Social Network Analysis

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Practical 1

<u>Aim:</u> Write a program to compute the following for a given a network:

- number of edges
- number of nodes
- degree of node
- node with lowest degree 2 the adjacency list
- matrix of the graph.

Software(s) used:

- R ver. 4.1.3
- RStudio ver. 2022.02.0+433

External packages required:

• igraph

Description:

• The igraph package: igraph is a library and R package for network analysis.

The main goals of the igraph library is to provide a set of data types and functions for:

- pain-free implementation of graph algorithms,
- fast handling of large graphs, with millions of vertices and edges, allowing rapid prototyping via high level languages like R.
- •library(): library() loads and attach add-on packages.

- •graph.formula():
 - Creating (small) graphs via a simple interface

• This function is useful if you want to create a small (named) graph quickly, it works for both directed and undirected graphs.

- •plot(): Use to plot any graph.
- •ecount (): Returns the count of number of edges in graph
- •vcount (): Returns the count of number of vertices in graph

●E ():

- Edges of a graph
- An edge sequence is a vector containing numeric edge ids, with a special class attribute that allows custom operations: selecting subsets of edges based on attributes, or graph structure, creating the intersection, union of edges, etc.

●V():

- Vertices of a graph
- Create a vertex sequence (vs) containing all vertices of a graph.

•degree():

- Degree and degree distribution of the vertices
- The degree of a vertex is its most basic structural property, the number of its adjacent edges.
- Mode-Character string, "out" for out-degree, "in" for indegree or "total" for the sum of the two. For undirected graphs this argument is ignored. "all" is a synonym of "total".

•max() and min():

Maxima and Minima

• Returns the (regular or parallel) maxima and minima of the input values.

- •get.adjacency(): Convert a graph to an adjacency matrix
- •get.adjlist():
 - Adjacency lists
 - Create adjacency lists from a graph, either for adjacent edges or for neighboring vertices

Source Code:

```
library(igraph)
u graph <- graph.formula(A - B, A - C, A - D, B - C, B -
F, C D, C - E, C - F, D - E, E - F, F - G, G - H)
d graph <- graph.formula(A <+ B, A <+ D, A -+ C, B -+ C,
B -+
E, B -+ F, C -+ D, C -+ F, D -+ E)
e count (u graph)
e count (d graph)
v count(u graph)
v count(d graph)
E(u graph)
E(d graph)
V(u graph)
V(d graph)
degree(u graph)
degree(u graph, mode =
```

```
"in") degree(u_graph, mode
= "out")

degree(d_graph)
degree(d_graph, mode =
"in") degree(d_graph, mode
= "out")

V(u_graph) $name[degree(u_graph) <= min(degree(u_graph))]
V(d_graph) $name[degree(d_graph, mode = "in") <=
min(degree(d_graph, mode = "in"))]
V(d_graph) $name[degree(d_graph, mode = "out") <=
min(degree(d_graph, mode = "out"))]

get.adjacency(u_graph)
get.adjlist(u_graph)
get.adjlist(u_graph)
get.adjlist(d_graph)</pre>
```

