# SPEC

3.00

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# 1 The Stepped Pressure Equilibrium Code

A PDF version of this manual is available: SPEC\_manual.pdf

- Github pages
- Subroutine documentations
- · SPEC on PPPL Theory Dept.
- MRxMHD website

# 2 Compilation hints for SPEC

In order to run SPEC, you need a copy of the HDF5 libraries installed which has both the Fortran interface and the parallel (MPI I/O) enabled.

### 2.1 Mac

See e.g. this document for more detailed instructions: https://support.hdfgroup.org/ftp/HD← F5/current/src/unpacked/release\_docs/INSTALL\_CMake.txt

In short:

- 1. download hdf5-1.10.5.tar.gz from https://www.hdfgroup.org/downloads/hdf5/source-code/
- 2. extract

```
tar xzf hdf5-1.10.5.tar.qz
```

1. cd into source folder

cd hdf5-1.10.5

1. make a build folder

mkdir build

1. cd into build folder

cd build

1. run cmake with options for parallel support and Fortran interface (parallel support and C++ interface are not compatible; so we have to disable the C++ interface)

1. actually build the HDF5 library

make

This should leave you with a file "hdf5-1.10.5.dmg" or similar, which you can install just as any other Mac application. During the build process of SPEC, you then only need to specify the HDF5 folder in the Makefile, which will likely be /Applications/HDF\_Group/HDF5/1.10.5.

## 3 Manual / Documentation

### 3.1 Poloidal flux and rotational transform

Given the canonical integrable form,  $\mathbf{A} = \psi \nabla \theta - \chi(\psi) \nabla \zeta$ , we can derive  $\mathbf{B} = \nabla \psi \times \nabla \theta + \nabla \zeta \times \nabla \psi \ \chi'$ . The poloidal flux is given by

$$\Psi_p = \iint \mathbf{B} \cdot \mathbf{e}_{\zeta} \times \mathbf{e}_{\psi} \ d\zeta d\psi = 2\pi \int \chi' d\psi. \tag{1}$$

The rotational-transform is

$$\pm = \frac{\mathbf{B} \cdot \nabla \theta}{\mathbf{B} \cdot \nabla \zeta} = \chi'.$$
(2)

The rotational-transform has the same sign as the poloidal flux.

The SPEC representation for the magnetic vector potential is

$$\mathbf{A} = A_{\theta} \nabla \theta + A_{\zeta} \nabla \zeta, \tag{3}$$

where we can see that  $A_{\zeta}=-\chi$ . The poloidal flux is

$$\int \mathbf{B} \cdot d\mathbf{s} = \oint A_{\zeta} d\zeta. \tag{4}$$

It would seem that the rotational-transform has opposite sign to  $A_{\zeta}$ . To be honest, I am a little confused regarding the sign.

### 3.2 Outline

This document is intended to organise the different potentially valuable improvements to the SPEC code, which could make it more robust, faster, and increase its capabilities.

The document is divided in two categories:

Numerical Improvements: independent improvements that are of numerical importance but have no added physics value *per se*, although they may allow new or better physics investigations.

Physics Applications: research topics that could be addressed with the code, either in its present form or after the completion of one or more topics listed in Numerical Improvements.

## 3.3 Numerical Improvements

## 3.3.1 Compile code with GCC for error checking

Has been implemented in Makefile for most platforms. Checks against Intel version show small differences on the order of  $10^{-15}$  relative deviation, which are likely due so slighly different optimization strategies.

## 3.3.2 Profile code with gprof to find inefficient lines of code

# 3.3.3 Run code with Valgrind to identify memory leaks

### 3.3.4 De-NAG-ification

Compilation of SPEC does not rely on NAG anymore; some functionality (e.g. SQP in ma02aa.f90) might need replacements for the NAG routines to be re-enabled.

### 3.3.5 Revision of spectral-constraints

This is bit of a mess. All the mathematics is standard, and all that is required is for someone to calmly go through lots of algebra. This task should be high priority, as SRH suspects that the spectral constraints as presently enforced result in an ill-conditioned force vector, which means that the code is overly sensitive to the initial guess and does not converge robustly. Potential speed improvements are tremendous.

### 3.3.6 Extension to arbitrary toroidal angle

This can further reduce the required Fourier resolution, and so this can reduce the computation. SRH is particularly interested in this as it will allow for exotic configurations (knots, figure-8, etc.) that cannot presently be computed.

### 3.3.7 Exploit symmetry of the metric

This is easy, but somewhat tedious. Take a look at ma00aa() to see what is required. Potential speed improvement is considerable.

### 3.3.8 symmetry of "local" Beltrami matrices

This is easy. Take a look at matrix(), which constructs the Beltrami matrices, and mp00ac(), which performs the inversion. Potential speed improvement is considerable.

### 3.3.9 Exploit block tri-diagonal structure of "global" linearized force balance matrix

This requires an efficient subroutine. SRH believes that Hirshman constructed such a routine (Hirshman et al. (2010) [4]). The potential speed improvement is tremendous. See newton() for where the tri-diagonal, linearized force-balance matrix is inverted.

### 3.3.10 Enforce Helicity constraint

This will allow investigation of different, arguably more-physical classes of equilibria. See ma02aa().

## 3.3.11 Establish test-cases

A suite of test cases should be constructed, with different geometries etc., that run fast, and that can be benchmarked to machine precision. In the InputFiles/TestCases directory, some input files for SPEC are available for this purpose. One should write routines which execute these input files and compare the output data against a publicy-available set of output files to check SPEC before a new release is made.

## 3.3.12 Verify free-boundary

This is almost complete. The corresponding publication is being written. The virtual casing routines need to be investigated and made more efficient. The virtual casing routine in slab geometry needs revision (because of an integral over an infinite domain).

## 3.3.13 Enforcement of toroidal current profile

Adjust  $\mu$ 's, fluxes and/or rotational transform to obtain desired current profile (without singular currents). This is implemented and needs to be merged into the master branch. An additional routine is required to iterate on the helicity multipliers etc. as required *after* the local Beltrami fields have been calculated and *before* the global force balance iterations proceed.

### 3.3.14 Interpret eigenvectors and eigenvalues of Hessian

This is already completed: see <a href="hesian">hesian</a>(). However, this actually computes the force gradient matrix. For toroidal geometry there is a complication; namely that the hessian matrix includes the derivatives of the spectral constraints. For Cartesian geometry, it is ready to go. SRH will begin writing a paper on the stability of slab MRxMHD equilibria.

## 3.4 Physics Applications

## 3.4.1 Calculate high-resolution equilibria, e.g. W7-X

requires: Exploit symmetry of the metric, symmetry of "local" Beltrami matrices, and other improvements that can make the code faster at high Fourier resolution

### 3.4.2 Calculate equilibria by conserving helicity and fluxes

Applications to saturated island studies, sawteeth, etc. requires: Calculate equilibria by conserving helicity and fluxes

### 3.4.3 Calculate free-boundary stellarator equilibria

to predict scrape-off-layer (SOL) topologies and  $\beta$ -limits. requires: Verify free-boundary Mostly complete.

## 3.4.4 Evaluate stability of MRxMHD equilibria

perhaps starting from simplest system (slab tearing). requires: Interpret eigenvectors and eigenvalues of Hessian

# 3.5 Revision of coordinate singularity: axisymmetric; polar coordinates

· Consider a general, magnetic vector potential given in Cartesian coordinates,

$$\mathbf{A} = A_x \nabla x + A_y \nabla y + A_z \nabla z + \nabla g \tag{5}$$

where  $A_x$ ,  $A_y$ ,  $A_z$ , and the as-yet-arbitrary gauge function, g, are regular at (x,y)=(0,0), i.e. they can be expanded as a Taylor series, e.g.

$$A_x = \sum_{i,j} \alpha_{i,j} x^i y^j, \qquad A_y = \sum_{i,j} \beta_{i,j} x^i y^j, \qquad A_z = \sum_{i,j} \gamma_{i,j} x^i y^j, \qquad g = \sum_{i,j} \delta_{i,j} x^i y^j, \tag{6}$$

for small x and small y.

• Note that we have restricted attention to the "axisymmetric" case, as there is no dependence on z.

The "polar" coordinate transformation,

$$x = r \cos \theta,$$

$$y = r \sin \theta,$$

$$z = \zeta,$$
(7)

induces the vector transformation

$$\nabla x = \cos \theta \, \nabla r - r \sin \theta \, \nabla \theta \qquad ,$$

$$\nabla y = \sin \theta \, \nabla r + r \cos \theta \, \nabla \theta \qquad ,$$

$$\nabla z = \qquad \qquad \nabla \zeta \qquad .$$
(8)

• By repeated applications of the double-angle formula, the expressions for  $A_x$ ,  $A_y$  and g can be cast as functions of  $(r, \theta)$ ,

$$A_x = \sum_{m} r^m [a_{m,0} + a_{m,1} r^2 + a_{m,2} r^4 + \dots] \sin(m\theta),$$
 (9)

$$A_y = \sum_{m} r^m [b_{m,0} + b_{m,1} r^2 + b_{m,2} r^4 + \dots] \cos(m\theta), \tag{10}$$

$$A_z = \sum_m r^m [c_{m,0} + c_{m,1} r^2 + c_{m,2} r^4 + \dots] \cos(m\theta), \tag{11}$$

$$g = \sum_{m} r^{m} [g_{m,0} + g_{m,1} r^{2} + g_{m,2} r^{4} + ...] \sin(m\theta),$$
 (12)

where attention is restricted to stellarator symmetric geometry, but similar expressions hold for the non-stellarator symmetric terms.

· Collecting these expressions, the vector potential can be expressed

$$\mathbf{A} = A_r \nabla r + A_\theta \nabla \theta + A_\zeta \nabla \zeta + \partial_r g \nabla r + \partial_\theta g \nabla \theta, \tag{13}$$

where

(Note: Mathematica was used to perform the algebraic manipulations, and the relevant notebook was included as part of the SPEC CVS repository.)

• There is precisely enough gauge freedom so that we may choose  $A_r = 0$ . For example, the choice

$$g_{1,0} = - \qquad b_{0,0} \qquad ,$$

$$g_{2,0} = - ( a_{1,0}/2 + b_{1,0}/2 ) / 2 ,$$

$$g_{3,0} = - ( a_{2,0}/2 + b_{2,0}/2 ) / 3 ,$$

$$... = ...$$
(15)

eliminates the lowest order r dependence in each harmonic.

- By working through the algebra (again, using Mathematica) the expressions for  $A_{\theta}$  and  $A_{\zeta}$  become

$$A_{\theta} = r^2 f_0(\rho) + r^3 f_1(\rho) \cos(\theta) + r^4 f_2(\rho) \cos(2\theta) + r^5 f_3(\rho) \cos(3\theta) + \dots$$
 (16)

$$A_{\zeta} = q_0(\rho) + r^1 q_1(\rho) \cos(\theta) + r^2 q_2(\rho) \cos(2\theta) + r^3 q_3(\rho) \cos(3\theta) + \dots$$
 (17)

where  $\rho \equiv r^2$  and the  $f_m(\rho)$  and  $g_m(\rho)$  are abitrary polynomials in  $\rho$ . [The expression for  $A_{\zeta}$  is unchanged from Eqn. (11).]

## 3.5.1 somewhat generally, ...

· For stellarator-symmetric configurations,

$$\mathbf{A} = \sum_{m,n} A_{\theta,m,n} \cos(m\theta - n\zeta) \nabla \theta + \sum_{m,n} A_{\zeta,m,n} \cos(m\theta - n\zeta) \nabla \zeta, \tag{18}$$

where now the dependence on  $\zeta$  is included, and the angles are arbitrary.

• The near-origin behaviour of  $A_{\theta}$  and  $A_{\zeta}$  given in Eqn. (16) and Eqn. (17) are flippantly generalized to

$$A_{\theta,m,n} = r^{m+2} f_{m,n}(\rho),$$
 (19)  
 $A_{\zeta,m,n} = r^m g_{m,n}(\rho),$  (20)

$$A_{\zeta,m,n} = r^m \quad g_{m,n}(\rho), \tag{20}$$

where the  $f_{m,n}(\rho)$  and  $g_{m,n}(\rho)$  are arbitrary polynomials in  $\rho$ .

• Additional gauge freedom can be exploited: including an additional gauge term  $\nabla h$  where h only depends on  $\zeta$ , e.g.

$$h(\zeta) = h_{0,0} \zeta + \sum h_{0,n} \sin(-n\zeta),$$
 (21)

does not change the magnetic field and does not change any of the above discussion.

• The representation for the  $A_{\theta,m,n}$  does not change, but we must clarify that Eqn. (20) holds for only the  $m \neq 0$  harmonics:

$$A_{\zeta,m,n} = r^m \quad g_{m,n}(\rho), \quad \text{for} \quad m \neq 0.$$
 (22)

• For the  $m=0,\,n\neq 0$  harmonics of  $A_\zeta$ , including the additional gauge gives  $A_{\zeta,0,n}=g_{0,n}(\rho)+n\,h_{0,n}.$  Recall that  $g_{0,n}(\rho)=g_{0,n,0}+g_{0,n,1}\rho+g_{0,n,2}\rho^2+...$ , and we can choose  $h_{0,n}=-g_{0,n,0}/n$  to obtain

$$A_{\zeta,m,n} = r^m \ g_{m,n}(\rho), \text{ for } m = 0, n \neq 0, \text{ with } g_{m,n}(0) = 0.$$
 (23)

• For the m=0, n=0 harmonic of  $A_{\zeta}$ , we have  $A_{\zeta,0,0}=g_{0,0}(\rho)+h_{0,0}$ . Similarly, choose  $h_{0,0}=-g_{0,n,0}$ to obtain

$$A_{\zeta,m,m} = r^m \ g_{m,n}(\rho), \text{ for } m = 0, n = 0, \text{ with } g_{m,n}(0) = 0.$$
 (24)

- To simplify the algorithmic implementation of these conditions, we shall introduce a "regularization" factor,  $\rho^{m/2} = r^m$ .
- Note that the representation for  $A_{\theta,m,n}$  given in Eqn. (19), with an arbitrary polynomial  $f_{m,n}(\rho)=f_{m,n,0}+f_{m,n,1}\rho+f_{m,n,2}\rho^2+...$ , is equivalent to  $A_{\theta,m,n}=\rho^{m/2}\alpha_{m,n}(\rho)$  where  $\alpha_{m,n}(\rho)$  is an arbitrary polynomial with the constraint  $\alpha_{m,n}(0) = 0$ .
- · We can write the vector potential as

$$A_{\theta,m,n} = \rho^{m/2} \alpha_{m,n}(\rho), \text{ with } \alpha_{m,n}(0) = 0 \text{ for all } (m,n),$$
 (25)

$$A_{\zeta,m,n} = \rho^{m/2} \beta_{m,n}(\rho), \text{ with } \beta_{m,n}(0) = 0 \text{ for } m = 0.$$
 (26)

# 3.5.2 non-stellarator symmetric terms

· Just guessing, for the non-stellarator-symmetric configurations,

$$A_{\theta,m,n} = \rho^{m/2} \alpha_{m,n}(\rho), \text{ with } \alpha_{m,n}(0) = 0 \text{ for all } (m,n),$$
 (27)

$$A_{\zeta,m,n} = \rho^{m/2} \beta_{m,n}(\rho), \text{ with } \beta_{m,n}(0) = 0 \text{ for } m = 0.$$
 (28)

4 Todo List 7

## 4 Todo List

### Subprogram bnorml (mn, Ntz, efmn, ofmn)

There is a very clumsy attempt to parallelize this which could be greatly improved.

### Subprogram casing (teta, zeta, gBn, icasing)

It would be MUCH faster to only require the tangential field on a regular grid!!!

Please check why  $B_s$  is not computed. Is it because  $B_s \nabla s \times \mathbf{n} = 0$  ?

This needs to be revised.

### Subprogram curent (Ivol, mn, Nt, Nz, iflag, IdltGp)

Perhaps this can be proved analytically; in any case it should be confirmed numerically.

### Subprogram inputlist::lconstraint

if Lconstraint==2, under reconstruction.

# Subprogram inputlist::wbuild\_vector\_potential

: what is this?

### Subprogram ma02aa (Ivol, NN)

If Lconstraint = 2, then  $\mu = \mu_1$  is varied in order to satisfy the helicity constraint, and  $\Delta \psi_p = \mu_2$  is not varied, and Nxdof=1. (under re-construction)

### Subprogram pc00aa (NGdof, position, Nvol, mn, ie04dgf)

Unfortunately, E04DGF seems to require approximately 3N function evaluations before proceeding to minimize the energy functional, where there are N degrees of freedom. I don't know how to turn this off!

