

## packxi

Packs, and unpacks, geometrical degrees of freedom; and sets coordinate axis.

[called by: [dforce](#), [global](#), [hesian](#), [newton](#) and [xspech](#).]

[calls: [rzaxis](#).]

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### 1.1 geometrical degrees of freedom

1. The geometrical degrees-of-freedom, namely the  $R_{j,v}$  and  $Z_{j,v}$  where  $v$  labels the interface and  $j$  labels the Fourier harmonic, must be “packxi”, and “unpackxi”, into a single vector,  $\xi$ , so that standard numerical routines can be called to find solutions to force-balance, i.e.  $\mathbf{F}[\xi] = 0$ .

2. A coordinate “pre-conditioning” factor is included:

$$\xi_k \equiv \frac{R_{j,v}}{\Psi_{j,v}}, \quad (1)$$

where  $\Psi_{j,v} \equiv \text{psifactor}(j,v)$ , which is defined in [global](#).

### 1.2 coordinate axis

1. The coordinate axis is not an independent degree-of-freedom of the geometry. It is constructed by extrapolating the geometry of the innermost interface down to a line.
2. Note that if the coordinate axis depends only on the geometry of the innermost interface then the block tridiagonal structure of the the force-derivative matrix is preserved.
3. Define the arc-length weighted averages,

$$R_0(\zeta) \equiv \frac{\int_0^{2\pi} R_1(\theta, \zeta) dl}{L(\zeta)}, \quad Z_0(\zeta) \equiv \frac{\int_0^{2\pi} Z_1(\theta, \zeta) dl}{L(\zeta)}, \quad (2)$$

where  $L(\zeta) \equiv \int_0^{2\pi} dl$  and  $dl \equiv \sqrt{\partial_\theta R_1(\theta, \zeta)^2 + \partial_\theta Z_1(\theta, \zeta)^2} d\theta$ .

4. Note that if  $dl$  does not depend on  $\theta$ , i.e. if  $\theta$  is the equal arc-length angle, then the expressions simplify.
5. Note that the geometry of the coordinate axis thus constructed only depends on the geometry of the innermost interface, by which I mean that the geometry of the coordinate axis is independent of the angle parameterization.

### 1.3 some numerical comments

1. First, the differential poloidal length,  $dl \equiv \sqrt{R_\theta^2 + Z_\theta^2}$ , is computed in real space using an inverse FFT the from Fourier harmonics of  $R$  and  $Z$ .
2. Second, the Fourier harmonics of the  $dl$  are computed using an FFT. The integration over  $\theta$  to construct  $L \equiv \int dl$  is now trivial: just multiply the  $m = 0$  harmonics of  $dl$  by  $2\pi$ . The `ajk(1:mn)` variable is used.
3. Next, the weighted  $Rdl$  and  $Zdl$  are computed in real space, and the poloidal integral is similarly taken.
4. Lastly, the Fourier harmonics are constructed using an FFT after dividing in real space.