Proving Logical Atomicity using Lock Invariants

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- The proofs are significantly more complicated than those that use TaDA-style lock specifications.
- This is the first foundational verification of a C implementation against logically atomic specifications.

Two Styles of Lock Specification

Lock Invariant Style [Gotsman et al., 2007; Hobor et al., 2008]

$$\{\ell \boxdot\!\!\rightarrow R\}\ \texttt{acquire}(\ell)\ \{R * \ell \boxdot\!\!\rightarrow R\}$$

$$\{R * \ell \boxdot\!\!\rightarrow R\}\ \texttt{release}(\ell)\ \{\ell \boxdot\!\!\rightarrow R\}$$

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TaDA Style [da Rocha Pinto et al., 2014]

$$\langle b.\ (\mathsf{L}(\ell) \wedge \neg b) \vee (\mathsf{U}(\ell) \wedge b)\rangle\ \texttt{acquire}(\ell)\ \langle \mathsf{L}(\ell) \wedge b\rangle$$

$$\langle \mathsf{L}(\ell)\rangle\ \texttt{release}(\ell)\ \langle \mathsf{U}(\ell)\rangle$$

Logically Atomic Specifications

Atomic Triple

$$\langle a.\ P_l \mid P_p(a) \rangle\ c\ \langle Q_l \mid Q_p(a) \rangle$$

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$$\langle s.\ \mathsf{is\_stack}_g\ p \mid \mathsf{stack}_g\ s \rangle\ \mathsf{push}(v)\ \langle \mathsf{is\_stack}_g\ p \mid \mathsf{stack}_g\ (v :: s) \rangle$$

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VST and Iris

Verified Software Toolchain

Iris

Concurrent Binary Search Tree (BST) as an Example

Atomic Specifications

$$\langle m.\,\mathsf{bst\_ref}\ p \mid \mathsf{bst\_abs}\ m\rangle\ \mathtt{insert}(p, k, v)\ \langle \mathsf{bst\_ref}\ p \mid \mathsf{bst\_abs}\ (m[k \mapsto v])\rangle$$

$$\langle m.\,\mathsf{bst\_ref}\ p \mid \mathsf{bst\_abs}\ m\rangle\ \mathtt{lookup}(p, k)\ \langle v.\,\mathsf{bst\_ref}\ p \mid \mathsf{bst\_abs}\ m \wedge m(k) = v\rangle$$

$$\langle m.\,\mathsf{bst\_ref}\ p \mid \mathsf{bst\_abs}\ m\rangle\ \mathtt{delete}(p, k)\ \langle \mathsf{bst\_ref}\ p \mid \mathsf{bst\_abs}\ (m[k \mapsto \_])\rangle$$

Coarse-Grained Locking

Definitions of Data Structure Assertions

$$R_{cg} \triangleq \exists t.\ \mathsf{BST}\ p\ t * \mathsf{ghost\_bst}_{.5}\ t$$

$$\mathsf{bst\_ref}\ l \triangleq l \mapsto_{\pi} R_{cg}$$

$$\mathsf{bst\_abs}\ t \triangleq \mathsf{ghost\_bst}_{.5}\ t$$

Sketch Proof of Coarse-Grained Insertion of BST

$$\langle l \boxdot\!\!\rightarrow_{\pi} R_{cg} \mid \text{ghost\_bst}_{.5}\, t \rangle$$

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acquire(l);
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- There are multiple locks on different pieces of the data structure
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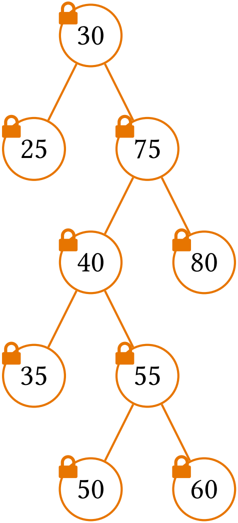
- The abstract state of the data structure is derived from the composition of the states of the locked components
- Each locked component c has a piece of ghost state ghost_c that is split between the lock invariant and the top-level abstract state
- The lock invariant for each component must also carefully account for the ownership of both that component's lock and the locks of related nodes

Hand-over-Hand Locking: BST Example

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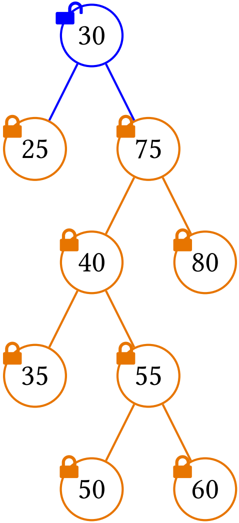