### Subprogram pc00ab (mode, NGdof, Position, Energy, Gradient, nstate, iuser, ruser)

IT IS VERY LIKELY THAT FFTs WOULD BE FASTER!!!

## Subprogram stzxyz (Ivol, stz, RpZ)

Please see co01aa() for documentation.

### Subprogram ending

The following belongs to the docs of the program xspech, not to the ending() subroutine. If you know how to attach the docs to the program xspech, please fix this.

If Lminimize.eq.1, call pc00aa() to find minimum of energy functional using quasi-Newton, preconditioned conjugate gradient method, E04DGF

# 5 Module Index

### 5.1 Modules

Here is a list of all modules:

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# 6 Data Type Index

# 6.1 Data Types List

Here are the data types with brief descriptions:

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# 7 File Index

# 7.1 File List

Here is a list of all documented files with brief descriptions:

bfield.f90 Returns $\dot{s}\equiv B^s/B^\zeta$ and $\dot{\theta}\equiv B^\theta/B^\zeta$	175
bnorml.f90 $ \hbox{Computes $B_{Plasma}\cdot \mathbf{e}_{\theta}\times \mathbf{e}_{\zeta}$ on the computational boundary, $\partial\mathcal{D}$ }$	176
brcast.f90 Broadcasts Beltrami fields, profiles,	176
casing.f90 Constructs the field created by the plasma currents, at an arbitrary, external location using virtual casing	176
coords.f90   Calculates coordinates, ${\bf x}(s,\theta,\zeta)\equiv R{\bf e}_R+Z{\bf e}_Z$ , and metrics, using FFTs	177
curent.f90 Computes the plasma current, $I\equiv\int B_{\theta}d\theta$ , and the "linking" current, $G\equiv\int B_{\zeta}d\zeta$	177
df00ab.f90 Evaluates volume integrals, and their derivatives w.r.t. interface geometry, using "packed" format	177

global.f90 Defines input namelists and global variables, and opens some output files	178
hesian.f90 Computes eigenvalues and eigenvectors of derivative matrix, $\nabla_{\xi}\mathbf{F}$	198
jo00aa.f90	198
Iforce.f90 Computes $B^2$ , and the spectral condensation constraints if required, on the interfaces, $\mathcal{I}_i$	198
ma00aa.f90 Calculates volume integrals of Chebyshev polynomials and metric element products	198
ma02aa.f90  Constructs Beltrami field in given volume consistent with flux, helicity, rotational-transform and/or parallel-current constraints	199
manual.f90 Code development issues and future physics applications	199
matrix.f90 Constructs energy and helicity matrices that represent the Beltrami linear system	199
metrix.f90 Calculates the metric quantities, $\sqrt{g}g^{\mu\nu}$ , which are required for the energy and helicity integral	s 200
mp00ac.f90 Solves Beltrami/vacuum (linear) system, given matrices	200
newton.f90 Employs Newton method to find ${\bf F}({\bf x})=0$ , where ${\bf x}\equiv\{{\rm geometry}\}$ and ${\bf F}$ is defined in dforce()	200
numrec.f90 Various miscellaneous "numerical" routines	201
packab.f90 Packs, and unpacks, Beltrami field solution vector; $\mathbf{a} \equiv \{A_{\theta,e,i,l},A_{\zeta,e,i,l},\text{etc.}\}$	202
packxi.f90 Packs, and unpacks, geometrical degrees of freedom; and sets coordinate axis	202
pc00aa.f90 Use preconditioned conjugate gradient method to find minimum of energy functional	202
pc00ab.f90 Returns the energy functional and it's derivatives with respect to geometry	203
pp00aa.f90 Constructs Poincaré plot and "approximate" rotational-transform (driver)	203
pp00ab.f90 Follows magnetic fieldline using ode-integration routine from rksuite.f	203
preset.f90 Allocates and initializes internal arrays	203
ra00aa.f90 Writes vector potential to .ext.sp.A	204

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stzxyz.f90	
Calculates coordinates, $\mathbf{x}(s,\theta,\zeta)\equiv R\mathbf{e}_R+Z\mathbf{e}_Z$ , and metrics, at given $(s,\theta,\zeta)$	204
tr00ab.f90	
Calculates rotational transform given an arbitrary tangential field	204
volume.f90	
Computes volume of each region; and, if required, the derivatives of the volume with respect to the interface geometry	205
wa00aa.f90	
Constructs smooth approximation to wall	205
xspech.f90	
Main program	206

# 8 Module Documentation

# 8.1 Diagnostics to check the code

### **Functions/Subroutines**

• subroutine bfield (zeta, st, Bst)

Compute the magnetic field.

• subroutine hesian (NGdof, position, Mvol, mn, LGdof)

Computes eigenvalues and eigenvectors of derivative matrix,  $\nabla_{\xi}\mathbf{F}$ .

• subroutine jo00aa (Ivol, Ntz, Iquad, mn)

Measures error in Beltrami equation,  $\nabla \times \mathbf{B} - \mu \mathbf{B}$ .

• subroutine pp00aa

Constructs Poincaré plot and "approximate" rotational-transform (driver).

• subroutine pp00ab (Ivol, sti, Nz, nPpts, poincaredata, fittedtransform, utflag)

Constructs Poincaré plot and "approximate" rotational-transform (for single field line).

• subroutine stzxyz (Ivol, stz, RpZ)

Calculates coordinates,  $\mathbf{x}(s, \theta, \zeta) \equiv R \mathbf{e}_R + Z \mathbf{e}_Z$ , and metrics, at given  $(s, \theta, \zeta)$ .

# 8.1.1 Detailed Description

# 8.1.2 Function/Subroutine Documentation

Compute the magnetic field.

Returns the magnetic field line equations,  $d\mathbf{x}/d\phi = \mathbf{B}/B^{\phi}$ .

### Equations of field line flow

· The equations for the fieldlines are normalized to the toroidal field, i.e.

$$\dot{s} \equiv \frac{B^s}{B^{\zeta}}, \qquad \dot{\theta} \equiv \frac{B^{\theta}}{B^{\zeta}}.$$
 (29)

### Representation of magnetic field

• The components of the vector potential,  $\mathbf{A} = A_{\theta} \nabla + A_{\zeta} \nabla \zeta$ , are

$$A_{\theta}(s,\theta,\zeta) = \sum_{i,l} A_{\theta,e,i,l} \overline{T}_{l,i}(s) \cos \alpha_i + \sum_{i,l} A_{\theta,o,i,l} \overline{T}_{l,i}(s) \sin \alpha_i, \tag{30}$$

$$A_{\zeta}(s,\theta,\zeta) = \sum_{i,l} A_{\zeta,e,i,l} \, \overline{T}_{l,i}(s) \cos \alpha_i + \sum_{i,l} A_{\zeta,o,i,l} \, \overline{T}_{l,i}(s) \sin \alpha_i, \tag{31}$$

where  $\overline{T}_{l,i}(s) \equiv \overline{s}^{m_i/2} T_l(s)$ ,  $T_l(s)$  is the Chebyshev polynomial, and  $\alpha_j \equiv m_j \theta - n_j \zeta$ . The regularity factor,  $\overline{s}^{m_i/2}$ , where  $\overline{s} \equiv (1+s)/2$ , is only included if there is a coordinate singularity in the domain (i.e. only in the innermost volume, and only in cylindrical and toroidal geometry.)

• The magnetic field,  $\sqrt{g} \mathbf{B} = \sqrt{g} B^s \mathbf{e}_s + \sqrt{g} B^\theta \mathbf{e}_\theta + \sqrt{g} B^\zeta \mathbf{e}_\zeta$ , is

$$\sqrt{g} \mathbf{B} = \mathbf{e}_{s} \sum_{i,l} \left[ (-m_{i} A_{\zeta,e,i,l} - n_{i} A_{\theta,e,i,l}) \overline{T}_{l,i} \sin \alpha_{i} + (+m_{i} A_{\zeta,o,i,l} + n_{i} A_{\theta,o,i,l}) \overline{T}_{l,i} \cos \alpha_{i} \right] 
+ \mathbf{e}_{\theta} \sum_{i,l} \left[ (-m_{i} A_{\zeta,e,i,l} - n_{i} A_{\theta,e,i,l}) \overline{T}'_{l,i} \cos \alpha_{i} + (-m_{i} A_{\zeta,o,i,l} + n_{i} A_{\theta,o,i,l}) \overline{T}'_{l,i} \sin \alpha_{i} \right]$$

$$+ \mathbf{e}_{\zeta} \sum_{i,l} \left[ (-m_{i} A_{\zeta,e,i,l} - n_{i} A_{\theta,e,i,l}) \overline{T}'_{l,i} \cos \alpha_{i} + (-m_{i} A_{\zeta,o,i,l} + n_{i} A_{\theta,o,i,l}) \overline{T}'_{l,i} \sin \alpha_{i} \right]$$

$$+ \mathbf{e}_{\zeta} \sum_{i,l} \left[ (-m_{i} A_{\zeta,e,i,l} - n_{i} A_{\theta,e,i,l}) \overline{T}'_{l,i} \cos \alpha_{i} + (-m_{i} A_{\zeta,o,i,l} + n_{i} A_{\theta,o,i,l}) \overline{T}'_{l,i} \sin \alpha_{i} \right]$$

$$+ \mathbf{e}_{\zeta} \sum_{i,l} \left[ (-m_{i} A_{\zeta,e,i,l} - n_{i} A_{\theta,e,i,l}) \overline{T}'_{l,i} \cos \alpha_{i} + (-m_{i} A_{\zeta,o,i,l} - n_{i} A_{\theta,o,i,l}) \overline{T}'_{l,i} \sin \alpha_{i} \right]$$

• In Eqn. (29), the coordinate Jacobian,  $\sqrt{g}$ , cancels. No coordinate metric information is required to construct the fieldline equations from the magnetic vector potential.

# IT IS REQUIRED TO SET IVOL THROUGH GLOBAL MEMORY BEFORE CALLING BFIELD.

The format of this subroutine is constrained by the NAG ode integration routines.

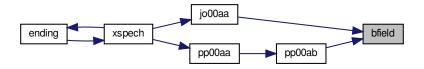
### **Parameters**

in	zeta	reta toroidal angle $\zeta$	
in	$st$ radial coordinate $s$ and poloidal angle $\theta$		
out	out $Bst$ tangential magnetic field directions $B_s$ , $I$		

References allglobal::ate, allglobal::ato, allglobal::aze, allglobal::doordinatesingularity, inputlist::lrad, allglobal::mn, inputlist::mpol, allglobal::mvol, allglobal::myid, allglobal::ncpu, allglobal::node, allglobal::notstellsym, constants::one, fileunits::ounit, allglobal::regumm, numerical::small, constants::two, numerical::vsmall, inputlist::wmacros, and constants::zero.

Referenced by jo00aa(), and pp00ab().

Here is the caller graph for this function:



Computes eigenvalues and eigenvectors of derivative matrix,  $\nabla_{\xi} \mathbf{F}$ .

### **Parameters**

in	NGdof	number of global degrees of freedom
in,out	position	internal geometrical degrees of freedom
in	Mvol	total number of volumes in computation
in	mn	number of Fourier harmonics
in	LGdof	what is this?

### construction of Hessian matrix

- The routine dforce() is used to compute the derivatives, with respect to interface geometry, of the force imbalance harmonics,  $[[p+B^2/2]]_j$ , which may be considered to be the "physical" constraints, and if Igeometry==3 then also the derivatives of the "artificial" spectral constraints,  $I_j \equiv (R_\theta X + Z_\theta Y)_j$ .
- The input variable Lconstraint determines how the enclosed fluxes,  $\Delta \psi_t$  and  $\Delta \psi_p$ , and the helicity multiplier,  $\mu$ , vary as the geometry is varied; see global.f90 and mp00ac() for more details.

## construction of eigenvalues and eigenvectors

- If LHevalues==T then the eigenvalues of the Hessian are computed using the NAG routine F02EBF.
- If LHevectors==T then the eigenvalues and the eigenvectors of the Hessian are computed.
- Note that if Igeometry==3, then the derivative-matrix also contains information regarding how the "artificial" spectral constraints vary with geometry; so, the eigenvalues and eigenvectors are not purely "physical".

The eigenvalues and eigenvectors (if required) are written to the file .ext.GF.ev as follows:

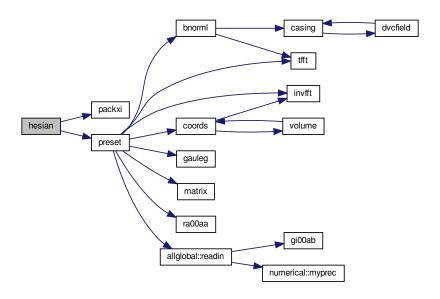
```
open (hunit, file="."//trim(ext)//".GF.ev", status="unknown", form="unformatted") write(hunit)ngdof,ldvr,ldvi ! integers: if only the city of the control of
                                                                                                                                                                                                    integers; if only the eigenvalues were computed then Ldvr=Ldvi=1;
write(hunit)evalr(1:ngdof)
                                                                                                                                                                                            ! reals
                                                                                                                                                                                                                                          ; real
                                                                                                                                                                                                                                                                                                             part of eigenvalues;
write(hunit)evali(1:ngdof)
                                                                                                                                                                                                   reals
                                                                                                                                                                                                                                                ; imaginary part of eigenvalues;
write(hunit)evecr(1:ngdof,1:ngdof) ! reals
                                                                                                                                                                                                                                                                                                               part of eigenvalues; only if Ldvr=NGdof;
                                                                                                                                                                                                                                               : real
write(hunit)eveci(1:ngdof,1:ngdof) ! reals
                                                                                                                                                                                                                                                ; imaginary part of eigenvalues; only if Ldvi=NGdof;
```

· The eigenvectors are saved in columns of evecr, eveci, as described by the NAG documentation for F02EBF.

References allglobal::cpus, allglobal::dbbdmp, allglobal::dbbdrz, allglobal::dessian, allglobal::dffdrz, allglobal ← ::dmupfdx, inputlist::dpp, inputlist::dqq, allglobal::drbc, allglobal::drbs, allglobal::dzbc, allglobal::dzbs, allglobal ::energy, inputlist::ext, constants::half, inputlist::helicity, allglobal::hessian, fileunits::hunit, inputlist::igeometry, allglobal::im, allglobal::irbc, allglobal::irbc, allglobal::irbc, allglobal::izbc, allglobal::izbc, allglobal::ibbintegral, inputlist::Icheck, inputlist::Ifindzero, inputlist::Ifreebound, allglobal::Ihessianallocated, inputlist::Ihevalues, inputlist ::lhevectors, inputlist::lhmatrix, allglobal::localconstraint, inputlist::lperturbed, inputlist::mu, fileunits::munit, allglobal::myid, allglobal::ncpu, allglobal::notstellsym, inputlist::nvol, constants::one, fileunits::ounit, packxi(), inputlist::pflux, preset(), allglobal::psifactor, numerical::small, numerical::sqrtmachprec, constants::ten, constants ← ::two, numerical::vsmall, inputlist::wmacros, allglobal::yesstellsym, and constants::zero.

Referenced by xspech().

Here is the call graph for this function:



Here is the caller graph for this function:



Measures error in Beltrami equation,  $\nabla \times \mathbf{B} - \mu \mathbf{B}$ .

This routine is called by xspech() as a post diagnostic and only if Lcheck==1.

# construction of current, $\mathbf{j} \equiv \nabla \times \nabla \times \mathbf{A}$

• The components of the vector potential,  $\mathbf{A} = A_{\theta} \nabla + A_{\zeta} \nabla \zeta$ , are

$$A_{\theta}(s,\theta,\zeta) = \sum_{i,l} A_{\theta,e,i,l} \overline{T}_{l,i}(s) \cos \alpha_i + \sum_{i,l} A_{\theta,o,i,l} \overline{T}_{l,i}(s) \sin \alpha_i,$$
 (33)

$$A_{\zeta}(s,\theta,\zeta) = \sum_{i,l} A_{\zeta,e,i,l} \, \overline{T}_{l,i}(s) \cos \alpha_i + \sum_{i,l} A_{\zeta,o,i,l} \, \overline{T}_{l,i}(s) \sin \alpha_i, \tag{34}$$

where  $\overline{T}_{l,i}(s) \equiv \overline{s}^{m_i/2} T_l(s)$ ,  $T_l(s)$  is the Chebyshev polynomial, and  $\alpha_j \equiv m_j \theta - n_j \zeta$ . The regularity factor,  $\overline{s}^{m_i/2}$ , where  $\overline{s} \equiv (1+s)/2$ , is only included if there is a coordinate singularity in the domain (i.e. only in the innermost volume, and only in cylindrical and toroidal geometry.)

