Spherical trigonometry

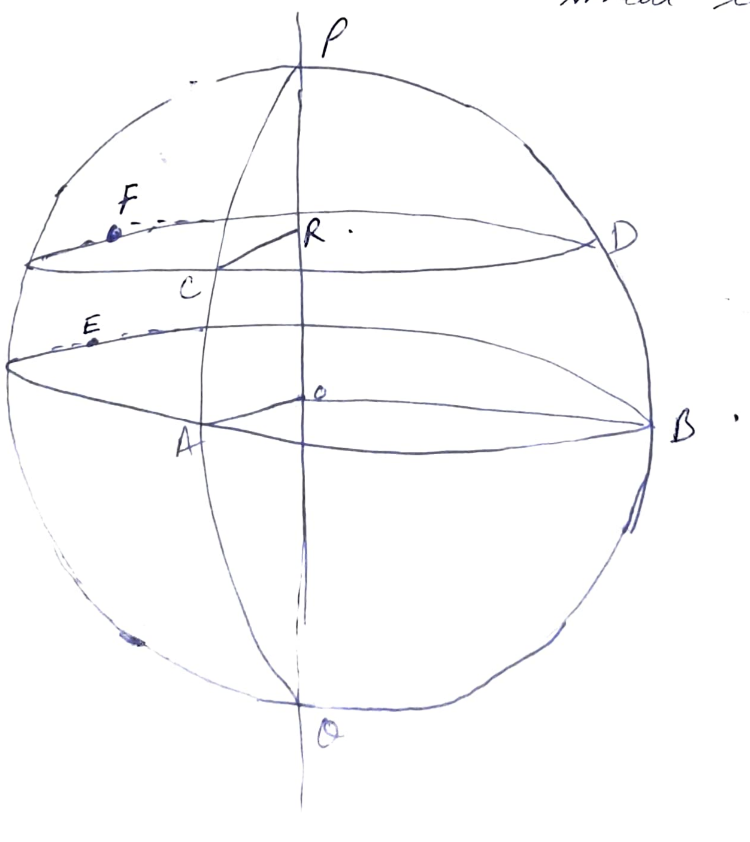
Spherical Astronomy is concerned essentially with the direction in which the stars are viewed.

great circle

Any plane passing through the center of a sphere cuts the surface in a circle which is called as great circle.

Small circle.

Any other plane intersecting the sphere but not passing through the centre will also cut the surface in a circle, which, in this case, is called a small circle.

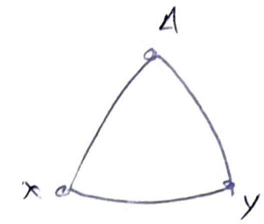


PCD is not a spherical Δ.
as, CD is not a great circle.

Any three points on the surface of a sphere, then the sphere can be bisected so that all three points lie in the same hemisphere.

and if the points are joined by great circle arcs all lying on this hemisphere, the figure obtained is called a spherical triangle.

for spherical triangle AXY



AX, AY, XY are the sides

and, spherical angles at X, Y, A are the angle of spherical triangle.

So, If R is the radius of the sphere.

the length of spherical arc AY is given by

AY = R × angle AOY.

(AOY is in radians).

<u>Length of small circle Arc</u>

Consider the small circle arc. CD.

length CD CD = RC × angle CRD.

also, AD = OA × angle of AOB.

Since the plane FCD is parallel to the plane EAB.

then, $C\hat{R}D = A\hat{O}B$.

for $RC \parallel OA$ and, $RD \parallel OB$

$\Rightarrow \quad CD = \frac{RC}{OA} AB.$

$OA = OC$ (radii of sphere)

$CD = \frac{RC}{OC} . AB$

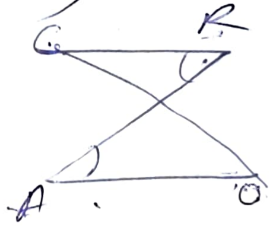
we have, $RC \perp OR$.

$\Rightarrow \quad RC = OC \cos R\hat{C}O$.

