Evaluation of Zernike Basis Functions

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The Zernike basis polynomials $R^m_{m+2j}(r)$ can be defined using a recurrence relation as follows for $j = 0, 1, 2, \ldots$. For given n and m, j has to go up to a maximum value of $j_{\max} = (n-m)/2$. This requires that the difference $(n-m)$ is positive and even. The following recurrence relation is used:

$$R^m_{n+2}(r) = \frac{n+2}{(n+2)^2 - m^2} \left\{ \left[4(n+1)r^2 - \frac{(n+m)^2}{n} - \frac{(n-m+2)^2}{n+2} \right] R^m_n - \frac{n^2-m^2}{n} R^m_{n-2} \right\} . \tag{1}$$

An index shift by 2 in n is performed:

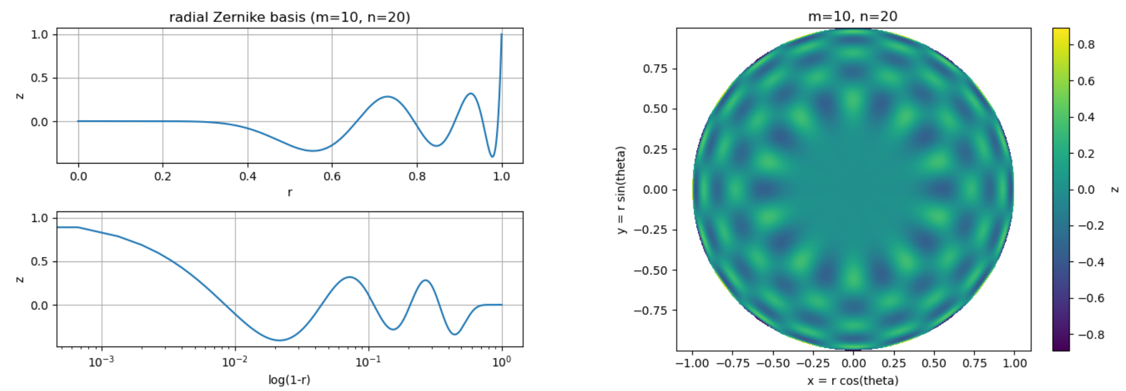
$$R^m_n(r) = \frac{n}{n^2-m^2} \left\{ \left[4(n-1)r^2 - \frac{(n-2+m)^2}{n-2} - \frac{(n-m)^2}{n} \right] \underbrace{R^m_{n-2}}_{=R^m_{j-1}} - \frac{(n-2)^2-m^2}{n-2} \underbrace{R^m_{n-4}}_{=R^m_{j-2}} \right\} . \tag{2}$$

The first two elements are computed separately:

$$R^m_m(r) = r^m \quad : j = 0 \tag{3}$$

$$R^m_{m+2}(r) = (m+2)r^{m+2} - (m+1)r^m \quad : j = 1 \, . \tag{4}$$

Unfortunately, I don't know where I found this. If the reader recognizes this reformulation from a citable source, please let the author[1] know. A demo implementation of this "method" is provided in the file `Zernike.py` along with this document. In Fig. 1 a demo output from this method is shown.



(a) Radial component of Zernike basis.

(b) Two-dimensional Zernike basis.

Figure 1: Evaluated Zernike basis function using the method presented in this document.

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