• The magnetic field,  $\sqrt{g} \mathbf{B} = \sqrt{g} B^s \mathbf{e}_s + \sqrt{g} B^\theta \mathbf{e}_\theta + \sqrt{g} B^\zeta \mathbf{e}_\zeta$ , is

$$\sqrt{g} \mathbf{B} = \mathbf{e}_{s} \sum_{i,l} \left[ (-m_{i} A_{\zeta,e,i,l} - n_{i} A_{\theta,e,i,l}) \overline{T}_{l,i} \sin \alpha_{i} + (+m_{i} A_{\zeta,o,i,l} + n_{i} A_{\theta,o,i,l}) \overline{T}_{l,i} \cos \alpha_{i} \right] 
+ \mathbf{e}_{\theta} \sum_{i,l} \left[ (-m_{i} A_{\zeta,e,i,l}) \overline{T}'_{l,i} \cos \alpha_{i} + (-m_{i} A_{\zeta,o,i,l}) \overline{T}'_{l,i} \sin \alpha_{i} \right]$$

$$+ \mathbf{e}_{\zeta} \sum_{i,l} \left[ (-m_{i} A_{\zeta,e,i,l}) \overline{T}'_{l,i} \cos \alpha_{i} + (-m_{i} A_{\zeta,o,i,l}) \overline{T}'_{l,i} \sin \alpha_{i} \right]$$

$$+ \mathbf{e}_{\zeta} \sum_{i,l} \left[ (-m_{i} A_{\zeta,e,i,l}) \overline{T}'_{l,i} \cos \alpha_{i} + (-m_{i} A_{\zeta,o,i,l}) \overline{T}'_{l,i} \sin \alpha_{i} \right]$$

$$+ \mathbf{e}_{\zeta} \sum_{i,l} \left[ (-m_{i} A_{\zeta,e,i,l}) \overline{T}'_{l,i} \cos \alpha_{i} + (-m_{i} A_{\zeta,o,i,l}) \overline{T}'_{l,i} \sin \alpha_{i} \right]$$

$$+ \mathbf{e}_{\zeta} \sum_{i,l} \left[ (-m_{i} A_{\zeta,e,i,l}) \overline{T}'_{l,i} \cos \alpha_{i} + (-m_{i} A_{\zeta,o,i,l}) \overline{T}'_{l,i} \sin \alpha_{i} \right]$$

· The current is

$$\sqrt{g}\mathbf{j} = (\partial_{\theta}B_{\zeta} - \partial_{\zeta}B_{\theta})\mathbf{e}_{s} + (\partial_{\zeta}B_{s} - \partial_{s}B_{\zeta})\mathbf{e}_{\theta} + (\partial_{s}B_{\theta} - \partial_{\theta}B_{s})\mathbf{e}_{\zeta}, \tag{36}$$

where (for computational convenience) the covariant components of  ${\bf B}$  are computed as

$$B_s = (\sqrt{g}B^s) g_{ss} / \sqrt{g} + (\sqrt{g}B^\theta) g_{s\theta} / \sqrt{g} + (\sqrt{g}B^\zeta) g_{s\zeta} / \sqrt{g}, \tag{37}$$

$$B_{\theta} = (\sqrt{g}B^{s}) g_{s\theta} / \sqrt{g} + (\sqrt{g}B^{\theta}) g_{\theta\theta} / \sqrt{g} + (\sqrt{g}B^{\zeta}) g_{\theta\zeta} / \sqrt{g}, \tag{38}$$

$$B_{\zeta} = (\sqrt{g}B^{s}) g_{s\zeta} / \sqrt{g} + (\sqrt{g}B^{\theta}) g_{\theta\zeta} / \sqrt{g} + (\sqrt{g}B^{\zeta}) g_{\zeta\zeta} / \sqrt{g}. \tag{39}$$

### quantification of the error

· The measures of the error are

$$||(\mathbf{j} - \mu \mathbf{B}) \cdot \nabla s|| \equiv \int ds \oint d\theta d\zeta ||\nabla g \mathbf{j} \cdot \nabla s - \mu \sqrt{g} \mathbf{B} \cdot \nabla s|, \qquad (40)$$

$$||(\mathbf{j} - \mu \mathbf{B}) \cdot \nabla \theta|| \equiv \int ds \oint d\theta d\zeta ||\sqrt{g} \mathbf{j} \cdot \nabla \theta - \mu \sqrt{g} \mathbf{B} \cdot \nabla \theta|, \qquad (41)$$

$$||(\mathbf{j} - \mu \mathbf{B}) \cdot \nabla \zeta|| \equiv \int ds \oint d\theta d\zeta ||\sqrt{g} \,\mathbf{j} \cdot \nabla \zeta - \mu \,\sqrt{g} \,\mathbf{B} \cdot \nabla \zeta|. \tag{42}$$

## comments

• Is there a better definition and quantification of the error? For example, should we employ an error measure that is dimensionless?

- If the coordinate singularity is in the domain, then  $|\nabla \theta| \to \infty$  at the coordinate origin. What then happens to  $||(\mathbf{j} \mu \mathbf{B}) \cdot \nabla \theta||$  as defined in Eqn. (41)?
- What is the predicted scaling of the error in the Chebyshev-Fourier representation scale with numerical resolution? Note that the predicted error scaling for  $E^s$ ,  $E^\theta$  and  $E^\zeta$  may not be standard, as various radial derivatives are taken to compute the components of  $\mathbf{j}$ . (See for example the discussion in Sec.IV.C in Hudson et al. (2011) [5], where the expected scaling of the error for a finite-element implementation is confirmed numerically.)
- Instead of using Gaussian integration to compute the integral over s, an adaptive quadrature algorithm may be preferable.

#### **Parameters**

in	Ivol	in which volume should the Beltrami error be computed	
in	Ntz	number of grid points in $\theta$ and $\zeta$	
in	Iquad	degree of Gaussian quadrature	
in	mn	number of Fourier harmonics	

### details of the numerics

- The integration over s is performed using Gaussian integration, e.g.,  $\int f(s)ds \approx \sum_k \omega_k f(s_k)$ ; with the abscissae,  $s_k$ , and the weights,  $\omega_k$ , for k=1, Iquad v, determined by CDGQF. The resolution, N  $\equiv$  Iquad v, is determined by Nquad (see global.f90 and preset()). A fatal error is enforced by jo00aa() if CDGQF returns an ifail  $\neq 0$ .
- Inside the Gaussian quadrature loop, i.e. for each  $s_k$ ,
  - The metric elements,  $g_{\mu,\nu}\equiv \text{gij}\,(1:6,0,1:\text{Ntz})$ , and the Jacobian,  $\sqrt{g}\equiv \text{sg}\,(0,1:\text{Ntz})$ , are calculated on a regular angular grid,  $(\theta_i,\zeta_j)$ , in coords(). The derivatives  $\partial_i g_{\mu,\nu}\equiv \text{gij}\,(1:6,\text{i},1\leftrightarrow\text{i})$ . Ntz) and  $\partial_i\sqrt{g}\equiv \text{sg}\,(\text{i},1:\text{Ntz})$ , with respect to  $i\in\{s,\theta,\zeta\}$  are also returned.
  - The Fourier components of the vector potential given in Eqn. (33) and Eqn. (34), and their first and second radial derivatives, are summed.
  - The quantities  $\sqrt{g}B^s$ ,  $\sqrt{g}B^\theta$  and  $\sqrt{g}B^\zeta$ , and their first and second derivatives with respect to  $(s, \theta, \zeta)$ , are computed on the regular angular grid (using FFTs).
  - The following quantities are then computed on the regular angular grid

$$\sqrt{g}j^{s} = \sum_{u} \left[ \partial_{\theta}(\sqrt{g}B^{u}) g_{u,\zeta} + (\sqrt{g}B^{u}) \partial_{\theta}g_{u,\zeta} - (\sqrt{g}B^{u})g_{u,\zeta} \partial_{\theta}\sqrt{g}/\sqrt{g} \right] / \sqrt{g}$$

$$- \sum_{u} \left[ \partial_{\zeta}(\sqrt{g}B^{u}) g_{u,\theta} + (\sqrt{g}B^{u}) \partial_{\zeta}g_{u,\theta} - (\sqrt{g}B^{u})g_{u,\theta} \partial_{\zeta}\sqrt{g}/\sqrt{g} \right] / \sqrt{g}, \quad (43)$$

$$\sqrt{g}j^{\theta} = \sum_{u} \left[ \partial_{\zeta}(\sqrt{g}B^{u}) g_{u,s} + (\sqrt{g}B^{u}) \partial_{\zeta}g_{u,s} - (\sqrt{g}B^{u})g_{u,s} \partial_{\zeta}\sqrt{g}/\sqrt{g} \right] / \sqrt{g}$$

$$- \sum_{u} \left[ \partial_{s}(\sqrt{g}B^{u}) g_{u,\zeta} + (\sqrt{g}B^{u}) \partial_{s}g_{u,\zeta} - (\sqrt{g}B^{u})g_{u,\zeta} \partial_{s}\sqrt{g}/\sqrt{g} \right] / \sqrt{g}, \quad (44)$$

$$\sqrt{g}j^{\zeta} = \sum_{u} \left[ \partial_{s}(\sqrt{g}B^{u}) g_{u,\theta} + (\sqrt{g}B^{u}) \partial_{s}g_{u,\theta} - (\sqrt{g}B^{u})g_{u,\theta} \partial_{s}\sqrt{g}/\sqrt{g} \right] / \sqrt{g}$$

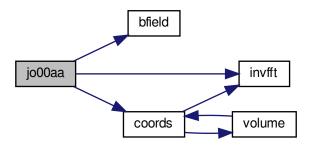
$$- \sum_{u} \left[ \partial_{\theta}(\sqrt{g}B^{u}) g_{u,s} + (\sqrt{g}B^{u}) \partial_{\theta}g_{u,s} - (\sqrt{g}B^{u})g_{u,s} \partial_{\theta}\sqrt{g}/\sqrt{g} \right] / \sqrt{g}. \quad (45)$$

• The error is stored into an array called beltramierror which is then written to the HDF5 file in hdfint().

References allglobal::ate, allglobal::ato, allglobal::aze, allglobal::azo, allglobal::beltramierror, bfield(), allglobal::cfmn, allglobal::cheby, coords(), allglobal::cpus, allglobal::dpflux, allglobal::dtflux, allglobal::efmn, inputlist::ext, allglobal::gbzeta, allglobal::guvij, constants::half, inputlist::igeometry, allglobal::im, allglobal::in, invfft(), allglobal-:ivol, allglobal::lcoordinatesingularity, inputlist::lerrortype, inputlist::lrad, inputlist::mpol, inputlist::mu, allglobal-::mvol, allglobal::myid, inputlist::nfp, allglobal::node, allglobal::notstellsym, allglobal::nt, inputlist::nvol, allglobal-::raz, allglobal::ofmn, constants::one, fileunits::ounit, constants::pi2, allglobal::pi2nfp, allglobal::regumm, allglobal-::rij, allglobal::rtt, allglobal::sfmn, allglobal::sg, allglobal::tt, constants::two, inputlist::wmacros, allglobal::zernike, constants::zero, and allglobal::zij.

Referenced by xspech().

Here is the call graph for this function:



Here is the caller graph for this function:



## 8.1.2.4 pp00aa() subroutine pp00aa

Constructs Poincaré plot and "approximate" rotational-transform (driver).

## relevant input variables

- · The resolution of Poincaré plot is controlled by
  - nPtraj trajectories will be located in each volume;
  - nPpts iterations per trajectory;
  - odetol o.d.e. integration tolerance;
- The magnetic field is given by bfield() .
- The approximate rotational transform is determined, in pp00ab(), by fieldline integration.

# format of output: Poincaré

• The Poincaré data is written to .ext.poincare:xxxx , where xxxx is an integer indicating the volume. The format of this file is as follows:

- $\theta \equiv \text{data}(1, k, j)$  is the poloidal angle,
- $s \equiv \text{data}(2, k, j)$  is the radial coordinate,
- $R \equiv \text{data}(3, k, j)$  is the cylindrical R,
- $Z \equiv \text{data}(4, k, j)$  is the cylindrical Z,
- The integer k=0,Nz-1 labels toroidal planes, so that  $\phi = (2\pi/\text{Nfp})(k/\text{Nz})$ ,
- The integer j=1,nPpts labels toroidal iterations.
- Usually (if no fieldline integration errors are encountered) the number of fieldlines followed in volume lvol is given by N+1, where the radial resolution,  $N \equiv Ni \ (lvol)$ , is given on input. This will be over-ruled by if  $nPtrj \ (lvol)$ , given on input, is non-negative.
- The starting location for the fieldline integrations are equally spaced in the radial coordinate  $s_i = s_{l-1} + i(s_l s_{l-1})/N$  for i = 0, N, along the line  $\theta = 0, \zeta = 0$ .

### format of output: rotational-transform

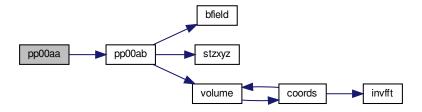
• The rotational-transform data is written to .ext.transform:xxxx , where xxxx is an integer indicating the volume. The format of this file is as follows:

```
open(lunit+myid, file="."//trim(ext)//".sp.t."//svol, status="unknown", form="unformatted")
write(lunit+myid) lnptrj-ioff+1 ! integer
write(lunit+myid) diotadxup(0:1,0,lvol) ! doubles
write(lunit+myid) ( fiota(itrj,1:2), itrj = ioff, lnptrj ) ! doubles
close(lunit+myid)
```

References allglobal::cpus, allglobal::diotadxup, inputlist::ext, constants::half, inputlist::igeometry, inputlist:-iota, allglobal::ivol, inputlist::lconstraint, allglobal::lcoordinatesingularity, allglobal::lplasmaregion, inputlist::lrad, allglobal::lvacuumregion, allglobal::mvol, allglobal::myid, allglobal::ncpu, inputlist::npts, inputlist::nptrj, inputlist::vol, allglobal::nz, inputlist::odetol, inputlist::oita, constants::one, fileunits::ounit, constants::pi, allglobal::pi2nfp, pp00ab(), inputlist::ppts, constants::two, inputlist::wmacros, and constants::zero.

Referenced by xspech().

Here is the call graph for this function:



Here is the caller graph for this function:



```
8.1.2.5 pp00ab() subroutine pp00ab (
    integer, intent(in) lvol,
    real, dimension(1:2) sti,
    integer, intent(in) Nz,
    integer, intent(in) nPpts,
    real, dimension(1:4,0:nz-1,1:nppts) poincaredata,
    real, dimension(1:2) fittedtransform,
    integer, intent(out) utflag)
```

Constructs Poincaré plot and "approximate" rotational-transform (for single field line).

### relevant input variables

- · The resolution of Poincaré plot is controlled by
  - nPpts iterations per trajectory;
  - odetol o.d.e. integration tolerance;

The magnetic field is given by bfield().

## rotational-transform

• The approximate rotational transform is determined by field line integration. This is constructed by fitting a least squares fit to the field line trajectory.

## **Parameters**

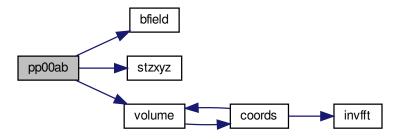
in	Ivol	
	sti	
in	Nz	
in	nPpts	
	poincaredata	
	fittedtransform	
out	utflag	

References bfield(), allglobal::cpus, allglobal::ivol, allglobal::mvol, allglobal::myid, allglobal::ncpu, al

inputlist::nvol, inputlist::odetol, constants::one, fileunits::ounit, constants::pi2, allglobal::pi2nfp, numerical::small, stzxyz(), constants::two, volume(), and constants::zero.

Referenced by pp00aa().

Here is the call graph for this function:



Here is the caller graph for this function:



Calculates coordinates,  $\mathbf{x}(s, \theta, \zeta) \equiv R \mathbf{e}_R + Z \mathbf{e}_Z$ , and metrics, at given  $(s, \theta, \zeta)$ .