(as, $RC \parallel OA$)

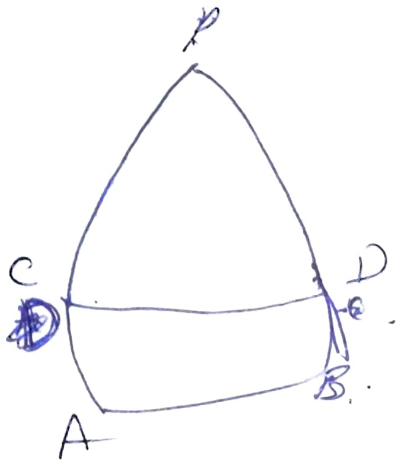
$CD = AB \cos R\hat{C}O$.

$= AB \cos A\hat{O}C$.



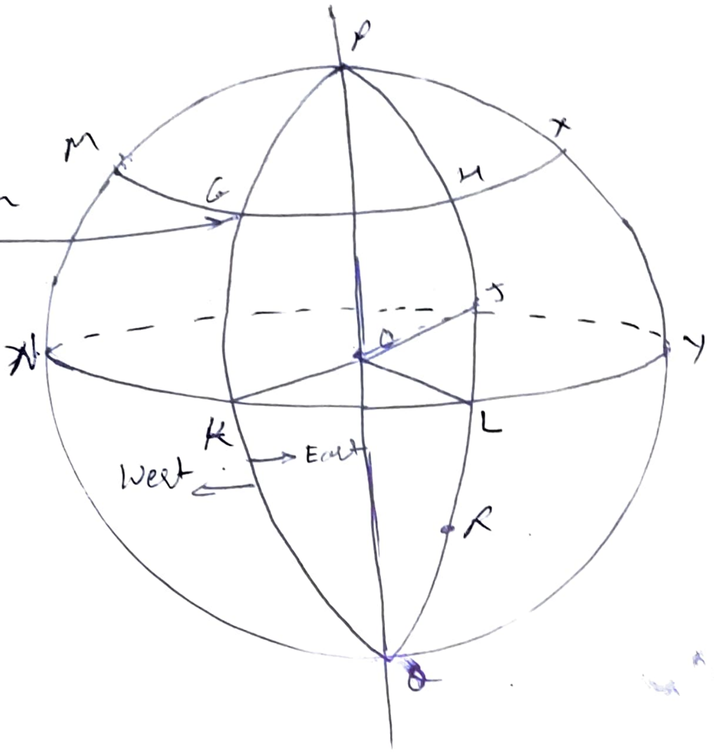
AOC is angle subtended at centre of the sphere by great circle arc. AC.

thus, $CD = AB \cos AC$



Since, $PA = 90°$

$CD = AB \sin PC \quad \text{---} (1)$.



Any semi-great circle terminated by P and Q is a meridian.

Longitudes are measured from 0° to 180° east of greenwhich meridian.
and 0° to 180° West

meridian through J cuts the equator in L and the angle LOJ or great circle arc LJ, is called the Latitude of J.

If J is b/w equator and the north pole P, Lattitude is said to be north (N).

Places, R b/w equator and south pole Q, is said to be, in South latitude.

Let ϕ denote the latitude of J: then, $L\hat{O}J$ or $LJ = \phi$

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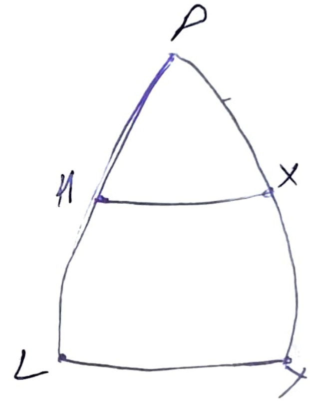
Since OP is $\perp$ to equator, $P\hat{O}L = 90^\circ$

and therefore

$$POJ = 90^\circ - \phi.$$

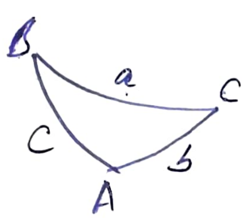
$\angle POJ$ or, spherical arc PJ ~~arc~~ is the Colatitude of J.