Output:

```
> library(igraph)
> u_graph ← graph.formula(A - B, A - C, A - D, B - C, B - F, C - D, C - E, C - F, D - E, E - F, F - G, G - H)
> d_graph ← graph.formula(A ++ B, A ++ D, A -+ C, B -+ C, B -+ E, B -+ F, C -+ D, C -+ F, D -+ E)
> ecount(d_graph)
[1] 11
> ecount(d_graph)
[1] 11
> vcount(u_graph)
[1] 8
> vcount(d_graph)
[1] 6
> E(u_graph)
+ 12/12 edges from 535b776 (vertex names):
[1] A - B A -- C A -- D B -- C B -- F C -- D C -- F C -- E D -- E F -- E F -- G G -- H
> E(d_graph)
+ 11/11 edges from 5365b0b (vertex names):
[1] A -> B A -> D A -> C B -> A B -> C B -> E B -> F D -> A D -> E C -> D C -> F
```

```
V(u_graph)
+ 8/8 vertices, named, from 535b776:
[1] A B C D F E G H
> V(d_graph)
+ 6/6 vertices, named, from 5365b0b:
[1] A B D C E F
 degree(u_graph)
ABCDFEGH
 3 5 3 4 3 2 1
 degree(u_graph, mode = "in")
ABCDFEGH
 3 5 3 4 3 2 1
 degree(u_graph, mode = "out")
ABCDFEGH
 3 5 3 4 3 2 1
3
 degree(d_graph)
ABDCEF
5 5 4 4 2 2
```

```
> degree(d_graph, mode = "in")
A B D C E F
2 1 2 2 2 2
> degree(d_graph, mode = "out")
A B D C E F
3 4 2 2 0 0
>
> V(u_graph)$name[degree(u_graph) == min(degree(u_graph))]
[1] "H"
> V(d_graph)$name[degree(d_graph, mode = "in") == min(degree(d_graph, mode = "in"))]
[1] "B"
> V(d_graph)$name[degree(d_graph, mode = "out") == min(degree(d_graph, mode = "out"))]
[1] "E" "F"
>
```

```
> get.adjacency(u_graph)
8 x 8 sparse Matrix of class "dgCMatrix"
    A B C D F E G H
A . 1 1 1 . . . .
B 1 . 1 . 1 1 . . .
C 1 1 . 1 1 1 . .
D 1 . 1 . . 1 1 .
E . . 1 1 1 . . .
G . . . . 1 . . 1
H . . . . . . 1 .
> get.adjacency(d_graph)
6 x 6 sparse Matrix of class "dgCMatrix"
    A B D C E F
A . 1 1 1 . .
B 1 . . 1 1 1
D 1 . . . 1 .
C . . 1 . . 1
```

```
> get.adjlist(u_graph)
$A
+ 3/8 vertices, named, from 535b776:
[1] B C D

$B
+ 3/8 vertices, named, from 535b776:
[1] A C F

$C
+ 5/8 vertices, named, from 535b776:
[1] A B D F E

$D
+ 3/8 vertices, named, from 535b776:
[1] A C E

$F
+ 4/8 vertices, named, from 535b776:
[1] B C E G
```

```
$E
+ 3/8 vertices, named, from 535b776:
[1] C D F
$G
+ 2/8 vertices, named, from 535b776:
[1] F H
$H
+ 1/8 vertex, named, from 535b776:
[1] G
> get.adjlist(d_graph)
$A
+ 5/6 vertices, named, from 5365b0b:
[1] B B D D C
$B
+ 5/6 vertices, named, from 5365b0b:
[1] A A C E F
```

```
$D
+ 4/6 vertices, named, from 5365b0b:
[1] A A C E

$C
+ 4/6 vertices, named, from 5365b0b:
[1] A B D F

$E
+ 2/6 vertices, named, from 5365b0b:
[1] B D

$F
+ 2/6 vertices, named, from 5365b0b:
[1] B C
```

Practical 2

<u>Aim:</u> Perform following tasks:

- View data collection forms and/or import onemode/two-mode datasets.
- Basic Networks matrices transformations.

Software(s) used:

- R ver. 4.1.3
- RStudio ver. 2022.02.0+433

External packages required:

• igraph

Description:

- The igraph package: igraph is a library and R package for network analysis.
 - The main goals of the igraph library is to provide a set of data types and functions for:
- pain-free implementation of graph algorithms,
- fast handling of large graphs, with millions of vertices and edges, allowing rapid prototyping via high level languages like R.
 - •getwd(): Used to get the absolute filepath of the current R session.
 - •require():library() and require() load and attach addon packages.

•read.csv(): Reads a file in table format and creates a data frame from it, with cases corresponding to lines and variables to fields in the file.

- •head(): Returns the first part of a vector, matrix, table, data frame or function. Since head() and tail() are generic functions, they may also have been extended to other classes.
- •graph.data.frame(): This function creates an igraph graph from one or two data frames containing the (symbolic) edge list and edge/vertex attributes.
- •get.adjacency(): Sometimes it is useful to work with a standard representation of a graph, like an adjacency matrix.
- •plot(): Draw a scatter plot with decorations such as axes and titles in the active graphics window.

