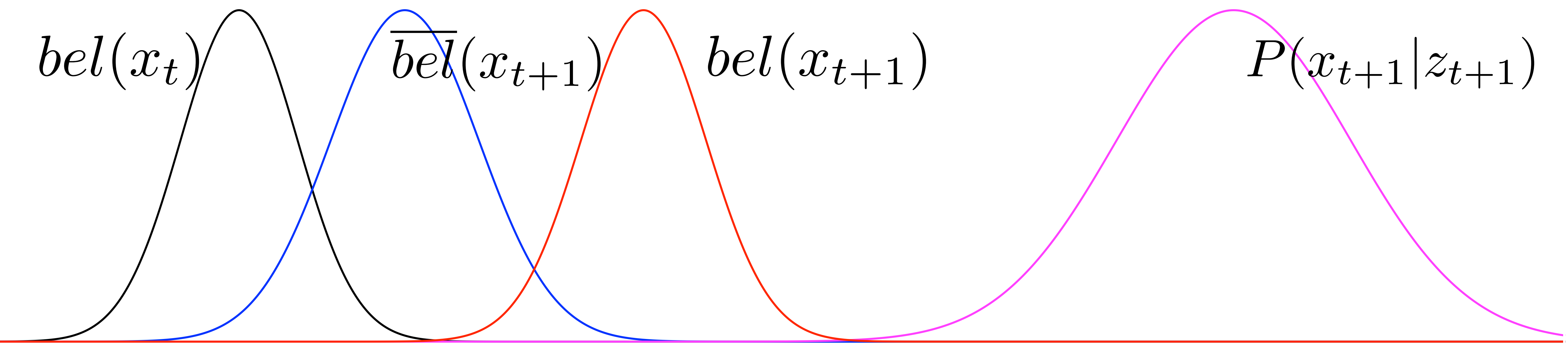
Today's objective

1. Understand the need for non-parametric filtering when faced with complex pdf in continuous space.

2. Importance sampling as an effective tool for dealing with complex pdf

Why can't we just use parametric filters?

Everything is a Gaussian - prior, motion, observation, posterior!



$$bel(x_t) = \eta P({\color{magenta}z_t}|x_t) \int P(x_t|x_{t-1}, {\color{blue}u_t})\, bel(x_{t-1}) dx_{t-1}$$

(Gaussian) (Gaussian) (Gaussian) (Gaussian)

Good things about parametric filters

We have so far been thinking about parametric filter (Kalman)

1. They are exact (when correct model)

E.g. Kalman Filter

2. They are efficient to compute

E.g. Sparse matrix inversion

Problems with parametric filters

1. Posterior has to have a fixed functional form (e.g. Gaussian)

    - even if our prior was a Gaussian, if control/measurement model is non-linear, posterior is NOT a Gaussian

2. We can always approximate with parametric belief (e.g. EKF)

    - what if true posterior was multi-modal? danger of losing a mode completely

How can we realize Bayes filters in a non-parametric fashion?

Tracking a landing pad with laser only

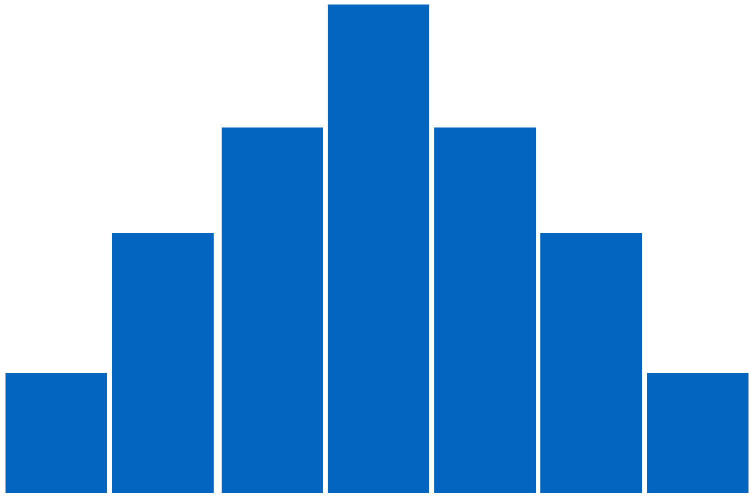
(Arora et al.)