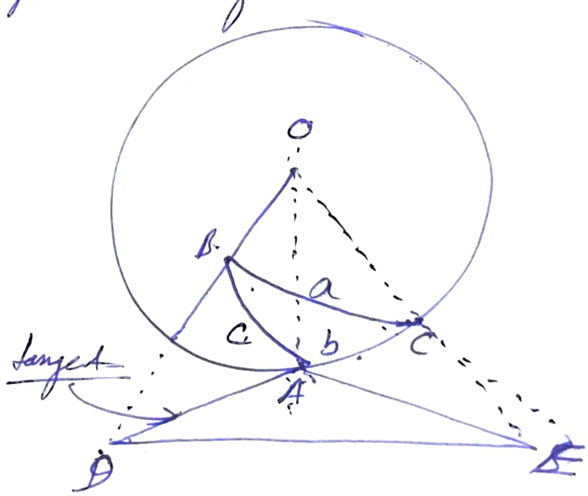
$$\text{Colat} = 90^\circ - \text{Lat}$$



Then,
$$\begin{aligned} HX &= LY \sin PH \\ &= LY \sin(90 - HL) \\ &= LY \cos HL \\ &= LY \cos\theta \quad \text{---- (2)} \end{aligned}$$

(3). fundamental formula for spherical trigo.





Let AD be the tangent to the great circle AC at A and AE be the tangent to BC at great circle BA.

$$\angle BAC = D\hat{A}E = A.$$

Now in plane triangle OAD, $O\hat{A}D$ is 90° and $A\hat{O}D$.

$$\angle AOD = \angle A\hat{O}C$$

Then,

$$AD = OA \tan c ; \quad OD = OA \sec c . \quad \cdots (3)$$

from the plane triangle OAE.

we have.

$AE = OA \tan b$. ~~$OE = OA\ OE$~~ $\cdots (4)$.

and $OE = OA \sec b$.

from plane triangle DAE.

$$DE^2 = AD^2 + AE^2 - 2\,AD.AE \cos(D\hat{A}E).$$

$$= (OA \tan c)^2 + (OA \tan b)^2 - 2 \cancel{\tan} (OA) \tan c (OA) \sec b \cos A .$$

$$= OA^2 \left[\tan^2 c + \tan^2 b - 2 \tan b \tan c \cos A \right]. \quad \cdots (5).$$

from the plane triangle DOE.

$$DE^2 = OD^2 + OE^2 - 2(OD)(OE) \cos D\hat{O}E$$

But, $D\hat{O}E = B\hat{O}C = a$.

$$= \left[(OA \sec c)^2 + (OA \sec b)^2 - 2(OA)^2 \sec c \sec b \cos a \right]$$

$$DE^2 = (OA)^2 \left[\sec^2 c + \sec^2 b - 2 \sec c \sec b \cos a \right]. \quad \cdots (6).$$

$\Rightarrow$ $6 = 5$

$$\tan^2 c + \tan^2 b - 2 \tan b \tan c \cos A$$

$$= \sec^2 c + \sec^2 b - 2 \sec b \sec c \cos a$$

Now, $\sec^2 c = 1 + \tan^2 c$, $\sec^2 b = 1 + \tan^2 b$.

So.

$$-2\tan b \tan c \cos A = 1 + 1 - 2\sec b \sec c \cos a$$

$$\cancel{2}\sec b \sec c \cos a = \cancel{2}(1 + \tan b \tan c \cos A)$$

$$\cos a = \frac{1}{\sec b \sec c} + \frac{\tan b \tan c \cos A}{\sec b \sec c}$$

$$\boxed{\cos a = \cos b \cos c + \sin b \sin c \cos A} \quad \cdots \text{(A)}$$

cosine formula.

fundamental formula of spherical trigonometry.

So,

$$\cos b = \cos c \cos a + \sin c \sin a \cos B \quad \cdots (7)$$

$$\cos c = \cos a \cos b + \sin a \sin b \cos C \quad \cdots (8)$$

Two direct practical solution.

(1) if two sides, e.g. b and c and the included angle A of a spherical triangle ABC are known, third side a can be calculated.

(2) if all three sides are known, the angles of the ~~spherical~~ triangle can be found successively by (7) (8).

$\cos a = \cos b \cos c + \sin b \sin c \cos A$.

we need $\cos A$.