Source Code:

```
require("igraph")

edges <-
read.csv("C:/Temp/input_files/edges.csv") nodes
<- read.csv("C:/Temp/input_files/nodes.csv")

head(edges)
head(nodes)

dir_graph <- graph.data.frame(d = edges, vertices = nodes,
directed = T)

get.adjacency(dir_graph)

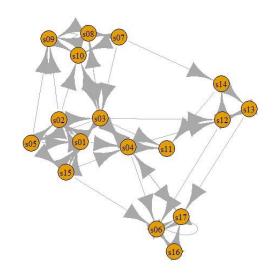
plot(dir_graph)</pre>
```

Output:

```
> require("igraph")
> head(edges)
 from to weight type
1 s01 s02 10 hyperlink
2 s01 s02
              12 hyperlink
3 s01 s03
             22 hyperlink
4 s01 s04 21 hyperlink
5 s04 s11 22 mention
6 s05 s15
                   mention
             21
  id
                   media media.type type.label audience.size
1 s01
                NY Times
                                  1 Newspaper
                                                         20
2 s02 Washington Post
                                  1 Newspaper
                                                         25
3 s03 Wall Street Journal
                                                         30
                                 1 Newspaper
               USA Today
                                 1 Newspaper
4 s04
                                                         32
5 s05
               LA Times
                                 1 Newspaper
                                                         20
                                 1 Newspaper
6 s06
           New York Post
                                                         50
```

```
> dir_graph ← graph.data.frame(d = edges, vertices = nodes, directed = T)
> get.adjacency(dir_graph)
17 x 17 sparse Matrix of class "dgCMatrix"
   [[ suppressing 17 column names 's01', 's02', 's03' ... ]]
s01 . 2 1 1 . . . . . . . . . . . 1 . . s02 1 . 1 . . . . . . . 1 1 . . . . . . .
s03 1 . . 1 1 . . 1 . 1 1 1 1 . . . . .
s04 . . 1 . . 1 . . . . 1 1 . . . . 1
s05 1 1 . . . . . . 1 . . . . . 1 .
s06 . . . . . 1 . . . . . . . . . . 1
s08 . . 1 . . . 1 . 2 . . . . . . .
s11 . . . . . . . . . . . . . . . . .
s12 . . . . . 1 . . . . . . 1 1 . . .
s13 . . . . . . . . . . . . .
s14 . . . . . . . . . . 1 . 1 . . . .
s15 2 . . 1 . 1 . . . . . . . . . .
s16 . . . . . 1 . . . . . . . .
```

```
> plot(dir_graph)
> |
```



Practical 3

<u>Aim:</u> Compute the following node level measures:

- Density
- Degree
- Reciprocity
- Transitivity
- Centralization 2 Clustering.

Software(s) used:

- R ver. 4.1.3
- RStudio ver. 2022.02.0+433

External packages required:

• igraph

Description:

- The igraph package: igraph is a library and R package for network analysis.
 - The main goals of the igraph library is to provide a set of data types and functions for:
- pain-free implementation of graph algorithms,
- fast handling of large graphs, with millions of vertices and edges, allowing rapid prototyping via high level languages like R.
 - •library(): library() and require() load and attach addon packages.
 - •graph.famous(): Create an igraph graph from a list of edges, or a notable graph.

•ecount (): Returns the count of number of edges in graph

- •vcount (): Returns the count of number of vertices in graph
- •graph.formula(): This function is useful if you want to create a small (named) graph quickly, it works for both directed and undirected graphs.
- plot (): Draw a scatter plot with decorations such as axes and titles in the active graphics window.
- •reciprocity(): Calculates the reciprocity of a directed graph.
- •dyad.census(): Classify dyads in a directed graphs. The relationship between each pair of vertices is measured. It can be in three states: mutual, asymmetric or non-existent.
- •adjacent.triangles(): Count how many triangles a vertex is part of, in a graph, or just list the triangles of a graph.
- •transitivity(): Transitivity measures the probability that the adjacent vertices of a vertex are connected. This is sometimes also called the clustering coefficient.
- •degree (): The degree of a vertex is its most basic structural property, the number of its adjacent edges.
 - •barabasi.game(): The BA-model is a very simple stochastic algorithm for building a graph.
- •watts.strogatz.game(): Generate a graph according to the Watts-Strogatz network model.