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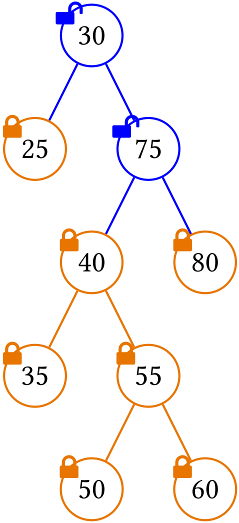


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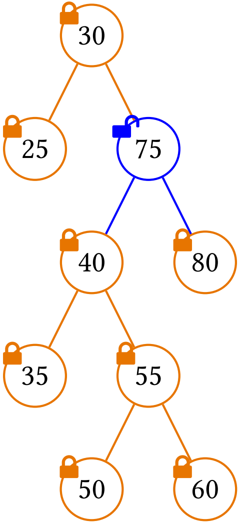


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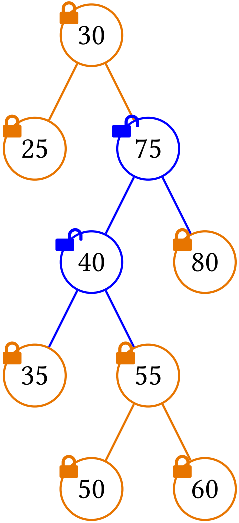


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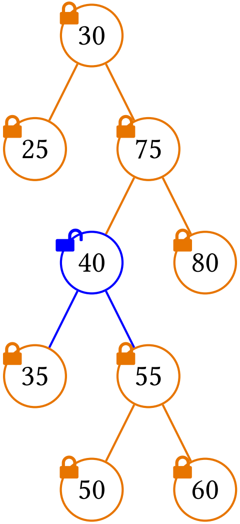


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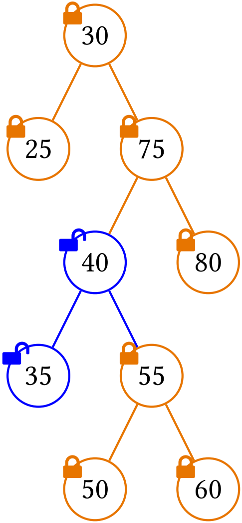


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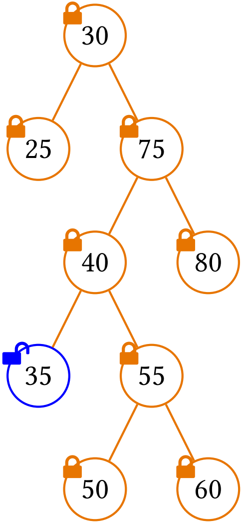


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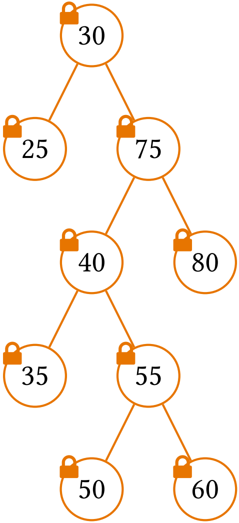


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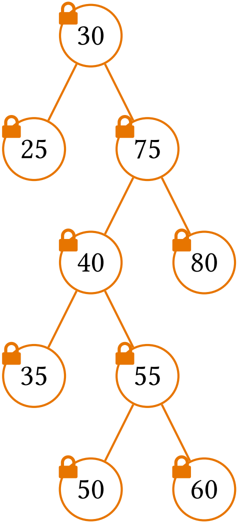

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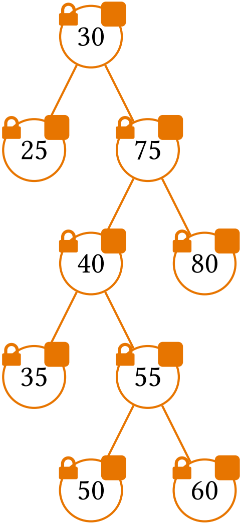

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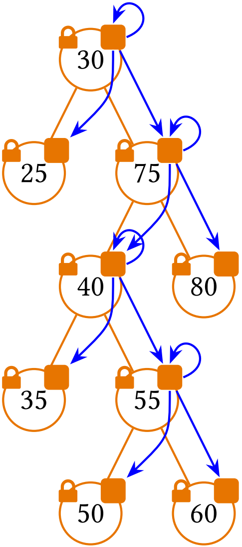

