

## curent

briefly

[called by: [mp00ac](#), [dforce](#).]

[calls: [coords](#).]

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### 1.1 enclosed currents

1. In the vacuum region, the enclosed currents are given by either surface integrals of the current density or line integrals of the magnetic field,

$$\int_S \mathbf{j} \cdot d\mathbf{s} = \int_{\partial S} \mathbf{B} \cdot d\mathbf{l}, \quad (1)$$

and line integrals are usually easier to compute than surface integrals . . .

2. The magnetic field is given by the curl of the magnetic vector potential, as described in e.g. [bfield](#).
3. The toroidal, plasma current is obtained by taking a “poloidal” loop,  $d\mathbf{l} = \mathbf{e}_\theta d\theta$ , on the plasma boundary, where  $B^s = 0$ , to obtain

$$I \equiv \int_0^{2\pi} \mathbf{B} \cdot \mathbf{e}_\theta d\theta = \int_0^{2\pi} (-\partial_s A_\zeta \bar{g}_{\theta\theta} + \partial_s A_\theta \bar{g}_{\theta\zeta}) d\theta, \quad (2)$$

where  $\bar{g}_{\mu\nu} \equiv g_{\mu\nu} / \sqrt{g}$ .

4. The poloidal, “linking” current through the torus is obtained by taking a “toroidal” loop,  $d\mathbf{l} = \mathbf{e}_\zeta d\zeta$ , on the plasma boundary to obtain

$$G \equiv \int_0^{2\pi} \mathbf{B} \cdot \mathbf{e}_\zeta d\zeta = \int_0^{2\pi} (-\partial_s A_\zeta \bar{g}_{\theta\zeta} + \partial_s A_\theta \bar{g}_{\zeta\zeta}) d\zeta. \quad (3)$$

### 1.2 “Fourier integration”

1. Using  $f \equiv -\partial_s A_\zeta \bar{g}_{\theta\theta} + \partial_s A_\theta \bar{g}_{\theta\zeta}$ , the integral for the plasma current is

$$I = \sum_i' f_i \cos(n_i \zeta) 2\pi, \quad (4)$$

where  $\sum'$  includes only the  $m_i = 0$  harmonics.

2. Using  $g \equiv -\partial_s A_\zeta \bar{g}_{\theta\zeta} + \partial_s A_\theta \bar{g}_{\zeta\zeta}$ , the integral for the linking current is

$$G = \sum_i' g_i \cos(m_i \zeta) 2\pi, \quad (5)$$

where  $\sum'$  includes only the  $n_i = 0$  harmonics.

3. The plasma current, Eqn.(4), should be independent of  $\zeta$ , and the linking current, Eqn.(5), should be independent of  $\theta$ . (Perhaps this can be proved analytically; in any case it should be confirmed numerically.)