•graph.union(): The union of two or more graphs are created. The graphs may have identical or overlapping vertex sets.

•simplify(): Simple graphs are graphs which do not contain loop and multiple edges.

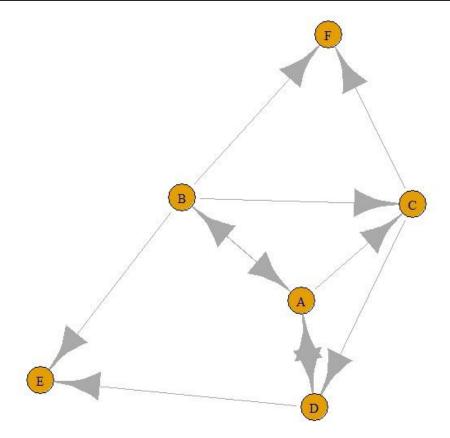
Source Code:

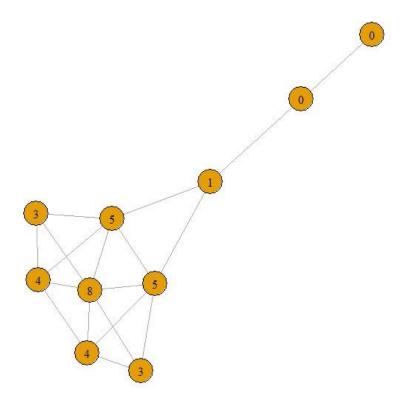
```
library("igraph")
kite <-
graph.famous("Krackhardt Kite")
vcount(kite) ecount(kite)
ecount(kite) / (vcount(kite) * (vcount(kite) - 1) / 2)
dir graph <- graph.formula(A <+ B, A <+ D, A -+ C, B -+
C, B -+ E, B -+ F, C -+ D, C -+ F, D -+ E)
plot(dir graph) reciprocity(dir graph)
dyad.census(dir graph)
mutual <- dyad.census(dir graph)$mut</pre>
mutual / (ecount(dir graph))
atri <- adjacent.triangles(kite)</pre>
plot(kite, vertex.label = atri)
transitivity(kite, type = "local")
```

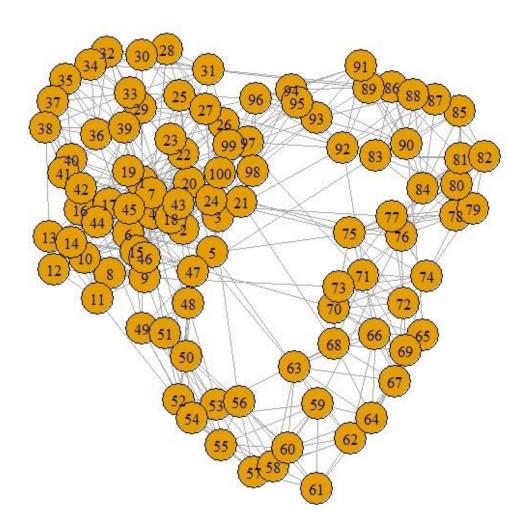
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plot(graph) Output:

```
> library("igraph")
> kite ← graph.famous("Krackhardt_Kite")
> vcount(kite)
[1] 10
> ecount(kite)
[1] 18
> ecount(kite) / (vcount(kite) * (vcount(kite) - 1) / 2)
[1] 0.4
> dir_graph ← graph.formula(A ++ B, A ++ D, A -+ C, B -+ C, B -+ E, B -+ F, C -+ D, C -+ F, D -+ E)
> plot(dir_graph)
> reciprocity(dir_graph)
[1] 0.3636364
> reciprocity(dir_graph)
[1] 0.3636364
> dyad.census(dir_graph)
$mut
[1] 2
$asym
[1] 7
$null
[1] 6
> mutual ← dyad.census(dir_graph)$mut
> mutual / (ecount(dir_graph))
[1] 0.1818182
> atri ← adjacent.triangles(kite)
> plot(kite, vertex.label = atri)
> transitivity(kite, type = "local")
[1] 0.66666667 0.66666667 1.00000000 0.53333333 1.00000000 0.50000000 0.3333333 0.000000000
[10] NaN
```







Practical 4

<u>Aim:</u> For a given network find the following:

- Length of the shortest path from a given node to another node
- The density of the graph
- Draw egocentric network of node G with chosen configuration parameters.