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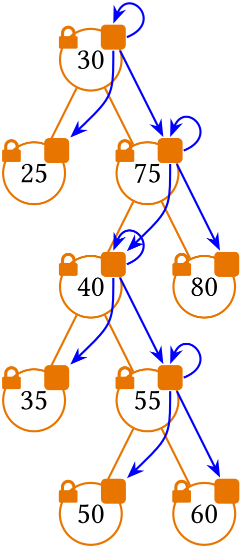

Lock Invariant: $\blacksquare_g(p)$

$\blacksquare_g(p) \triangleq \exists i\, j\, c.\ \text{👻}^{.5}_g((i,j),c) *$

$\quad p.\texttt{lock} \boxdot\!\!\longmapsto_{\pi_1} \blacksquare_g *$

$\quad \text{match } c \text{ with}$

$\quad |\ \text{None} \Rightarrow p.\texttt{t} = \texttt{NULL}$

$\quad |\ \text{Some } (k, v, g_l, g_r) \Rightarrow k \in (i,j) \land$

$\qquad \exists p_l, p_r.\ p.\texttt{t} \mapsto (k, v, p_l, p_r) *$

$\qquad p_l.\texttt{lock} \boxdot\!\!\longmapsto_{\pi_2} \blacksquare_{g_l} *$

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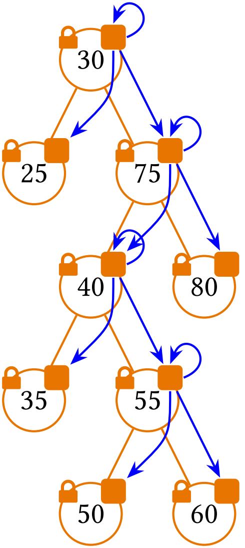

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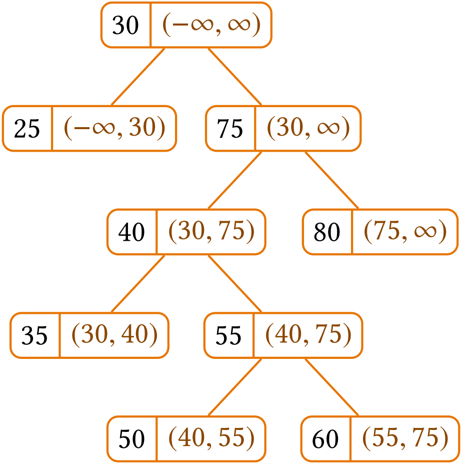
$$p_r.\mathtt{lock} \mapsto_{\pi_2} \blacksquare_{g_r}$$

Per-Thread Handle to the BST

$$\text{bst\_ref}_g\ b \triangleq \exists p.\ b \mapsto p * p.\mathtt{lock} \mapsto \blacksquare_g(p)$$

Global Ghost State: $\mathsf{bst\_abs}_g$



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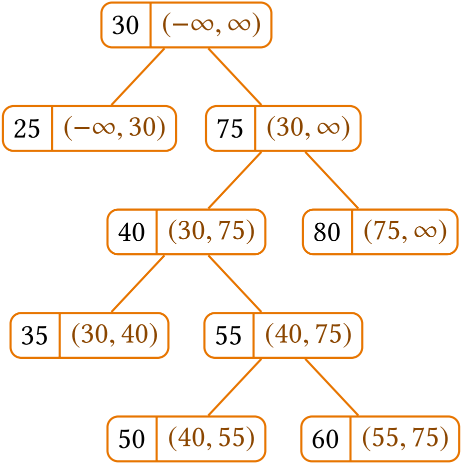

$\square_g\ (t, (i, j)) \triangleq$

match t with

| Leaf $\Rightarrow$ $\text{👻}_g^{.5}\ ((i, j), \mathrm{None})$

| Node $(k, v, t_l, g_l, t_r, g_r) \Rightarrow$

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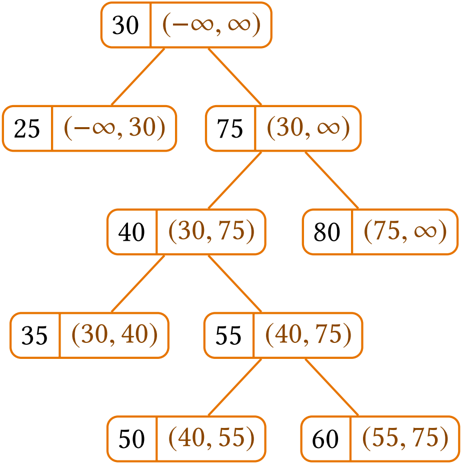
$\square_{g_l}\ (t_l, (i, k)) \ast \square_{g_r}\ (t_r, (k, j))$

