IKS INTERNSHIP PROGRAM

PROJECT REPORT

ON

STUDY OF SPHERICAL ASTRONOMY AND ECLIPSE FORMATION FROM INDIAN TEXTS

SUBMITTED BY

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Prashant Sharma

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Table of Contents

Abstract

Detailed calculations, method, and clear idea regarding spherical trigonometry has provided here with the introduction containing a short intro to the project giving scope, and the methodology followed. Eclipses are astronomical phenomena that periodically occur in nature. The AIM here is to Measure their recurrence over a finite period of time. and It could also be used to mark calendars in ancient India. Lunar and solar eclipses are two different forms of eclipses. To find the occurrence of the eclipse, different mathematical methods are used. In Indian classical siddhāntic texts, eclipse computation is based on the true positions and longitudes of the Sun, the Moon, and the ascending or descending node. Both Indian and Modern computations of eclipses are mathematically compared and analyzed. In the present, we discuss the mathematical tool spherical trigonometry and the algorithm for computation on MATLAB for the lunar eclipse on the basis of Suryasidhanta. Here we consider the path of the moon or sun in an eclipse and draw it on an imaginary sphere, and the measured coordinate change gives the time interval of the eclipse. For the current projects, the theorems and methods studied from the book "Spherical Astronomy - W.M.SMART" were used

general problems and ideal conditions, such as considering every orbit to be spherical and follow spherical geometry.

गोलाकार त्रिकोणमिति के बारे में विस्तृत गणना, विधि और स्पष्ट विचार यहां परियोजना के लिए एक संक्षिप्त परिचय के साथ परिचय प्रदान किया गया है जिसमें गुंजाइश और कार्यप्रणाली का पालन किया गया है

ग्रहण खगोलीय घटनाएं हैं जो समय-समय पर प्रकृति में घटित होती हैं। समय की एक सीमित अवधि में उनकी पुनरावृत्ति को मापना।

Executive Summary

Eclipses are astronomical aspects that occur in nature periodically. Measuring their occurrence of them is considered a fixed period of time. And ancient Indian use it to mark the calendar. The two types of eclipses are lunar and solar eclipses. A lunar eclipse is more frequent than a solar eclipse. The computation of occurrence and periodicity of these eclipses are based on the positions/longitudes of the celestial bodies, the Sun, the Moon, and the Earth. Geometrically the 'conjunction' of the Sun and the Moon refers to the solar eclipse, and the 'opposition' of the Sun and the Moon refers to a lunar eclipse. To find the occurrence of the eclipse, different mathematical methods are used. In Indian classical siddhāntic texts, eclipse computation is based on the true positions and longitudes of the Sun, the Moon, and the ascending or descending node. In Indian parlance, it is called Rāhu or Ketu. Modern eclipse computation is based on the International Astronomical Union terms (IAU). Both Indian and Modern computation of eclipses is mathematically compared and analyzed. In the present, we discuss the mathematical tool spherical trigonometry and the algorithm for computation on MATLAB for the lunar eclipse on

the basis of Suryasidhanta. Here we consider the path of the moon or sun in an eclipse and draw it on an imaginary sphere, and the measured coordinate change gives the time interval of the eclipse. For calculations, the MATLAB program is written in this project. This program gives the coordinates output for spherical trigonometry hence The timings of the eclipse are compared, and it is mathematically analyzed. For the current projects, the theorems and methods studied in the book “Spherical Astronomy - W.M.SMART” were used
general problems and ideal conditions, such as considering every orbit to be spherical and following the spherical geometry.
Detailed calculations, method, and clear idea regarding spherical trigonometry has provided here with the introduction containing a short intro to the project giving scope and the methodology followed. Some background justification mentions the history and the problem in an elaborated manner, followed by a brief description of the project where we discuss Surya Siddhanta and spherical trigonometry methods to solve the problems on the surface of the sphere. Then for the calculation, the calculator approach with pen and paper, thereafter Writing a Program to solve all the problems given in the textbook. Problems in the book prepare us to solve the Spherical triangle problems. Now the only task we have to do is the measure the coordinates of celestial bodies and then draw their path of it. Boom… we now have the date and time calculator that tells the position of the stars, moon, or any other celestial bodies, so the MATLAB program has a crucial role in the project while writing the code, many bugs and failures were faced. Since it is paid Software, we could access the software with the Institute's permission.

Introduction
the project was conducted to figure out the relation between the ancient algorithms to predict the date and time in the calendar a good spherical trigonometry background is a must.
So we started with learning the basics of spherical geometry.
Then we solved for the spherical trigonometry in deep derived all the relations and formulas required and found all the methods. These all are the basic prerequisite for the project. And our first month leads us to prepare the basic tools we need to compare our main goal, i.e., to find a calendar for a given period of time and compare it with the ancient method for the calendar.
As we move, we figure out that the problems in the real world are with more complicated numbers and consist of thousands of data points. So we need a program to solve this problem using computers as we have very less time to solve it manually. So a MATLAB script was built to solve the problems, and verification for the manual calculation was done.
These tools will help us to solve the position of the stars and the celestial bodies in the latter stage of the project.
All of the work was done with the help of the INTERNET, which allowed us to find many resources such as books and software. To learn the concepts, we need not go outside, and a computer was a must to solve all the problems, so the project is still going on the computers under the guidance of our mentor.

Predicting Solar and Lunar eclipses in the past was primarily a matter of keen observation- sometimes extending for a period of centuries- and then matching the results to a proper mathematical calculation.

Our Vedic sages were truly curious people who tried to comprehend nature. They considered these beautiful and awe-inspiring. For instance, in the "Panchavisa Brahmana," a part of Samaveda, there is a poetical description of the various stages of a solar eclipse. Later Aryabhata (circa 500 AD) provided a detailed eclipse analysis with detailed calculations. (see link (4) in reference).Such research on eclipse was continued by other great scientists such as Bhaskara-2. However, we have given up that scientific bent and have started believing in whatever stupid belief that people around us say. Such ignorance is doing justice neither to science nor to religion. We need to be inspired by the great cultures of Indus Valley, Maurya, Guptan, Chola, and various other cultures that all sparked various scholarly thought. However, in recent years, we have gotten backward in approaching nature and the world around us.

In short, be as curious as our Vedic ancestors and enjoy the beauties of nature. Document your observations and discuss them. Constantly question the things around us and keep an open mind. That is what our Vedic sages who wrote the Upanishads wanted. You don't need to take their facts as true, but instead, use their frameworks to approach the world. Our observations might change, but the way we approach those observations to need not.

असतो मा सद्गमय ।
तमसो मा ज्योतिर्गमय ।
मृत्योर्मा अमृतं गमय ।
ॐ शान्तिः शान्तिः शान्तिः ॥
– बृहदारण्यक उपनिषद् 1.3.28

From untruth lead us to Truth.
From darkness lead us to Light.
From death lead us to Immortality.
Om Peace, Peace, Peace.

From the "Brihadaranyaka Upanishad"

Background

In the early Indian age, the Vedas and Puranas were used as references for keeping records of the time and position of the stars.

We use calendars daily to see our schedule, recall significant occasions, and, most importantly, reflect on the past and plan for the future. We have been doing this since the dawn of civilization for humans. There is extensive written evidence from before humans began counting days, but the calculation has always been accurate. Ancient Indian astronomers are still the pride of the whole world. The introduction of astronomical calculations based on spherical trigonometry marked the beginning of a new age of siddhantic astronomy, then Mayasura's Surya Siddhanta later revolutionized Indian astronomy. Mayasura observed rare conjunction of the sun and the moon, and all planets except their nodes and subsidies were in conjunction with Aries (Mesha) on the new moon day of Chaitra month at the end of the 28th Krita yuga. Software simulations using the JPL Horizons (a web application) and the ephemeris system established that such conjunction on Chaitra Shukla Pratipada occurred only once in the last 16000 years, that is, on 22nd February 678 BCE. It is thus established that Mayasura wrote Surya Siddhanta in 6778 BCE. The calendar of Surya Siddhanta completes 8800 years in the year 2022.

All these calculations were done way back in time without using computers or advanced astronomical equipment. If our ancestors could do this back in time, why not us? So, this project aims to figure out how Surya Siddhanta can be helpful in quickly figuring out the dates and then verifying their accuracy and relatedness with the modern calendar.

The first requirement for understanding the concepts in Surya Siddhanta is to learn Spherical Trigonometry, which is the initial step of this project. Then we would explore Surya Siddhanta in depth. As we know, Astronomy is a field of study that encompasses everything outside of the surface of the earth. Every planetary motion and the solar event takes place in three-dimensional physical space, and most planets and their orbits are spherical or elliptical. However, certain planets have odd but well-defined orbits. With the aid of contemporary technology, it is possible to correctly predict objects' past and future positions by treating these motions as motion in spherical dimensions. We would calculate and verify the calendar system by determining the planetary motions and positions. Based on Mayasura's Surya Siddhanta from 6778 BCE, Lata Deva wrote the Surya Siddhanta, now known as A Synopsis of Mayasura's Surya Siddhanta. The primary distinction between these two Siddhantas is that Lata Deva described a deva yuga of 4 lakh 32,000 years, whereas Mayasura described an asura yuga of 1 lakh 80 000 years. Surya Siddhanta was written by Lata Deva on Chaitra Prapada, or February 18th, 3101 BCE, while observing the approximate conjunction of the planets. Here we will figure out which of the Siddhanta is more accurate. Learning and solving spherical trigonometry will help us see how some Ancient civilizations had the concept of periods that were truly enormous by contemporary standards at that point in time. Apart from the Mayans, the ancient Hindus appear to be the only people who dared to think beyond a few thousand years.

In the past, predicting solar and lunar eclipses mostly involved careful observation, which may last for centuries at a time. The observations were then compared to accurate mathematical calculations.

The first reliable mentions of people ‘predicting’ eclipses come from the Yajur Veda, but the verses escape me at present. The first known depiction of the requisite mathematical calculations is from the works of the 3rd century BC musician, grammarian, and mathematician Pingala who apparently considered the Universe to have begun as Music sung by the gods and thus applied his ideas on Musical theory to predict eclipses successfully. He used equations of up to three degrees to predict these eclipses and found very accurate results

Description

(a)Surya Siddhanta and Spherical Trigonometry

Suryasidhanta is still one of the methods that we can fit on many of the spherical trigonometry problems perhaps it's an ancient method to calculate the position of the stars it works.

Here we have solved the problems and understand the derivation of the formulas that are given in the book by W.M. Smart. Based on the formulation, algorithms have been made to solve the problems for different conditions. All the problems were done using the normal user computer, and manual verification using a simple calculator was done.

There's a lot of difference between the manual calculations and the pre-programmed solution we are getting after attempting eight times to solve the problems using manual methods, it was still not as accurate as it should be, but in the end, we found that all the errors were due to unit conversion that is very difficult for a normal human so only the MATLAB program was giving the right and accurate solution here is the output we are getting in the FIGURE-1.