- This routine is a "copy" of co01aa(), which calculates the coordinate information on a regular, discrete grid in  $\theta$  and  $\zeta$  at given s whereas stzxyz() calculates the coordinate information at a single point  $(s, \theta, \zeta)$ .
- Todo Please see co01aa() for documentation.

### **Parameters**

in	Ivol	
in	stz	
out	RpZ	

References allglobal::cpus, constants::half, allglobal::halfmm, inputlist::igeometry, allglobal::im, allglobal::im, allglobal::irbs, allglobal::irbs, allglobal::izbs, allglobal::izbs, allglobal::lcoordinatesingularity, allglobal::mvol, allglobal::mvol, allglobal::notstellsym, inputlist::ntor, inputlist::nvol, constants::one, fileunits::ounit, numerical ::vsmall, and constants::zero.

Referenced by pp00ab().

Here is the caller graph for this function:



# 8.2 Free-Boundary Computation

### **Functions/Subroutines**

```
• subroutine bnorml (mn, Ntz, efmn, ofmn)  \textit{Computes } \mathbf{B}_{Plasma} \cdot \mathbf{e}_{\theta} \times \mathbf{e}_{\zeta} \ \ \textit{on the computational boundary, } \partial \mathcal{D}.
```

subroutine casing (teta, zeta, gBn, icasing)

Constructs the field created by the plasma currents, at an arbitrary, external location using virtual casing.

• subroutine dvcfield (Ndim, tz, Nfun, vcintegrand)

Differential virtual casing integrand.

## 8.2.1 Detailed Description

### 8.2.2 Function/Subroutine Documentation

Computes  $\mathbf{B}_{Plasma} \cdot \mathbf{e}_{\theta} \times \mathbf{e}_{\zeta}$  on the computational boundary,  $\partial \mathcal{D}$ .

## free-boundary constraint

- The normal field at the computational boundary,  $\partial \mathcal{D}$ , should be equal to  $(\mathbf{B}_P + \mathbf{B}_C) \cdot \mathbf{e}_\theta \times \mathbf{e}_\zeta$ , where  $\mathbf{B}_P$  is the "plasma" field (produced by internal plasma currents) and is computed using virtual casing, and  $\mathbf{B}_C$  is the "vacuum" field (produced by the external coils) and is given on input.
- The plasma field,  $\mathbf{B}_P$ , can only be computed after the equilibrium is determined, but this information is required to compute the equilibrium to begin with; and so there is an iteration involved.
- Suggested values of the vacuum field can be self generated; see xspech() for more documentation on this.

## compute the normal field on a regular grid on the computational boundary

- For each point on the computational boundary, casing() is called to compute the normal field produced by the plasma currents.
- Todo There is a very clumsy attempt to parallelize this which could be greatly improved.
- An FFT gives the required Fourier harmonics.

See also

casing.f90

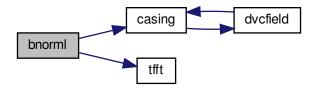
### **Parameters**

in	mn	total number of Fourier harmonics	
in	Ntz	total number of grid points in $\boldsymbol{\theta}$ and $zeta$	
out	efmn	even Fourier coefficients	
out	ofmn	odd Fouier coefficients	

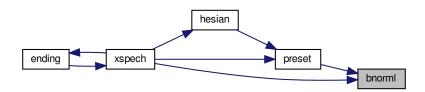
References allglobal::ate, allglobal::ato, allglobal::aze, allglobal::azo, casing(), allglobal::cfmn, allglobal::cpus, allglobal::dxyz, allglobal::global; allglobal::gteta, allglobal::guvij, allglobal::gzeta, constants::half, inputlist ::igeometry, allglobal::jiimag, allglobal::ijireal, allglobal::im, allglobal::jiimag, allglobal::jireal, inputlist::lcheck, allglobal::lcoordinatesingularity, inputlist::lrad, fileunits::lunit, allglobal::mvol, allglobal::myid, allglobal::mcpu, allglobal::notstellsym, allglobal::nt, allglobal::nxyz, allglobal::nz, constants::one, fileunits::ounit, constants ::pi, constants::pi2, allglobal::pi2nfp, allglobal::rij, allglobal::sfmn, allglobal::sg, numerical::small, constants ::ten, allglobal::tetazeta, tfft(), allglobal::tt, constants::two, inputlist::vcasingper, inputlist::vcasingtol, allglobal ::virtualcasingfactor, inputlist::wmacros, constants::zero, and allglobal::zij.

Referenced by preset(), and xspech().

Here is the call graph for this function:



Here is the caller graph for this function:



Constructs the field created by the plasma currents, at an arbitrary, external location using virtual casing.

Compute the external magnetic field using virtual casing.

### Theory and numerics

Required inputs to this subroutine are the geometry of the plasma boundary,

$$\mathbf{x}(\theta,\zeta) \equiv x(\theta,\zeta)\mathbf{i} + y(\theta,\zeta)\mathbf{j} + z(\theta,\zeta)\mathbf{k},\tag{46}$$

and the tangential field on this boundary,

$$\mathbf{B}_s = B^{\theta} \mathbf{e}_{\theta} + B^{\zeta} \mathbf{e}_{\zeta},\tag{47}$$

where  $\theta$  and  $\zeta$  are arbitrary poloidal and toroidal angles, and  $\mathbf{e}_{\theta} \equiv \partial \mathbf{x}/\partial \theta$ ,  $\mathbf{e}_{\zeta} \equiv \partial \mathbf{x}/\partial \zeta$ . This routine assumes that the plasma boundary is a flux surface, i.e.  $\mathbf{B} \cdot \mathbf{e}_{\theta} \times \mathbf{e}_{\zeta} = 0$ .

• The virtual casing principle (Shafranov & Zakharov (1972) [8], Lazerson (2012) [6] and Hanson (2015) [1]) shows that the field outside/inside the plasma arising from plasma currents inside/outside the boundary is equivalent to the field generated by a surface current,

$$\mathbf{j} = \mathbf{B}_s \times \mathbf{n},\tag{48}$$

where  ${\bf n}$  is normal to the surface.

• The field at some arbitrary point,  $\bar{\mathbf{x}}$ , created by this surface current is given by

$$\mathbf{B}(\bar{\mathbf{x}}) = -\frac{1}{4\pi} \int_{\mathcal{S}} \frac{(\mathbf{B}_s \times d\mathbf{s}) \times \hat{\mathbf{r}}}{r^2},\tag{49}$$

where  $d\mathbf{s} \equiv \mathbf{e}_{\theta} \times \mathbf{e}_{\zeta} \ d\theta d\zeta$ .

· For ease of notation introduce

$$\mathbf{J} \equiv \mathbf{B}_{\mathbf{s}} \times d\mathbf{s} = \alpha \, \mathbf{e}_{\theta} - \beta \, \mathbf{e}_{\zeta}, \tag{50}$$

where  $\alpha \equiv B_{\zeta} = B^{\theta} g_{\theta\zeta} + B^{\zeta} g_{\zeta\zeta}$  and  $\beta \equiv B_{\theta} = B^{\theta} g_{\theta\theta} + B^{\zeta} g_{\theta\zeta}$ .

• We may write in Cartesian coordinates  ${f J}=j_x\ {f i}+j_y\ {f j}+j_z\ {f k},$  where

$$j_x = \alpha x_\theta - \beta x_\zeta \tag{51}$$

$$j_y = \alpha y_\theta - \beta y_\zeta \tag{52}$$

$$j_z = \alpha z_\theta - \beta z_\zeta. \tag{53}$$

· Requiring that the current,

$$\mathbf{j} \quad \equiv \quad \nabla \times \mathbf{B} = \sqrt{g}^{-1} (\partial_{\theta} B_{\zeta} - \partial_{\zeta} B_{\theta}) \ \mathbf{e}_{s} + \sqrt{g}^{-1} (\partial_{\zeta} B_{s} - \partial_{s} B_{\zeta}) \ \mathbf{e}_{\theta} + \sqrt{g}^{-1} (\partial_{s} B_{\theta} - \partial_{\theta} B_{s}) \ \mathbf{e}_{\zeta} \tag{54}$$

has no normal component to the surface, i.e.  $\mathbf{j}\cdot\nabla s=0$ , we obtain the condition  $\partial_{\theta}B_{\zeta}=\partial_{\zeta}B_{\theta}$ , or  $\partial_{\theta}\alpha=\partial_{\zeta}\beta$ . In axisymmetric configurations, where  $\partial_{\zeta}\beta=0$ , we must have  $\partial_{\theta}\alpha=0$ .

• The displacement from an arbitrary point, (X,Y,Z), to a point, (x,y,z), that lies on the surface is given

$$\mathbf{r} \equiv r_x \,\mathbf{i} + r_y \,\mathbf{j} + r_z \,\mathbf{k} = (X - x) \,\mathbf{i} + (Y - y) \,\mathbf{j} + (Z - z) \,\mathbf{k}. \tag{55}$$

The components of the magnetic field produced by the surface current are then

$$B^{x} = \oint \!\! \oint \! d\theta d\zeta \ (j_{y}r_{z} - j_{z}r_{y})/r^{3}, \tag{56}$$

$$B^{y} = \oint \!\! \oint \! d\theta d\zeta \ (j_{z}r_{x} - j_{x}r_{z})/r^{3}, \tag{57}$$

$$B^{z} = \oint \!\! \oint \! d\theta d\zeta \ (j_{x}r_{y} - j_{y}r_{x})/r^{3} \tag{58}$$

up to a scaling factor virtualcasing factor  $=-1/4\pi$  that is taken into account at the end.

· When all is said and done, this routine calculates

$$\int_0^{2\pi} \int_0^{2\pi} \text{vcintegrand } d\theta d\zeta \tag{59}$$

for a given (X, Y, Z), where vcintegrand is given in Eqn. (61).

• The surface integral is performed using DCUHRE, which uses an adaptive subdivision strategy and also computes absolute error estimates. The absolute and relative accuracy required are provided by the inputvar vcasingtol. The minimum number of function evaluations is provided by the inputvar vcasingits.

### Calculation of integrand

• An adaptive integration is used to compute the integrals. Consequently, the magnetic field tangential to the plasma boundary is required at an arbitrary point. This is computed, as always, from  $\mathbf{B} = \nabla \times \mathbf{A}$ , and this provides  $\mathbf{B} = B^{\theta} \mathbf{e}_{\theta} + B^{\zeta} \mathbf{e}_{\zeta}$ . Recall that  $B^{s} = 0$  by construction on the plasma boundary.

Todo It would be MUCH faster to only require the tangential field on a regular grid!!!

• Then, the metric elements  $g_{\theta\theta}$ ,  $g_{\theta\zeta}$  and  $g_{\zeta\zeta}$  are computed. These are used to "lower" the components of the magnetic field,  $\mathbf{B} = B_{\theta} \nabla \theta + B_{\zeta} \nabla \zeta$ .

**Todo** Please check why  $B_s$  is not computed. Is it because  $B_s \nabla s \times \mathbf{n} = 0$  ?

- The distance between the "evaluate" point, (X,Y,Z), and the given point on the surface, (x,y,z) is computed.
- If the computational boundary becomes too close to the plasma boundary, the distance is small and this causes problems for the numerics. I have tried to regularize this problem by introducing  $\epsilon$  =inputvar vcasingeps. Let the "distance" be

$$D \equiv \sqrt{(X-x)^2 + (Y-y)^2 + (Z-Z)^2} + \epsilon^2.$$
 (60)

• On taking the limit that  $\epsilon \to 0$ , the virtual casing integrand is

vcintegrand 
$$\equiv (B_x n_x + B_y n_y + B_z n_z)(1 + 3\epsilon^2/D^2)/D^3$$
, (61)

where the normal vector is  $\mathbf{n} \equiv n_x \mathbf{i} + n_y \mathbf{j} + n_z \mathbf{k}$ . The normal vector, Nxyz, to the computational boundary (which does not change) is computed in preset().

Todo This needs to be revised.

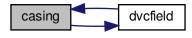
### **Parameters**

in	teta	$\theta$
in	zeta	ζ
out	gBn	$\sqrt{g}\mathbf{B}\cdot\mathbf{n}$
out	icasing	return flag from dcuhre()

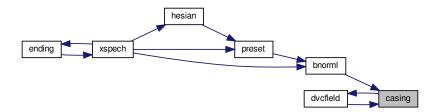
References allglobal::cpus, dvcfield(), allglobal::dxyz, inputlist::ext, allglobal::globaljk, allglobal::myid, allglobal::ncpu, allglobal::nxyz, fileunits::ounit, constants::pi, constants::pi2, inputlist::vcasingits, inputlist::vcasingper, inputlist::vcasingtol, fileunits::vunit, inputlist::wmacros, and constants::zero.

Referenced by bnorml(), and dvcfield().

Here is the call graph for this function:



Here is the caller graph for this function:



Differential virtual casing integrand.

Differential virtual casing integrand

### **Parameters**

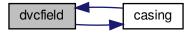
in	Ndim	number of parameters (==2)
in	tz	$ heta$ and $\zeta$
in	Nfun	number of function values (==3)
out	vcintegrand	cartesian components of magnetic field

References allglobal::ate, allglobal::ate, allglobal::aze, allglobal::aze, allglobal::aze, allglobal::dxyz,

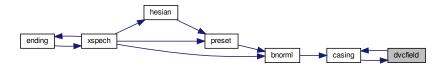
allglobal::first\_free\_bound, constants::four, allglobal::globaljk, constants::half, inputlist::igeometry, allglobal::im, allglobal::irbc, allglobal::irbc, allglobal::irbc, allglobal::izbc, allglobal::izbc, inputlist::lrad, allglobal::mn, allglobal::mvol, allglobal::mvol, allglobal::ncpu, allglobal::notstellsym, inputlist::nvol, allglobal::nxyz, constants::one, fileunits::ounit, allglobal::pi2nfp, numerical::small, constants::three, allglobal::tt, inputlist::vcasingeps, fileunits::vunit, allglobal::yesstellsym, and constants::zero.

Referenced by casing().

Here is the call graph for this function:



Here is the caller graph for this function:



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## 8.3 Parallelization

### **Functions/Subroutines**

subroutine brcast (Ivol)
 Broadcasts Beltrami fields, profiles, . . .

## 8.3.1 Detailed Description

### 8.3.2 Function/Subroutine Documentation

```
8.3.2.1 brcast() subroutine brcast ( integer, intent(in) lvol )
```

Broadcasts Beltrami fields, profiles, . . .

## broadcasting

- The construction of the Beltrami fields is distributed on separate cpus.
- · All "local" information needs to be broadcast so that the "global" force vector,

$$\mathbf{F}_i \equiv [[p + B^2/2]]_i = (p + B^2/2)_{v,i} - (p + B^2/2)_{v-1,i}$$
(62)

can be constructed, and so that restart and output files can be saved to file.

## **Parameters**

	in	Ivol	index of nested volume
--	----	------	------------------------

References allglobal::ate, allglobal::ato, allglobal::aze, allglobal::azo, allglobal::bemn, allglobal::bomn, allglobal::bomn, allglobal::cpus, inputlist::curpol, inputlist::curror, allglobal::diotadxup, allglobal::ditgpdxtp, allglobal::dpflux, allglobal::dtflux, inputlist::helicity, allglobal::iemn, allglobal::imagneticok, allglobal::iomn, inputlist::lconstraint, inputlist::lfindzero, inputlist::lrad, allglobal::mn, inputlist::mnvol, inputlist::mu, allglobal::mvol, allglobal::myid, allglobal::ncpu, allglobal::emn, inputlist::nvol, fileunits::ounit, allglobal::pemn, allglobal::pomn, allglobal::semn, allglobal::somn, inputlist::wmacros, and constants::zero.

# 8.4 Geometry

#### **Functions/Subroutines**

• subroutine coords (Ivol, Iss, Lcurvature, Ntz, mn) Calculates coordinates,  $\mathbf{x}(s,\theta,\zeta) \equiv R\,\mathbf{e}_R + Z\,\mathbf{e}_Z$ , and metrics, using FFTs.

## 8.4.1 Detailed Description

## 8.4.2 Function/Subroutine Documentation

Calculates coordinates,  $\mathbf{x}(s, \theta, \zeta) \equiv R \mathbf{e}_R + Z \mathbf{e}_Z$ , and metrics, using FFTs.