Global Ghost State: $\mathrm{bst\_abs}_g$



$\square_g\,(t, (i, j)) \triangleq$
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| Leaf $\Rightarrow$ 👻$^{.5}_{g}\,((i, j), \mathrm{None})$
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$\mathrm{bst\_abs}_g\ m \triangleq\ \exists t.\ t\ \mathrm{impl}\ m \wedge$
$\square_g\,(t, (-\infty, +\infty)) \ast$
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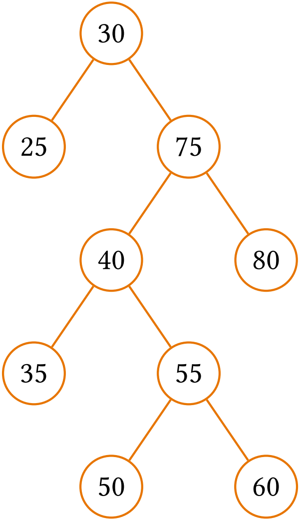
BST Rotation



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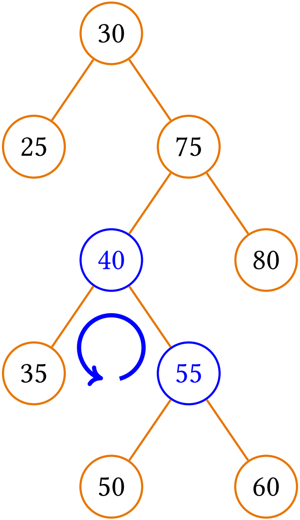

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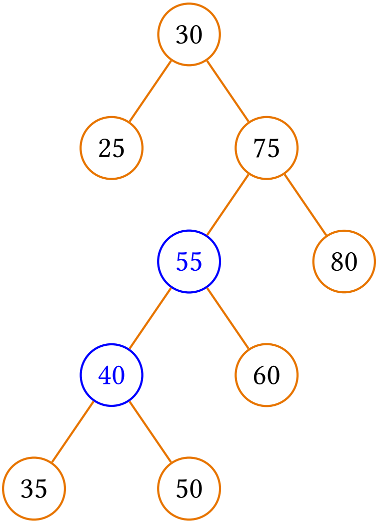

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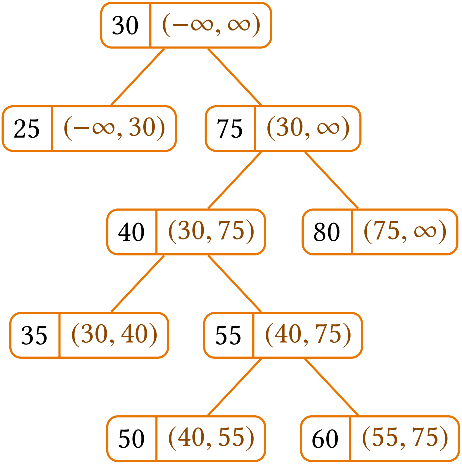

$\square_g\,(t, (i, j)) \triangleq$
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Concurrent Binary Search Tree (BST) as an Example

Atomic Specifications

$$\langle m.\,\mathrm{bst\_ref}\ p \mid \mathrm{bst\_abs}\ m\rangle\ \texttt{insert}(p, k, v)\ \langle \mathrm{bst\_ref}\ p \mid \mathrm{bst\_abs}\ (m[k \mapsto v])\rangle$$

$$\langle m.\,\mathrm{bst\_ref}\ p \mid \mathrm{bst\_abs}\ m\rangle\ \texttt{lookup}(p, k)\ \langle v.\,\mathrm{bst\_ref}\ p \mid \mathrm{bst\_abs}\ m \wedge m(k) = v\rangle$$

$$\langle m.\,\mathrm{bst\_ref}\ p \mid \mathrm{bst\_abs}\ m\rangle\ \texttt{delete}(p, k)\ \langle \mathrm{bst\_ref}\ p \mid \mathrm{bst\_abs}\ (m[k \mapsto \_])\rangle$$

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- Compare to the template proofs for hand-over-hand locking, which use TaDA-style lock specs:
  - Still need per-node ghost state and connection to global ghost state
  - No share accounting: locks are part of the global ghost state and are accessed atomically, never owned by any thread

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- There might be systems where the old lock specs are necessary (e.g. in VST they were part of the soundness proof), so it's good to know that we can still prove atomic specs for data structures.
- We have just modified VST to support TaDA-style lock specs instead, and are looking forward to simpler atomicity proofs for fine-grained C programs.