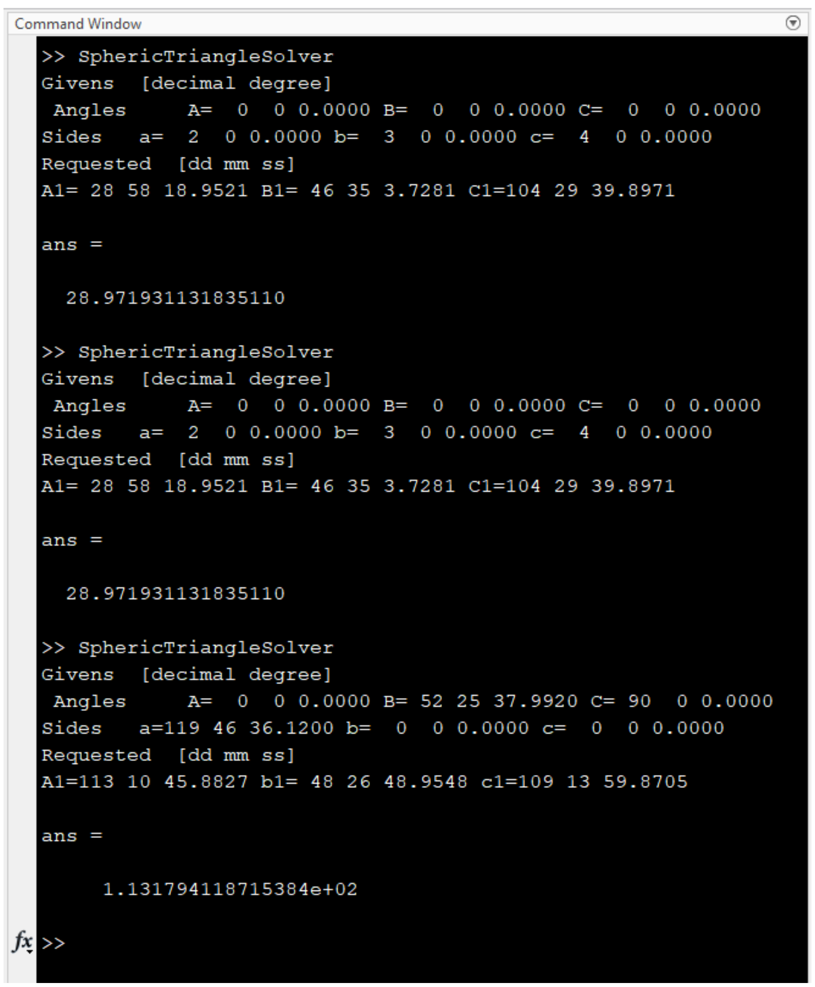

FIGURE-1

(b)Eclipse formation

As a result, anytime the Moon passes in front of the Sun in a way that the Moon circle crosses over the Sun's disc, an eclipse of the Sun takes place anywhere on Earth. Depending on whether the eclipse is complete or annular depends on how much of the Sun's disc the Moon's circle covers. If a new moon appears as the Sun is moving from position one to position one-fourth, it is clear from the top of the Moon's ascending node figure that a solar eclipse will occur. This period is the eclipse season; it starts 19 days before the Sun passes through a lunar node and ends 19 days thereafter. There are two complete eclipse seasons, one at each node, during a calendar year. The path of the formations can be found by drawing the arcs on the imaginary spherical surface, and by the slight change in the interval of observation, more coordinates can be observed, and the full path can be traced.

(c)Problem-solving methods

The most difficult part was writing the algorithm for all the conditions of spherical trigonometry problems.

So if we go into detail.

A spherical triangle is made up of the area enclosed by the three great circular arcs intersecting pairwise in three vertices on the surface of the sphere Figure-2

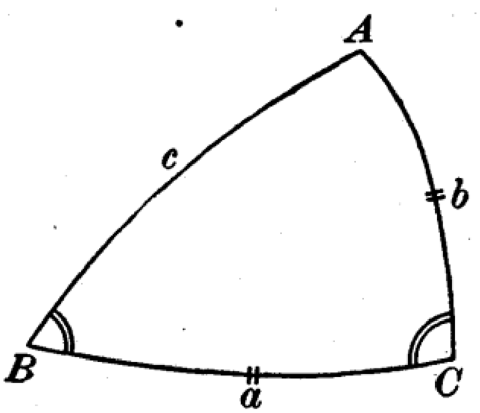

Figure-2

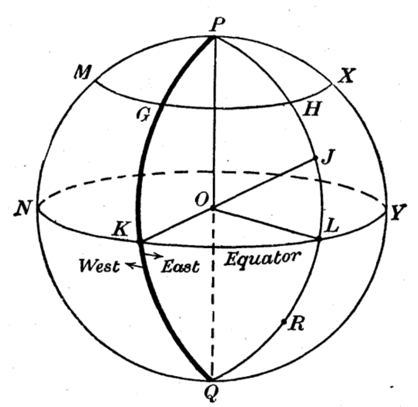

Figure-3

Some basic concepts from spherical trigonometry that played an important role in this project

- Intro to spherical trigonometry
- Spherical triangle
- Length of a small circle arc
- Terrestrial latitude and longitude
- The fundamental formula of trigonometry (Figure-3)
- The sine-formula
- Cosine-formula
- The Four-parts formula
- Right angled and quadrilateral triangle
- Polar formula

- Numerical example
- Haversine formula
- Trigonometrical ratios for small angles
- Delambre's and Napier's analogies.

Along with solving the problems, verification was done using coding on the MATLAB program one of the interesting problems was this one.

- **4. Two ships X and Y are steaming along the parallels of latitude 48° N and 15° S respectively, in such a way that at any given moment the two ships are on the same meridian of longitude. If the speed of X is 15 knots,* find the speed of Y.**

Here is one of the most important practical examples we can see in ancient times. When voyagers used to travel through the sea horizon, the only way to find their position and the time elapsed was by knowing the position of the stars. The same information helps them to determine the speed of the boat, and they can determine the end of their journey.

Another example-

6. The most southerly latitude reached by the great circle joining a place A on the equator to a place B in south latitude ϕ is ϕ_1. Prove that the difference of longitude between A and B is $90° + \cos^{-1}(\tan\phi \cot\phi_1)$.

Simple derivations and arrangements give the proof (the proof is attached to the assignment at the end of this report).

Methods

(a)Celestial Coordinates

An object's position on the Earth is marked by two positions: latitude (N-S) and longitude (E-W). Similarly, a star's position on the Celestial Sphere (the way the sky appears to us from Earth) is marked by Declination (abbreviation dec) and Right Ascension (abbreviation RA).

Declination is measured in degrees north (+) or south (-) of an imaginary line called the **Celestial Equator** (CE). The Celestial Equator is the projection of the Earth's Equator onto the Celestial Sphere. The CE has a declination of 0 degrees, by definition. At dec = +90 degrees (90 degrees N) is the **North Celestial Pole** (NCP), the projection of the Earth's North Pole onto the Celestial Sphere. The **South Celestial Pole** (SCP) is at dec = -90 degrees. The positions of landmarks change as you change latitude on the Earth, although their coordinates do not, as you move north or south. At the Earth's equator, the Celestial Equator is directly overhead, and the poles are on opposite sides of the horizon. All stars are visible as they rise, culminate at the meridian, and set. As you move north, the Celestial Equator mirrors your movement, moving south the same number of degrees away from the zenith (the straight-overhead point) as you moved north of the equator. So, by the time you reach Austin (30 degrees North of the Equator), the Celestial Equator has moved away from the zenith, 30 degrees to the south.

Now on the poles, The Celestial Equator has moved and has taken the poles with it. The NCP has risen 30 degrees into the sky as you moved 30 degrees north, and the SCP has sunk 30 degrees below the horizon. Had you moved south, the opposite would have occurred as the Celestial Equator moved north of the zenith. As we go north and observe the sky, we see that some stars circle the NCP incessantly, never sinking below the horizon, and that other stars vanish from view, never having an opportunity to ascend. The first group's stars are referred to as circumpolar stars. Since the NCP is 30 degrees from the horizon, it makes sense that stars close to the pole -- within 30 degrees -- will never drop below the horizon as they circle the NCP. Therefore, as seen from Austin, stars with declinations in the range of +60 degrees to +90 degrees are circumpolar. Similarly, stars with declinations ranging from -90 degrees (SCP) to -60 degrees never rise above Austin's horizon. The southernmost stars have declinations of about -60 degrees, although hills, buildings, and haze make -40 degrees a more practical limit. So we can say in general:

1. The distance of the Celestial Equator from the zenith is equal to your latitude. If the distance is south of the zenith, you are north of the equator, and vice versa.
2. The NCP's altitude (distance above the horizon) is equal to your latitude north of the equator; ditto for the SCP in the Earth's southern hemisphere.
3. The circumpolar stars are within (latitude) degrees of the celestial pole. They have declinations greater than +(90-latitude) degrees for the northern hemisphere.
4. The southernmost star visible from your location in the northern hemisphere is at declination -(90-latitude) degrees. The opposite situation applies in the southern hemisphere.
5. Those stars with declinations equal to your latitude pass directly overhead during the night.

(b)Representation of spherical surface

To determine the positions of stars and planets in the sky in an absolute sense, we project the Earth's spherical surface onto the sky and call it a celestial sphere.

The celestial sphere has a north and south celestial pole as well as a celestial equator which are projected reference points to the same positions on the Earth's surface. Right Ascension and Declination serve as an absolute coordinate system fixed on the sky rather than a relative system like the zenith/horizon system. Right Ascension is the equivalent of longitude, only measured in hours, minutes, and seconds (since the Earth rotates in the same units).

Declination is the equivalent of latitude measured in degrees from the celestial equator (0 to 90). Any point of the celestial, i.e., the position of a star or planet, can be referenced with a unique Right Ascension and Declination.

Since the Earth turns on its axis once every 24 hours, the stars trace arcs through the sky parallel to the celestial equator. The appearance of this motion will vary depending on where you are located on the Earth's surface.

(c)Required formulas

haversine(θ) = $\sin^2(\theta/2)$. The haversine formula is a very accurate way of computing distances

$$\text{hav}\ \theta = \tfrac{1}{2}(1 - \cos\theta) = \sin^2\frac{\theta}{2}$$

between two points on the surface of a sphere using the latitude and longitude of the two points. Using the above equation, algorithms were made to solve any spherical trigonometry problem.

A spherical triangle has 6 basic elements: three angles (A, B, C) and three sides (a,b,c).FIGURE-4

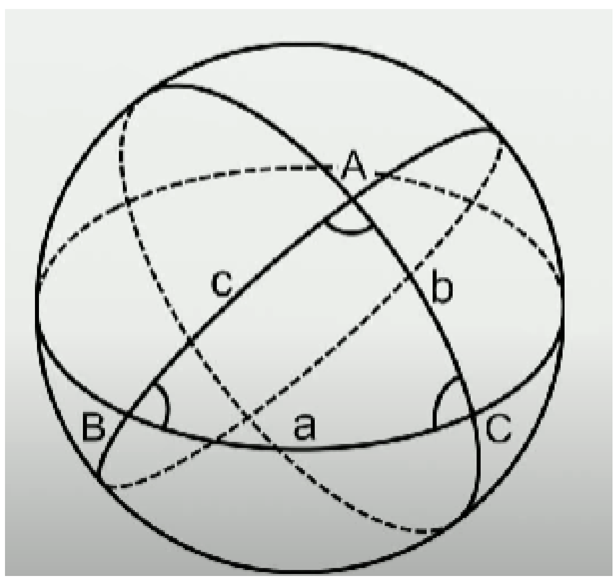

Figure-4

Given any three of the 6 basic elements, the function finds the other three. Usage of function for example, Given 3 elements of a spherical triangle:
a=2, b=3, C=104.4944158
[A1, B1, A2, B2,C1,C2,a1,b1,a2,b2,c1,c2] = SphericTriangleSolver(A,B,C,a,b,c)
[A1, B1, A2, B2,C1,C2,a1,b1,a2,b2,c1,c2] = SphericTriangleSolver(0,0,104.4944158,2,3,0)

NOTE the unknown (A, B, c) 3-elements of the spherical triangle should be replaced with zero.

(d)Intro to MATLAB[5]

It is a programming platform designed specifically for engineers and scientists to analyze and design systems and products that transform our world. The MATLAB language is a matrix-based language allowing the most natural expression of computational mathematics.
With MATLAB, we can Analyze data, Develop algorithms, and Create models and applications

(e)Prepare a Code to solve

The code consist of many ‘for’ loops and if-else statements a simple flow can be prepared before going on the actual code, which is called as pseudo-code.
A spherical triangle has 6 basic elements: three angles (A, B, C) and three sides (a,b,c).
Given any three of the 6 basic elements, the function finds the other three.
So the written algorithm works as follows
It takes the Input inside the code as

INPUT:

- A, B, C, a, b, c element of a spherical triangle. Only 3 values should be entered.
- The others must be zero.
- Inputs units must be [decimal degree]

And gives the OUTPUT in the terminal:

- A1, B1, C1, a1, b1, c1 are 1. solutions of the spherical triangle.
- A2, B2, C2, a2, b2, c2 are 2. solutions of the spherical triangle.
- Outputs units are degrees, minutes, and seconds [dd mm ss]

Now, going into further detail here is an algorithm that is working for now.
Firstly the program asks for input in the format of ANGLES and sides. Angles and sides must be given in degree decimal format. NOTE that the sides are nothing but the arc of the sphere so that it also must be written in terms of degree decimals format
After that, we store and assign the given values to some variables A, B, C, a, b and c.
These values are angles and arcs, respectively. And then, we look for the missing variables.
In order to solve for all other angles and sides, we need to have at least three of them known.
So the triangle now has different formats of inputs such as SSS, SAS, ASA, AAS, and AAA (S-side, A-angle) we classify them on the basis of these parameters and pass the argue to the function for solving the other values.
We have already derived and illustrated the formulas for the sine and cosine functions.