Then,

$$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c}$$

$$\cos A = \operatorname{cosec} b \operatorname{cosec} c \left[\cos a - \cos b \cos c\right]. \quad \text{---} (9)$$

Since,

$$\cos A = 1 - 2\sin^2 \frac{A}{2}$$

$$\cos a = \cos b \cos c + \sin b \sin c \left[1 - 2\sin^2 \frac{A}{2}\right].$$

$$\cos a = \cos(b-c) - 2\sin b \sin c \sin^2 \frac{A}{2}.$$

So, $\cos(b-c) - \cos(a) = 2\sin b \sin c \sin^2\left(\frac{A}{2}\right)$.

$$2\sin\left(\frac{a+b-c}{2}\right)\sin\left(\frac{a-(b-c)}{2}\right) = 2\sin b \sin c \sin^2 \frac{A}{2}.$$

Let, s is, $2s = a+b+c$.

Then, $a+b-c = 2(s-c)$

and $a-b+c = 2(s-b)$.

~~Then~~ Hence

$$\cancel{2}\sin(s-c)\sin(s-b) = \cancel{2}\sin b \sin c \sin^2 \frac{A}{2}.$$

now

$$\sin\frac{A}{2} = \sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin b \sin c}} \quad \ldots (11).$$

$$\sin\frac{B}{2} = \sqrt{\frac{\sin(s-c)\sin(s-a)}{\sin c \sin a}}.$$

$$\sin\frac{C}{2} = \sqrt{\frac{\sin(s-a)\sin(s-b)}{\sin a \sin b}}$$

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if we are.

$$\cos A = 2\cos^2\frac{A}{2} - 1.$$

then

$$\cos a = \cos b\cos c + \sin b\sin c\cos A$$

$$= \cos b\cos c + \sin b\sin c\left(2\cos^2\frac{A}{2} - 1\right)$$

$$\cos a = \cos(b+c) + 2\cos^2\frac{A}{2}\sin b\sin c.$$

~~cos a − cos bc~~

$$2\cos^2\frac{A}{2}\sin b\sin c = \cos(b+c) - \cos a$$

$$= 2\sin\left(\frac{b+c+a}{2}\right)\sin\left(\frac{b+c-a}{2}\right)$$

$$2\cos^2\frac{A}{2}\sin b\sin c = 2\sin(s)\sin(s-a)$$

$$\cos\frac{A}{2} = \sqrt{\frac{\sin(s)\sin(s-a)}{\sin b\sin c}} \quad (12).$$

11ly

$$\cos\frac{B}{2} = \sqrt{\frac{\sin(s)\sin(s-b)}{\sin a\sin c}}.$$

$$\cos\frac{C}{2} = \sqrt{\frac{\sin s\sin(s-c)}{\sin a\sin b}}$$

$$\tan\frac{A}{2} = \sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin(s)\sin(s-a)}}$$

so similarly

$$\tan\frac{B}{2} = \sqrt{\frac{\sin(s-c)\sin(s-a)}{\sin s\sin(s-b)}}$$

$$\tan\frac{C}{2} = \sqrt{\frac{\sin(s-a)(s-b)}{\sin(s)\sin(s-c)}}.$$

(b) sine formula.

$\cos a = \cos b\cos c + \sin b\sin c\cos A.$

$\sin b\sin c\cos A = \cos a - \cos b\cos c.$

$\sin^2 b\sin^2 c\cos^2 A. = \cancel{\cos^2 a}\ \cos^2 a + \cos^2 b\cos^2 c.$

$- 2\cos a\cos b\cos c.$

see L.H.S.

$\sin^2 b\sin^2 c(1-\sin^2 A) = \sin^2 b\sin^2 c\cos^2 A.$

$\sin^2 b\sin^2 c - \sin^2 b\sin^2 c\sin^2 A.$

$= \sin^2 b - \sin^2 b\cos^2 c - \sin^2 b\sin^2 c\sin^2 A.$

$= 1-\cos^2 b - \cancel{\cos^2 c} + \cos^2 b\cos^2 c - \sin^2 b\sin^2 c\sin^2 A$

so $\cancel{\sin^2 b\sin^2 c\sin^2 A} =$

$\sin^2 b \sin^2 c \sin^2 A.$

$$= 1 - \cos^2 b - \overset{\cos^2 c}{\cancel{\sin^2 b}} + \cos^2 b \cos^2 c - \sin^2 b \sin^2 c \sin^2 A.$$

$$= \cos^2 a + \cos^2 b \cos^2 c - 2(\cos a \cos b \cos c)$$

$\cancel{1 - \cos^2 a - \sin^2 b \rightarrow}$

$$1 - \cos^2 a - \cos^2 b - \cos^2 c + 2(\cos a \cos b \cos c)$$
$$= \sin^2 b \sin^2 c \sin^2 A.$$

