

DAY = 12

## Find GCD [Great Common Divisor]

### Naive to Euclid

→  $\text{gcd}(m, n)$

→ largest  $k$  such that  $k$  divides  
 $m$  and  $k$  divides  $n$

→ e.g.  $\text{gcd}(8, 12) = 4$

→  $\text{gcd}(18, 25) = 1$

→ 1 divides every number.

→ at least one common divisor  
for every  $m, n$

### Computing $\text{gcd}(m, n)$

→ List out factors of  $m$

→ List out factors of  $n$

→ Report the largest number  
that appears in both.

\* No 76 :- May not be good  
for all two questions.

→ Factors of  $m$  must be between 1 and  $m$ .

→ Test each number in this range.

→ If it divides  $m$  without remainder, add it to set of factors.

→ Example: gcd(14, 63)

→ Factors of 14

1	2	X	X	X	7	X	X	14	X	14	X	14
---	---	---	---	---	---	---	---	----	---	----	---	----

⇒

1	2	7	14
---	---	---	----

→ Factors of 63:

1	X	3	X	7	X	9	X	21	X	63	X
---	---	---	---	---	---	---	---	----	---	----	---

1	3	7	9	21	63
---	---	---	---	----	----

→ Construct list of common factors

→ For each factor of 14,  
check if it is a factor of 63.

1	7
---	---

→ Return the largest factor in the list:

7
---

→ This is the gcd.

An algorithm for  $\text{gcd}(m, n)$

- Use  $f_m, f_n$  for list of factors of  $m, n$
  - For each  $i$  from 1 to  $m$ ,  
add  $i$  to  $f_m$  if  $i$  divides  $m$
  - For each  $j$  from 1 to  $n$ ,  
add  $j$  to  $f_n$  if  $j$  divides  $n$
  - Use  $c_f$  for list of common factors
    - For each  $f$  in  $f_m$ ,  
add  $f$  to  $c_f$  if  $f$  also appears in  $f_n$
- ⇒ Return largest (rightmost) if ascending order  
value in  $c_f$

D. ~~backward~~

def  $\text{gcd}(m, n)$ :

$f_m = []$

for  $i$  in range(1,  $m+1$ ):  
if ( $m \% i == 0$ ):

$f_m.append(i)$

$f_n = []$

for  $j$  in range(1,  $n+1$ ):

if ( $n \% j == 0$ ):

$f_n.append(j)$

```
cf = []
for f in fm:
    if f in fo:
        cf.append(f)
return cf[-1]
```

## Improving Naïve

- With singlescan from 1 to  $\min(m, n)$ .
- No need to compute two lists and then comparing.
  - if i divides both  $m, n$   
then add it to cf.
- any common factor must be less than  $\min(m, n)$ 
  - will go with o for each i  
in 1 to  $\min(m, n)$ ,

Instead of 1 to  $\max(m, n)$ .

② def gcd(m, n):

```
cf = []
for i in range(1, min(m, n) + 1):
    if (m % i == 0) and (n % i == 0):
        cf.append(i)
return cf[-1]
```

## Without Lists:

```
def gcd(m,n):  
    for i in range (1,min(m,n)+1):  
        if (m%i)==0 and (n%i)==0:  
            mrcf = i #mrcf - most recent  
            # common factor  
    return mrcf.
```

## Scan Backwards

→ When scan backwards,  
may get the result faster than before.

```
def gcd (m,n):  
    i = min(m,n).  
    while i>0:  
        if (m%i)==0 and (n%i)==0:  
            return(i)  
        else:  
            i = i-1
```

## Euclid's Algo

→ Suppose d divides both m and n ; ~~and m>n~~

→ Then  $m=ad$ ,  $n=bd$

→ So  $m-n = ad-bd = (a-b)d$

→ d divides  $m-n$  as well.

→ So  $\gcd(m,n) = \gcd(n,m-n)$

to cache.

- consider  $\text{gcd}(m,n)$  with  $m > n$
- If  $n$  divides  $m$ , return  $n$
- otherwise, compute  $\text{gcd}(n, m-n)$  and return that value.

def gcd(m,n):

if  $m < n$ :

$$m, n = n, m$$

if  $m \mod n = 0$

return ( $n$ )

else:

$$\text{diff} = m - n$$

return ( $\text{gcd}(\min(m,n), \text{diff}) \times \min(n, \text{diff})$ )

Still, It Can Be Improved;

I Left here for you to  
Practice.