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$$

Results

Spherical trigonometry was studied from the book "Spherical Astronomy -W.M Smart," and exercise problems were solved manually and also with MATLAB. A working algorithm is designed to solve the spherical trigonometry problem and equip us for the future working of the project. Now, this algorithm can be used to solve the position of the stars and verify the calendar (interval of eclipses and other translations in celestial bodies)with the one determined by the ancient Surya Siddhanta method.

Discussion

It is clear from the result that the accuracy of calculation and positioning of the instrument is the key role in finding the date and time other than keeping the record of the dates. Spherical trigonometry must be the tool required to calculate the future position of celestial bodies. Approximations for the small angles in sin and cosine functions lead to a bit error in the calculation that we need to consider if we looking for accurate date and time calculations, and also, the four decimal point is not that accurate we need to have in rocket landing, but it is okay for calculating the time by observing the position of the stars in the sky

Conclusion and Recommendations

Indian ancestors has a very good knowledge of astronomical bodies not only that they can easily calculate the path and formation of the sun, moon, other planets, and some stars by just using the knowledge passed by their family that too carried from Purana Vedas and Siddhanta.

The results we found here using spherical trigonometry is true and absolute, but it amazes with the question of how they knew the coordinates and the angle with that much accuracy, and if they know some unique hidden or unrevealed method, then how did they calculate those huge numbers with trigonometry functions, it would be recommended to study khagol shashtra to know about these and more precisely the mathematical methods in past time such as geometry, and trigonometry. Furthermore, we can think of some there may be some other methods other than spherical trigonometry.

Acknowledgment

I would like to convey my gratitude to **Dr. Anil Kumar Gourishetty** for his patient supervision, enthusiastic encouragement, and guidance in my study endeavor. And will always be grateful to the Indian Knowledge System and the Ministry of Education Government of INDIA for providing me with this marvelous opportunity to learn and explore ancient Indian knowledge and for funding this project.

I would also like to express my gratitude to Adhavan A and Samarpita Sahu for their suggestions and helpful ideas to keep my progress on time.

Finally, I'd like to thank them for their inspiration and support during my academic career.

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Appendices

While preparing the function, Rody P.S. Oldenhuis subfunctions are used.
Data, Calculations, Other Supporting Information, and additional Details on the scope of work can be found here.
Include data, calculations, other supporting information, and additional detail on the scope of work, pertinent background information, and methods and procedures.
The following are the efforts and background performed during the project