Let x be defined by.

$$x^2 \sin^2 a \sin^2 b \sin^2 c = 1 - \cos^2 a - \cos^2 b - \cos^2 c + 2 \cos a \cos b \cos c.$$

$$\frac{x^2 \sin^2 a \cancel{\sin^2 b \sin^2 c}}{\cancel{\sin^2 b \sin^2 c} \sin^2 A} = 1$$

$$x^2 = \frac{\sin^2 A}{\sin^2 a}$$

$$x = \pm \frac{\sin A}{\sin a}$$

We can find,

$$x = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c} = \frac{\sin A}{\sin a}.$$

Suppose two sides a and b and the angles are given then,

$$\cancel{\sin B = \sin c} \quad \sin A = \frac{\sin a \sin B}{\sin b}$$

eqⁿ - 7 .

$\cos b = \cos a \cos c + \sin a \sin c \cos B$.

So,

$\sin a \sin c \cos B = \cos b - \cos a \cos c$.

$= \cos b - \cos c(\cos b \cos c + \sin b \sin c \cos A)$.

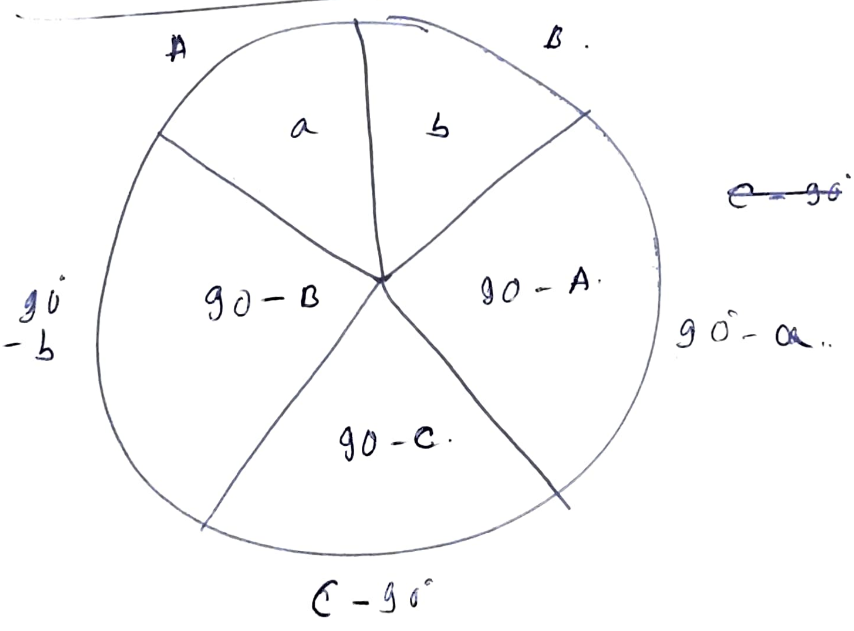
$= \sin^2 c \cos b - \sin b \sin c \cos c \cos A$.

$\sin a \cos \underline{B} = \sin c \cos \underline{b} - \sin \underline{b} \cos c \cos A$ . . . . (c)

relation involving all three sides

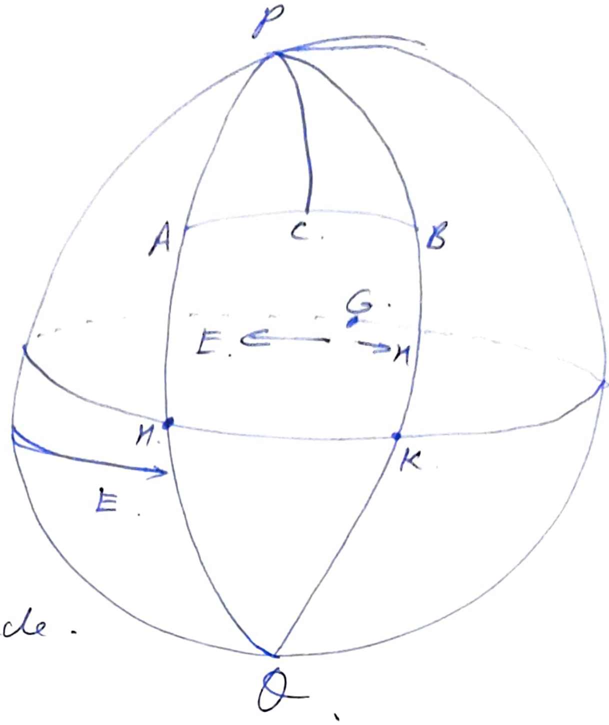
we can write.

