coords

Calculates coordinate transformation, and metric elements and curvatures if required, using FFTs.

[called by: global, bnorml, lforce, dforce, curent, jo00aa, metrix, sc00aa.]

[calls: .]

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1.1 coordinates

- 1. We work in coordinates, (s, θ, ζ) , which are be defined inversely via a transformation to Cartesian coordinates, (x, y, z).
- 2. The toroidal angle, ζ , is identical to the cylindrical angle, $\zeta \equiv \phi$.
- 3. The radial coordinate, s, is not a global variable: it only needs to be defined in each volume, and in each volume $s \in [-1, 1]$.
- 4. The choice of poloidal angle, θ , does not affect the following.

1.2 geometry

- 1. The geometry of the "ideal"-interfaces, $\mathbf{x}_{v}(\theta,\zeta)$, is given by $R(\theta,\zeta)$ and $Z(\theta,\zeta)$ as follows:
 - Igeometry=1 : Cartesian

$$\mathbf{x} \equiv \theta \,\hat{\mathbf{i}} + \zeta \,\hat{\mathbf{j}} + R \,\hat{\mathbf{k}} \tag{1}$$

• Igeometry=2 : Cylindrical

$$\mathbf{x} = R \cos \theta \,\hat{\mathbf{i}} + R \sin \theta \,\hat{\mathbf{j}} + \zeta \,\hat{\mathbf{k}} \tag{2}$$

• Igeometry=3 : Toroidal

$$\mathbf{x} \equiv R \,\hat{\mathbf{r}} + Z \,\hat{\mathbf{k}} \tag{3}$$

where $\hat{\mathbf{r}} \equiv \cos \phi \, \hat{\mathbf{i}} + \sin \phi \, \hat{\mathbf{j}} \, \text{and} \, \hat{\phi} \equiv -\sin \phi \, \hat{\mathbf{i}} + \cos \phi \, \hat{\mathbf{j}}$

2. The geometry of the ideal interfaces is given as Fourier summation: e.g., for stellarator-symmetry

$$R_v(\theta,\zeta) \equiv \sum_j R_{j,v} \cos \alpha_j,$$
 (4)

$$Z_v(\theta,\zeta) \equiv \sum_j Z_{j,v} \sin \alpha_j,$$
 (5)

where $\alpha_j \equiv m_j \theta - n_j \zeta$.

1.3 interpolation between interfaces

- 1. The "coordinate" functions, $R(s, \theta, \zeta)$ and $Z(s, \theta, \zeta)$, are constructed by radially interpolating the Fourier representations of the ideal-interfaces.
- 2. The v-th volume is bounded by \mathbf{x}_{v-1} and \mathbf{x}_v .
- 3. In each <u>annular</u> volume, the coordinates are constructed by linear interpolation:

$$R(s,\theta,\zeta) \equiv \sum_{j} \left[\frac{(1-s)}{2} R_{j,v-1} + \frac{(1+s)}{2} R_{j,v} \right] \cos \alpha_{j},$$

$$Z(s,\theta,\zeta) \equiv \sum_{j} \left[\frac{(1-s)}{2} Z_{j,v-1} + \frac{(1+s)}{2} Z_{j,v} \right] \sin \alpha_{j},$$

$$(6)$$

1.3.1 coordinate singularity: regularized extrapolation

- 1. For cylindrical or toroidal geometry, in the innermost, "simple-torus" volume, the coordinates are constructed by an interpolation that "encourages" the interpolated coordinate surfaces to not intersect.
- 2. Introduce $\bar{s} \equiv (s+1)/2$, so that in each volume $\bar{s} \in [0,1]$, then

$$R_j(s) = R_{j,0} + (R_{j,1} - R_{j,0})f_j, (7)$$

$$Z_{j}(s) = Z_{j,0} + (Z_{j,1} - Z_{j,0})f_{j}, (8)$$

where, in toroidal geometry,

$$f_j \equiv \begin{cases} \bar{s} &, \text{ for } m_j = 0, \\ \bar{s}^{m_j/2} &, \text{ otherwise.} \end{cases}$$
 (9)

3. Note: The location of the coordinate axis, i.e. the $R_{j,0}$ and $Z_{j,0}$, is set in the coordinate "packing" and "unpacking" routine, packxi.

1.4 Jacobian

- 1. The coordinate Jacobian (and some other metric information) is given by
 - Igeometry=1 : Cartesian

$$\mathbf{e}_{\theta} \times \mathbf{e}_{\zeta} = -R_{\theta} \,\hat{\mathbf{i}} - R_{\zeta} \,\hat{\mathbf{j}} + \hat{\mathbf{k}} \tag{10}$$

$$\boldsymbol{\xi} \cdot \mathbf{e}_{\theta} \times \mathbf{e}_{\zeta} = \delta R \tag{11}$$

$$\sqrt{g} = R_s \tag{12}$$

• Igeometry=2 : Cylindrical

$$\mathbf{e}_{\theta} \times \mathbf{e}_{\zeta} = (R_{\theta} \sin \theta + R \cos \theta) \,\hat{\mathbf{i}} + (R \sin \theta - R_{\theta} \cos \theta) \,\hat{\mathbf{j}} - RR_{\zeta} \,\hat{\mathbf{k}}$$
(13)

$$\boldsymbol{\xi} \cdot \mathbf{e}_{\theta} \times \mathbf{e}_{\zeta} = \delta R R \tag{14}$$

$$\sqrt{g} = R_s R \tag{15}$$

• Igeometry=3 : Toroidal

$$\mathbf{e}_{\theta} \times \mathbf{e}_{\zeta} = -R Z_{\theta} \,\hat{r} + (Z_{\theta} R_{\zeta} - R_{\theta} Z_{\zeta}) \hat{\phi} + R R_{\theta} \,\hat{z} \tag{16}$$

$$\boldsymbol{\xi} \cdot \mathbf{e}_{\theta} \times \mathbf{e}_{\zeta} = R(\delta Z R_{\theta} - \delta R Z_{\theta}) \tag{17}$$

$$\sqrt{g} = R(Z_s R_\theta - R_s Z_\theta) \tag{18}$$

1.4.1 cylindrical metrics

1. The cylindrical metrics and Jacobian are

$$\sqrt{g} = R_s R, \quad g_{ss} = R_s R_s, \quad g_{s\theta} = R_s R_{\theta}, \quad g_{s\zeta} = R_s R_{\zeta}, \quad g_{\theta\theta} = R_{\theta} R_{\theta} + R^2, \quad g_{\theta\zeta} = R_{\theta} R_{\zeta}, \quad g_{\zeta\zeta} = R_{\zeta} R_{\zeta} + 1 \tag{19}$$

1.5 logical control

1. The logical control is provided by Lcurvature as follows:

Lcurvature=0 : only the coordinate transformation is computed, i.e. only R and Z are calculated e.g. global

Lcurvature=1: the Jacobian, \sqrt{g} , and "lower" metrics, $g_{\mu,\nu}$, are calculated e.g. bnorml, lforce, curent, metrix, sc00aa

Lcurvature=2: the "curvature" terms are calculated, by which I mean the second derivatives of the position vector; this information is required for computing the current, $\mathbf{j} = \nabla \times \nabla \times \mathbf{A}$ e.g. jo00aa

Lcurvature=3: the derivative of the $g_{\mu,\nu}/\sqrt{g}$ w.r.t. the interface boundary geometry is calculated e.g. metrix, curent

Lcurvature=4: the derivative of the $g_{\mu,\nu}$ w.r.t. the interface boundary geometry is calculated e.g. dforce