Software(s) used:

• R ver. 4.1.3

• RStudio ver. 2022.02.0+433

External packages required:

• igraph

Description:

- The igraph package: igraph is a library and R package for network analysis.
 - The main goals of the igraph library is to provide a set of data types and functions for:
- pain-free implementation of graph algorithms,
- fast handling of large graphs, with millions of vertices and edges, allowing rapid prototyping via high level languages like R.
 - •library(): library() and require() load and attach addon packages.
 - •as.matrix():matrix() creates a matrix from the given set of values. as.matrix() attempts to turn its argument into a matrix.
 - •read.table(): Reads a file in table format and creates a data frame from it, with cases corresponding to lines and variables to fields in the file.
 - •colnames() and rownames(): Retrieve or set the row or column names of a matrix-like object.
 - •is.na(): The generic function is.na() indicates which elements are missing.

•graph.adjacency():
 graph_from_adjacency_matrix() is a flexible
 function for creating igraph graphs from adjacency matrices.

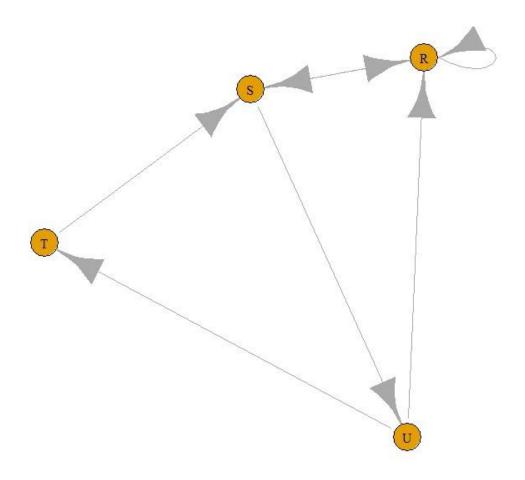
- •plot(): Draw a scatter plot with decorations such as axes and titles in the active graphics window.
- •shortest.paths(): shortest_paths() calculates one shortest path (the path itself, and not just its length) from or to the given vertex.
- •print():print() prints its argument and returns it invisibly.
- •graph.formula(): This function is useful if you want to create a small (named) graph quickly, it works for both directed and undirected graphs.
- •graph.density(): The density of a graph is the ratio of the number of edges and the number of possible edges.
 - •simplify(): Simple graphs are graphs which do not contain loop and multiple edges.

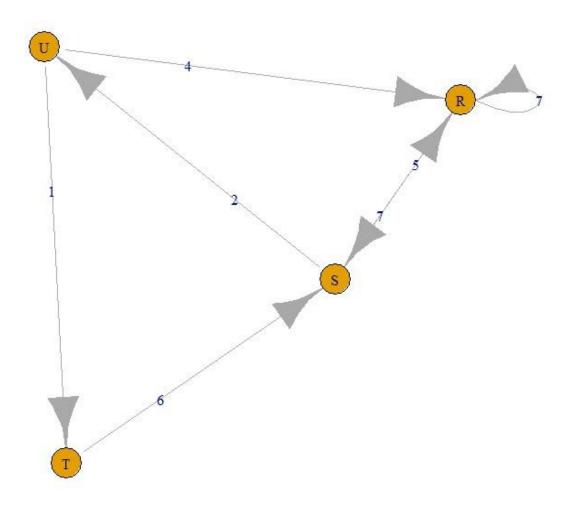
Source Code:

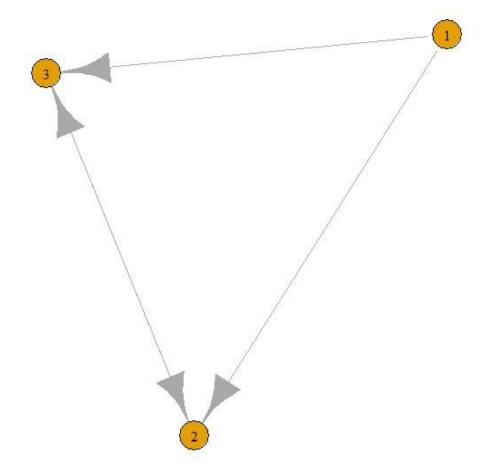
```
U 4 0 1 0", header=T))nms <- matt[, 1] matt <- matt[, -1]
colnames(matt) <- rownames(matt) <- nms</pre>
matt[is.na(matt)] <- 0</pre>
g <- graph.adjacency(matt, weighted=TRUE)</pre>
plot(g)
s.paths <- shortest.paths(g, algorithm = "dijkstra")</pre>
print(s.paths)
shortest.paths(g, v="R", to="S")
plot(g, edge.label=E(g)$weight) dg
<- graph.formula(1-+2, 1-+3, 2<+3)
plot(dg)
graph.density(dg, loops=TRUE)
graph.density(simplify(dg), loops=FALSE)
```

Output:

```
> plot(g)
  RSTU
R 0 5 5 4
S 5 0 3 2
T 5 3 0 1
U 4 2 1 0
  S
> plot(g, edge.label=E(g)$weight)
> plot(dg)
[1] 0.4444444
[1] 0.6666667
```







Practical 5

<u>Aim:</u> Write a program to distinguish between a network as a matrix, a network as an edge list and a network as a sociogram (or "network graph") using 3 distinct networks representatives of each.

Software(s) used:

- R ver. 4.1.3
- RStudio ver. 2022.02.0+433

External packages required:

• igraph

Description:

- The igraph package: igraph is a library and R package for network analysis.
 - The main goals of the igraph library is to provide a set of data types and functions for:
- pain-free implementation of graph algorithms,
- fast handling of large graphs, with millions of vertices and edges, allowing rapid prototyping via high level languages like R.
 - •library(): library() and require() load and attach addon packages.
 - •graph.formula(): This function is useful if you want to create a small (named) graph quickly, it works for both directed and undirected graphs.

•plot(): Draw a scatter plot with decorations such as axes and titles in the active graphics window.

- •get.adjacency(): Sometimes it is useful to work with a standard representation of a graph, like an adjacency matrix.
- •E (): An edge sequence is a vector containing numeric edge ids, with a special class attribute that allows custom operations: selecting subsets of edges based on attributes, or graph structure, creating the intersection, union of edges, etc.
- •get.adjedgelist(): Create adjacency lists from a graph, either for adjacent edges or for neighboring vertices.

Source Code:

```
plot(sociogram)

get.adjacency(sociogram)

E(sociogram)

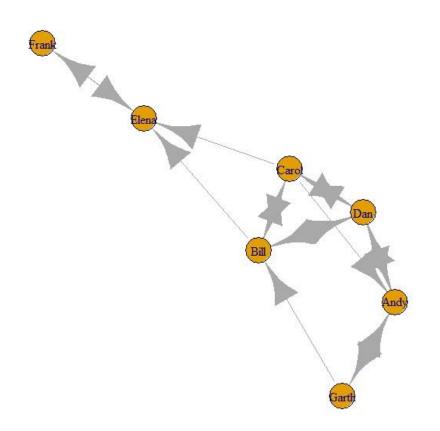
get.adjedgelist(sociogram)
```