$\sin a \cos \underline{C} = \cos \underline{c} \sin b - \sin \underline{c} \cos b \cos A$ . . . . (14)



Latitude $24°18'N$.
$36°47'N$.

Longitude $133°39'E$.
$125°24 W$.



find:
① Length of the great circle arc AB.

② $\angle PAB$, P being the north pole,

③. most nearby northerly point on the great circle AB.

PAMQ - meridian through A. cutting the equator at M.

and MA - latitude of A, ~~i.e, MA =~~.

i.e, $MA = 24°18'$.

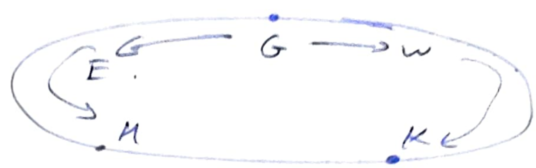
and, PA is colatitude, of A,

$\therefore PA = 90° - 24°18' = 65°42'$.

$|||^{ly}$, $PB = \cancel{90° - 53°} \; 90° - 36°47' = 53°13'$
colatitude

Let the greenwhich meridian intersects the equator in G.

Then.



$GH = \text{Long}(E)$ of A

$= 133^\circ 39'$.

$GK = \text{long}(W)$ of $B = 125^\circ 24'$ ~~tot~~.

So the great circle arc. $AB = HK$ (great circle)

$= GH + GK$.

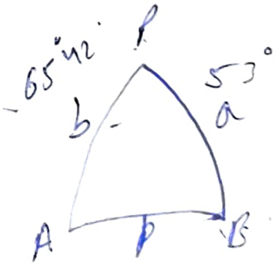
$= 133^\circ 39' + 125^\circ 24'$

$= 259^\circ 03'$

HK (shorter arc) $= 360^\circ - 259^\circ 03$

$= 100^\circ 57'$.

See triangle, APB. PA and PB are given

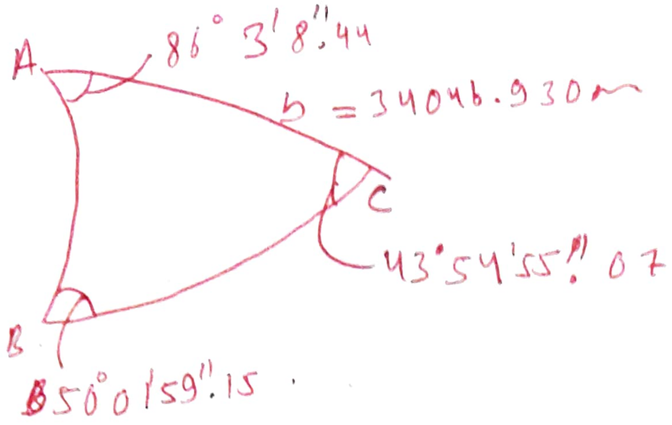


$\cos a = \cos p \cos b + \sin p \sin b \cos A$.

$\cos p = \cos a \cos b + \sin a \sin b \cos P$.

$\cos AB = \cos PB \cos PA + \sin PB \sin PA \cos APB$.

$= \cos 65^\circ 42' \cos 53^\circ 13' + \sin 65^\circ 42' \sin 53^\circ 13 \cos$



$A = 86°3'8''.44$

$B = 50°01'59''.15$

$C = 43°54'55''.07$

$b = 34'46.930$ m.

$R = 6370$ Km.