## Coordinates

- We work in coordinates,  $(s, \theta, \zeta)$ , which are be defined *inversely* via a transformation *to* Cartesian coordinates, (x, y, z).
- The toroidal angle,  $\zeta$ , is identical to the cylindrical angle,  $\zeta \equiv \phi$ .
- The radial coordinate, s, is *not* a global variable: it only needs to be defined in each volume, and in each volume  $s \in [-1, 1]$ .
- The choice of poloidal angle,  $\theta$ , does not affect the following.

## Geometry

- The geometry of the "ideal"-interfaces,  $\mathbf{x}_v(\theta,\zeta)$ , is given by  $R(\theta,\zeta)$  and  $Z(\theta,\zeta)$  as follows:
  - Igeometry=1: Cartesian

$$\mathbf{x} \equiv r_{pol}\theta \,\,\hat{\mathbf{i}} + r_{tor}\zeta \,\,\hat{\mathbf{j}} + R \,\,\hat{\mathbf{k}} \tag{63}$$

where  $r_{pol}$  and  $r_{tor}$  are inputs and  $r_{pol}=r_{tor}=1$  by default.

- Igeometry=2: Cylindrical

$$\mathbf{x} = R \cos \theta \,\hat{\mathbf{i}} + R \sin \theta \,\hat{\mathbf{j}} + \zeta \,\hat{\mathbf{k}} \tag{64}$$

- Igeometry=3: Toroidal

$$\mathbf{x} \equiv R \,\hat{\mathbf{r}} + Z \,\hat{\mathbf{k}} \tag{65}$$

where  $\hat{\mathbf{r}} \equiv \cos \phi \, \hat{\mathbf{i}} + \sin \phi \, \hat{\mathbf{j}}$  and  $\hat{\phi} \equiv -\sin \phi \, \hat{\mathbf{i}} + \cos \phi \, \hat{\mathbf{j}}$ .

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The geometry of the ideal interfaces is given as Fourier summation: e.g., for stellarator-symmetry

$$R_v(\theta,\zeta) \equiv \sum_j R_{j,v} \cos \alpha_j,$$
 (66)

$$Z_v(\theta,\zeta) \equiv \sum_j Z_{j,v} \sin \alpha_j,$$
 (67)

where  $\alpha_j \equiv m_j \theta - n_j \zeta$ .

## interpolation between interfaces

- The "coordinate" functions,  $R(s,\theta,\zeta)$  and  $Z(s,\theta,\zeta)$ , are constructed by radially interpolating the Fourier representations of the ideal-interfaces.
- The v-th volume is bounded by  $\mathbf{x}_{v-1}$  and  $\mathbf{x}_{v}$ .
- In each annular volume, the coordinates are constructed by linear interpolation:

$$R(s,\theta,\zeta) \equiv \sum_{j} \left[ \frac{(1-s)}{2} R_{j,v-1} + \frac{(1+s)}{2} R_{j,v} \right] \cos \alpha_{j},$$

$$Z(s,\theta,\zeta) \equiv \sum_{j} \left[ \frac{(1-s)}{2} Z_{j,v-1} + \frac{(1+s)}{2} Z_{j,v} \right] \sin \alpha_{j},$$

$$(68)$$

### coordinate singularity: regularized extrapolation

- · For cylindrical or toroidal geometry, in the innermost, "simple-torus" volume, the coordinates are constructed by an interpolation that "encourages" the interpolated coordinate surfaces to not intersect.
- Introduce  $\bar{s} \equiv (s+1)/2$ , so that in each volume  $\bar{s} \in [0,1]$ , then

$$R_i(s) = R_{i,0} + (R_{i,1} - R_{i,0})f_i,$$
 (69)

$$Z_j(s) = Z_{j,0} + (Z_{j,1} - Z_{j,0})f_j,$$
 (70)

where, in toroidal geometry,

$$f_j \equiv \left\{ \begin{array}{ll} \bar{s} & , & \text{for } m_j = 0, \\ \bar{s}^{m_j} & , & \text{otherwise.} \end{array} \right\}. \tag{71}$$

• Note: The location of the coordinate axis, i.e. the  $R_{j,0}$  and  $Z_{j,0}$ , is set in the coordinate "packing" and "unpacking" routine, packxi().

# Jacobian

- The coordinate Jacobian (and some other metric information) is given by
  - Igeometry=1: Cartesian

$$\mathbf{e}_{\theta} \times \mathbf{e}_{\zeta} = -r_{tor} R_{\theta} \,\hat{\mathbf{i}} - r_{pol} R_{\zeta} \,\hat{\mathbf{j}} + r_{pol} r_{tor} \hat{\mathbf{k}}$$

$$\boldsymbol{\xi} \cdot \mathbf{e}_{\theta} \times \mathbf{e}_{\zeta} = \delta R$$
(72)

$$\boldsymbol{\xi} \cdot \mathbf{e}_{\theta} \times \mathbf{e}_{\zeta} = \delta R \tag{73}$$

$$\sqrt{g} = R_s \, r_{pol} \, r_{tor} \tag{73}$$

- Igeometry=2: Cylindrical

$$\mathbf{e}_{\theta} \times \mathbf{e}_{\zeta} = (R_{\theta} \sin \theta + R \cos \theta) \,\hat{\mathbf{i}} + (R \sin \theta - R_{\theta} \cos \theta) \,\hat{\mathbf{j}} - RR_{\zeta} \,\hat{\mathbf{k}}$$
 (75)

$$\boldsymbol{\xi} \cdot \mathbf{e}_{\theta} \times \mathbf{e}_{\zeta} = \delta R R \tag{76}$$

$$\sqrt{g} = R_s R \tag{77}$$

- Igeometry=3: Toroidal

$$\mathbf{e}_{\theta} \times \mathbf{e}_{\zeta} = -R Z_{\theta} \,\hat{r} + (Z_{\theta} R_{\zeta} - R_{\theta} Z_{\zeta}) \hat{\phi} + R R_{\theta} \,\hat{z} \tag{78}$$

$$\boldsymbol{\xi} \cdot \mathbf{e}_{\theta} \times \mathbf{e}_{\zeta} = R(\delta Z R_{\theta} - \delta R Z_{\theta}) \tag{79}$$

$$\sqrt{g} = R(Z_s R_\theta - R_s Z_\theta) \tag{80}$$

#### cartesian metrics

· The cartesian metrics are

$$g_{ss}=R_sR_s,\quad g_{s\theta}=R_sR_{\theta},\quad g_{s\zeta}=R_sR_{\zeta},\quad g_{\theta\theta}=R_{\theta}R_{\theta}+r_{pol}^2,\quad g_{\theta\zeta}=R_{\theta}R_{\zeta},\quad g_{\zeta\zeta}=R_{\zeta}R_{\zeta}+r_{tor}^2 \quad \text{(81)}$$

## cylindrical metrics

· The cylindrical metrics are

$$g_{ss}=R_sR_s,\quad g_{s\theta}=R_sR_\theta,\quad g_{s\zeta}=R_sR_\zeta,\quad g_{\theta\theta}=R_\theta R_\theta+R^2,\quad g_{\theta\zeta}=R_\theta R_\zeta,\quad g_{\zeta\zeta}=R_\zeta R_\zeta+1 \quad \text{(82)}$$

## logical control

- The logical control is provided by Lcurvature as follows:
  - Lcurvature=0 : only the coordinate transformation is computed, i.e. only R and Z are calculated, e.g. global()
  - Lcurvature=1 : the Jacobian,  $\sqrt{g}$ , and "lower" metrics,  $g_{\mu,\nu}$ , are calculated, e.g. bnorml(), lforce(), curent(), metrix(), sc00aa()
  - Lcurvature=2: the "curvature" terms are calculated, by which I mean the second derivatives of the position vector; this information is required for computing the current,  $\mathbf{j} = \nabla \times \nabla \times \mathbf{A}$ , e.g. jo00aa()
  - Lcurvature=3 : the derivative of the  $g_{\mu,\nu}/\sqrt{g}$  w.r.t. the interface boundary geometry is calculated, e.g. metrix(), curent()
  - Lcurvature=4 : the derivative of the  $g_{\mu,\nu}$  w.r.t. the interface boundary geometry is calculated, e.g. dforce()
  - Lcurvature=5 : the derivative of  $\sqrt{g}$  w.r.t. the interface boundary geometry is calculated, e.g. rzaxis()

### **Parameters**

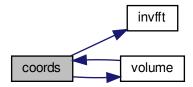
in	Ivol	specified in which volume to compute coordinates
in	Iss	radial coordinate $s$
in	Lcurvature	logical control flag
in	Ntz	number of points in $\theta$ and $\zeta$
in	mn	number of Fourier harmonics

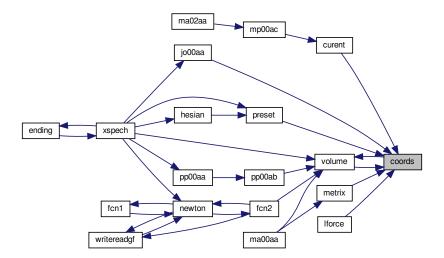
References allglobal::cosi, allglobal::cpus, allglobal::dbdx, allglobal::guvij, constants::half, allglobal::halfmm, inputlist::igeometry, allglobal::im, allglobal::in, invfft(), allglobal::irbc, allglobal::irbs, allglobal::irbs, allglobal::izbc, allglobal::izbc, allglobal::irbs, allglobal::notstellsym, allglobal::nt, inputlist::ntor, allglobal::nz, constants::one, fileunits::ounit, constants::pi2, allglobal::pi2nfp, allglobal::rij, inputlist::rpol, inputlist::rtor, allglobal::sg, allglobal::sini, numerical::small, constants::two, volume(), numerical::vsmall, constants::zero, and allglobal::zij.

Referenced by curent(), jo00aa(), lforce(), metrix(), preset(), and volume().

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Here is the call graph for this function:





## 8.5 Plasma Currents

#### **Functions/Subroutines**

• subroutine curent (Ivol, mn, Nt, Nz, iflag, IdItGp) Computes the plasma current,  $I \equiv \int B_{\theta} \, d\theta$ , and the "linking" current,  $G \equiv \int B_{\zeta} \, d\zeta$ .

## 8.5.1 Detailed Description

### 8.5.2 Function/Subroutine Documentation

```
8.5.2.1 curent() subroutine curent (
    integer, intent(in) lvol,
    integer, intent(in) mn,
    integer, intent(in) Nt,
    integer, intent(in) Nz,
    integer, intent(in) iflag,
    real, dimension(0:1,-1:2), intent(out) ldItGp )
```

Computes the plasma current,  $I \equiv \int B_{\theta} d\theta$ , and the "linking" current,  $G \equiv \int B_{\zeta} d\zeta$ .

#### enclosed currents

• In the vacuum region, the enclosed currents are given by either surface integrals of the current density or line integrals of the magnetic field,

$$\int_{S} \mathbf{j} \cdot d\mathbf{s} = \int_{\partial S} \mathbf{B} \cdot d\mathbf{l},\tag{83}$$

and line integrals are usually easier to compute than surface integrals.

- The magnetic field is given by the curl of the magnetic vector potential, as described in e.g. bfield() .
- The toroidal, plasma current is obtained by taking a "poloidal" loop,  $d\mathbf{l}=\mathbf{e}_{\theta}\,d\theta$ , on the plasma boundary, where  $B^s=0$ , to obtain

$$I \equiv \int_0^{2\pi} \mathbf{B} \cdot \mathbf{e}_{\theta} \, d\theta = \int_0^{2\pi} (-\partial_s A_{\zeta} \, \bar{g}_{\theta\theta} + \partial_s A_{\theta} \, \bar{g}_{\theta\zeta}) \, d\theta, \tag{84}$$

where  $\bar{g}_{\mu\nu} \equiv g_{\mu\nu}/\sqrt{g}$ .

• The poloidal, "linking" current through the torus is obtained by taking a "toroidal" loop,  $d{f l}={f e}_\zeta\,d\zeta$ , on the plasma boundary to obtain

$$G \equiv \int_0^{2\pi} \mathbf{B} \cdot \mathbf{e}_{\zeta} \, d\zeta = \int_0^{2\pi} \left( -\partial_s A_{\zeta} \, \bar{g}_{\theta\zeta} + \partial_s A_{\theta} \, \bar{g}_{\zeta\zeta} \right) \, d\zeta. \tag{85}$$

## Fourier integration

• Using  $f\equiv -\partial_s A_\zeta \; \bar{g}_{\theta\theta} + \partial_s A_\theta \; \bar{g}_{\theta\zeta}$ , the integral for the plasma current is

$$I = \sum_{i}' f_i \cos(n_i \zeta) 2\pi, \tag{86}$$

where  $\sum'$  includes only the  $m_i=0$  harmonics.

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• Using  $g\equiv -\partial_s A_\zeta \; \bar{g}_{\theta\zeta} + \partial_s A_\theta \; \bar{g}_{\zeta\zeta}$ , the integral for the linking current is

$$G = \sum_{i}' g_i \cos(m_i \zeta) 2\pi, \tag{87}$$

where  $\sum'$  includes only the  $n_i=0$  harmonics.

• The plasma current, Eqn. (86), should be independent of  $\zeta$ , and the linking current, Eqn. (87), should be independent of  $\theta$ .

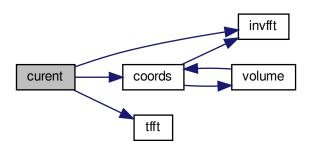
**Todo** Perhaps this can be proved analytically; in any case it should be confirmed numerically.

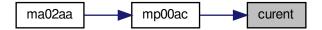
#### **Parameters**

in	Ivol	index of volume
in	mn number of Fourier harmon	
in	Nt	number of grid points along $\theta$
in	Nz number of grid points along ζ	
in iflag some integer flag		some integer flag
out	ldltGp	plasma and linking current

References allglobal::ate, allglobal::ato, allglobal::aze, allglobal::azo, allglobal::cfmn, allglobal::comn, coords(), allglobal::cpus, allglobal::efmn, allglobal::evmn, allglobal::guvij, allglobal::jiimag, allglobal::jireal, allglobal::im, allglobal::im, allglobal::ine, invfft(), allglobal::jiimag, allglobal::jireal, inputlist::lrad, allglobal::mne, allglobal::mvol, allglobal::mvol, allglobal::ncpu, allglobal::notstellsym, allglobal::ntz, allglobal::odmn, allglobal::ofmn, constants::one, fileunits::ounit, constants::pi2, allglobal::sfmn, allglobal::sg, allglobal::simn, tfft(), allglobal::tt, constants::two, inputlist::wmacros, allglobal::yesstellsym, and constants::zero.

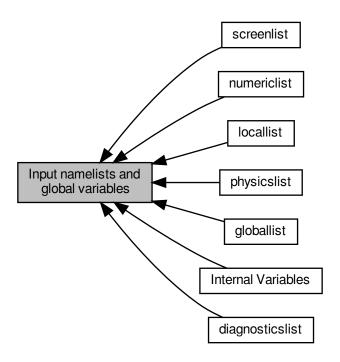
Referenced by mp00ac().





# 8.6 Input namelists and global variables

Collaboration diagram for Input namelists and global variables:



# Modules

· physicslist

The name list physicslist controls the geometry, profiles, and numerical resolution.

· numericlist

The namelist numericlist controls internal resolution parameters that the user rarely needs to consider.

locallist

The name list locallist controls the construction of the Beltrami fields in each volume.

· globallist

The namelist globallist controls the search for global force-balance.

diagnosticslist

The namelist diagnosticslist controls post-processor diagnostics, such as Poincaré plot resolution, etc.

· screenlist

The namelist screenlist controls screen output. Every subroutine, e.g. xy00aa.h, has its own write flag, Wxy00aa.

Internal Variables

## **Modules**

· module constants

some constants used throughout the code

module numerical

platform-dependant numerical resolution

· module fileunits

central definition of file units to avoid conflicts

module cputiming

timing variables

· module typedefns

type definitions for custom datatypes

· module allglobal

global variable storage used as "workspace" throughout the code

#### **Functions/Subroutines**

- subroutine allglobal::build\_vector\_potential (Ivol, iocons, aderiv, tderiv)
- subroutine allglobal::whichcpuid (vvol, cpu\_id)

Returns which MPI node is associated to a given volume.

#### **Variables**

· character inputlist::ext

The input file is , ext.sp , where ext\*100 or ext.sp\*100 is given as command line input.

• integer, parameter inputlist::mnvol = 256

The maximum value of Nvol is MNvol=256.

• integer, parameter inputlist::mmpol = 64

The maximum value of Mpol is MNpol=64.

• integer, parameter inputlist::mntor = 64

The maximum value of Ntor is MNtor=64.

