tr00ab

Calculates rotational transform given an arbitrary tangential field.

[called by: dforce and mp00ac.]

contents

1.0.1 constructing straight field line angle on interfaces

1. The algorithm stems from introducing a straight field line angle $\theta_s = \theta + \lambda(\theta, \zeta)$, where

$$\lambda = \sum_j \lambda_{o,j} \sin(m_j \theta - n_j \zeta) + \sum_j \lambda_{e,j} \cos(m_j \theta - n_j \zeta) \tag{1}$$

and insisting that

$$\frac{\mathbf{B} \cdot \nabla \theta_s}{\mathbf{B} \cdot \nabla \zeta} = \dot{\theta}(1 + \lambda_\theta) + \lambda_\zeta = \iota\!\!\!-, \tag{2}$$

where $\iota\!\!\!-$ is a constant that is to be determined.

2. Writing $\dot{\theta} = -\partial_s A_\zeta / \partial_s A_\theta$, we have

$$\partial_s A_\theta \; \iota\!\!\!- + \partial_s A_\zeta \, \lambda_\theta - \partial_s A_\theta \, \lambda_\zeta = -\partial_s A_\zeta \tag{3}$$

3. Expanding this equation we obtain

$$\begin{aligned}
& \left(A'_{\theta,e,k} \cos \alpha_k + A'_{\theta,o,k} \sin \alpha_k\right) \; \iota\!\!\!- \\
+ & \left(A'_{\zeta,e,k} \cos \alpha_k + A'_{\zeta,o,k} \sin \alpha_k\right) \left(+m_j \lambda_{o,j} \cos \alpha_j - m_j \lambda_{e,j} \sin \alpha_j\right) \\
- & \left(A'_{\theta,e,k} \cos \alpha_k + A'_{\theta,o,k} \sin \alpha_k\right) \left(-n_j \lambda_{o,j} \cos \alpha_j + n_j \lambda_{e,j} \sin \alpha_j\right) \\
= \; - & \left(A'_{\zeta,e,k} \cos \alpha_k + A'_{\zeta,o,k} \sin \alpha_k\right),
\end{aligned} \tag{4}$$

where summation over $k = 1$, `mn` and $j = 2$, `mns` is implied

4. After applying double angle formulae,

$$\begin{aligned}
& \left(A'_{\theta,e,k} \cos \alpha_k + A'_{\theta,o,k} \sin \alpha_k\right) \; \iota\!\!\!- \\
+ & \; \lambda_{o,j} \left(+m_j A'_{\zeta,e,k} + n_j A'_{\theta,e,k}\right) \left[+\cos(\alpha_k + \alpha_j) + \cos(\alpha_k - \alpha_j)\right]/2 \\
+ & \; \lambda_{e,j} \left(-m_j A'_{\zeta,e,k} - n_j A'_{\theta,e,k}\right) \left[+\sin(\alpha_k + \alpha_j) - \sin(\alpha_k - \alpha_j)\right]/2 \\
+ & \; \lambda_{o,j} \left(+m_j A'_{\zeta,o,k} + n_j A'_{\theta,o,k}\right) \left[+\sin(\alpha_k + \alpha_j) + \sin(\alpha_k - \alpha_j)\right]/2 \\
+ & \; \lambda_{e,j} \left(-m_j A'_{\zeta,o,k} - n_j A'_{\theta,o,k}\right) \left[-\cos(\alpha_k + \alpha_j) + \cos(\alpha_k - \alpha_j)\right]/2 \\
= \; - & \left(A'_{\zeta,e,k} \cos \alpha_k + A'_{\zeta,o,k} \sin \alpha_k\right),
\end{aligned} \tag{5}$$

and equating coefficients, an equation of the form $\mathbf{A} \cdot \mathbf{x} = \mathbf{b}$ is obtained, where

$$\mathbf{x} = (\underbrace{\iota\!\!\!-}_{\texttt{x[1]}} , \underbrace{\lambda_{o,2}, \lambda_{o,3}, \ldots}_{\texttt{x[2: N]}} , \underbrace{\lambda_{e,2}, \lambda_{e,3}, \ldots}_{\texttt{x[N+1:2N-1]}})^T. \tag{6}$$

1.0.2 alternative iterative method

1. Consider the equation $\dot{\theta}(1 + \lambda_\theta) + \lambda_\zeta = \iota\!\!\!-$, where $\lambda = \sum_j \lambda_j \sin \alpha_j$, given on a grid

$$\dot{\theta}_i + \dot{\theta}_i \sum_j m_j \cos \alpha_{i,j} \lambda_j - \sum_j n_j \cos \alpha_{i,j} \lambda_j = \iota\!\!\!-, \tag{7}$$

where i labels the grid point.

2. This is a matrix equation . . .