MATLAB Code
Assignment
Presentation report

```
%
% This function solves any type of spherical triangle
% A spherical triangle has 6 basic elements: three angles (A,B,C) and↙
three sides (a,b,c).
% Given any three of the 6 basic elements, the function finds the other↙
three.
%
% Usage of function
% for example, Given 3 elements of a spherical triangle: a=2, b=3,↙
C=104.4944158
%[A1, B1, A2, B2,C1,C2,a1,b1,a2,b2,c1,c2] = SphericTriangleSolver(A,B,C,↙
a,b,c)
%[A1, B1, A2, B2,C1,C2,a1,b1,a2,b2,c1,c2] = SphericTriangleSolver↙
(0,0,104.4944158,2,3,0)

% Attention, the unknown (A,B,c) 3 elements of the spherical triangle↙
should be replaced with zero.

% A spherical triangle has 6 basic elements: three angles (A,B,C) and↙
three sides (a,b,c).
% Given any three of the 6 basic elements, the function finds the other↙
three.
%
%
% INPUT:
% A,B,C,a,b,c   element of spherical triangle. Only 3 values should be↙
entered.
% the others must be zero.
% Inputs units must be [decimal degree]



%OUTPUT:
%
%  A1, B1,C1,a1,b1,c1  are 1. solutions of the spherical triangle.
%  A2, B2,C2,a2,b2,c2  are 2. solutions of the spherical triangle.
% outputs units are degrees minutes seconds [dd mm ss]

% Usage of function
%[A1, B1, A2, B2,C1,C2,a1,b1,a2,b2,c1,c2] = SphericTriangleSolver↙
(0,0,104.4944158,2,3,0)
```

```
% Attention, the unknown, desired 3 elements of the spherical triangle should be replaced with zero.

% While preparing this function, Rody P.S. Oldenhuis subfunctions are used.


function [A1, B1, A2, B2,C1,C2,a1,b1,a2,b2,c1,c2] = SphericTriangleSolver(A,B,C,a,b,c)

format long
%
ro=180/pi;
A=0;,B=0;,C=0;     a=2;,b=3;,c=4;
%A=28.97193110;,B=46.5843689140;,C=104.4944158550;     a=0;,b=0;,c=0;
%A=0;,B=46.5843689140;,C=0;     a=2;,b=0;,c=4;
%A=0;,B=0;,C=104.49441585500;     a=0;,b=3;,c=4;
%A=0;,B=46.58436891400;,C=104.4944158550;     a=0;,b=0;,c=4;

%A=112.62944444;,B=65.30972222220;,C=0;     a=0;,b=70.874166666;,c=0;
%A=0;,B=0;,C=0;     a=73.701111110;,b=70.874166666;,c=7.0355555550;
%A=112.62944444;,B=65.30972222220;,C=0;     a=0;,b=70.874166666;,c=0;

%A=120.57333330;,B=0;,C=0;     a=105.4811111110;,b=75.7294444440;,c=0;
%A=112.62944444;,B=65.30972222220;,C=0;     a=0;,b=70.874166666;,c=0;

%A=60.46011667;,B=42.55760065;,C=80.6425750;     a=0;,b=0;,c=0;

%A=25;,B=0;,C=0;     a=0;,b=50;,c=70;
%A=76.255;,B=115.795;,C=0;     a=0;,b=0;,c=81.4367777770;
%A=30;,B=0;,C=0;     a=70;,b=0;,c=80;

%a=148.573333;,b=142.1933333;c=0;     A=153.12666666;,B=0;,C=0;
%A=132.695;,B=107.1163889;,C=0;     a=146.345833333;,b=0;,c=0;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
A1=0;A2=0;B1=0;B2=0 ;C1=0;C2=0;a1=0;a2=0;b1=0;b2=0 ;c1=0;c2=0;

fprintf('Givens  [decimal degree]   \n')
fprintf(' Angles     A=%3d %2.0f %6.4f B=%3d %2.0f %6.4f C=%3d %2.0f %6.4f\n', degrees2dms(A), degrees2dms(B), degrees2dms(C))
```

```
fprintf('Sides   a=%3d %2.0f %6.4f b=%3d %2.0f %6.4f c=%3d %2.0f %6.4↙
f\n', degrees2dms(a), degrees2dms(b), degrees2dms(c))
fprintf('Requested  [dd mm ss]   \n')
A=A/ro;,B=B/ro;,C=C/ro;  a=a/ro;,b=b/ro;,c=c/ro;


%1.option KKK


if (A==0 & B==0 & C==0)
[A1,B1, C1, A2,B2,C2] = sss(a,b,c);
A1=ro*A1;A2=ro*A2;B1=ro*B1;B2=ro*B2 ;C1=ro*C1;C2=ro*C2;a1=ro*a1;↙
a2=ro*a2;b1=ro*b1;b2=ro*b2 ;c1=ro*c1;c2=ro*c2;

fprintf('A1=%3d %2.0f %6.4f B1=%3d %2.0f %6.4f C1=%3d %2.0f %6.4f\n',↙
degrees2dms(A1), degrees2dms(B1), degrees2dms(C1))
%fprintf('A2=%3d %2.0f %6.4f B2=%3d %2.0f %6.4f C2=%3d %2.0f %6.4f\n',↙
degrees2dms(A2), degrees2dms(B2), degrees2dms(C2))

end
%2.option AAA

if (a==0 & b==0 & c==0)
[a1, b1, c1, a2, b2, c2] = aaa(A,B,C);
A1=ro*A1;A2=ro*A2;B1=ro*B1;B2=ro*B2 ;C1=ro*C1;C2=ro*C2;a1=ro*a1;↙
a2=ro*a2;b1=ro*b1;b2=ro*b2 ;c1=ro*c1;c2=ro*c2;

fprintf('a1=%3d %2.0f %6.4f b1=%3d %2.0f %6.4f c1=%3d %2.0f %6.4f\n',↙
degrees2dms(a1), degrees2dms(b1), degrees2dms(c1))
%fprintf('a2=%3d %2.0f %6.4f b2=%3d %2.0f %6.4f c2=%3d %2.0f %6.4f\n',↙
degrees2dms(a2), degrees2dms(b2), degrees2dms(c2))

end

%3.option KAK  [c1, A1, B1, c2, A2, B2] = sas(a, C, b)

if (c==0 & A==0 & B==0)
[c1, A1, B1, c2, A2, B2] = sas(a,C,b);
A1=ro*A1;A2=ro*A2;B1=ro*B1;B2=ro*B2 ;C1=ro*C1;C2=ro*C2;a1=ro*a1;↙
a2=ro*a2;b1=ro*b1;b2=ro*b2 ;c1=ro*c1;c2=ro*c2;
```

```
fprintf('c1=%3d %2.0f %6.4f A1=%3d %2.0f %6.4f B1=%3d %2.0f %6.4f\n',↙
degrees2dms(c1), degrees2dms(A1), degrees2dms(B1))
%fprintf('c2=%3d %2.0f %6.4f A2=%3d %2.0f %6.4f B2=%3d %2.0f %6.4f\n',↙
degrees2dms(c2), degrees2dms(A2), degrees2dms(B2))

end

if (a==0 & B==0 & C==0)
[a1, B1, C1, a2, B2, C2] = sas(b,A,c);
A1=ro*A1;A2=ro*A2;B1=ro*B1;B2=ro*B2 ;C1=ro*C1;C2=ro*C2;a1=ro*a1;↙
a2=ro*a2;b1=ro*b1;b2=ro*b2 ;c1=ro*c1;c2=ro*c2;

fprintf('a1=%3d %2.0f %6.4f B1=%3d %2.0f %6.4f C1=%3d %2.0f %6.4f\n',↙
degrees2dms(a1), degrees2dms(B1), degrees2dms(C1))
%fprintf('a2=%3d %2.0f %6.4f B2=%3d %2.0f %6.4f C2=%3d %2.0f %6.4f\n',↙
degrees2dms(a2), degrees2dms(B2), degrees2dms(C2))
end
%[c1, A1, B1, c2, A2, B2] = sas(a, C, b)

if (b==0 & A==0 & C==0)
[b1,C1, A1, b2, C2, A2] = sas(c,B,a);
A1=ro*A1;A2=ro*A2;B1=ro*B1;B2=ro*B2 ;C1=ro*C1;C2=ro*C2;a1=ro*a1;↙
a2=ro*a2;b1=ro*b1;b2=ro*b2 ;c1=ro*c1;c2=ro*c2;

fprintf('b1=%3d %2.0f %6.4f C1=%3d %2.0f %6.4f A1=%3d %2.0f %6.4f\n',↙
degrees2dms(b1), degrees2dms(C1), degrees2dms(A1))
%fprintf('b2=%3d %2.0f %6.4f C2=%3d %2.0f %6.4f A2=%3d %2.0f %6.4f\n',↙
degrees2dms(b2), degrees2dms(C2), degrees2dms(A2))

end

%4.option AKA   [C1, a1, b1, C2, a2, b2] = asa(A, B, c)

if (C==0 & a==0 & b==0)
[C1,a1, b1, C2 a2, b2] = asa(A,B,c);
A1=ro*A1;A2=ro*A2;B1=ro*B1;B2=ro*B2 ;C1=ro*C1;C2=ro*C2;a1=ro*a1;↙
a2=ro*a2;b1=ro*b1;b2=ro*b2 ;c1=ro*c1;c2=ro*c2;

fprintf('C1=%3d %2.0f %6.4f a1=%3d %2.0f %6.4f b1=%3d %2.0f %6.4f\n',↙
degrees2dms(C1), degrees2dms(a1), degrees2dms(b1))
%fprintf('C2=%3d %2.0f %6.4f a2=%3d %2.0f %6.4f b2=%3d %2.0f %6.4f\n',↙
```

```
degrees2dms(C2), degrees2dms(a2), degrees2dms(b2))

end

if (A==0 & b==0 & c==0)
[A1, b1, c1, A2, b2, c2] = asa(B,C,a);
A1=ro*A1;A2=ro*A2;B1=ro*B1;B2=ro*B2 ;C1=ro*C1;C2=ro*C2;a1=ro*a1;↙
a2=ro*a2;b1=ro*b1;b2=ro*b2 ;c1=ro*c1;c2=ro*c2;

fprintf('A1=%3d %2.0f %6.4f b1=%3d %2.0f %6.4f c1=%3d %2.0f %6.4f\n',↙
degrees2dms(A1), degrees2dms(b1), degrees2dms(c1))
%fprintf('A2=%3d %2.0f %6.4f b2=%3d %2.0f %6.4f c2=%3d %2.0f %6.4f\n',↙
degrees2dms(A2), degrees2dms(b2), degrees2dms(c2))

end

if (B==0 & a==0 & c==0)
[B1,  c1,a1, B2,c2, a2] = asa(C,A,b);[B1,  c1,a1, B2,c2, a2] =[B1,  c1,↙
a1, B2,c2, a2] *ro;
fprintf('a1=%3d %2.0f %6.4f c1=%3d %2.0f %6.4f B1=%3d %2.0f %6.4f\n',↙
degrees2dms(a1), degrees2dms(c1), degrees2dms(B1))
%fprintf('a2=%3d %2.0f %6.4f c2=%3d %2.0f %6.4f B2=%3d %2.0f %6.4f\n',↙
degrees2dms(a2), degrees2dms(c2), degrees2dms(B2))

end

%5.option KKA

if (c==0 & B==0 & C==0)
[c1, B1, C1, c2, B2, C2] = ssa(a,b,A);
A1=ro*A1;A2=ro*A2;B1=ro*B1;B2=ro*B2 ;C1=ro*C1;C2=ro*C2;a1=ro*a1;↙
a2=ro*a2;b1=ro*b1;b2=ro*b2 ;c1=ro*c1;c2=ro*c2;

fprintf('B1=%3d %2.0f %6.4f C1=%3d %2.0f %6.4f c1=%3d %2.0f %6.4f\n',↙
degrees2dms(c1), degrees2dms(B1), degrees2dms(C1))
fprintf('B2=%3d %2.0f %6.4f C2=%3d %2.0f %6.4f c2=%3d %2.0f %6.4f\n',↙
degrees2dms(c2), degrees2dms(B2), degrees2dms(C2))

end

if (a==0 & A==0 & C==0)
```

```
[a1,A1, C1, a2, A2, C2] = ssa(b,c,B);
A1=ro*A1;A2=ro*A2;B1=ro*B1;B2=ro*B2 ;C1=ro*C1;C2=ro*C2;a1=ro*a1;↙
a2=ro*a2;b1=ro*b1;b2=ro*b2 ;c1=ro*c1;c2=ro*c2;


fprintf('C1=%3d %2.0f %6.4f A1=%3d %2.0f %6.4f a1=%3d %2.0f %6.4f\n',↙
degrees2dms(a1), degrees2dms(A1), degrees2dms(C1))
fprintf('C2=%3d %2.0f %6.4f A2=%3d %2.0f %6.4f a2=%3d %2.0f %6.4f\n',↙
degrees2dms(a2), degrees2dms(A2), degrees2dms(C2))

end

if (b==0 & A==0 & B==0)
[A1, B1, b1, A2, B2, b2] = ssa(c,a,C);
A1=ro*A1;A2=ro*A2;B1=ro*B1;B2=ro*B2 ;C1=ro*C1;C2=ro*C2;a1=ro*a1;↙
a2=ro*a2;b1=ro*b1;b2=ro*b2 ;c1=ro*c1;c2=ro*c2;

fprintf('b1=%3d %2.0f %6.4f A1=%3d %2.0f %6.4f B1=%3d %2.0f %6.4f\n',↙
degrees2dms(b1), degrees2dms(A1), degrees2dms(B1))
fprintf('b2=%3d %2.0f %6.4f A2=%3d %2.0f %6.4f B2=%3d %2.0f %6.4f\n',↙
degrees2dms(b2), degrees2dms(A2), degrees2dms(B2))

end

%6.option AAK

if (C==0 & c==0 & b==0)
[b1, c1, C1, b2, c2, C2] = aas(A,B,a);
A1=ro*A1;A2=ro*A2;B1=ro*B1;B2=ro*B2 ;C1=ro*C1;C2=ro*C2;a1=ro*a1;↙
a2=ro*a2;b1=ro*b1;b2=ro*b2 ;c1=ro*c1;c2=ro*c2;

fprintf('c1=%3d %2.0f %6.4f b1=%3d %2.0f %6.4f C1=%3d %2.0f %6.4f\n',↙
degrees2dms(c1), degrees2dms(b1), degrees2dms(C1))
fprintf('c2=%3d %2.0f %6.4f b2=%3d %2.0f %6.4f C2=%3d %2.0f %6.4f\n',↙
degrees2dms(c2), degrees2dms(b2), degrees2dms(C2))

end

if (A==0 & a==0 & c==0)
[A1, a1, c1, A2, a2, c2] = aas(B,C,b);
A1=ro*A1;A2=ro*A2;B1=ro*B1;B2=ro*B2 ;C1=ro*C1;C2=ro*C2;a1=ro*a1;↙
```

```
a2=ro*a2;b1=ro*b1;b2=ro*b2 ;c1=ro*c1;c2=ro*c2;

fprintf('A1=%3d %2.0f %6.4f c1=%3d %2.0f %6.4f a1=%3d %2.0f %6.4f\n',↙
degrees2dms(c1), degrees2dms(A1), degrees2dms(a1))
fprintf('A2=%3d %2.0f %6.4f c2=%3d %2.0f %6.4f a2=%3d %2.0f %6.4f\n',↙
degrees2dms(c2), degrees2dms(A2), degrees2dms(a2))

end

if (B==0 & a==0 & b==0)
[a1, B1, b1, a2, B2, b2] = aas(C,A,c);
A1=ro*A1;A2=ro*A2;B1=ro*B1;B2=ro*B2 ;C1=ro*C1;C2=ro*C2;a1=ro*a1;↙
a2=ro*a2;b1=ro*b1;b2=ro*b2 ;c1=ro*c1;c2=ro*c2;

fprintf('a1=%3d %2.0f %6.4f B1=%3d %2.0f %6.4f c1=%3d %2.0f %6.4f\n',↙
degrees2dms(a1), degrees2dms(b1), degrees2dms(B1))
fprintf('a2=%3d %2.0f %6.4f B2=%3d %2.0f %6.4f c2=%3d %2.0f %6.4f\n',↙
degrees2dms(a2), degrees2dms(b2), degrees2dms(B2))

end

%%%%%==================================================
%7.option  AKK

if (b==0 & B==0 & C==0)
[c1, B1, C1, c2, B2, C2] = ssa(a,c,A);
A1=ro*A1;A2=ro*A2;B1=ro*B1;B2=ro*B2 ;C1=ro*C1;C2=ro*C2;a1=ro*a1;↙
a2=ro*a2;b1=ro*b1;b2=ro*b2 ;c1=ro*c1;c2=ro*c2;
fprintf('C1=%3d %2.0f %6.4f B1=%3d %2.0f %6.4f b1=%3d %2.0f %6.4f\n',↙
degrees2dms(c1), degrees2dms(B1), degrees2dms(C1))
fprintf('C2=%3d %2.0f %6.4f B2=%3d %2.0f %6.4f b2=%3d %2.0f %6.4f\n',↙
degrees2dms(c2), degrees2dms(B2), degrees2dms(C2))
end

if (c==0 & A==0 & C==0)
[c1, A1, C1, c2, A2, C2] = ssa(b,a,B);
A1=ro*A1;A2=ro*A2;B1=ro*B1;B2=ro*B2 ;C1=ro*C1;C2=ro*C2;a1=ro*a1;↙
a2=ro*a2;b1=ro*b1;b2=ro*b2 ;c1=ro*c1;c2=ro*c2;

fprintf('A1=%3d %2.0f %6.4f c1=%3d %2.0f %6.4f C1=%3d %2.0f %6.4f\n',↙
degrees2dms(c1), degrees2dms(C1), degrees2dms(A1))
```

```
fprintf('A2=%3d %2.0f %6.4f c2=%3d %2.0f %6.4f C2=%3d %2.0f %6.4f\n',↙
degrees2dms(c2), degrees2dms(C2), degrees2dms(A2))
end

if (a==0 & A==0 & B==0)
[a1, A1, B1, a2, A2, B2] = ssa(c,b,C);
A1=ro*A1;A2=ro*A2;B1=ro*B1;B2=ro*B2 ;C1=ro*C1;C2=ro*C2;a1=ro*a1;↙
a2=ro*a2;b1=ro*b1;b2=ro*b2 ;c1=ro*c1;c2=ro*c2;

fprintf('B1=%3d %2.0f %6.4f a1=%3d %2.0f %6.4f A1=%3d %2.0f %6.4f\n',↙
degrees2dms(a1), degrees2dms(B1), degrees2dms(A1))
fprintf('B2=%3d %2.0f %6.4f a2=%3d %2.0f %6.4f A2=%3d %2.0f %6.4f\n',↙
degrees2dms(a2), degrees2dms(B2), degrees2dms(A2))
end


% 8.option  KAA

if (c==0 & B==0 & b==0)
[c1, b1, B1, c2, b2,B2] = aas2(A,C,a);
A1=ro*A1;A2=ro*A2;B1=ro*B1;B2=ro*B2 ;C1=ro*C1;C2=ro*C2;a1=ro*a1;↙
a2=ro*a2;b1=ro*b1;b2=ro*b2 ;c1=ro*c1;c2=ro*c2;

fprintf('c1=%3d %2.0f %6.4f B1=%3d %2.0f %6.4f b1=%3d %2.0f %6.4f\n',↙
degrees2dms(c1), degrees2dms(B1), degrees2dms(b1))
fprintf('c2=%3d %2.0f %6.4f B2=%3d %2.0f %6.4f b2=%3d %2.0f %6.4f\n',↙
degrees2dms(c2), degrees2dms(B2), degrees2dms(b2))
end

if (c==0 & a==0 & C==0)
[c1, a1, C1, c2, a2,C2] = aas2(A,B,b);
A1=ro*A1;A2=ro*A2;B1=ro*B1;B2=ro*B2 ;C1=ro*C1;C2=ro*C2;a1=ro*a1;↙
a2=ro*a2;b1=ro*b1;b2=ro*b2 ;c1=ro*c1;c2=ro*c2;

fprintf('c1=%3d %2.0f %6.4f a1=%3d %2.0f %6.4f C1=%3d %2.0f %6.4f\n',↙
degrees2dms(a1), degrees2dms(c1), degrees2dms(C1))
fprintf('c2=%3d %2.0f %6.4f a2=%3d %2.0f %6.4f C2=%3d %2.0f %6.4f\n',↙
degrees2dms(a2), degrees2dms(c2), degrees2dms(C2))
end

if (b==0 & a==0 & A==0)
```

```
[b1, a1, A1, b2, a2,A2] = aas2(C,B,c);
A1=ro*A1;A2=ro*A2;B1=ro*B1;B2=ro*B2 ;C1=ro*C1;C2=ro*C2;a1=ro*a1;
a2=ro*a2;b1=ro*b1;b2=ro*b2 ;c1=ro*c1;c2=ro*c2;

fprintf('a1=%3d %2.0f %6.4f A1=%3d %2.0f %6.4f b1=%3d %2.0f %6.4f\n',
degrees2dms(a1), degrees2dms(A1), degrees2dms(b1))
fprintf('a2=%3d %2.0f %6.4f A2=%3d %2.0f %6.4f b2=%3d %2.0f %6.4f\n',
degrees2dms(a2), degrees2dms(A2), degrees2dms(b2))
end

%a1, B1, c1degrees2dms
function [a1, b1, c1, a2, b2, c2] = aaa(A, B, C)
%AAA  gives both solutions to the angle-angle-angle problem, in radians.
%
%   AAA(A, B, C) will result in NaNs if the existence condition
%   |pi - |A|-|B|| <= |C| <= pi - ||A| - |B||
%   is not met.
%


    % first solution
    a1 = acos2( (cos(A) + cos(B).*cos(C)) ./ (sin(B).*sin(C)),  A );
    b1 = acos2( (cos(B) + cos(A).*cos(C)) ./ (sin(A).*sin(C)),  B );
    c1 = acos2( (cos(C) + cos(A).*cos(B)) ./ (sin(A).*sin(B)),  C );

    % second solution
    a2 = 2*pi - a1;
    b2 = 2*pi - b1;
    c2 = 2*pi - c1;

    % check constraints
    indices = ( ...
        abs(pi - abs(A)-abs(B)) > abs(C) | ...
        abs(C) > pi - abs(abs(A)-abs(B)) );
    a1(indices) = NaN;    a2(indices) = NaN;
    b1(indices) = NaN;    b2(indices) = NaN;
    c1(indices) = NaN;    c2(indices) = NaN;

end
function [b1, c1, C1, b2, c2, C2] = aas(A, B, a)
%AAS   gives both solutions to the angle-angle-side problem, in radians.
```

```
%
%   AAS(A, B, a) may result in a vector filled with NaNs if the↙
existence
%   condition |sin(B)sin(a)| <= |sin(A)| is not met. This function uses↙
the
%   Middle Side Law function MSL.m and Middle Angle Law function MAL.m↙
to
%   determine the solutions.
%

    % first solution
    b0 = asin( (sin(B).*sin(a))./sin(A) );
    b0(imag(b0) ~= 0) = NaN;

    b1 = mod(b0, 2*pi);
    c1 = msl(a, b1, A, B);
    C1 = mal(A, B, a, b1);

    % second solution
    b2 = mod(pi - b1, 2*pi);
    c2 = msl(a, b2, A, B);
    C2 = mal(A, B, a, b2);




    % check constraints
    indices = ( abs(sin(B).*sin(a)) > abs(sin(A)) );
    b1(indices) = NaN;    c1(indices) = NaN;
    C1(indices) = NaN;    b2(indices) = NaN;
    c2(indices) = NaN;    C2(indices) = NaN;

end


% Middle-angle-law
function C = mal(A, B, a, b)
%MAL    Computes the missing angle in a spherical triangle, in radians.
%
%   MAL(A, B, a, b) is the implementation of the Middle Angle Law, and
%   returns the missing angle C.
```

```
%
%   See also MSL, MALD.

    % sine & cosine of C
    % NOTE: denomenator not needed
    sinC  =  sin(A).*cos(B).*cos(b) + sin(B).*cos(A).*cos(a);
    cosC  = -cos(A).*cos(B) + sin(A).*sin(B).*cos(a).*cos(b);

    % C is the arctangent of the ratio of these two
    C = mod( atan2(sinC, cosC), 2*pi);

end

% Middle-side-law
function c = msl(a, b, A, B)
%MSL    Computes the missing side in a spherical triangle, in radians.
%
%   MSL(a, b, A, B) is the implementation of the Middle Side Law, and
%   returns the missing angular side c.

    sinc  = (sin(a).*cos(b).*cos(B) + sin(b).*cos(a).*cos(A));
    cosc  = (cos(a).*cos(b) - sin(a).*sin(b).*cos(A).*cos(B));

    % c is the arctangent of the sine over the cosine
    c = mod( atan2(sinc, cosc), 2*pi);

end


function signedcos = acos2(alpha, beta)
%ACOS2      4-quadrant arccosine function, in radians.
%
%   ACOS2(alpha, beta) computes the four-quadrant arccosine of the amgle
%   [alpha]. For arguments |alpha| > 1, the result is NaN. The resulting
%   angle is not uniquely determined by alpha, nor by the lengths or
%   order of the sides of the triangle (as in ATAN2), so an additional
%   argument [beta] is required. If [beta] < pi/2, the small angle
%   (0 <= alpha <= pi/2) is returned. If [beta] > pi/2, the large angle
%   (pi/2 < alpha < pi) is returned.
%
%   See also acos2d.
```

```
    H = 2*( mod(beta, 2*pi) < pi ) - 1;
    H(~isreal(H)) = NaN;

    % compute signed arc-cosine
    signedcos = H .* acos(alpha);

    % set complex results to NaN & take the modulus
    signedcos(imag(signedcos) ~= 0) = NaN;
    signedcos = mod(signedcos, 2*pi);

    % determine alphaues for zero-alphaued acos
    ind1 = (signedcos == 0);
    ind2     = (H < 0);                   ind3     = (H > 0);
    indices1 = ((ind1 + ind2) == 2);      indices2 = ((ind1 + ind3) == 2);
    signedcos(indices1) = pi;             signedcos(indices2) = 0;

end

function [C1, a1, b1, C2, a2, b2] = asa(A, B, c)
%ASA   gives both solutions to the angle-side-angle problem, in radians.
%
%   ASA(A, B, c) returns the missing values C, a, b. It uses the
%   four-quadrant arccosine function ACOS2 to determine these values.
%

    % first solution
    % NOTE: normal acos (in stead of acos2) is indeed correct.
    C1 = acos( -cos(A) .*cos(B) + sin(A).*sin(B).*cos(c));
    a1 = acos( (cos(A) + cos(B).*cos(C1)) ./ (sin(B).*sin(C1)));
    b1 = acos( (cos(B) + cos(A).*cos(C1)) ./ (sin(A).*sin(C1)));

    C1(imag(C1) ~= 0) = NaN;
    a1(imag(a1) ~= 0) = NaN;
    b1(imag(b1) ~= 0) = NaN;

    % second solution
    C2 = 2*pi - C1;
    a2 = mod(a1 + pi, 2*pi);
```

```
    b2 = mod(b1 + pi, 2*pi);

end
function [c1, A1, B1, c2, A2, B2] = sas(a, C, b)
%SAS   gives both solutions to the side-angle-side problem, in radians.
%
%   SAS(a, C, b) returns the remaining unknowns of the spherical↙
triangle,
%   [c1, A1, B1, c2, A2, B2].
%



    % first solution
    c1 = acos2( cos(a).*cos(b) + sin(a).*sin(b).*cos(C),  C );
    A1 = acos2( (cos(a) - cos(b).*cos(c1))./(sin(b).*sin(c1)),  a );
    B1 = acos2( (cos(b) - cos(a).*cos(c1))./(sin(a).*sin(c1)),  b );

    % second solution
    c2 = 2*pi - c1;
    A2 = mod(A1 + pi, 2*pi);
    B2 = mod(B1 + pi, 2*pi);

end

function [B1, C1, c1, B2, C2, c2] = ssa(a, b, A)
%SSA   gives both solutions to the side-side-angle problem, in radians.
%
%   SSA(a, b, A) will result in NaNs if the existence condition
%   |sin b * sin A| <= | sin a |  is not met.
%
%


    % first solution
    B0 = asin(sin(b).*sin(A)./sin(a));
    B0(imag(B0) ~= 0) = NaN;

    B1 = mod(B0, 2*pi);
    C1 = mal(A, B1, a, b);
    c1 = msl(a, b, A, B1);

    % second solution
```

```
    B2 = mod(pi - B1, 2*pi);
    C2 = mal(A, B2, a, b);
    c2 = msl(a, b, A, B2);

    % check constraints
    indices = ( abs(sin(b).*sin(A)) > abs(sin(a)) );
    B1(indices) = NaN;    C1(indices) = NaN;
    c1(indices) = NaN;    B2(indices) = NaN;
    C2(indices) = NaN;    c2(indices) = NaN;

end




function [A1, B1, C1, A2, B2, C2] = sss(a, b, c)
%SSS   gives both solutions to the side-side-side problem, in radians.
%
%   SSS(a, b, c) results in NaNs for those indices where the existence
%   condition |pi - a| - |pi - b| <= |pi - c| <= |pi - a| + |pi -b| is↙
not
%   met.
%

    % first solution
    A1 = acos2( (cos(a) - cos(b).*cos(c))./(sin(b).*sin(c)), a);
    B1 = acos2( (cos(b) - cos(a).*cos(c))./(sin(a).*sin(c)), b);
    C1 = acos2( (cos(c) - cos(a).*cos(b))./(sin(a).*sin(b)), c);

    % second solution
    A2 = 2*pi - A1;
    B2 = 2*pi - B1;
    C2 = 2*pi - C1;

    % check constraints
    indices = ( ...
        (abs(pi-a) - abs(pi-b)) > abs(pi-c) | ...
        abs(pi-c) > (abs(pi-a) + abs(pi-b)) );
    A1(indices) = NaN;   B1(indices) = NaN;   C1(indices) = NaN;
    A2(indices) = NaN;   B2(indices) = NaN;   C2(indices) = NaN;
```

```
end
function [a1, c1, C1, a2, c2, C2] = aas2(A, B, b)

%benimki    aas2

a1=asin(sin(A)/sin(B)*sin(b));
a2=pi-a1;
%1.çözüm

C1=2* atan(cos((a1-b)/2)/tan((A+B)/2)/cos((a1+b)/2));
C2=2* atan(cos((a2-b)/2)/tan((A+B)/2)/cos((a2+b)/2));

c1=2* atan(tan((a1+b)/2)*cos((A+B)/2)/cos((A-B)/2));
c2=2* atan(tan((a2+b)/2)*cos((A+B)/2)/cos((A-B)/2));


end
end
```