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

$1° = 111177.4734$ m.

$$\varepsilon = \frac{ab \sin C}{2R^2 \sin 1''} = \frac{c^2 \sin A \sin B}{2R^2 \sin C \sin 1''}$$

Spherical ε

$$= \frac{a^2 \sin B \sin C}{2R^2 \sin A \sin 1''} = \frac{b^2 \sin A \sin C}{2R^2 \sin B \sin 1''}$$

Then b in arc.

$$b = \frac{34046.930\text{m} \times 1°}{111177.4734\text{m}} = \cancel{0.306239}$$

$$= \cancel{34046.9}\ \frac{34046°55'48''}{111177.4734} = 0.30623946523$$

$$= 0°18'22.46''$$

$b = 34046.930\text{m} = 0°18'22.46''$

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B}$$

$$\sin a = \frac{\sin b \sin A}{\sin B} = \frac{\sin(0°18'22.46'')\sin 86°3'8''.44}{\sin 50°\ 01'\ 59''.15}$$

$$= \frac{5.3321 \times 10^{-3}}{\sin(50°01'59''.15)} = 6.9572 \times 10^{-3}$$

$$\sin a = 3.98626143 \times 10^{-1}\ °$$

$$a = 0°23'55.05'' = \sin^{-1}(\quad)$$

$$\frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

$$\Rightarrow \sin c = \frac{\sin b \sin C}{\sin B}$$

$$= \frac{\sin(0°18'22.46'')\sin(43°54'55.07'')}{\sin(50°01'59.15'')}$$

$$\sin c = 4.837 \times 10^{-3}$$

$$c = \sin^{-1}(4.837 \times 10^{-3})$$

$$= z. = \cancel{0°0} \quad 0°16'37.71''$$

$$= 0°16'37.71''$$

$$E = \frac{ab \sin C}{2R^2 \sin 1''}$$

$\sin 1'' = 4.848136 \times 10^{-6}$

$$= \frac{(0^\circ 23' 55.05'')(0^\circ 18' 22.46'') \sin(43^\circ 54' 55''.07)}{2 (6370 \times 10^3)^2 \, 4.848136}$$

$= 2.152021 \times 10^2.$

$= 215^\circ 12' 7.71''$

haversine $(\theta) = \sin^2\left(\frac{\theta}{2}\right)$

$hav(\theta) = \frac{1}{2}(1 - \cos\theta) = \sin^2\left(\frac{\theta}{2}\right).$

$\cos\theta = 1 - 2\sin^2\frac{\theta}{2}$

$= 1 - 2\, hav\,\theta.$

$\cos a = \cos b \cos c + \sin b \sin c \cos A.$

we have, $1 - 2\,hav\,\theta = \cos\theta.$

$\cos a = 1 - 2\,hav\,a.$

$1 - 2\,hav\,a = \cos b \cos c + \sin b \sin c\,(1 - 2\,hav\,A)$

$1 - 2\,hav\,a = \cos(b-c) - 2\sin b \sin c\, hav\,A.$

$1 - 2\,hav\,a = 1 - 2\,hav(b-c) - 2\sin b \sin c\, hav\,A.$

$hav\,a = hav(b-c) + hav\,A \sin b \sin c$

hav a = hav(b-c) + sin b sin c hav A.

haversines.

trigonometrical ratios for small angles.

- if θ - small angle.
  expressed in circular measure.

Approximation we have

$\sin\theta = \theta$ radians | $\cos\theta = 1$ | $\tan\theta = \theta$ radian |

Now

1 radian $= 57^\circ 17' 45''$.

$= 3437\frac{3}{4}'$

$= 206265''$.

so that, $1'' = \frac{1}{206265}$ rad.

and $1' = \frac{1}{3438}$ (approx)

$\sin 1'' = \frac{1}{206265}$, ... 36

$\sin 1' = \frac{1}{3438}$. 37

$\sin\theta = \frac{\theta''}{2026285}$.

$\therefore \sin\theta'' = \theta'' \sin 1''$

$\sin\theta' = \theta' \sin 1'$.

$\tan\theta'' = \theta'' \sin 1''$

24 hour = 360°

1 hour = 15°

$1^{min} = 15'$

1 sec = 15''

daubt

$\sin 1^m = \sin 15' = 15 \sin 1'$ ... 41

$\sin 1^s = \sin 15'' = 15 \sin 1''$ ... 42

if H is a small angle.