Output:

```
library(igraph)
7 x 7 sparse Matrix of class "dgCMatrix"
     Andy Garth Bill Elena Frank Carol Dan
Andy
             1
Garth
        1
                   1
Bill
                                         1
                         1
                                     1
Elena
                               1
Frank
                         1
Carol
        1
                   1
                                         1
                         1
Dan
```

```
E(sociogram)
+ 16/16 edges from f262a60 (vertex names):
 [1] Andy →Garth Andy →Dan Garth→Andy Garth→Bill
[5] Bill →Elena Bill →Carol Bill →Dan
                                             Elena→Frank
[9] Frank→Elena Carol→Andy Carol→Bill Carol→Elena
[13] Carol\rightarrowDan Dan \rightarrowAndy Dan \rightarrowBill Dan \rightarrowCarol
$Andv
+ 5/16 edges from f262a60 (vertex names):
[1] Andy →Garth Andy →Dan Garth→Andy Carol→Andy
[5] Dan →Andy
$Garth
+ 3/16 edges from f262a60 (vertex names):
[1] Garth→Andy Garth→Bill Andy →Garth
$Bill
+ 6/16 edges from f262a60 (vertex names):
[1] Bill →Elena Bill →Carol Bill →Dan
                                             Garth→Bill
[5] Carol\rightarrowBill Dan \rightarrowBill
```

```
$Elena
+ 4/16 edges from f262a60 (vertex names):
[1] Elena→Frank Bill →Elena Frank→Elena Carol→Elena
$Frank
+ 2/16 edges from f262a60 (vertex names):
[1] Frank→Elena Elena→Frank
$Carol
+ 6/16 edges from f262a60 (vertex names):
[1] Carol→Andy Carol→Bill Carol→Elena Carol→Dan
[5] Bill →Carol Dan →Carol
$Dan
+ 6/16 edges from f262a60 (vertex names):
[1] Dan →Andy Dan →Bill Dan →Carol Andy →Dan
[5] Bill →Dan Carol→Dan
```



Practical 6

<u>Aim:</u> Write a program to exhibit structural equivalence, automatic equivalence, and regular equivalence from a network.

Software(s) used:

- R ver. 4.1.3
- RStudio ver. 2022.02.0+433

External packages required:

- igraph
- sna

Description:

- The igraph package: igraph is a library and R package for network analysis.
 - The main goals of the igraph library is to provide a set of data types and functions for:
- pain-free implementation of graph algorithms,
- fast handling of large graphs, with millions of vertices and edges, allowing rapid prototyping via high level languages like R.
 - The sna package: sna is a package containing a range of tools for social network analysis. Supported functionality includes node and graph-level indices, structural distance and covariance methods, structural equivalence detection, p* modeling, random graph generation, and 2D/3D network visualization (among other things).

•library(): library() and require() load and attach addon packages.

- read.csv(): Reads a file in table format and creates a data frame from it, with cases corresponding to lines and variables to fields in the file.
- •equiv.clust():equiv.clust() uses a definition of approximate equivalence (equiv.fun()) to form a hierarchical clustering of network positions.
- •plot(): Draw a scatter plot with decorations such as axes and titles in the active graphics window.
- •sedist(): sedist() uses the graphs indicated by g in the arguments to assess the extent to which each vertex is structurally equivalent.
- •cmdscale(): Classical multidimensional scaling (MDS) of a data matrix.
- •as.dist(): This function computes and returns the distance matrix computed by using the specified distance measure to compute the distances between the rows of a data matrix.
- •blockmodel (): Given a set of equivalence classes and one or more graphs, blockmodel will form a blockmodel of the input graph(s) based on the classes in question

Source Code:

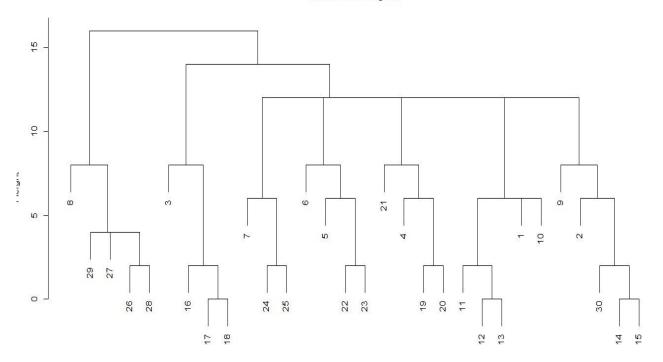
```
library(sna)
library(igraph)
```

```
links2 <- read.csv("C:/Temp/input files/edges1.csv",</pre>
header=T, row.names=1) eq<-equiv.clust(links2) plot(eq)</pre>
g.se<-sedist(links2)</pre>
plot(cmdscale(as.dist(g.se)))
b<-blockmodel(links2,eq,h=10)
plot(b)
```

