volume

Computes volume of each region; and, if required, the derivatives of the volume with respect to the interface geometry.

[called by: dforce and xspech.] [calls: .]

contents

1.1 volume integral

1. The volume enclosed by the v-th interface is given by the integral

$$V = \int_{\mathcal{V}} dv = \frac{1}{3}\int_{\mathcal{V}} \nabla\cdot\mathbf{x}\, dv = \frac{1}{3}\int_{\mathcal{S}} \mathbf{x}\cdot d\mathbf{s} = \frac{1}{3}\int_0^{2\pi} d\theta \int_0^{2\pi/N} d\zeta \quad \mathbf{x}\cdot\mathbf{x}_\theta\times\mathbf{x}_\zeta|^s \tag{1}$$

where we have used $\nabla\cdot\mathbf{x} = 3$, and have assumed that the domain is periodic in the angles.

1.2 representation of surfaces

1. The coordinate functions are

$$R(\theta,\zeta) = \sum_i R_{e,i}\, \cos\alpha_i + \sum_i R_{o,i}\, \sin\alpha_i \tag{2}$$

$$Z(\theta,\zeta) = \sum_i Z_{e,i}\, \cos\alpha_i + \sum_i Z_{o,i}\, \sin\alpha_i, \tag{3}$$

where $\alpha_i \equiv m_i\theta - n_i\zeta$.

1.3 geometry

1. The geometry is controlled by the input parameter `Igeometry` as follows:

2. `Igeometry.eq.1` : Cartesian : $\sqrt{g} = R_s$

$$\begin{aligned} V &= \int_0^{2\pi} d\theta \int_0^{2\pi/N} d\zeta\; R \\ &= 2\pi\, \frac{2\pi}{N}\, R_{e,1} \end{aligned} \tag{4}$$

3. `Igeometry.eq.2` : cylindrical : $\sqrt{g} = RR_s = \frac{1}{2}\partial_s(R^2)$

$$\begin{aligned} V &= \frac{1}{2}\int_0^{2\pi} d\theta \int_0^{2\pi/N} d\zeta\; R^2 \\ &= \frac{1}{2}\, 2\pi\, \frac{2\pi}{N}\, \frac{1}{2}\, \sum_i\sum_j R_{e,i}R_{e,j}\,[\cos(\alpha_i-\alpha_j)+\cos(\alpha_i+\alpha_j)] \\ &+ \frac{1}{2}\, 2\pi\, \frac{2\pi}{N}\, \frac{1}{2}\, \sum_i\sum_j R_{o,i}R_{o,j}\,[\cos(\alpha_i-\alpha_j)-\cos(\alpha_i+\alpha_j)] \end{aligned} \tag{5}$$

4. `Igeometry.eq.3` : toroidal : $\mathbf{x}\cdot\mathbf{e}_\theta\times\mathbf{e}_\zeta = R(ZR_\theta - RZ_\theta)$

$$\begin{aligned} V &= \frac{1}{3}\int_0^{2\pi} d\theta \int_0^{2\pi/N} d\zeta\; R\,(ZR_\theta - RZ_\theta) \\ &= \frac{1}{3}\sum_i\sum_j\sum_k R_{e,i}\,(Z_{e,j}R_{o,k} - R_{e,j}Z_{o,k})\,(+m_k)\iint d\theta d\zeta\;\cos\alpha_i\cos\alpha_j\cos\alpha_k \\ &+ \frac{1}{3}\sum_i\sum_j\sum_k R_{e,i}\,(Z_{o,j}R_{e,k} - R_{o,j}Z_{e,k})\,(-m_k)\iint d\theta d\zeta\;\cos\alpha_i\sin\alpha_j\sin\alpha_k \\ &+ \frac{1}{3}\sum_i\sum_j\sum_k R_{o,i}\,(Z_{e,j}R_{e,k} - R_{e,j}Z_{e,k})\,(-m_k)\iint d\theta d\zeta\;\sin\alpha_i\cos\alpha_j\sin\alpha_k \\ &+ \frac{1}{3}\sum_i\sum_j\sum_k R_{o,i}\,(Z_{o,j}R_{o,k} - R_{o,j}Z_{o,k})\,(+m_k)\iint d\theta d\zeta\;\sin\alpha_i\sin\alpha_j\cos\alpha_k \end{aligned} \tag{6}$$

5. (Recall that the integral over an odd function is zero, so various terms in the above expansion have been ignored.)