· integer allglobal::ncpu

number of MPI tasks

· integer allglobal::ismyvolumevalue

flag to indicate if a CPU is operating on its assigned volume

• real allglobal::cpus

initial time

· real allglobal::pi2nfp

pi2/nfp; assigned in readin()

• real allglobal::pi2pi2nfp

 $4\pi^2 Nfp$ 

· real allglobal::pi2pi2nfphalf

 $2\pi^2 Nfp$ 

· real allglobal::pi2pi2nfpquart

 $\pi^2 Nfp$ 

· real allglobal::forceerr

total force-imbalance

· real allglobal::energy

MHD energy.

• real, dimension(:), allocatable allglobal::ipdt

Toroidal pressure-driven current.

• real, dimension(:,:), allocatable allglobal::ipdtdpf

Toroidal pressure-driven current.

· integer allglobal::mvol

number of total volumes; equal to Nvol for fixed-boundary; equal to Nvol+1 for free-boundary

· logical allglobal::yesstellsym

internal shorthand copies of Istellsym, which is an integer input;

logical allglobal::notstellsym

internal shorthand copies of Istellsym, which is an integer input;

- logical allglobal::yesmatrixfree
- · logical allglobal::notmatrixfree

to use matrix-free method or not

real, dimension(:,:), allocatable allglobal::cheby

local workspace for evaluation of Chebychev polynomials

real, dimension(:,:,:), allocatable allglobal::zernike

local workspace for evaluation of Zernike polynomials

• real, dimension(:,:,:), allocatable allglobal::tt

derivatives of Chebyshev polynomials at the inner and outer interfaces;

real, dimension(:,:,:,:), allocatable allglobal::rtt

derivatives of Zernike polynomials at the inner and outer interfaces;

• real, dimension(:,:), allocatable allglobal::rtm

 $r^m$  term of Zernike polynomials at the origin

· real, dimension(:), allocatable allglobal::zernikedof

Zernike degree of freedom for each m.

• logical, dimension(:), allocatable allglobal::imagneticok

used to indicate if Beltrami fields have been correctly constructed;

· logical allglobal::iconstraintok

Used to break iteration loops of slaves in the global constraint minimization.

• real, dimension(:,:), allocatable allglobal::beltramierror

to store the integral of |curlB-mu\*B| computed by jo00aa;

# 8.6.1 Detailed Description

Input namelists.

## 8.7 "local" force

#### **Functions/Subroutines**

subroutine Iforce (Ivol, iocons, ideriv, Ntz, dBB, XX, YY, length, DDI, MMI, iflag)
 Computes B<sup>2</sup>, and the spectral condensation constraints if required, on the interfaces, \( \mathcal{I}\_i \).

## 8.7.1 Detailed Description

#### 8.7.2 Function/Subroutine Documentation

Computes  $B^2$ , and the spectral condensation constraints if required, on the interfaces,  $\mathcal{I}_i$ .

# field strength

- The field strength is given by  $B^2 = B^s B_s + B^\theta B_\theta + B^\zeta B_\zeta$ , and on the interfaces  $B^s = 0$  by construction.
- The magnetic field is  $\sqrt{g} \mathbf{B} = (\partial_{\theta} A_{\zeta} \partial_{\zeta} A_{\theta}) \mathbf{e}_s \partial_s A_{\zeta} \mathbf{e}_{\theta} + \partial_s A_{\theta} \mathbf{e}_{\zeta}.$
- The covariant components of the field are computed via  $B_{\theta} = B^{\theta}g_{\theta\theta} + B^{\zeta}g_{\theta\zeta}$  and  $B_{\zeta} = B^{\theta}g_{\theta\zeta} + B^{\zeta}g_{\zeta\zeta}$ .
- The expression for  $B^2$  is

$$(\sqrt{g})^2 B^2 = A'_{\zeta} A'_{\zeta} g_{\theta\theta} - 2 A'_{\zeta} A'_{\theta} g_{\theta\zeta} + A'_{\theta} A'_{\theta} g_{\zeta\zeta}, \tag{88}$$

where the " $\prime$ " denotes derivative with respect to s.

· The quantity returned is

$$F \equiv \text{pscale} \times \frac{P}{V^{\gamma}} + \frac{B^2}{2}, \tag{89}$$

where  $P \equiv \text{adiabatic}$  and  $V \equiv \text{volume}$ .

### spectral constraints

• In addition to the physical-force-balance constraints, namely that  $[[p+B^2/2]]=0$  across the interfaces, additional angle constraints are required to obtain a unique Fourier representation of the interface geometry.

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• Introducing the angle functional: a weighted combination of the "polar" constraint; the normalized, poloidal, spectral width (Hirshman & Meier (1985) [3], Hirshman & Breslau (1998) [2]) the poloidal-angle origin constraint; and the "length" of the angle curves

where i labels the interfaces, and

$$\Theta_{i,\theta} \equiv \frac{x y_{\theta} - x_{\theta} y}{x^2 + y^2}, \tag{91}$$

$$M_i \equiv \frac{\sum_j m_j^p (R_{j,i}^2 + Z_{j,i}^2)}{\sum_j (R_{j,i}^2 + Z_{j,i}^2)},$$
(92)

$$L_i \equiv \sqrt{[R_i(\theta,\zeta) - R_{i-1}(\theta,\zeta)]^2 + [Z_i(\theta,\zeta) - Z_{i-1}(\theta,\zeta)]^2},$$
(93)

and where j labels the Fourier harmonics. The  $\alpha_i$ ,  $\beta_i$ ,  $\gamma_i$  and  $\delta_i \equiv \text{sweight}$  are user-supplied weight factors.

• The polar constraint is derived from defining  $\tan \Theta \equiv y/x$ , where

$$x(\theta,\zeta) \equiv R_i(\theta,\zeta) - R_{i,0}(\zeta), \tag{94}$$

$$y(\theta,\zeta) \equiv Z_i(\theta,\zeta) - Z_{i,0}(\zeta),$$
 (95)

and where the geometric center of each interface is given by the arc-length weighted integrals, see rzaxis(),

$$R_{i,0} \equiv \int_0^{2\pi} d\theta \ R_i(\theta,\zeta) \sqrt{R_{i,\theta}(\theta,\zeta)^2 + Z_{i,\theta}(\theta,\zeta)^2}, \tag{96}$$

$$Z_{i,0} \equiv \int_0^{2\pi} d\theta \ Z_i(\theta,\zeta) \sqrt{R_{i,\theta}(\theta,\zeta)^2 + Z_{i,\theta}(\theta,\zeta)^2}, \tag{97}$$

and  $\cos\Theta = x/\sqrt{x^2+y^2}$  has been used to simplify the expressions and to avoid divide-by-zero.

Only "poloidal tangential" variations will be allowed to find the extremum of F, which are described by

$$\delta R_i(\theta,\zeta) \equiv R_{i,\theta}(\theta,\zeta) \, \delta u_i(\theta,\zeta), \tag{98}$$

$$\delta Z_i(\theta,\zeta) \equiv Z_{i,\theta}(\theta,\zeta) \, \delta u_i(\theta,\zeta), \tag{99}$$

from which it follows that the variation in each Fourier harmonic is

$$\delta R_{j,i} = \oint \!\! \oint \! d\theta d\zeta \ R_{i,\theta}(\theta,\zeta) \, \delta u_i(\theta,\zeta) \, \cos(m_j \theta - n_j \zeta), \tag{100}$$

$$\delta Z_{j,i} = \oint \!\! \int \!\! d\theta d\zeta \ Z_{i,\theta}(\theta,\zeta) \, \delta u_i(\theta,\zeta) \, \sin(m_j \theta - n_j \zeta), \tag{101}$$

and

$$\delta R_{i,\theta}(\theta,\zeta) \equiv R_{i,\theta\theta}(\theta,\zeta) \, \delta u_i(\theta,\zeta) + R_{i,\theta}(\theta,\zeta) \, \delta u_{i,\theta}(\theta,\zeta) \tag{102}$$

$$\delta Z_{i,\theta}(\theta,\zeta) \equiv Z_{i,\theta\theta}(\theta,\zeta) \, \delta u_i(\theta,\zeta) + Z_{i,\theta}(\theta,\zeta) \, \delta u_{i,\theta}(\theta,\zeta) \tag{103}$$

• The variation in F is

$$\begin{split} \delta F &= \sum_{i=1}^{N-1} \alpha_i \quad \oint \!\!\!\! \oint \! d\theta d\zeta \, \left( \frac{-2\Theta_{i,\theta\theta}}{\Theta_{i,\theta}^2} \right) \delta u_i \\ &+ \sum_{i=1}^{N-1} \beta_i \quad \oint \!\!\!\!\! \oint \! d\theta d\zeta \, \left( R_{i,\theta} X_i + Z_{i,\theta} Y_i \right) \delta u_i \\ &+ \sum_{i=1}^{N-1} \gamma_i \quad \int \! d\zeta \, \left( Z_i(0,\zeta) - Z_{i,0} \right) Z_{i,\theta} \, \delta u_i \end{split}$$

$$+ \sum_{i=1}^{N-1} \delta_{i} \oint d\theta d\zeta \left(\frac{\Delta R_{i} R_{i,\theta} + \Delta Z_{i} Z_{i,\theta}}{L_{i}}\right) \delta u_{i}$$

$$- \sum_{i=1}^{N-1} \delta_{i+1} \oint d\theta d\zeta \left(\frac{\Delta R_{i+1} R_{i,\theta} + \Delta Z_{i+1} Z_{i,\theta}}{L_{i+1}}\right) \delta u_{i}$$
(104)

where, for the stellarator symmetric case,

$$X_i \equiv \sum_{j} (m_j^p - M_i) R_{j,i} \cos(m_j \theta - n_j \zeta), \tag{105}$$

$$Y_i \equiv \sum_{j} (m_j^p - M_i) Z_{j,i} \sin(m_j \theta - n_j \zeta), \tag{106}$$

and

$$\Delta R_i \equiv R_i(\theta, \zeta) - R_{i-1}(\theta, \zeta),$$
 (107)

$$\Delta Z_i \equiv Z_i(\theta, \zeta) - Z_{i-1}(\theta, \zeta), \tag{108}$$

• The spectral constraints derived from Eqn. (104) are

$$I_{i}(\theta,\zeta) \equiv -2\alpha_{i}\frac{\Theta_{i,\theta\theta}}{\Theta_{i,\theta}^{2}} + \beta_{i}\left(R_{i,\theta}X_{i} + Z_{i,\theta}Y_{i}\right) + \gamma_{i}\left(Z_{i}(0,\zeta) - Z_{i,0}\right)Z_{i,\theta}(0,\zeta)$$

$$+ \delta_{i}\frac{\Delta R_{i}R_{i,\theta} + \Delta Z_{i}Z_{i,\theta}}{L_{i}} - \delta_{i+1}\frac{\Delta R_{i+1}R_{i,\theta} + \Delta Z_{i+1}Z_{i,\theta}}{L_{i+1}}$$

$$(109)$$

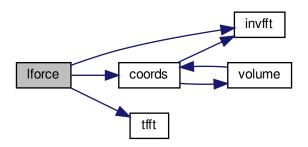
- Note that choosing p=2 gives  $X=-R_{\theta\theta}$  and  $Y=-Z_{\theta\theta}$ , and the spectrally condensed angle constraint,  $R_{\theta}X+Z_{\theta}Y=0$ , becomes  $\partial_{\theta}(R_{\theta}^2+Z_{\theta}^2)=0$ , which defines the equal arc length angle.
- The poloidal-angle origin term, namely  $\gamma_i \left( Z_i(0,\zeta) Z_{i,0} \right) Z_{i,\theta}(0,\zeta)$  is only used to constrain the  $m_j = 0$  harmonics.
- The construction of the angle functional was influenced by the following considerations:
  - The minimal spectral width constraint is very desirable as it reduces the required Fourier resolution, but it does not constrain the m=0 harmonics and the minimizing spectral-width poloidal-angle may not be consistent with the poloidal angle used on adjacent interfaces.
  - The regularization of the vector potential and the coordinate interpolation near the coordinate origin (see elsewhere) assumes that the poloidal angle is the polar angle.
  - The user will provide the Fourier harmonics of the boundary, and thus the user will implicitly define the
    poloidal angle used on the boundary.
  - Minimizing the length term will ensure that the poloidal angle used on each interface is smoothly connected to the poloidal angle used on adjacent interfaces.
- A suitable choice of the weight factors,  $\alpha_i$ ,  $\beta_i$ ,  $\gamma_i$  and  $\delta_i$ , will ensure that the polar constraint dominates for the innermost surfaces and that this constraint rapidly becomes insignificant away from the origin; that the minimal spectral constraint dominates in the "middle"; and that the minimizing length constraint will be significant near the origin and dominant near the edge, so that the minimizing spectral width angle will be continuously connected to the polar angle on the innermost surfaces and the user-implied angle at the plasma boundary. The length constraint should not be insignificant where the spectral constraint is dominant (so that the m=0 harmonics are constrained).
- The polar constraint does not need normalization. The spectral width constraint has already been normalized.
   The length constraint is not yet normalized, but perhaps it should be.
- The spectral constraints given in Eqn. (109) need to be differentiated with respect to the interface Fourier harmonics,  $R_{j,i}$  and  $Z_{j,i}$ . The first and second terms lead to a block diagonal hessian, and the length term leads to a block tri-diagonal hessian.
- Including the poloidal-angle origin constraint means that the polar angle constraint can probably be ignored, i.e.  $\alpha_i = 0$ .

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#### **Parameters**

in	Ivol	
in	iocons	
in	ideriv	
in	Ntz	
	dBB	
	XX	
	YY	
	length	
	DDI	
	MMI	
in	iflag	

References inputlist::adiabatic, allglobal::ate, allglobal::ato, allglobal::aze, allglobal::azo, allglobal::bemn, allglobal::bemn, allglobal::cfmn, allglobal::comn, coords(), allglobal::cpus, allglobal::drij, allglobal::dzij, allglobal::dzij, allglobal::dzij, allglobal::iemn, inputlist::igeometry, allglobal::ijimag, allglobal::ijreal, allglobal::im, allglobal::in, invfft(), allglobal::iomn, allglobal::irbc, allglobal::irbc, allglobal::izbc, allglobal::izbs, allglobal::izij, allglobal::jimag, allglobal::jireal, inputlist::lcheck, allglobal::lcoordinatesingularity, inputlist::lrad, allglobal::mmpp, allglobal::mvol, allglobal::myid, allglobal::ncpu, allglobal::notstellsym, allglobal::nt, inputlist::nvol, allglobal::nz, allglobal::odmn, allglobal::rtj, allglobal::rtt, allglobal::semn, allglobal::sfmn, allglobal::sg, allglobal::simn, allglobal::somn, tfft(), allglobal::trij, allglobal::tt, constants::two, allglobal::tzij, allglobal::vvolume, allglobal::yesstellsym, and constants::zero.



# 8.8 Integrals

#### **Functions/Subroutines**

- subroutine df00ab (pNN, xi, Fxi, DFxi, Ldfjac, iflag)
  - Evaluates volume integrals, and their derivatives w.r.t. interface geometry, using "packed" format.
- subroutine ma00aa (Iquad, mn, Ivol, Irad)

Calculates volume integrals of Chebyshev polynomials and metric element products.

# 8.8.1 Detailed Description

#### 8.8.2 Function/Subroutine Documentation

```
8.8.2.1 df00ab() subroutine df00ab (
    integer, intent(in) pNN,
    real, dimension(0:pnn-1), intent(in) xi,
    real, dimension(0:pnn-1), intent(out) Fxi,
    real, dimension(0:ldfjac-1,0:pnn-1), intent(out) DFxi,
    integer, intent(in) Ldfjac,
    integer, intent(in), value iflag)
```

Evaluates volume integrals, and their derivatives w.r.t. interface geometry, using "packed" format.

# Parameters

in	pNN
in	xi
out	Fxi
out	DFxi
in	Ldfjac
in	iflag

References allglobal::cpus, allglobal::dma, allglobal::dmd, constants::half, inputlist::helicity, allglobal::myid, inputlist::nvol, constants::one, fileunits::ounit, numerical::small, constants::two, and constants::zero.

Referenced by ma02aa().



