

Problem 1, Comparing Growth Rates

$2/N = O(1/N) = O(N^{-1})$
 $128 = O(1)$ constant time $= O(N^0)$
 $\log N = O(\log N)$
 $\sqrt{N} = O(\sqrt{N}) = O(N^{1/2})$
 $23N = O(N) = O(N^1)$
 $N \log N = O(N \log N)$
 $N^2 = O(N^2)$
 $42N^3 = O(N^3)$
 $2^n = O(2^n)$
 $2^{n+1} = O(2^n)$
 $3^n = O(3^n)$
 $N! = O(N^N)$

Problem 2, Running Time Analysis

Example 1: The runtime is $O(N^2)$ because the inner loop takes at most $O(N)$ time and is executed N times.

Example 2: The runtime is $O(N)$ because the inner loop takes $O(N)$ time and is executed 23 times.

Example 3: The runtime is $O(\log_k x)$ because this recursive function divides x by k repeatedly.

Problem 3, Rearranging Train Cars

- a. Move 3 to s3, move 6 to s2, move 9 to s1, move 2 to s3, move 4 to s2, move 7 to s1, move 1 to output track, move 2 to output, move 3 to output, move 4 to OP, move 8 to s3, move 5 to OP, move 6 to OP, move 7 to OP, move 8 to OP and move 9 to OP.
- b. The three holding tracks must be ordered from biggest to smallest, with the biggest on the bottom. Otherwise, you have a larger number that will be moved to the output before a smaller number. This does not work for every sequence, for example: [1, 9, 8, 7, 6, 5, 4, 3, 2]. After moving 2 to s3, 3 to s2 and 4 to s1, you would be forced to place a larger number on top of a smaller number.