Spherical trigonometry

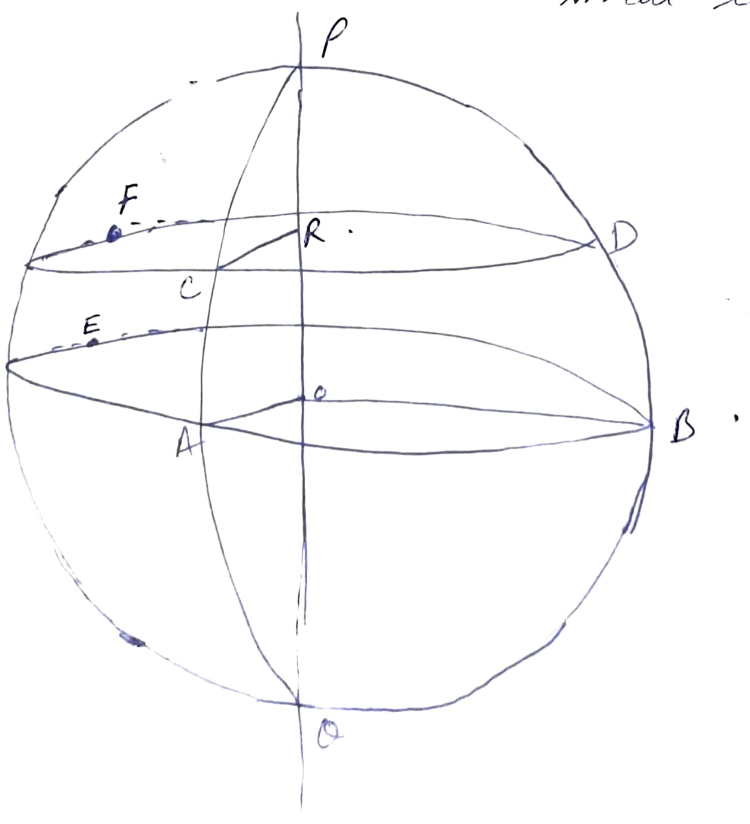
Spherical Astronomy is concerned essentially with the direction in which the stars are viewed.

great circle

Any plane passing through the center of a sphere cuts the surface in a circle which is called as great circle.

small circle.

Any other plane intersecting the sphere but not passing through the centre will also cut the surface in a circle, which, in this case, is ~~a~~ called a small circle.

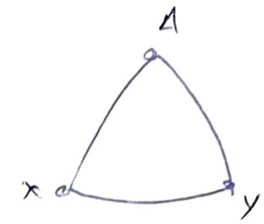


PCD is not a spherical Δ.
as, CD is not a great circle.

Any three points on the surface of a sphere, then the sphere can be bisected so that all three points lie in the same hemisphere.

and if the points are joined by great circle arcs all lying on this hemisphere, the figure obtained is called a spherical triangle.

for spherical triangle AXY



AX, AY, XY are the sides

and, spherical angles at X, Y, A are the angle of spherical triangle.

So, If R is the radius of the sphere, the length of spherical arc AY is given by

$$AY = R \times \text{angle } AOY.$$

(AOY is in radians).