Question: What are our options for non-parametric belief representations?

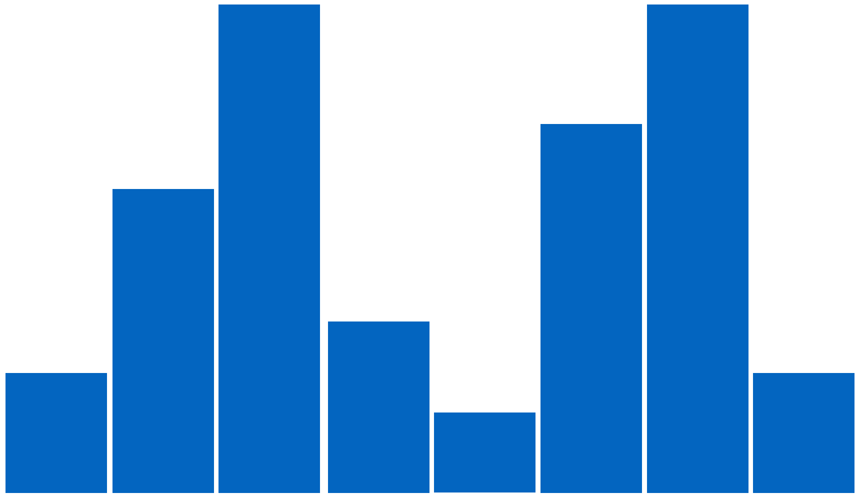
1. Histogram filter
2. Normalized importance sampling
3. Particle filter

Approach 1: Histogram filter

Simplest approach - discretize the space!