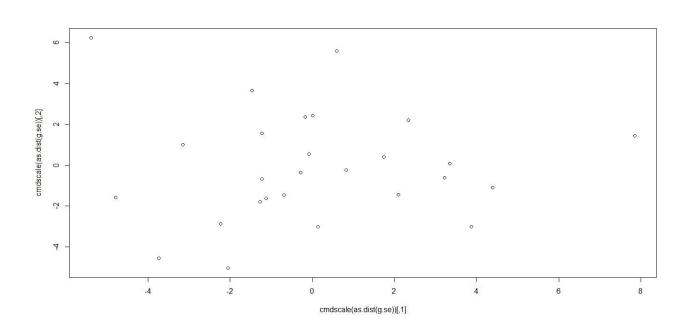
Output:

```
library(sna)
library(igraph)
links2 ← read.csv("C:/Temp/input_files/edges1.csv", header=T, row.names=1)
eq←equiv.clust(links2)
plot(eq)
 plot(cmdscale(as.dist(g.se)))
b—blockmodel(links2,eq,h=10)
```

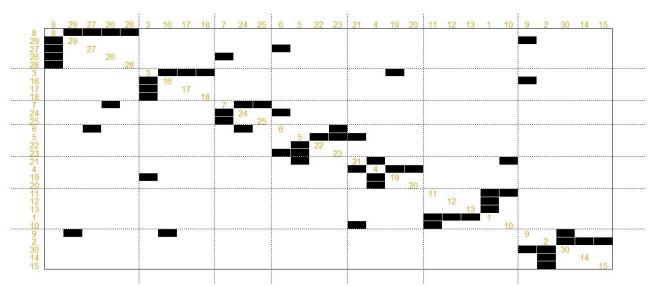
Cluster Dendrogram



as.dist(equiv.dist) hclust (*, "complete")



Relation - 1



Practical 7

<u>Aim:</u> Perform SVD analysis of a network.

Software(s) used:

- R ver. 4.1.3
- RStudio ver. 2022.02.0+433

External packages required:

• igraph

Description:

- The igraph package: igraph is a library and R package for network analysis.
 - The main goals of the igraph library is to provide a set of data types and functions for:
- pain-free implementation of graph algorithms,
- fast handling of large graphs, with millions of vertices and edges, allowing rapid prototyping via high level languages like R.
 - •matrix():matrix() creates a matrix from the given set of values.
 - ●c (): Combines values into a vector or list.
 - •print():print() prints its argument and returns it invisibly.
 - •svd(): Compute the singular-value decomposition of a rectangular matrix.

Source Code:

Output:

```
[,1] [,2] [,3] [,4]
           0
      1
         0
           0
      1
         0
   1
      0
         1
           0
   1
      0
         1
           0
   1
      0
         1
           0
   1
         0
      0
           1
   1
      0
         0
           1
      0
           1
```

```
> svd(a)
$d
[1] 3.464102e+00 1.732051e+00 1.732051e+00 1.922963e-16
$u
 [,1] [,2] [,3] [,4] [1,] -0.3333333  0.4714045 -1.741269e-16  7.760882e-01
                                   [,3]
 [2,] -0.3333333  0.4714045 -3.692621e-16 -1.683504e-01
 [3,] -0.3333333 0.4714045 -5.301858e-17 -6.077378e-01
 [4,] -0.3333333 -0.2357023 -4.082483e-01 6.774193e-17
 [5,] -0.3333333 -0.2357023 -4.082483e-01 6.774193e-17
 [6,] -0.3333333 -0.2357023 -4.082483e-01 6.774193e-17
 [7,] -0.3333333 -0.2357023 4.082483e-01 5.194768e-17
 [8,] -0.3333333 -0.2357023 4.082483e-01 5.194768e-17
 [9,] -0.3333333 -0.2357023 4.082483e-01 5.194768e-17
$v
          [,1]
                    [,2]
                                  [,3] [,4]
[2,] -0.2886751    0.8164966    -2.509507e-16    -0.5
[3,] -0.2886751 -0.4082483 -7.071068e-01 -0.5
[4,] -0.2886751 -0.4082483 7.071068e-01 -0.5
```