6. The trigonometric terms are

$$
\begin{aligned}
4\ \cos\alpha_i \cos\alpha_j \cos\alpha_k &= +\ \cos(\alpha_i+\alpha_j+\alpha_k)\ +\ \cos(\alpha_i+\alpha_j-\alpha_k)\ +\ \cos(\alpha_i-\alpha_j+\alpha_k)\ +\ \cos(\alpha_i-\alpha_j-\alpha_k)\\
4\ \cos\alpha_i \sin\alpha_j \sin\alpha_k &= -\ \cos(\alpha_i+\alpha_j+\alpha_k)\ +\ \cos(\alpha_i+\alpha_j-\alpha_k)\ +\ \cos(\alpha_i-\alpha_j+\alpha_k)\ -\ \cos(\alpha_i-\alpha_j-\alpha_k)\\
4\ \sin\alpha_i \cos\alpha_j \sin\alpha_k &= -\ \cos(\alpha_i+\alpha_j+\alpha_k)\ +\ \cos(\alpha_i+\alpha_j-\alpha_k)\ -\ \cos(\alpha_i-\alpha_j+\alpha_k)\ +\ \cos(\alpha_i-\alpha_j-\alpha_k)\\
4\ \sin\alpha_i \sin\alpha_j \cos\alpha_k &= -\ \cos(\alpha_i+\alpha_j+\alpha_k)\ -\ \cos(\alpha_i+\alpha_j-\alpha_k)\ +\ \cos(\alpha_i-\alpha_j+\alpha_k)\ +\ \cos(\alpha_i-\alpha_j-\alpha_k)
\end{aligned}
\tag{7}
$$

7. The required derivatives are

$$
\begin{aligned}
3\frac{\partial V}{\partial R_{e,i}} &= (+Z_{e,j}R_{o,k}m_k - R_{e,j}Z_{o,k}m_k - R_{e,j}Z_{o.k}m_k) && \iint d\theta d\zeta\ \cos\alpha_i \cos\alpha_j \cos\alpha_k\\
&+ (-Z_{o,j}R_{e,k}m_k + R_{o,j}Z_{e,k}m_k + R_{o,j}Z_{e,k}m_k) && \iint d\theta d\zeta\ \cos\alpha_i \sin\alpha_j \sin\alpha_k\\
&+ (-R_{o,k}Z_{e,j}m_i) && \iint d\theta d\zeta\ \sin\alpha_i \cos\alpha_j \sin\alpha_k\\
&+ (-R_{e,k}Z_{o,j}m_i) && \iint d\theta d\zeta\ \sin\alpha_i \sin\alpha_j \cos\alpha_k
\end{aligned}
\tag{8}
$$

$$
\begin{aligned}
3\frac{\partial V}{\partial Z_{o,i}} &= (-R_{e,k}R_{e,j}m_i) && \iint d\theta d\zeta\ \cos\alpha_i \cos\alpha_j \cos\alpha_k\\
&+ (-R_{o,k}R_{o,j}m_i) && \iint d\theta d\zeta\ \cos\alpha_i \sin\alpha_j \sin\alpha_k\\
&+ (-R_{e,j}R_{e,k}m_k) && \iint d\theta d\zeta\ \sin\alpha_i \cos\alpha_j \sin\alpha_k\\
&+ (+R_{o,j}R_{o,k}m_k) && \iint d\theta d\zeta\ \sin\alpha_i \sin\alpha_j \cos\alpha_k
\end{aligned}
\tag{9}
$$

volume.h last modified on 18-07-26 10:50:26.7; SPEC subroutines;