Length of small circle Arc

Consider the small circle arc. CD.

length CD $\quad CD = RC \times \text{angle } CRD.$

also, $\quad AD = OA \times \text{angle of } AOB.$

Since the plane FCD is parallel to the plane EAB.

then, $C\hat{R}D = A\hat{O}B$.

for $RC \parallel OA$ and, $RD \parallel OB$

$$\Rightarrow \quad CD = \frac{RC}{OA} AB.$$

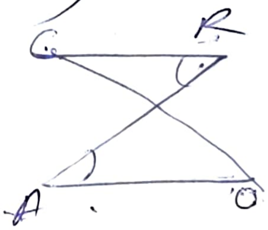
$OA = OC$ (radii of sphere)

$$CD = \frac{RC}{OC} \cdot AB$$

we have, $RC \perp OR$.

$$\Rightarrow \quad RC = OC \cos R\hat{C}O.$$

(as, $RC \parallel OA$)

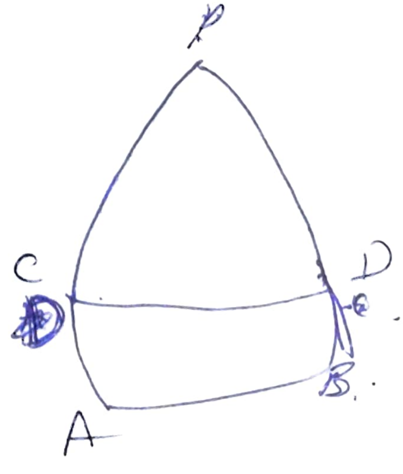


$$CD = AB \cos R\hat{C}O.$$

$$= AB \cos A\hat{O}C.$$

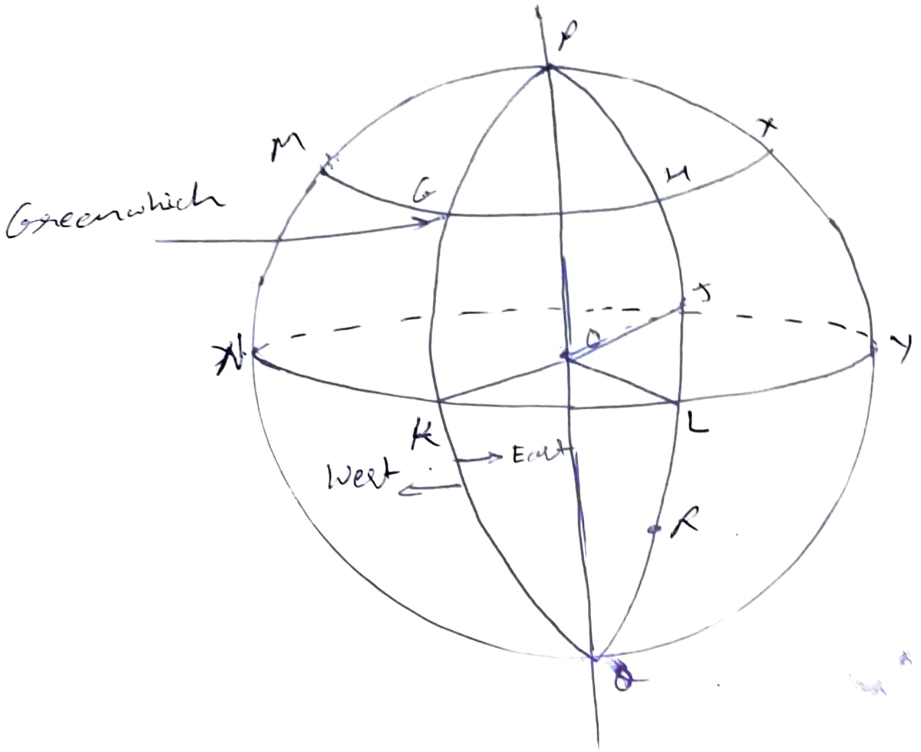
AOC is angle subtended at centre of the sphere by great circle arc AC.

then, $CD = AB \cos AC$.



Since, $PA = 90^\circ$

$$CD = AB \sin PC \quad \text{---} (1).$$



Any semi-great circle terminated by P and Q is a meridian.

Longitudes are measured from 0° to 180° east of greenwhich meridian.
and 0° to 180° West

meridian through J cuts the equator in L and the angle LOJ or great circle arc LJ, is called the Latitude of J.
If J is b/w equator and the north pole P, Lattitude is said to be north (N).
Places R b/w equator and south pole Q, is said to be, in South latitude.
Let ϕ denote the latitude of J: then, $L\hat{O}J$ or $LJ = ($

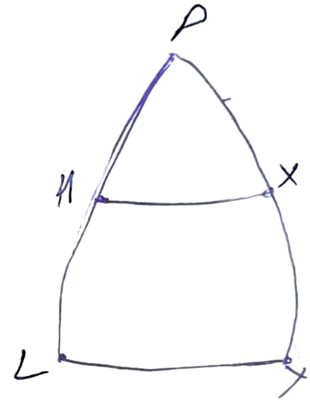
Since OP is ⊥ to equator, $P\hat{O}L = 90^\circ$

and therefore

$$POJ = 90^\circ - \phi.$$

$\angle POJ$ or, spherical arc PJ ~~arc~~ is the Colatitude of J.

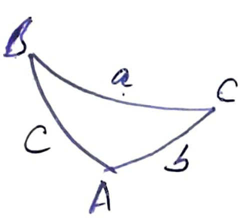
$$\text{Colat} = 90^\circ - \text{Lat}:$$

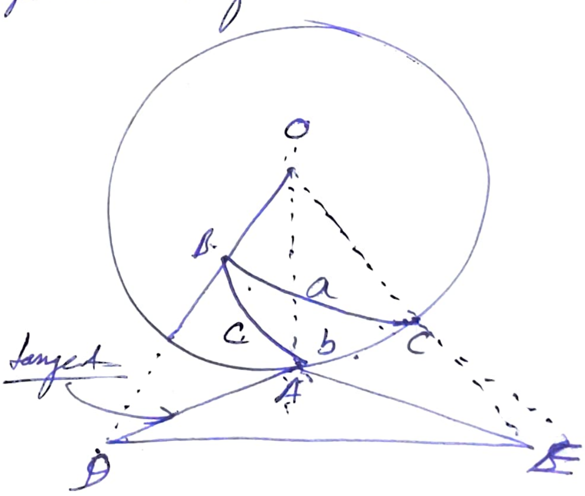


Then,

$$HX = LY\sin PH.$$
$$= LY\sin(90 - HL)$$
$$= LY\cos HL.$$
$$= LY\cos\theta \quad \text{------} (2)$$

(3). fundamental formula for spherical trigo.





Let AD be the tangent to the great circle AC at A and AE be the tangent to BC at great circle BA.

$$\angle BAC = D\hat{A}E = A.$$

Now in plane triangle OAD, $O\hat{A}D$ is 90° and. $A\hat{O}D$.

$$\angle AOD = \angle A\hat{O}C.$$

Then,

$$AD = OA \tan c; \qquad OD = OA \sec c. \quad \text{--- (3)}$$

from the plane triangle OAE.

we have.

$AE = OA \tan b$. ~~OE = OA OE~~

and $\quad OE = OA \sec b$. --- (4).

from plane triangle. DAE.

$$DE^2 = AD^2 + AE^2 - 2\, AD.AE \cos(D\hat{A}E).$$

$$= (OA \tan c)^2 + (OA \tan b)^2 - 2 (OA)\tan c (OA)\sec b \cos A.$$

$$= OA^2 \left[\tan^2 c + \tan^2 b - 2 \tan b \tan c \cos A\right]. \quad \text{--- (5)}.$$

from the plane triangle DOE.

$$DE^2 = OD^2 + OE^2 - 2(OD)(OE) \cos D\hat{O}E$$

But, $\quad D\hat{O}E = B\hat{O}C = a.$

$$= \left[(OA \sec c)^2 + (OA \sec b)^2 - 2(OA)^2 \sec c \sec b \cos a\right]$$

$$DE^2 = (OA)^2 \left[\sec^2 c + \sec^2 b - 2 \sec c \sec b \cos a\right]. \quad \text{--- (6)}.$$

$\Rightarrow \quad 6 = 5$

$$\tan^2 c + \tan^2 b - 2 \tan b \tan c \cos A$$

$$= \sec^2 c + \sec^2 b - 2 \sec b \sec c \cos a$$

Now, $\quad \sec^2 c = 1 + \tan^2 c, \; \sec^2 b = 1 + \tan^2 b.$

So.

$$-2\tan b \tan c \cos A = 1 + 1 - 2\sec b \sec c \cos a$$

$$\cancel{2}\sec b \sec c \cos a = \cancel{2}(1 + \tan b \tan c \cos A)$$

$$\cos a = \frac{1}{\sec b \sec c} + \frac{\tan b \tan c \cos A}{\sec b \sec c}.$$

$$\boxed{\cos a = \cos b \cos c + \sin b \sin c \cos A} \quad \ldots \text{(A)}$$

cosine formula.

fundamental formula of spherical trigonometry.

so,

$$\cos b = \cos c \cos a + \sin c \sin a \cos B \quad \ldots (7)$$

$$\cos c = \cos a \cos b + \sin a \sin b \cos C \quad \ldots (8)$$

---

two direct practical solution.

1. if two sides, e.g., b and c and ~~or~~ the included angle A. of a spherical triangle ABC are known. third side a can be calculated.

2. if all three sides are known. the angles of the ~~spherical~~ triangle can be found successively by (7) (8).

$\cos a = \cos b \cos c + \sin b \sin c \cos A$.

we need $\cos A$.

then,

$$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c}.$$

$$\cos A = \operatorname{cosec} b \operatorname{cosec} c \left[\cos a - \cos b \cos c\right]. \quad \text{--- (9)}$$

Since,

$$\cos A = 1 - 2\sin^2\frac{A}{2}.$$

$$\cos a = \cos b \cos c + \sin b \sin c\left[1 - 2\sin^2\frac{A}{2}\right].$$

$$\cos a = \cos(b-c) - 2\sin b \sin c \sin^2\frac{A}{2}.$$

So, $\cos(b-c) - \cos(a) = 2\sin b \sin c \sin^2\left(\frac{A}{2}\right)$.

$$2\sin\left(\frac{a+b-c}{2}\right)\sin\left(\frac{a-(b-c)}{2}\right) = 2\sin b \sin c \sin^2\frac{A}{2}.$$

Let, s is, $2s = a+b+c$.

then, $a+b-c = 2(s-c)$

and $a-b+c = 2(s-b)$.

~~then~~ Hence

$$\cancel{2}\sin(s-c)\sin(s-b) = \cancel{2}\sin b \sin c \sin^2\frac{A}{2}.$$

now

$$\sin\frac{A}{2} = \sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin b\sin c}} \quad \ldots (11).$$

$$\sin\frac{B}{2} = \sqrt{\frac{\sin(s-c)\sin(s-a)}{\sin c\sin a}}.$$

$$\sin\frac{C}{2} = \sqrt{\frac{\sin(s-a)\sin(s-b)}{\sin a\sin b}}$$

---

if we use.

$$\cos A = 2\cos^2\frac{A}{2} - 1.$$

then

$$\cos a = \cos b\cos c + \sin b\sin c\cos A$$

$$= \cos b\cos c + \sin b\sin c\left(2\cos^2\frac{A}{2} - 1\right)$$

$$\cos a = \cos(b+c) + 2\cos^2\frac{A}{2}\sin b\sin c.$$

~~$\cos a - \cos bc$~~

$$2\cos^2\frac{A}{2}\sin b\sin c = \cos(b+c) - \cos a$$

$$= 2\sin\left(\frac{b+c+a}{2}\right)\sin\left(\frac{b+c-a}{2}\right)$$

$$2\cos^2\frac{A}{2}\sin b\sin c = 2\sin(s)\sin(s-a)$$

$$\cos\frac{A}{2} = \sqrt{\frac{\sin(s)\sin(s-a)}{\sin b\sin c}} \quad (12).$$

$||^{ly}$

$$\cos\frac{B}{2} = \sqrt{\frac{\sin(s)\sin(s-b)}{\sin a\sin c}}.$$

$$\cos\frac{C}{2} = \sqrt{\frac{\sin s\sin(s-c)}{\sin a\sin b}}$$

$$\tan\frac{A}{2} = \sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin(s)\sin(s-a)}}$$

so similarly.

$$\tan\frac{B}{2} = \sqrt{\frac{\sin(s-c)\sin(s-a)}{\sin s \sin(s-b)}}$$

$$\tan\frac{C}{2} = \sqrt{\frac{\sin(s-a)(s-b)}{\sin(s)\sin(s-c)}}.$$

---

(6) Sine formula.