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```
8.8.2.2 ma00aa() subroutine ma00aa (
                integer, intent(in) lquad,
                integer, intent(in) mn,
                 integer, intent(in) lvol,
                 integer, intent(in) lrad )
```

Calculates volume integrals of Chebyshev polynomials and metric element products.

# **Chebyshev-metric information**

· The following quantities are calculated:

DToosc(l,p,i,j) 
$$\equiv \int ds \, \overline{T}'_{l,i} \, \overline{T}_{p,j} \, \oint \!\!\!\! \oint \!\!\! d\theta d\zeta \, \sin \alpha_i \cos \alpha_j$$
 (112)

DTooss(l,p,i,j) 
$$\equiv \int ds \, \overline{T}'_{l,i} \, \overline{T}_{p,j} \, \oint \!\!\!\! \int d\theta d\zeta \, \sin \alpha_i \sin \alpha_j$$
 (113)

TTsscc(l,p,i,j) 
$$\equiv \int ds \, \overline{T}_{l,i} \, \overline{T}_{p,j} \, \oint \!\!\!\! \oint \!\!\! d\theta d\zeta \, \cos \alpha_i \cos \alpha_j \, \bar{g}_{ss}$$
 (114)

TTsscs(l,p,i,j) 
$$\equiv \int ds \, \overline{T}_{l,i} \, \overline{T}_{p,j} \, \oint \!\!\!\! \oint \!\!\! d\theta d\zeta \, \cos \alpha_i \sin \alpha_j \, \overline{g}_{ss}$$
 (115)

TTsssc(l,p,i,j) 
$$\equiv \int ds \, \overline{T}_{l,i} \, \overline{T}_{p,j} \, \oint \!\!\!\! \int \!\!\!\! d\theta d\zeta \, \sin \alpha_i \cos \alpha_j \, \overline{g}_{ss}$$
 (116)

TTssss(l,p,i,j) 
$$\equiv \int ds \, \overline{T}_{l,i} \, \overline{T}_{p,j} \, \oint \!\!\!\! \int \!\!\! d\theta d\zeta \, \sin \alpha_i \sin \alpha_j \, \overline{g}_{ss}$$
 (117)

$$\mathsf{TDstcc}\left(\mathsf{l},\mathsf{p},\mathsf{i},\mathsf{j}\right) \ \equiv \ \int ds \ \overline{T}_{l,i} \ \overline{T}'_{p,j} \ \oint \!\!\!\!\! \oint \! d\theta d\zeta \ \cos\alpha_i \cos\alpha_j \ \bar{g}_{s\theta} \tag{118}$$

TDstcs(l,p,i,j) 
$$\equiv \int ds \, \overline{T}_{l,i} \, \overline{T}'_{p,j} \, \oint \!\!\!\! \int \!\!\!\! d\theta d\zeta \, \cos \alpha_i \sin \alpha_j \, \overline{g}_{s\theta}$$
 (119)

$$\mathsf{TDstcc}(\mathsf{l},\mathsf{p},\mathsf{i},\mathsf{j}) \equiv \int ds \, \overline{T}_{l,i} \, \overline{T}'_{p,j} \, \oint \!\!\!\!\! \oint \!\!\! d\theta d\zeta \, \cos\alpha_i \cos\alpha_j \, \bar{g}_{s\zeta} \tag{122}$$

TDstcs(l,p,i,j) 
$$\equiv \int ds \, \overline{T}_{l,i} \, \overline{T}'_{p,j} \, \oint \!\!\!\! \oint \!\!\! d\theta d\zeta \, \cos \alpha_i \sin \alpha_j \, \bar{g}_{s\zeta}$$
 (123)

TDstsc(l,p,i,j) 
$$\equiv \int ds \, \overline{T}_{l,i} \, \overline{T}'_{p,j} \, \oint \!\!\!\! \int \!\!\!\! d\theta \, d\zeta \, \sin \alpha_i \cos \alpha_j \, \overline{g}_{s\zeta}$$
 (124)

TDstss(l,p,i,j) 
$$\equiv \int ds \, \overline{T}_{l,i} \, \overline{T}'_{p,j} \, \oint \!\!\!\! \int \!\!\!\! d\theta \, d\zeta \, \sin \alpha_i \sin \alpha_j \, \bar{g}_{s\zeta}$$
 (125)

$$\text{DDstcc}(l,p,i,j) \equiv \int ds \, \overline{T}'_{l,i} \, \overline{T}'_{p,j} \, \oint \!\!\!\! \oint \!\! d\theta d\zeta \, \cos\alpha_i \cos\alpha_j \, \bar{g}_{\theta\theta} \tag{126}$$

$$\text{DDstsc}(l,p,i,j) \equiv \int ds \, \overline{T}'_{l,i} \, \overline{T}'_{p,j} \, \oint \!\!\!\! \oint \!\!\! d\theta d\zeta \, \sin\alpha_i \cos\alpha_j \, \bar{g}_{\theta\theta} \tag{128}$$

$$\mathsf{DDstcc}(\mathsf{l},\mathsf{p},\mathsf{i},\mathsf{j}) \equiv \int ds \, \overline{T}'_{l,i} \, \overline{T}'_{p,j} \, \oint \!\!\!\!\! \oint \!\!\! d\theta d\zeta \, \cos\alpha_i \cos\alpha_j \, \overline{g}_{\theta\zeta} \tag{130}$$

DDstcs(l,p,i,j) 
$$\equiv \int ds \, \overline{T}'_{l,i} \, \overline{T}'_{p,j} \, \oint \!\!\!\! \oint \!\!\! d\theta d\zeta \, \cos \alpha_i \sin \alpha_j \, \overline{g}_{\theta\zeta}$$
 (131)

$$\text{DDstsc}(1,p,i,j) \equiv \int ds \, \overline{T}'_{l,i} \, \overline{T}'_{p,j} \, \oint \!\!\!\! \oint \!\!\! d\theta d\zeta \, \sin \alpha_i \cos \alpha_j \, \overline{g}_{\theta\zeta} \tag{132}$$

DDstss(l,p,i,j) 
$$\equiv \int ds \, \overline{T}'_{l,i} \, \overline{T}'_{p,j} \, \oint \!\!\!\! \oint \!\!\! d\theta d\zeta \, \sin \alpha_i \sin \alpha_j \, \bar{g}_{\theta\zeta}$$
 (133)

$$\text{DDstcc}(1,p,i,j) \equiv \int ds \, \overline{T}'_{l,i} \, \overline{T}'_{p,j} \, \oint \!\!\!\! \oint \!\! d\theta d\zeta \, \cos\alpha_i \cos\alpha_j \, \bar{g}_{\zeta\zeta} \tag{134}$$

DDstcs(l,p,i,j) 
$$\equiv \int ds \, \overline{T}'_{l,i} \, \overline{T}'_{p,j} \, \oint \!\!\!\! \oint \!\!\! d\theta d\zeta \, \cos \alpha_i \sin \alpha_j \, \bar{g}_{\zeta\zeta}$$
 (135)

$$\text{DDstsc}(1,p,i,j) \equiv \int ds \, \overline{T}'_{l,i} \, \overline{T}'_{p,j} \, \oint \!\!\!\! \oint \!\! d\theta d\zeta \, \sin \alpha_i \cos \alpha_j \, \overline{g}_{\zeta\zeta} \tag{136}$$

DDstss(l,p,i,j) 
$$\equiv \int ds \, \overline{T}'_{l,i} \, \overline{T}'_{p,j} \, \oint \!\!\!\!\! \oint \!\!\!\! d\theta d\zeta \, \sin \alpha_i \sin \alpha_j \, \bar{g}_{\zeta\zeta}$$
 (137)

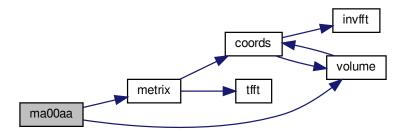
where  $\overline{T}_{l,i} \equiv T_l \, \overline{s}^{m_i/2}$  if the domain includes the coordinate singularity, and  $\overline{T}_{l,i} \equiv T_l$  if not; and  $\overline{g}_{\mu\nu} \equiv g_{\mu\nu}/\sqrt{g}$ .

• The double-angle formulae are used to reduce the above expressions to the Fourier harmonics of  $\bar{g}_{\mu\nu}$ : see kija and kijs, which are defined in preset.f90 .

### **Parameters**

in	Iquad	degree of quadrature	
in <i>mn</i> num		number of Fourier harmonics	
in	Ivol	index of nested volume	
in <i>lrad</i>		order of Chebychev polynomials	

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# 8.9 Solver/Driver

#### **Functions/Subroutines**

• subroutine ma02aa (Ivol, NN)

Constructs Beltrami field in given volume consistent with flux, helicity, rotational-transform and/or parallel-current constraints.

# 8.9.1 Detailed Description

#### 8.9.2 Function/Subroutine Documentation

Constructs Beltrami field in given volume consistent with flux, helicity, rotational-transform and/or parallel-current constraints.

#### **Parameters**

ĺ	in	Ivol	index of nested volume for which to run this	1
	in	NN	number of degrees of freedom in the (packed format) vector potential;	

## sequential quadratic programming

- Only relevant if LBsequad=T . See LBeltrami for details.
- Documentation on the implementation of  ${\tt E04UFF}$  is under construction.

# **Newton method**

• Only relevant if LBnewton=T . See LBeltrami for details.

## linear method

- Only relevant if LBlinear=T. See LBeltrami for details.
- The quantity  $\mu$  is *not* not treated as a "magnetic" degree-of-freedom equivalent to in the degrees-of-freedom in the magnetic vector potential (as it strictly should be, because it is a Lagrange multiplier introduced to enforce the helicity constraint).
- In this case, the Beltrami equation,  $\nabla \times \mathbf{B} = \mu \mathbf{B}$ , is *linear* in the magnetic degrees-of-freedom.
- · The algorithm proceeds as follows:

# plasma volumes

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– In addition to the enclosed toroidal flux,  $\Delta\psi_t$ , which is held constant in the plasma volumes, the Beltrami field in a given volume is assumed to be parameterized by  $\mu$  and  $\Delta\psi_p$ . (Note that  $\Delta\psi_p$  is not defined in a torus.)

- These are "packed" into an array, e.g.  $\mu \equiv (\mu, \Delta \psi_p)^T$ , so that standard library routines , e.g. C05PCF, can be used to (iteratively) find the appropriately-constrained Beltrami solution, i.e.  $\mathbf{f}(\mu) = 0$ .
- The function  $f(\mu)$ , which is computed by mp00ac(), is defined by the input parameter Lconstraint:
  - \* If Lconstraint = -1, 0, then  $\mu$  is not varied and Nxdof=0.
  - \* If Lconstraint = 1, then  $\mu$  is varied to satisfy the transform constraints; and Nxdof=1 in the simple torus and Nxdof=2 in the annular regions. (Note that in the "simple-torus" region, the enclosed poloidal flux  $\Delta \psi_p$  is not well-defined, and only  $\mu=\mu_1$  is varied in order to satisfy the transform constraint on the "outer" interface of that volume.)
  - \* Todo If Lconstraint = 2, then  $\mu=\mu_1$  is varied in order to satisfy the helicity constraint, and  $\Delta\psi_p=\mu_2$  is *not* varied, and Nxdof=1. (under re-construction)

#### vacuum volume

- In the vacuum,  $\mu=0$ , and the enclosed fluxes,  $\Delta\psi_t$  and  $\Delta\psi_p$ , are considered to parameterize the family of solutions. (These quantities may not be well-defined if  ${\bf B}\cdot{\bf n}\neq 0$  on the computational boundary.)
- These are "packed" into an array,  $\mu \equiv (\Delta \psi_t, \Delta \psi_p)^T$ , so that, as above, standard routines can be used to iteratively find the appropriately constrained solution, i.e.  $\mathbf{f}(\mu) = 0$ .
- The function  $f(\mu)$ , which is computed by mp00ac(), is defined by the input parameter Lconstraint:
  - \* If Lconstraint = -1, then  $\mu$  is not varied and Nxdof=0.
  - \* If Lconstraint = 0,2, then  $\mu$  is varied to satisfy the enclosed current constraints, and Nxdof=2.
  - \* If Lconstraint = 1, then  $\mu$  is varied to satisfy the constraint on the transform on the inner boundary  $\equiv$  plasma boundary and the "linking" current, and Nxdof=2.
- The Beltrami fields, and the rotational-transform and helicity etc. as required to determine the function  $f(\mu)$  are calculated in mp00ac().
- This routine, mp00ac(), is called iteratively if Nxdof>1 via C05PCF to determine the appropriately constrained Beltrami field,  $\mathbf{B}_{\mu}$ , so that  $\mathbf{f}(\mu) = 0$ .
- The input variables mupftol and mupfits control the required accuracy and maximum number of iterations.
- If Nxdof=1, then mp00ac() is called only once to provide the Beltrami fields with the given value of  $\mu$ .

## debugging: finite-difference confirmation of the derivatives of the rotational-transform

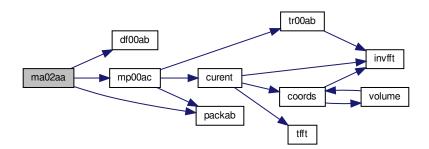
- Note that the rotational-transform (if required) is calculated by tr00ab(), which is called by mp00ac().
- If Lconstraint=1, then mp00ac() will ask tr00ab() to compute the derivatives of the transform with respect to variations in the helicity-multiplier,  $\mu$ , and the enclosed poloidal-flux,  $\Delta\psi_p$ , so that C05PCF may more efficiently find the solution.
- · The required derivatives are

$$\frac{\partial +}{\partial \mu}$$
 (138)

$$\frac{\partial t}{\partial \Delta \psi_n} \tag{139}$$

to improve the efficiency of the iterative search. A finite difference estimate of these derivatives is available; need DEBUG, Lcheck=2 and Lconstraint=1.

References allglobal::cpus, df00ab(), constants::half, inputlist::helicity, allglobal::im, allglobal::in, allglobal::lblinear, allglobal::lbnewton, allglobal::lbsequad, inputlist::lcheck, inputlist::lconstraint, inputlist::lrad, mp00ac(), inputlist::mu, inputlist::mupfits, inputlist::mupfits, inputlist::mupfits, inputlist::mupfits, inputlist::ounit, packab(), numerical::small, constants::ten, numerical::vsmall, inputlist::wmacros, and constants::zero.



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## 8.10 Build matrices

## **Functions/Subroutines**

• subroutine matrix (Ivol, mn, Irad)

Constructs energy and helicity matrices that represent the Beltrami linear system. gauge conditions

### 8.10.1 Detailed Description

#### 8.10.2 Function/Subroutine Documentation

Constructs energy and helicity matrices that represent the Beltrami linear system.

## gauge conditions

• In the v-th annulus, bounded by the (v-1)-th and v-th interfaces, a general covariant representation of the magnetic vector-potential is written