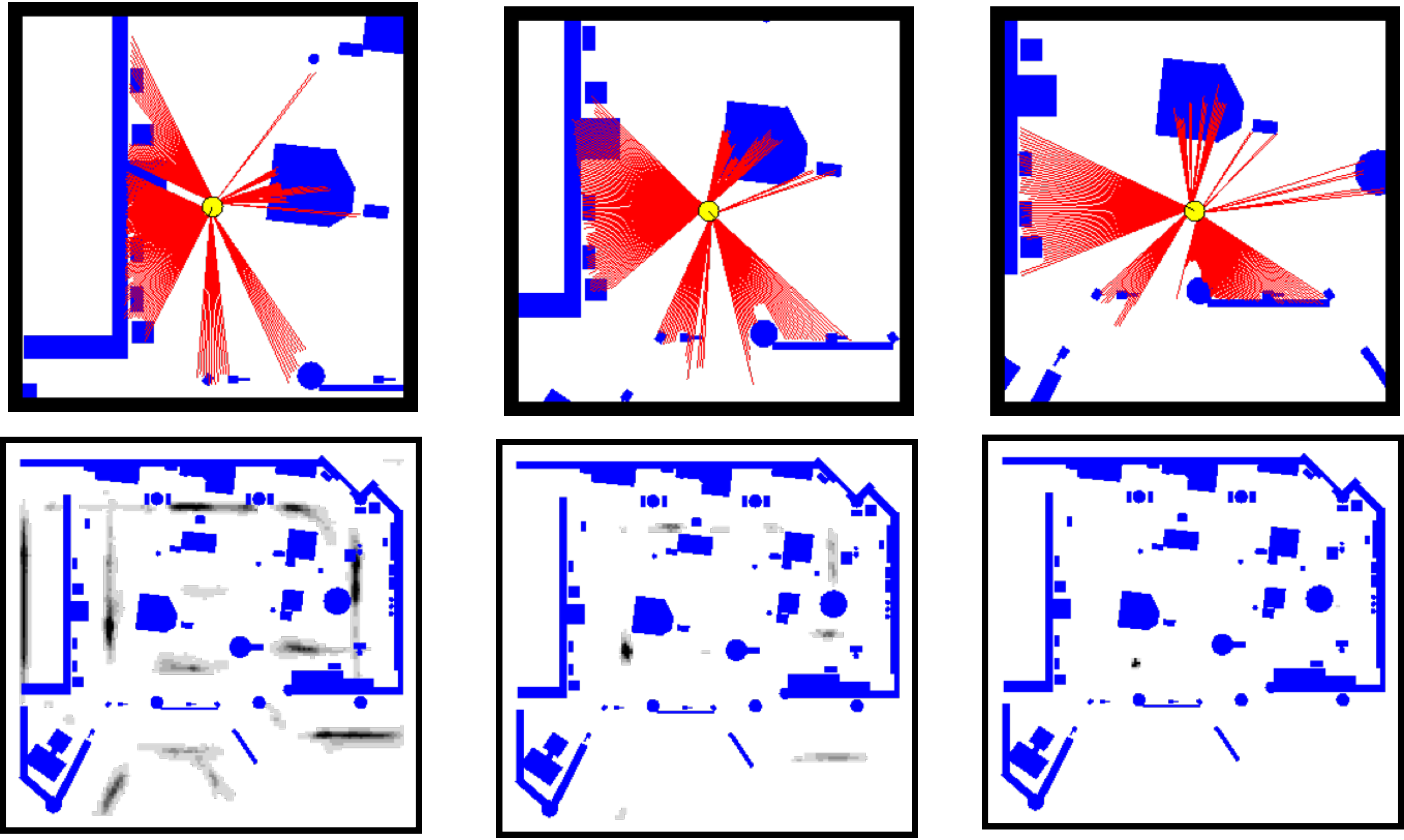


Prior $bel(x_t)$



Posterior $bel(x_{t+1})$

Example: Grid-based localization



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Issues with grid-based localization

1. Curse of dimensionality

   Remedy: Adaptive discretization

2. Wasted computational effort

   Remedy: Pre-cache measurements from cell centers

3. Wasted memory resources

   Remedy: Update a select number of cells only

If discretization is expensive,

can we sample?

Monte-Carlo method

Q: What do we intend to do with the belief $bel(x_{t+1})$?

Ans: Often times we will be evaluating the expected value

$$\mathbb{E}[f] = \int_x f(x) bel(x) dx$$

Mean position:	$f(x) \equiv x$
Probability of collision:	$f(x) \equiv \mathbb{I}(x \in \mathcal{O})$
Mean value / cost-to-go:	$f(x) \equiv V(x)$

Monte-Carlo method

Problem: Can't evaluate the integral below since we don't know bel

$$\mathbb{E}[f] = \int_x f(x)bel(x)dx$$

Solution: Sample from the distribution $x_1, \ldots, x_N \sim bel(x)$

$$\mathbb{E}[f] \approx \frac{1}{N}\sum_i^N f(x_i)$$

(originated in Los Alamos)

+ Incremental, any-time.

+ Converges to the true expectation under a mild set of assumptions

Lots of general applications!

Can we always sample?

$$bel(x_t) = \eta P(z_t|x_t) \int p(x_t|u_t, x_{t-1}) bel(x_{t-1}) dx_{t-1}$$

How can we sample from the product of two distributions?

Question:

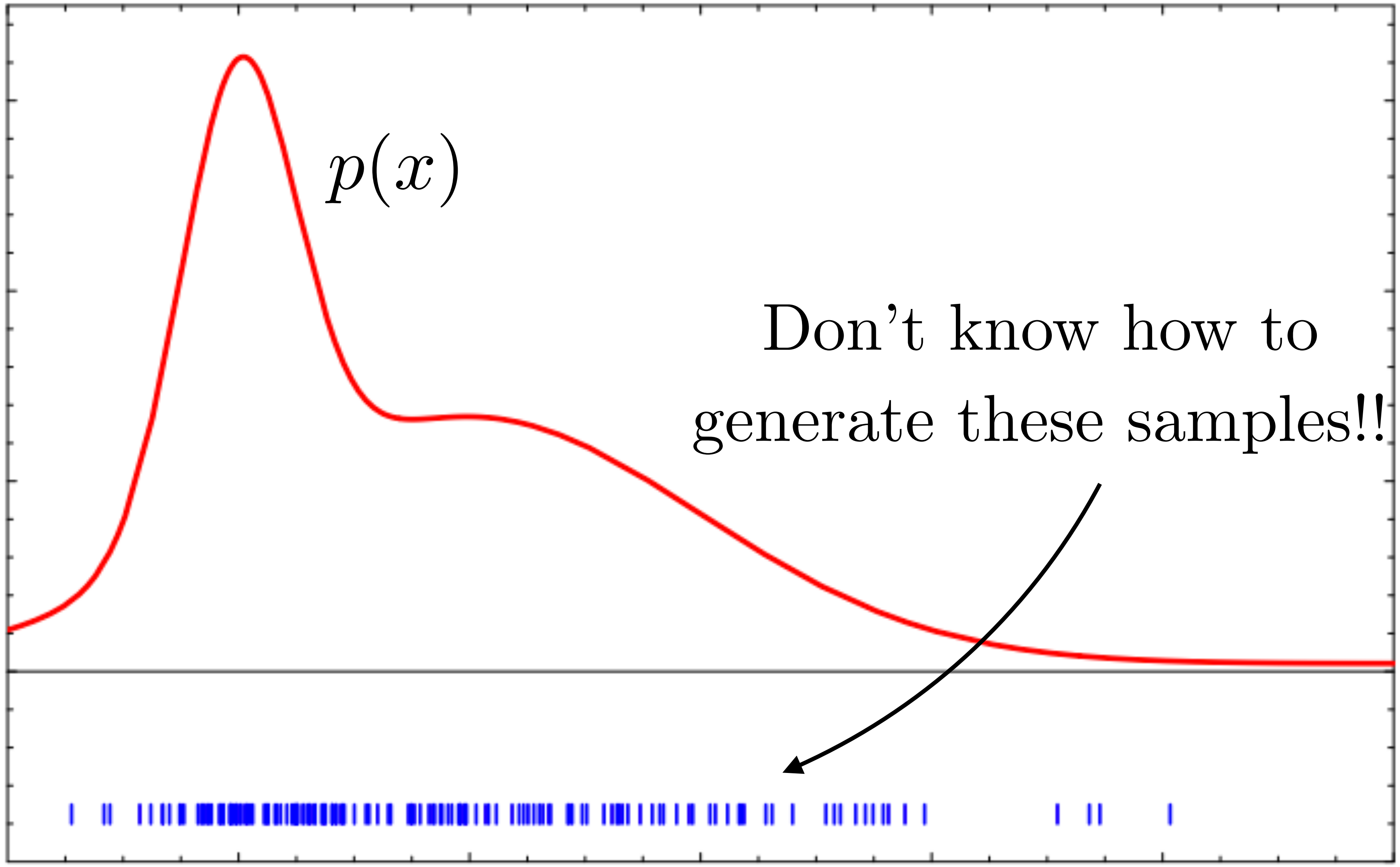
How can we sample from a complex distribution p(x)?

Solution: Importance sampling

Trick:

1. Sample from a proposal distribution (easy),
2. Reweigh samples to fix it!

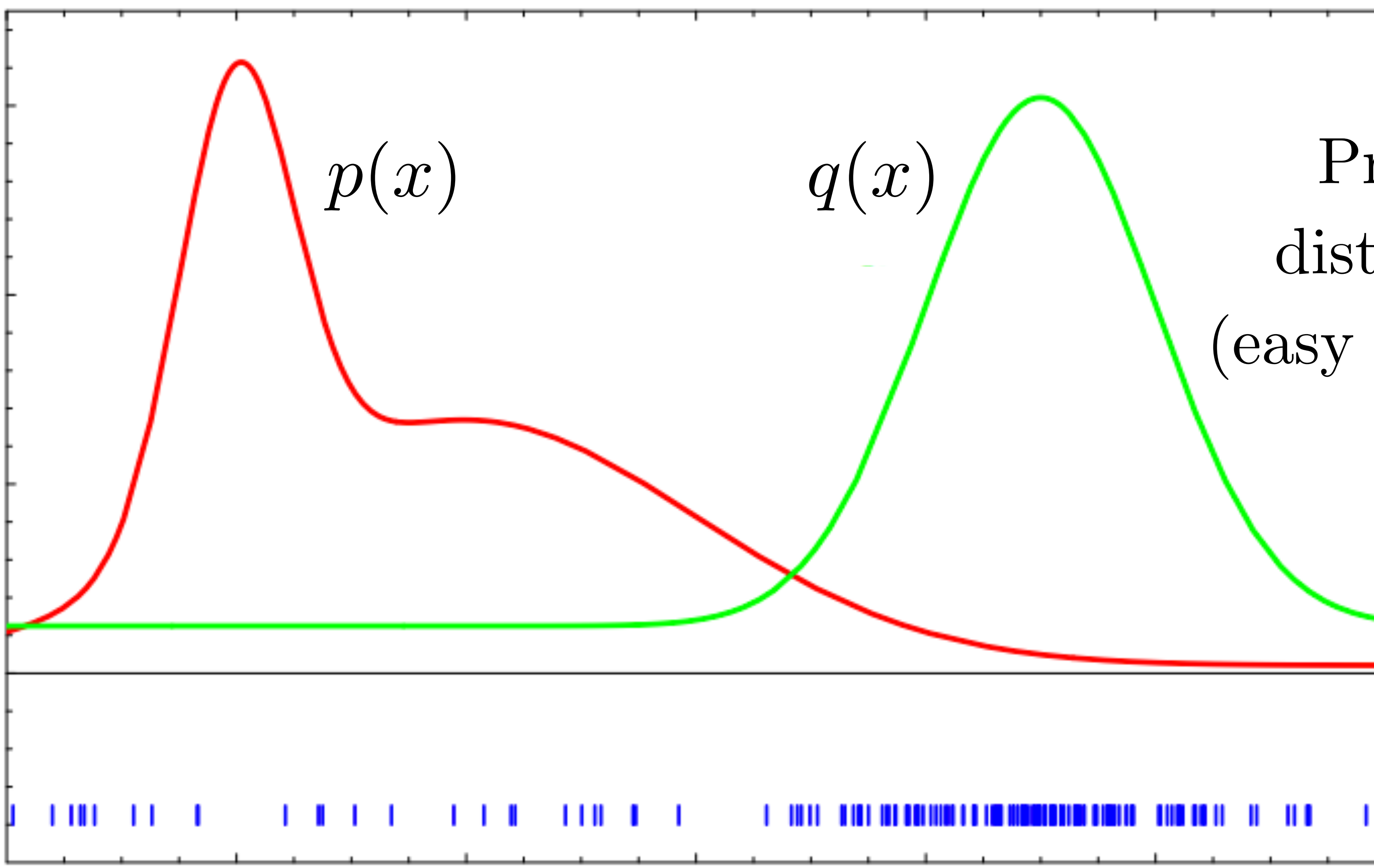
Solution: Importance sampling



Solution: Importance sampling

Trick:

1. Sample from a proposal distribution (easy),

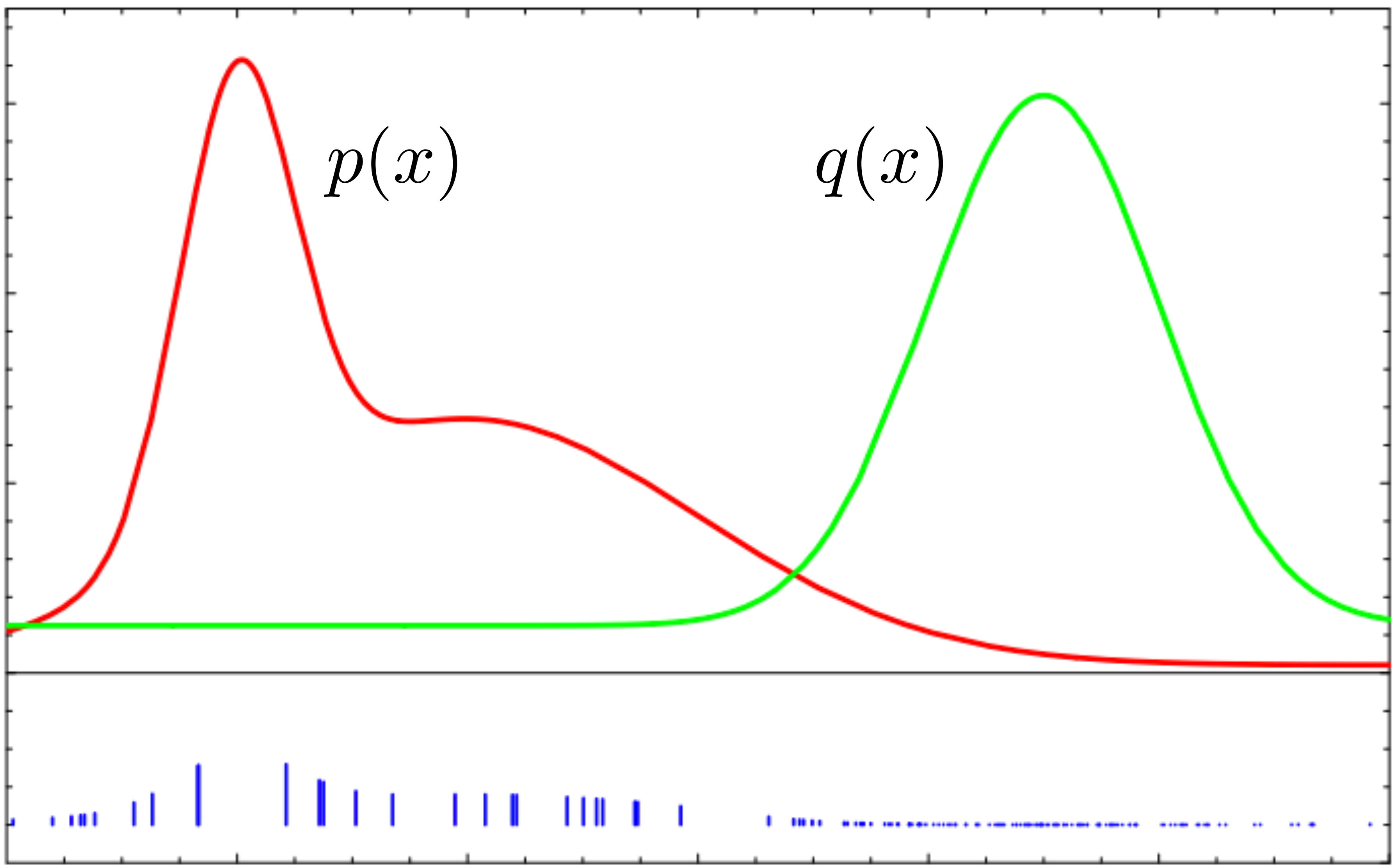


Proposal distribution (easy to sample)

Solution: Importance sampling

Trick:

1. Sample from a proposal distribution (easy),
2. Reweigh samples to fix it!



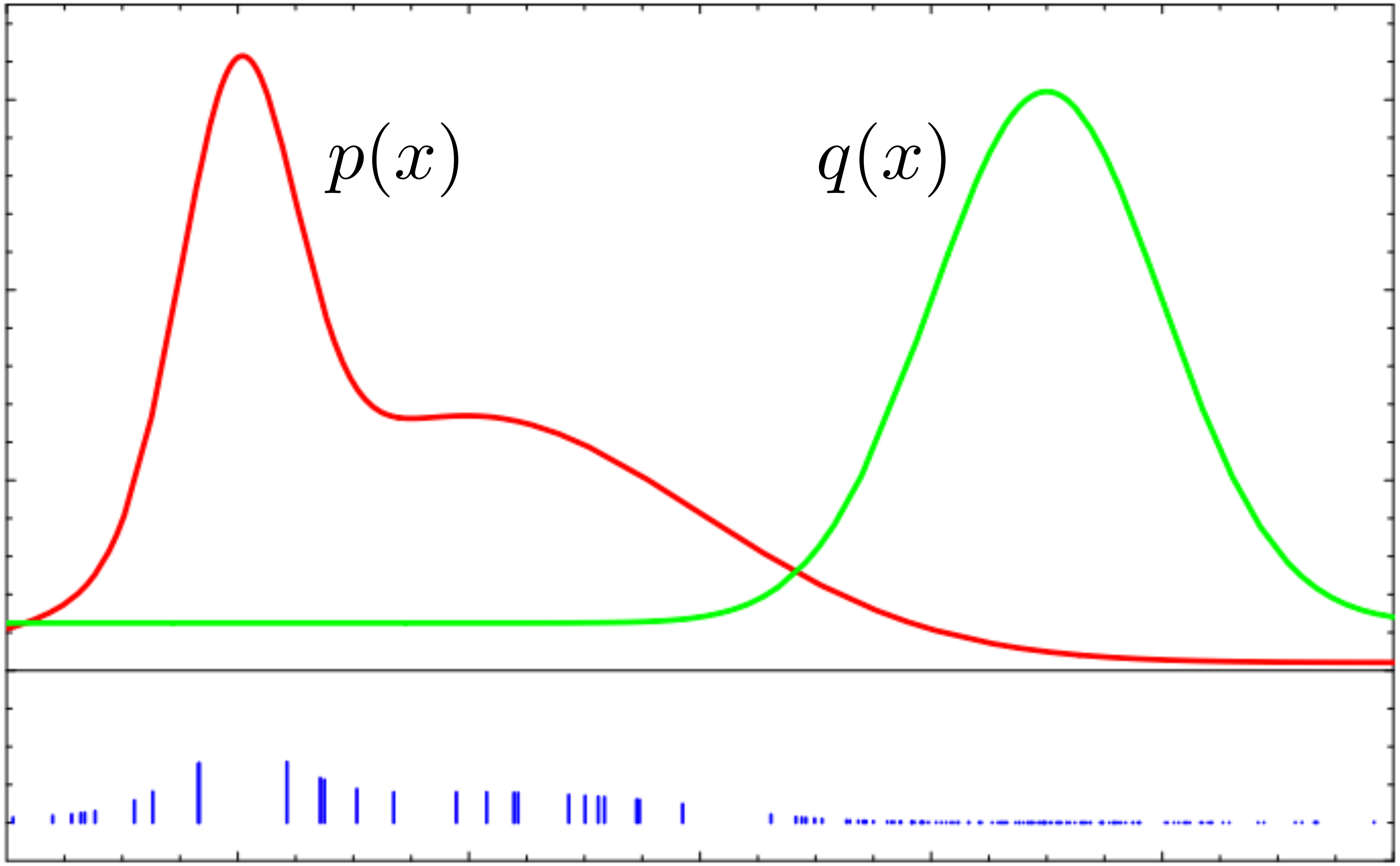
Solution: Importance sampling

Trick: Sample from a proposal distribution (easy), reweigh samples to fix it!

$$
\begin{aligned}
\mathop{\mathrm{E}}_{p(x)}[f(x)] &= \sum p(x) f(x) \\
&= \sum p(x) f(x) \frac{q(x)}{q(x)} \\
&= \sum q(x) \frac{p(x)}{q(x)} f(x) \\
&= \mathop{\mathrm{E}}_{q(x)}[\frac{p(x)}{q(x)} f(x)] \\
&\approx \frac{1}{N} \sum_{i=1}^{N} \frac{p(x_i)}{q(x_i)} f(x_i).
\end{aligned}
$$

For convergence, make sure support of target is a subset of proposal

Question: What makes a good proposal distribution?