$\cos a = \cos b \cos c + \sin b \sin c \cos A.$

$\sin b \sin c \cos A = \cos a - \cos b \cos c.$

$\sin^2 b \sin^2 c \cos^2 A. = \cancel{\cos^2 a} \cos^2 a + \cos^2 b \cos^2 c.$

$- 2\cos a \cos b \cos c.$

see L.H.S.

$\sin^2 b \sin^2 c (1 - \sin^2 A) = \sin^2 b \sin^2 c \cos^2 A.$

$\sin^2 b \sin^2 c - \sin^2 b \sin^2 c \sin^2 A.$

$= \sin^2 b - \sin^2 b \cos^2 c - \sin^2 b \sin^2 c \sin^2 A.$

$= 1 - \cos^2 b - \cancel{\cos^2 c} + \cos^2 b \cos^2 c - \sin^2 b \sin^2 c \sin^2 A$

so ~~$\sin^2 b \sin^2 c \sin^2 A$~~ =

$\sin^2 b \sin^2 c \cancel{\sin^2} A.$

$$= 1 - \cos^2 b - \cancel{\sin^2 b}^{\cos^2 c} + \cos^2 b \cos^2 c - \sin^2 b \sin^2 c \sin^2 A.$$

$$= \cos^2 a + \cos^2 b \cos^2 c - 2(\cos a \cos b \cos c)$$

$\cancel{1 - \cos^2 b - \sin^2 b \to}$

$$1 - \cos^2 a - \cos^2 b - \cos^2 c + 2(\cos a \cos b \cos c)$$
$$= \sin^2 b \sin^2 c \sin^2 A.$$

Let x be defined by.

$$x^2 \sin^2 a \sin^2 b \sin^2 c \Big| = 1 - \cos^2 a - \cos^2 b - \cos^2 c + 2\cos a \cos b \cos c.$$

$$\frac{x^2 \sin^2 a \cancel{\sin^2 b \sin^2 c}}{\cancel{\sin^2 b \sin^2 c} \sin^2 A} = 1$$

$$x^2 = \frac{\sin^2 A}{\sin^2 a}$$

$$x = \pm \frac{\sin A}{\sin a}$$

We can find,

$$x = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c} = \frac{\sin A}{\sin a}.$$

Suppose two sides a and b and the angles are given.

then,

$$\cancel{\sin B = \sin c} \quad \sin A = \frac{\sin a \sin B}{\sin b}$$

eq^n - (7).

$$\cos b = \cos a \cos c + \sin a \sin c \cos B.$$

So, $\sin a \sin c \cos B = \cos b - \cos a \cos c.$

$$= \cos b - \cos c(\cos b \cos c + \sin b \sin c \cos A).$$

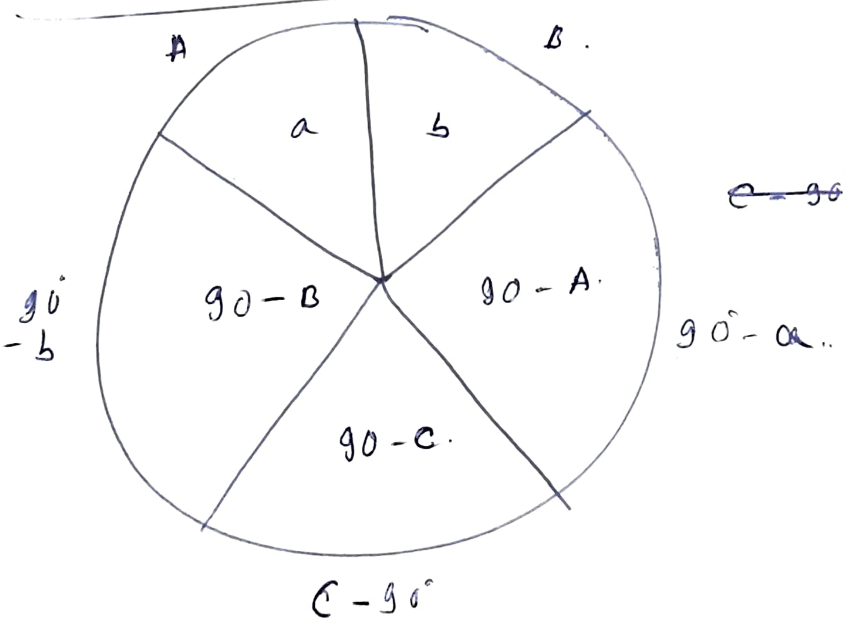
$$= \sin^2 c \cos b - \sin b \sin c \cos c \cos A.$$

$$\sin a \cos \underline{B} = \sin c \cos \underline{b} - \sin \underline{b} \cos c \cos A. \quad \ldots \quad (c)$$

relation ~~inf~~ involving all three sides

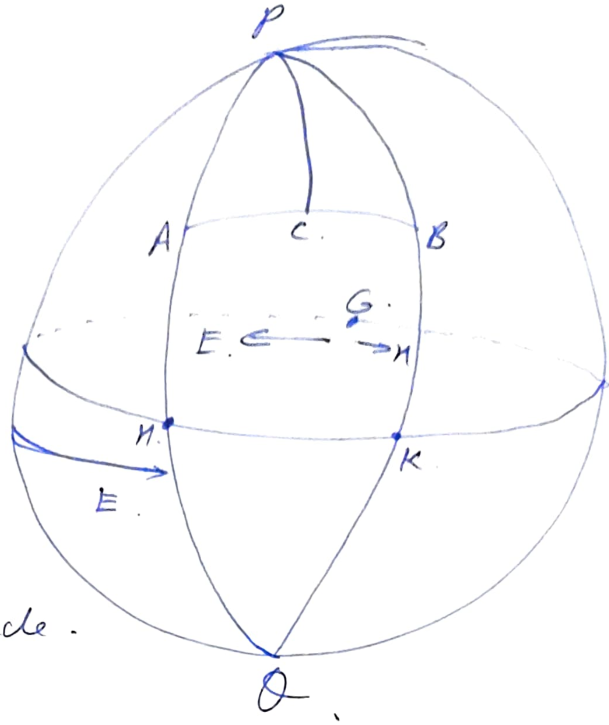
we can write.

$$\sin a \cos \underline{C} = \cos \underline{c} \sin b - \sin \underline{c} \cos b \cos A \quad \ldots \quad (14)$$



Latitude $24°18'N$.
$36°47'N$.

Longitude $133°39'E$.
$125°24W$.



find:
① Length of the great circle arc AB.

② $\angle PAB$, P being the north pole,

③. most nearby northerly point on the great circle AB.

PAHQ - meridian through A. cutting the equator at H.

and HA - latitude of A, ~~i.e, HA =~~.

i.e, $HA = 24°18'$.

and, PA is colatitude of A,

$\therefore PA = 90° - 24°18' = 65°42'$.

$|||^{ly}$, $PB = \cancel{90° - 53}\ 90° - 36°47' = 53°13'$

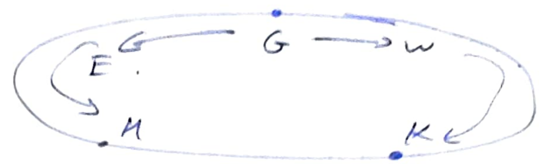
colatitude

Let the greenwhich meridian intersects the equator in G.

Then.

$GH = \text{Long}(E)$ of A

$= 133^\circ 39'$.



$GK = \text{long}(W)$ of $B = 125^\circ 24'$ ~~tot~~.

So the great circle arc. $AB = HK$ (great circle)

$= GH + GK$.

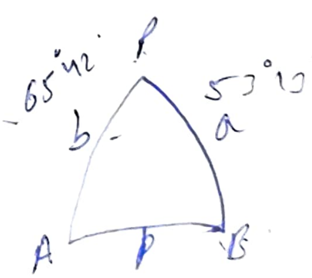
$= 133^\circ 39' + 125^\circ 24'$

$= 259^\circ 03'$

HK (shorter arc) $= 360^\circ - 259^\circ 03$

$= 100^\circ 57'$.

See triangle, APB. PA and PB are given

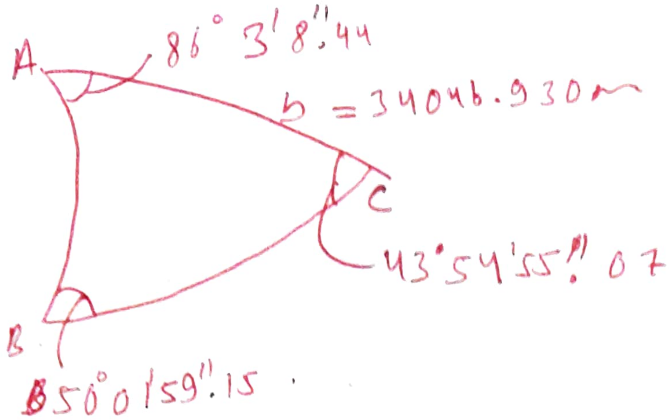


$\cos a = \cos p \cos b + \sin p \sin b \cos A$.

$\cos p = \cos a \cos b + \sin a \sin b \cos P$.

$\cos AB = \cos PB \cos PA + \sin PB \sin PA \cos A\cancel{P}B$.

$= \cos 65^\circ 42' \cos 53^\circ 13' + \sin 65^\circ 42'$ ~~sin~~ $53^\circ 13 \cos$



$A = 86° 3' 8''.44$

$B = 50° 01' 59''.~~15~~$

$C = 43° 54' 55''.07$

$b = 34046.930\ m$

$R = 6370\ Km$

$1° = 111177.4734\ m$

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

Spherical ~~ss~~

$$\varepsilon = \frac{ab \sin C}{2R^2 \sin 1''} = \frac{c^2 \sin A \sin B}{2R^2 \sin C \sin 1''}$$

$$= \frac{a^2 \sin B \sin C}{2R^2 \sin A \sin 1''} = \frac{b^2 \sin A \sin C}{2R^2 \sin B \sin 1''}$$

Then b in arc.

$$b = \frac{34046.930\,m \times 1°}{111177.4734\,m} = \cancel{0.306239}$$

$$= \cancel{34046.9} \ \frac{34046°55'48''}{111177.4734} = 0.30623946523$$

$= 0° 18' 22.46''$

$b = 34046.930m = 0° 18' 22.46''$

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B}$$

$$\sin a = \frac{\sin b \sin A}{\sin B} = \frac{\sin(0^\circ 18' 22.46'') \sin 86^\circ 3' 8''.44}{\sin 50^\circ 01' 59''.15}$$

$$= \frac{5.3321 \times 10^{-3}}{\sin(50^\circ 01' 59''.15)} = 6.9572 \times 10^{-3}$$

$$\sin a = 3.98626143 \times 10^{-1}$$

$$a = 0^\circ 23' 55.05'' = \sin^{-1}(\ )$$

$$\frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

$$\Rightarrow \sin c = \frac{\sin b \sin C}{\sin B}$$

$$= \frac{\sin(0^\circ 18' 22.46'') \sin(43^\circ 54' 55''.07)}{\sin(50^\circ 01' 59.15'')}$$

$$\sin c = 4.837 \times 10^{-3}$$

$$c = \sin^{-1}(4.837 \times 10^{-3})$$

$$= z. = \cancel{0^\circ 0} \quad 0^\circ 16' 37.71''$$

$$= 0^\circ 16' 37.71''$$

$$\varepsilon = \frac{ab \sin C}{2R^2 \sin 1''}$$

$\sin 1'' = 4.848136 \times 10^{-6}$

$$= \frac{(0^\circ 23' 55.05'')(0^\circ 18' 22.46'') \sin(43^\circ 54' 55''.07)}{2(6370 \times 10^3)^2\, 4.848136}$$

$= 2.152021 \times 10^2$

$= 215^\circ 12' 7.71''$

---

haversine $(\theta) = \sin^2\left(\frac{\theta}{2}\right)$

$\text{hav}(\theta) = \frac{1}{2}(1 - \cos\theta) = \sin^2\left(\frac{\theta}{2}\right)$.

$\cos\theta = 1 - 2\sin^2\frac{\theta}{2}$

$= 1 - 2\,\text{hav}\,\theta$.

$\cos a = \cos b \cos c + \sin b \sin c \cos A$.

we have, $1 - 2\,\text{hav}\,\theta = \cos\theta$.

