tr00ab

Calculates rotational transform given an arbitrary tangential field.

[called by: dforce and mp00ac.]

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1.0.1 constructing straight field line angle on interfaces

1. The algorithm stems from introducing a straight field line angle $\theta_s = \theta + \lambda(\theta, \zeta)$, where

$$\lambda = \sum_j \lambda_{o,j} \sin(m_j\theta - n_j\zeta) + \sum_j \lambda_{e,j} \cos(m_j\theta - n_j\zeta) \tag{1}$$

and insisting that

$$\frac{\mathbf{B}\cdot\nabla\theta_s}{\mathbf{B}\cdot\nabla\zeta} = \dot\theta(1+\lambda_\theta) + \lambda_\zeta = \text{ɨ}, \tag{2}$$

where ɨ is a constant that is to be determined.

2. Writing $\dot\theta = -\partial_s A_\zeta / \partial_s A_\theta$, we have

$$\partial_s A_\theta \; \text{ɨ} + \partial_s A_\zeta \, \lambda_\theta - \partial_s A_\theta \, \lambda_\zeta = -\partial_s A_\zeta \tag{3}$$

3. Expanding this equation we obtain

$$\begin{aligned} & & \left(A'_{\theta,e,k}\cos\alpha_k + A'_{\theta,o,k}\sin\alpha_k\right) \; \text{ɨ} \\ & + & \left(A'_{\zeta,e,k}\cos\alpha_k + A'_{\zeta,o,k}\sin\alpha_k\right) \left(+m_j\lambda_{o,j}\cos\alpha_j - m_j\lambda_{e,j}\sin\alpha_j\right) \\ & - & \left(A'_{\theta,e,k}\cos\alpha_k + A'_{\theta,o,k}\sin\alpha_k\right) \left(-n_j\lambda_{o,j}\cos\alpha_j + n_j\lambda_{e,j}\sin\alpha_j\right) \\ = & - & \left(A'_{\zeta,e,k}\cos\alpha_k + A'_{\zeta,o,k}\sin\alpha_k\right), \end{aligned} \tag{4}$$

where summation over $k = 1,$ `mn` and $j = 2,$ `mns` is implied

4. After applying double angle formulae,

$$\begin{aligned} & & \left(A'_{\theta,e,k}\cos\alpha_k + A'_{\theta,o,k}\sin\alpha_k\right) \; \text{ɨ} \\ & + & \lambda_{o,j}\left(+m_j A'_{\zeta,e,k} + n_j A'_{\theta,e,k}\right)\left[+\cos(\alpha_k+\alpha_j) + \cos(\alpha_k-\alpha_j)\right]/2 \\ & + & \lambda_{e,j}\left(-m_j A'_{\zeta,e,k} - n_j A'_{\theta,e,k}\right)\left[+\sin(\alpha_k+\alpha_j) - \sin(\alpha_k-\alpha_j)\right]/2 \\ & + & \lambda_{o,j}\left(+m_j A'_{\zeta,o,k} + n_j A'_{\theta,o,k}\right)\left[+\sin(\alpha_k+\alpha_j) + \sin(\alpha_k-\alpha_j)\right]/2 \\ & + & \lambda_{e,j}\left(-m_j A'_{\zeta,o,k} - n_j A'_{\theta,o,k}\right)\left[-\cos(\alpha_k+\alpha_j) + \cos(\alpha_k-\alpha_j)\right]/2 \\ = & - & \left(A'_{\zeta,e,k}\cos\alpha_k + A'_{\zeta,o,k}\sin\alpha_k\right), \end{aligned} \tag{5}$$

and equating coefficients, an equation of the form $\mathbf{A}\cdot\mathbf{x} = \mathbf{b}$ is obtained, where

$$\mathbf{x} = (\underbrace{\;\text{ɨ}\;}_{\texttt{x[1]}}, \underbrace{\lambda_{o,2}, \lambda_{o,3}, \ldots}_{\texttt{x[2: N]}}, \underbrace{\lambda_{e,2}, \lambda_{e,3}, \ldots}_{\texttt{x[N+1:2N-1]}})^T. \tag{6}$$

1.0.2 alternative iterative method

1. Consider the equation $\dot\theta(1+\lambda_\theta) + \lambda_\zeta = \text{ɨ}$, where $\lambda = \sum_j \lambda_j \sin\alpha_j$, given on a grid

$$\dot\theta_i + \dot\theta_i \sum_j m_j \cos\alpha_{i,j}\lambda_j - \sum_j n_j \cos\alpha_{i,j}\lambda_j = \text{ɨ}, \tag{7}$$

where i labels the grid point.

2. This is a matrix equation . . .