Applying importance sampling to Bayes filtering

Target distribution : Posterior

$$bel(x_t) = \eta P(z_t|x_t) \int p(x_t|u_t, x_{t-1}) bel(x_{t-1}) dx_{t-1}$$

Proposal distribution : After applying motion model

$$\overline{bel}(x_t) = \int p(x_t|u_t, x_{t-1}) bel(x_{t-1}) dx_{t-1}$$

Why is this easy to sample from?

Importance Ratio:

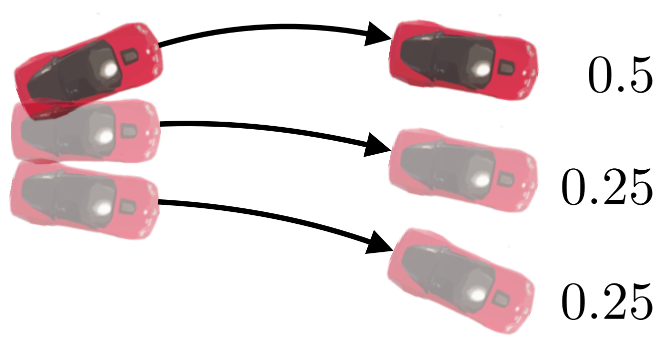
$$r = \frac{bel(x_t)}{\overline{bel}(x_t)} = \eta P(z_t|x_t)$$

Question: What are our options for non-parametric belief representations?

1. Histogram filter
2. Normalized importance sampling
3. Particle filter

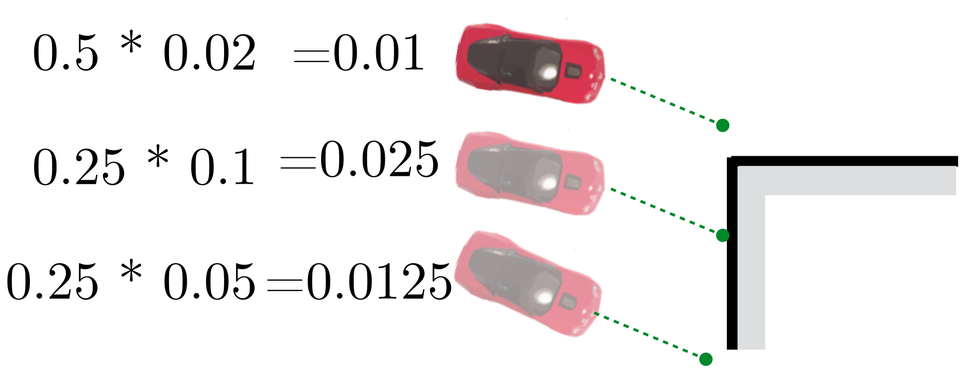
Approach 2: Normalized Importance Sampling

$$bel(x_{t-1}) = \left\{ \begin{matrix} x_{t-1}^1 \\ w_{t-1}^1 \end{matrix}, \begin{matrix} x_{t-1}^2 \\ w_{t-1}^2 \end{matrix}, \cdots, \begin{matrix} x_{t-1}^M \\ w_{t-1}^M \end{matrix} \right\}$$



for $i = 1$ to M

sample $\overline{x}_t^i \sim P(x_t | {\color{blue}u_t}, x_t^i)$



for $i = 1$ to M

$$w_t^i = P(z_t | \bar{x}_t^i) w_{t-1}^i$$

for $i = 1$ to M

$$w_t^i = \frac{w_t^i}{\sum_i w_t^i} \qquad bel(x_t) = \left\{ \begin{matrix} \bar{x}_t^1 \\ w_t^1 \end{matrix}, \cdots, \begin{matrix} \bar{x}_t^M \\ w_t^M \end{matrix} \right\}$$