$\cos a = 1 - 2\,\text{hav}\,a$.

$1 - 2\,\text{hav}\,a = \cos b \cos c + \sin b \sin c\,(1 - 2\,\text{hav}\,A)$

$1 - 2\,\text{hav}\,a = \cos(b-c) - 2 \sin b \sin c\,\text{hav}\,A$.

$1 - 2\,\text{hav}\,a = 1 - 2\,\text{hav}(b-c) - 2 \sin b \sin c\,\text{hav}\,A$.

$\text{hav}\,a = \text{hav}(b-c) + \text{hav}\,A \sin b \sin c$

hav a = hav(b − c) + sin b sin c hav A.

haversines..

trigonometrical ratios for small angles.

- if θ - small angle.
  expressed in circular measure.

Approximation we have

$\sin\theta = \theta$ radians | $\cos\theta = 1$ | $\tan\theta = \theta$ radian |

Now

1 radian $= 57°\,17'\,45''$.

$= 3437\frac{3}{4}'$

$= 206265''$.

so that, $1'' = \frac{1}{206265}$ rad.

and $1' = \frac{1}{3438}$ (approx)

$\sin 1'' = \frac{1}{206265}$, ... 36

$\sin 1' = \frac{1}{3438}$. 37

$\sin\theta = \frac{\theta''}{2026285}$

$\therefore \sin\theta'' = \theta'' \sin 1''$

$\sin\theta' = \theta' \sin 1'$.

$\tan\theta'' = \theta'' \sin 1''$

24 hour = 360°;

1 hour = 15°

$1^{min} = 15'$

1 sec = 15''

doubt

$\sin 1^m = \sin 15' = 15 \sin 1'$ ... 41

$\sin 1^s = \sin 15'' = 15 \sin 1''$ ... 42

if H is a small angle.

H^m ← expressed in minutes of time.

$\sin H = H^m \sin 1^m = 15 H^m \sin 1'$ ... 43

$\sin H = H^s \sin 1^s = 15 H^s \sin 1'$ ... 44

Delambre's and Napier's analogies.

$$\sin\tfrac{1}{2}c\,\sin\tfrac{1}{2}(A-B) = \cos\tfrac{1}{2}C\,\sin\tfrac{1}{2}(a-b)$$

$$\sin\tfrac{1}{2}c\,\cos\tfrac{1}{2}(A-B) = \sin\tfrac{1}{2}C\,\sin\tfrac{1}{2}(a+b).$$

$$\cos\tfrac{1}{2}c\,\sin\tfrac{1}{2}(A+B) = \cos\tfrac{1}{2}C\,\cos\tfrac{1}{2}(a-b).$$

$$\cos\tfrac{1}{2}c\,\cos\tfrac{1}{2}(A+B) = \sin\tfrac{1}{2}C\,\cos\tfrac{1}{2}(a+b)$$

Delambre's analogies.

Napier's analogies:

$$\tan\tfrac{1}{2}(a+b) = \frac{\cos\frac{1}{2}(A-B)}{\cos\frac{1}{2}(A+B)}\tan\tfrac{1}{2}c$$

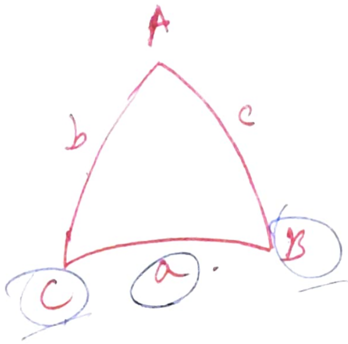
$$\tan\tfrac{1}{2}(a-b) = \frac{\sin\frac{1}{2}(A-B)}{\sin\frac{1}{2}(A+B)}\tan\tfrac{1}{2}c$$

$$\frac{\tan(A+B)}{\tan(A-B)} = \cancel{\sin\tfrac{1}{2}(a-b)}\quad \frac{\cos\frac{1}{2}(a-b)\cot\frac{1}{2}C}{\cos\frac{1}{2}(a+b)}$$

$$\tan\tfrac{1}{2}(A-B) = \frac{\sin\frac{1}{2}(a-b)}{\sin\frac{1}{2}(a+b)}\cot\tfrac{1}{2}C.$$

wrong ans

1)



$a = 119^\circ 46' 36'' = 119 + \frac{46}{60} + \frac{36}{360}$

$= 119.7767^\circ$

$B = 52^\circ 25' 38''$

$= 52.42722$

$C = 90^\circ$

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C = 1}$$

$$\sin A = \frac{\sin a \sin B}{\sin b} \qquad \sin A = \frac{\sin a \sin C}{\sin c}$$

$\Rightarrow \sin a$

$\cos a = \cos b \cos c + \sin b \sin c \cos A.$

$$\sin\frac{A}{2} = \sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin b \sin c}}$$

$\sin a \sin B \neq \sin A \sin b = \sin c$

$\sin(119^\circ 46' 36'') \sin(52^\circ 25' 38'') = \sin c.$

$\sin c = 6.87933 \times 10^{-1}.$

$c = 4.3466 \times 10^{1}$

$= 43^\circ 28' 0.27'' = c.$

$\sin b = \sin c \times \sin B.$

$= \sin(43^\circ 28' 0.27'') \sin(52^\circ 25' 38'')$

$b = 3.304 \times 10^{1}$

$= 33^\circ 2' 28.30''$

$\sin A = \dfrac{\sin a}{\sin c} = \dfrac{\sin(119^\circ 48' 36'')}{\sin(43^\circ 28' 0.27'')} = 1.261703.$

$\sin A. = 1^\circ 15' 42.13''$

doubt

Whenever the angle is very small
its difficult to calc. the cosine and other function

from the programm. — asa format.

$b = 48^\circ 26' 48.95''$ —

$c = 105^\circ 13\ 59.8705.$ —

$A = 113^\circ 10' 45.8827''.$ —

aaa
aas
asa.
sas

Ministry of Education
Government of India

STUDY OF SPHERICAL ASTRONOMY & ECLIPSE FORMATION FROM INDIAN TEXTS

A project by

Prashant Sharma

UNDER THE GUIDANCE OF

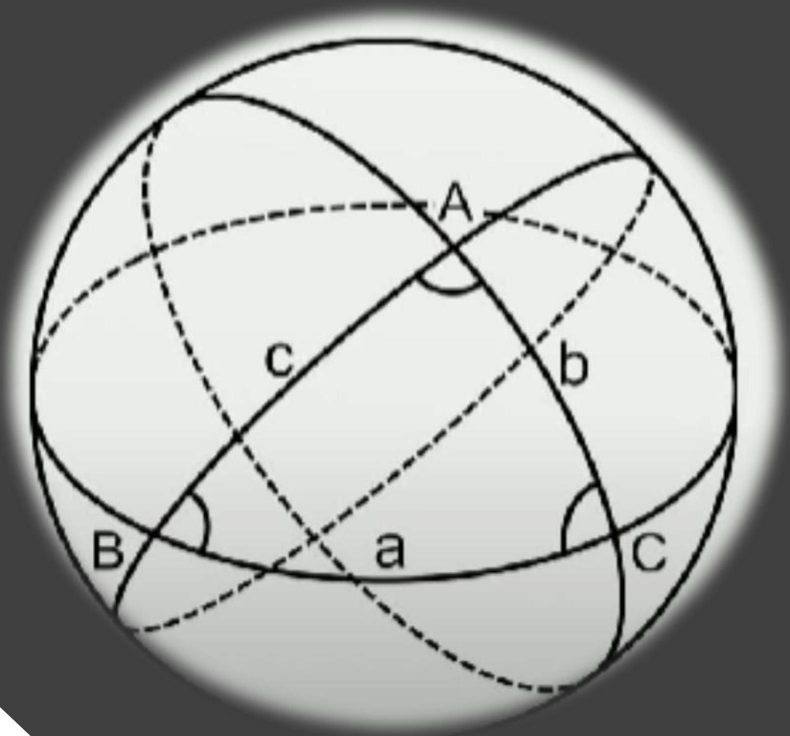
Dr. Anil Kumar Gourishetty

Time-Line

- **Week 1-2** — Reviewing spherical trigonometry theorems and methods
- **Week 3-4** — Problem-solving and verification using the computational software MATLAB.
- **Week 5** — Learning Surya Siddhanta spherical trigonometry
- **Week 6-7** — Understanding the Vedic and Puranic calendar, Surya Siddhanta to predict upcoming celestial events.
- **Week 8** — summarizing all previous work.

Spherical Trigonometry

- A spherical triangle has 6 basic elements: three angles (A,B,C) and three sides (a,b,c).



Arcs & angles

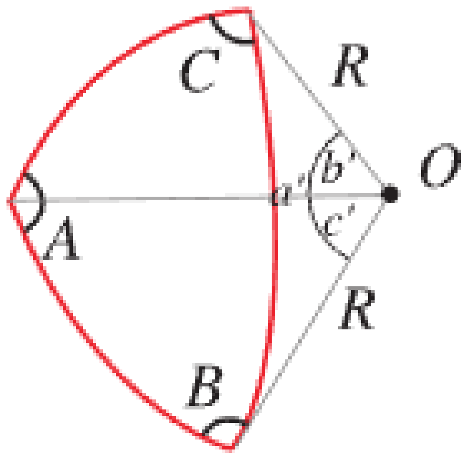
a = OA, **b** = OB, **c** = OC

Angular lengths of the sides (in radians)

a’ = BOC , b’ = COA , c’ = AOB

So Arc Length are

a = Ra’ , b = Rb’ , c = Rc’

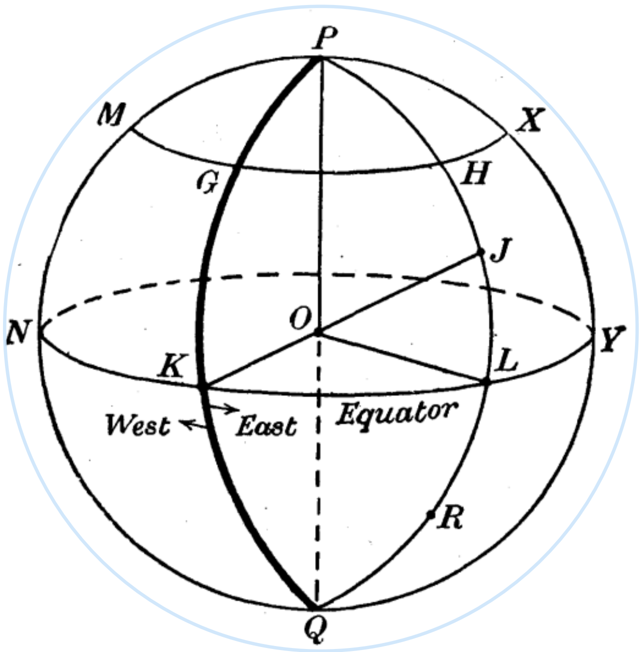


HX = LYsin(PH)

$= LYsin(90 - HL)$

But, HL = ϑ $(latitude)$

So $HX = LYcos\theta$



Fundamental formula of spherical trigonometry

- When **all three Sides** are given

  Angle of triangle can be found by **cosine-formula.**

- For **two sides and one angle**

  Say, b and c , and the included angle A of a spherical triangle ABC are known,

  Then **cosine-formula** can be used to calculate the third side a to be made.

Rules & Laws

1

Fundamental formula of spherical trigonometry

- Cosine-formula

2

Law of Sines

- $$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$$
- Relation between any two sides and two opposite sides

3

Haversine formula

- $\text{hav}(a) = hav(b - c) + \sin(b)\sin(c)\,hav(A)$
- $\text{hav}\theta = sin^2\frac{\theta}{2}$

4

Four parts formula

- $$\cos(\text{inner side}).\cos(\text{inner angle}) = \sin(\text{inner side}).\cot(\text{other side}) - \sin(\text{inner angle}).\cot(\text{other angle})$$

5

Delambre's & Napier's analogies

- Some further trigonometry calculations provides some useful results

Problem solving & verification

Difficult calculations

MATLAB

Command Window

```
>> SphericTriangleSolver
Givens   [decimal degree]
 Angles      A= 28 58 18.9520 B= 46 35 3.7281 C=104 29 39.8971
Sides    a=  0   0 0.0000 b=  0   0 0.0000 c=  0   0 0.0000
a1=  1 59 59.9977 b1=  2 59 59.9965 c1=  3 59 59.9953
```