H^m ← expressed in minutes of times.

$\sin H = H^m \sin 1^m = 15 H^m \sin 1'$ ... 43

$\sin H = H^s \sin 1^s = 15 H^s \sin 1'$ ... 44

Delambre's and Napier's analogies.

$$\sin\tfrac{1}{2}c \,\sin\tfrac{1}{2}(A-B) = \cos\tfrac{1}{2}C \,\sin\tfrac{1}{2}(a-b)$$

$$\sin\tfrac{1}{2}c \,\cos\tfrac{1}{2}(A-B) = \sin\tfrac{1}{2}C \,\sin\tfrac{1}{2}(a+b).$$

$$\cos\tfrac{1}{2}c \,\sin\tfrac{1}{2}(A+B) = \cos\tfrac{1}{2}c \,\cos\tfrac{1}{2}(a-b).$$

$$\cos\tfrac{1}{2}c \,\cos\tfrac{1}{2}(A+B) = \sin\tfrac{1}{2}C \,\cos\tfrac{1}{2}(a+b)$$

Delambre's analogies.

Napier's analogies:

$$\tan\tfrac{1}{2}(a+b) = \frac{\cos\frac{1}{2}(A-B)}{\cos\frac{1}{2}(A+B)}\tan\tfrac{1}{2}c$$

$$\tan\tfrac{1}{2}(a-b) = \frac{\sin\frac{1}{2}(A-B)}{\sin\frac{1}{2}(A+B)}\tan\tfrac{1}{2}c$$

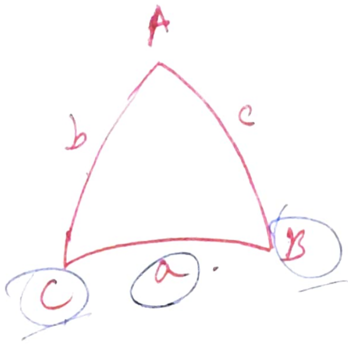
$$\frac{\tan(A+B)}{\tan(A-B)} = \cancel{\sin\tfrac{1}{2}(a-b)}$$

$$\frac{\cos\frac{1}{2}(a-b)\cot\frac{1}{2}C}{\cos\frac{1}{2}(a+b)}$$

$$\tan\tfrac{1}{2}(A-B) = \frac{\sin\frac{1}{2}(a-b)}{\sin\frac{1}{2}(a+b)}\cot\tfrac{1}{2}C.$$

(1)

wrong ans



$a = 119^\circ 46' 36'' = 119 + \frac{46}{60} + \frac{36}{360}$

$= 119.7767^\circ$

$B = 52^\circ 25' 38''$

$= 52.42722$

$C = 90^\circ$

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{(\sin C = 1)}$$

$$\sin A = \frac{\sin a \sin B}{\sin b} \quad \Big| \quad \sin A = \frac{\sin a \sin C}{\sin c}$$

$\Rightarrow \sin a$

$\cos a = \cos b \cos c + \sin b \sin c \cos A.$

$$\sin \frac{A}{2} = \sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin b \sin c}}$$

$\sin a \sin B \neq \sin A \sin b = \sin c$

$\sin(119^\circ 46' 36'') \sin(52^\circ 25' 38'') = \sin c.$

$\sin c = 6.87933 \times 10^{-1}.$

$c = 4.3466 \times 10^{1}$

$= 43^\circ 28' 0.27'' = \_\_$

$\sin b = \sin c \times \sin B.$

$= \sin(43^\circ 28' 0.27') \sin(52^\circ 25' 38'')$

$b = 3.304 \times 10^{1}$

$= 33^\circ 2' 28.30''$

$$\sin A = \frac{\sin a}{\sin c} = \frac{\sin(119^\circ 48' 36'')}{\sin(43^\circ 28' 0.27'')} = 1.261703.$$

$\sin A. = 1^\circ 15' 42.13''$

doubt

Whenever the angle is very small
its difficult to calc. the cosine and other function

from the programm. — asa format.

$b = 48^\circ 26' 48.95''$

$c = 105^\circ 13' 59.8705.$

$A = 113^\circ 10' 45.8827''.$

aaa
aas
asa.
sas