$$\bar{\mathbf{A}} = \bar{A}_s \nabla s + \bar{A}_\theta \nabla \theta + \bar{A}_\zeta \nabla \zeta eta. \tag{140}$$

• To this add  $\nabla g(s,\theta,\zeta)$ , where g satisfies

$$\begin{array}{lcl} \partial_{s}g(s,\theta,\zeta) & = & - & \bar{A}_{s}(s,\theta,\zeta) \\ \partial_{\theta}g(-1,\theta,\zeta) & = & - & \bar{A}_{\theta}(-1,\theta,\zeta) \\ \partial_{\zeta}g(-1,0,\zeta) & = & - & \bar{A}_{\zeta}(-1,0,\zeta). \end{array} \tag{141}$$

• Then  $\mathbf{A} = \bar{\mathbf{A}} + \nabla g$  is given by  $\mathbf{A} = A_{\theta} \nabla \theta + A_{\zeta} \nabla \zeta$  with

$$A_{\theta}(-1,\theta,\zeta) = 0 \tag{142}$$

$$A_{\zeta}(-1,0,\zeta) = 0 \tag{143}$$

- This specifies the gauge: to see this, notice that no gauge term can be added without violating the conditions in Eqn. (142) or Eqn. (143).
- Note that the gauge employed in each volume is distinct.

# boundary conditions

- The magnetic field is  $\sqrt{g} \mathbf{B} = (\partial_{\theta} A_{\zeta} \partial_{\zeta} A_{\theta}) \mathbf{e}_{s} \partial_{s} A_{\zeta} \mathbf{e}_{\theta} + \partial_{s} A_{\theta} \mathbf{e}_{\zeta}$ .
- In the annular volumes, the condition that the field is tangential to the inner interface,  $\sqrt{g}\mathbf{B}\cdot\nabla s=0$  at s=-1, gives  $\partial_{\theta}A_{\zeta}-\partial_{\zeta}A_{\theta}=0$ . With the above condition on  $A_{\theta}$  given in Eqn. (142), this gives  $\partial_{\theta}A_{\zeta}=0$ , which with Eqn. (143) gives

$$A_{\zeta}(-1,\theta,\zeta) = 0. \tag{144}$$

• The condition at the outer interface, s=+1, is that the field is  $\sqrt{g} \mathbf{B} \cdot \nabla s = \partial_{\theta} A_{\zeta} - \partial_{\zeta} A_{\theta} = b$ , where b is supplied by the user. For each of the plasma regions, b=0. For the vacuum region, generally  $b \neq 0$ .

#### enclosed fluxes

- In the plasma regions, the enclosed fluxes must be constrained.
- · The toroidal and poloidal fluxes enclosed in each volume are determined using

$$\int_{S} \mathbf{B} \cdot \mathbf{ds} = \int_{\partial S} \mathbf{A} \cdot \mathbf{dl}.$$
 (145)

## Fourier-Chebyshev representation

• The components of the vector potential,  $\mathbf{A} = A_{\theta} \nabla + A_{\zeta} \nabla \zeta eta$ , are

$$A_{\theta}(s,\theta,\zeta) = \sum_{i,l} A_{\theta,e,i,l} \, \overline{T}_{l,i}(s) \cos \alpha_i + \sum_{i,l} A_{\theta,o,i,l} \, \overline{T}_{l,i}(s) \sin \alpha_i, \tag{146}$$

$$A_{\zeta}(s,\theta,\zeta) = \sum_{i,l} A_{\zeta,e,i,l} \, \overline{T}_{l,i}(s) \cos \alpha_i + \sum_{i,l} A_{\zeta,o,i,l} \, \overline{T}_{l,i}(s) \sin \alpha_i, \tag{147}$$

where  $\overline{T}_{l,i}(s) \equiv \overline{s}^{m_i/2} T_l(s)$ ,  $T_l(s)$  is the Chebyshev polynomial, and  $\alpha_j \equiv m_j \theta - n_j \zeta$ . The regularity factor,  $\overline{s}^{m_i/2}$ , where  $\overline{s} \equiv (1+s)/2$ , is only included if there is a coordinate singularity in the domain (i.e. only in the innermost volume, and only in cylindrical and toroidal geometry.)

• The magnetic field,  $\sqrt{g} \mathbf{B} = \sqrt{g} B^s \mathbf{e}_s + \sqrt{g} B^\theta \mathbf{e}_\theta + \sqrt{g} B^\zeta \mathbf{e}_\zeta$ , is

$$\sqrt{g} \mathbf{B} = \mathbf{e}_{s} \sum_{i,l} \left[ (-m_{i} A_{\zeta,e,i,l} - n_{i} A_{\theta,e,i,l}) \overline{T}_{l,i} \sin \alpha_{i} + (+m_{i} A_{\zeta,o,i,l} + n_{i} A_{\theta,o,i,l}) \overline{T}_{l,i} \cos \alpha_{i} \right] 
+ \mathbf{e}_{\theta} \sum_{i,l} \left[ (-m_{i} A_{\zeta,e,i,l} - n_{i} A_{\theta,e,i,l}) \overline{T}'_{l,i} \cos \alpha_{i} + (-m_{i} A_{\zeta,o,i,l} + n_{i} A_{\theta,o,i,l}) \overline{T}'_{l,i} \sin \alpha_{i} \right] (148) 
+ \mathbf{e}_{\zeta} \sum_{i,l} \left[ (-m_{i} A_{\zeta,e,i,l} - n_{i} A_{\theta,e,i,l}) \overline{T}'_{l,i} \cos \alpha_{i} + (-m_{i} A_{\zeta,o,i,l} - n_{i} A_{\theta,o,i,l}) \overline{T}'_{l,i} \sin \alpha_{i} \right]$$

• The components of the velocity,  ${f v}\equiv v_s \nabla s + v_\theta \nabla \theta + v_\zeta \nabla \zeta eta$ , are

$$v_s(s,\theta,\zeta) = \sum_{i,l} v_{s,e,i,l} \overline{T}_{l,i}(s) \cos \alpha_i + \sum_{i,l} v_{s,o,i,l} \overline{T}_{l,i}(s) \sin \alpha_i, \tag{149}$$

$$v_{\theta}(s,\theta,\zeta) = \sum_{i,l} \frac{v_{\theta,e,i,l}}{T_{l,i}(s)} \cos \alpha_i + \sum_{i,l} \frac{v_{\theta,o,i,l}}{T_{l,i}(s)} \overline{T}_{l,i}(s) \sin \alpha_i, \tag{150}$$

$$v_{\zeta}(s,\theta,\zeta) = \sum_{i,l} v_{\zeta,e,i,l} \, \overline{T}_{l,i}(s) \cos \alpha_i + \sum_{i,l} v_{\zeta,o,i,l} \, \overline{T}_{l,i}(s) \sin \alpha_i. \tag{151}$$

### constrained energy functional

 The constrained energy functional in each volume depends on the vector potential and the Lagrange multipliers.

$$\mathcal{F} \equiv \mathcal{F}[A_{\theta,e,i,l}, A_{\zeta,e,i,l}, A_{\theta,o,i,l}, A_{\zeta,o,i,l}, v_{s,e,i,l}, v_{s,o,i,l}, v_{\theta,e,i,l}, v_{\phi,o,i,l}, v_{\zeta,e,i,l}, v_{\zeta,o,i,l}, \mu, a_i, b_i, c_i, d_i, e_i, f_i, g_1, h_1], (152)$$

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and is given by:

$$\mathcal{F} \equiv \int \mathbf{B} \cdot \mathbf{B} \, dv + \int \mathbf{v} \cdot \mathbf{v} \, dv - \mu \left[ \int \mathbf{A} \cdot \mathbf{B} \, dv - K \right]$$

$$+ \sum_{i=1} a_{i} \left[ \sum_{l} A_{\theta,e,i,l} T_{l}(-1) - 0 \right]$$

$$+ \sum_{i=1} b_{i} \left[ \sum_{l} A_{\xi,e,i,l} T_{l}(-1) - 0 \right]$$

$$+ \sum_{i=2} c_{i} \left[ \sum_{l} A_{\theta,o,i,l} T_{l}(-1) - 0 \right]$$

$$+ \sum_{i=2} d_{i} \left[ \sum_{l} A_{\zeta,o,i,l} T_{l}(-1) - 0 \right]$$

$$+ \sum_{i=2} e_{i} \left[ \sum_{l} (-m_{i} A_{\zeta,e,i,l} - n_{i} A_{\theta,e,i,l}) T_{l}(+1) - b_{s,i} \right]$$

$$+ \sum_{i=2} f_{i} \left[ \sum_{l} (+m_{i} A_{\zeta,o,i,l} + n_{i} A_{\theta,o,i,l}) T_{l}(+1) - b_{c,i} \right]$$

$$+ \sum_{i=2} A_{\theta,e,l,l} T_{l}(+1) - \Delta \psi_{l}$$

$$+ \sum_{l} A_{\zeta,e,l,l} T_{l}(+1) + \Delta \psi_{p}$$

where

- $a_i$ ,  $b_i$ ,  $c_i$  and  $d_i$  are Lagrange multipliers used to enforce the combined gauge and interface boundary condition on the inner interface,
- $e_i$  and  $f_i$  are Lagrange multipliers used to enforce the interface boundary condition on the outer interface, namely  $\sqrt{g} \mathbf{B} \cdot \nabla s = b$ ; and
- $g_1$  and  $h_1$  are Lagrange multipliers used to enforce the constraints on the enclosed fluxes.
- In each plasma volume the boundary condition on the outer interface is b=0.
- In the vacuum volume (only for free-boundary), we may set  $\mu=0$ .

## derivatives of magnetic energy integrals

• The first derivatives of  $\int dv \, \mathbf{B} \cdot \mathbf{B}$  with respect to  $A_{\theta,e,i,l}, A_{\theta,o,i,l}, A_{\zeta,e,i,l}$  and  $A_{\zeta,o,i,l}$  are

$$\begin{split} &\frac{\partial}{\partial A_{\theta,e,i,l}}\int\!\!dv\;\mathbf{B}\cdot\mathbf{B} &= 2\int\!\!dv\;\mathbf{B}\cdot\frac{\partial\mathbf{B}}{\partial A_{\theta,e,i,l}} = 2\int\!\!dv\;\mathbf{B}\cdot\left[-n_i\overline{T}_{l,i}\sin\alpha_i\,\mathbf{e}_s + \overline{T}'_{l,i}\cos\alpha_i\,\mathbf{e}_\zeta\right]/\sqrt{N_0}4)\\ &\frac{\partial}{\partial A_{\theta,o,i,l}}\int\!\!dv\;\mathbf{B}\cdot\mathbf{B} &= 2\int\!\!dv\;\mathbf{B}\cdot\frac{\partial\mathbf{B}}{\partial A_{\theta,o,i,l}} = 2\int\!\!dv\;\mathbf{B}\cdot\left[+n_i\overline{T}_{l,i}\cos\alpha_i\,\mathbf{e}_s + \overline{T}'_{l,i}\sin\alpha_i\,\mathbf{e}_\zeta\right]/\sqrt{N_0}5)\\ &\frac{\partial}{\partial A_{\zeta,e,i,l}}\int\!\!dv\;\mathbf{B}\cdot\mathbf{B} &= 2\int\!\!dv\;\mathbf{B}\cdot\frac{\partial\mathbf{B}}{\partial A_{\zeta,e,i,l}} = 2\int\!\!dv\;\mathbf{B}\cdot\left[-m_i\overline{T}_{l,i}\sin\alpha_i\,\mathbf{e}_s - \overline{T}'_{l,i}\cos\alpha_i\,\mathbf{e}_\theta\right]/\sqrt{N_0}6)\\ &\frac{\partial}{\partial A_{\zeta,o,i,l}}\int\!\!dv\;\mathbf{B}\cdot\mathbf{B} &= 2\int\!\!dv\;\mathbf{B}\cdot\frac{\partial\mathbf{B}}{\partial A_{\zeta,o,i,l}} = 2\int\!\!dv\;\mathbf{B}\cdot\left[+m_i\overline{T}_{l,i}\cos\alpha_i\,\mathbf{e}_s - \overline{T}'_{l,i}\sin\alpha_i\,\mathbf{e}_\theta\right]/\sqrt{N_0}7) \end{split}$$

• The second derivatives of  $\int dv \ \mathbf{B} \cdot \mathbf{B}$  with respect to  $A_{\theta,e,i,l}$ ,  $A_{\theta,o,i,l}$ ,  $A_{\zeta,e,i,l}$  and  $A_{\zeta,o,i,l}$  are

$$\frac{\partial}{\partial A_{\theta,e,j,p}} \frac{\partial}{\partial A_{\theta,e,i,l}} \int dv \ \mathbf{B} \cdot \mathbf{B} = 2 \int dv \ (+n_j n_i \overline{T}_{p,j} \overline{T}_{l,i} s_j s_i g_{ss} - n_j \overline{T}_{p,j} \overline{T}'_{l,i} s_j c_i g_{s\zeta} - n_i \overline{T}_{l,i} \overline{T}'_{p,j} s_i c_j g_{s\zeta} + \overline{T}'_{p,j} \overline{T}'_{l,i} c_j c_i g_{\zeta\zeta}) / \sqrt{2} \frac{\partial}{\partial A_{\theta,e,i,l}} \int dv \ \mathbf{B} \cdot \mathbf{B} = 2 \int dv \ (-n_j n_i \overline{T}_{p,j} \overline{T}_{l,i} c_j s_i g_{ss} + n_j \overline{T}_{p,j} \overline{T}'_{l,i} c_j c_i g_{s\zeta} - n_i \overline{T}_{l,i} \overline{T}'_{p,j} s_i s_j g_{s\zeta} + \overline{T}'_{p,j} \overline{T}'_{l,i} s_j c_i g_{\zeta\zeta}) / \sqrt{2} \frac{\partial}{\partial A_{\theta,e,i,l}} \int dv \ \mathbf{B} \cdot \mathbf{B} = 2 \int dv \ (+m_j n_i \overline{T}_{p,j} \overline{T}_{l,i} s_j s_i g_{ss} - m_j \overline{T}_{p,j} \overline{T}'_{l,i} s_j c_i g_{s\zeta} + n_i \overline{T}_{l,i} \overline{T}'_{p,j} s_i c_j g_{s\theta} - \overline{T}'_{p,j} \overline{T}'_{l,i} c_j c_i g_{\theta\zeta}) / \sqrt{2} \frac{\partial}{\partial A_{\theta,e,i,l}} \int dv \ \mathbf{B} \cdot \mathbf{B} = 2 \int dv \ (-m_j n_i \overline{T}_{p,j} \overline{T}_{l,i} c_j s_i g_{ss} + m_j \overline{T}_{p,j} \overline{T}'_{l,i} c_j c_i g_{s\zeta} + n_i \overline{T}_{l,i} \overline{T}'_{p,j} s_i s_j g_{s\theta} - \overline{T}'_{p,j} \overline{T}'_{l,i} s_j c_i g_{\theta\zeta}) / \sqrt{2} \frac{\partial}{\partial A_{\theta,e,i,l}} \int dv \ \mathbf{B} \cdot \mathbf{B} = 2 \int dv \ (-m_j n_i \overline{T}_{p,j} \overline{T}_{l,i} c_j s_i g_{ss} + m_j \overline{T}_{p,j} \overline{T}'_{l,i} c_j c_i g_{s\zeta} + n_i \overline{T}_{l,i} \overline{T}'_{p,j} s_i s_j g_{s\theta} - \overline{T}'_{p,j} \overline{T}'_{l,i} s_j c_i g_{\theta\zeta}) / \sqrt{2} \frac{\partial}{\partial A_{\theta,e,i,l}} \frac{\partial}{\partial A_{\theta,e,i,l}} \int dv \ \mathbf{B} \cdot \mathbf{B} = 2 \int dv \ (-m_j n_i \overline{T}_{p,j} \overline{T}_{l,i} c_j s_i g_{ss} + m_j \overline{T}_{p,j} \overline{T}'_{l,i} c_j c_i g_{s\zeta} + n_i \overline{T}_{l,i} \overline{T}'_{p,j} s_i s_j g_{s\theta} - \overline{T}'_{p,j} \overline{T}'_{l,i} s_j c_i g_{\theta\zeta}) / \sqrt{2} \frac{\partial}{\partial A_{\theta,e,i,l}} \frac{\partial}{\partial A_{\theta,e,i,l}}$$

$$\frac{\partial}{\partial A_{\theta,e,j,p}} \frac{\partial}{\partial A_{\theta,o,i,l}} \int dv \ \mathbf{B} \cdot \mathbf{B} = 2 \int dv \ (-n_{j}n_{i}\overline{T}_{p,j}\overline{T}_{l,i}s_{j}c_{i}g_{ss} - n_{j}\overline{T}_{p,j}\overline{T}'_{l,i}s_{j}s_{i}g_{s\zeta} + n_{i}\overline{T}_{l,i}\overline{T}'_{p,j}c_{i}c_{j}g_{s\zeta} + \overline{T}'_{p,j}\overline{T}'_{l,i}c_{j}s_{i}g_{\zeta\zeta})/\sqrt{2}$$

$$\frac{\partial}{\partial A_{\theta,o,j,p}} \frac{\partial}{\partial A_{\theta,o,i,l}} \int dv \ \mathbf{B} \cdot \mathbf{B} = 2 \int dv \ (+n_{j}n_{i}\overline{T}_{p,j}\overline{T}_{l,i}c_{j}c_{i}g_{ss} + n_{j}\overline{T}_{p,j}\overline{T}'_{l,i}c_{j}s_{i}g_{s\zeta} + n_{i}\overline{T}_{l,i}\overline{T}'_{p,j}c_{i}s_{j}g_{s\zeta} + \overline{T}'_{p,j}\overline{T}'_{l,i}s_{j}s_{i}g_{\zeta\zeta})/\sqrt{2}$$

$$\frac{\partial}{\partial A_{\phi,o,i,l}} \int dv \ \mathbf{B} \cdot \mathbf{B} = 2 \int dv \ (-m_{j}n_{i}\overline{T}_{p,j}\overline{T}_{l,i}s_{j}c_{i}g_{ss} - m_{j}\overline{T}_{p,j}\overline{T}'_{l,i}s_{j}s_{i}g_{s\zeta} - n_{i}\overline{T}_{l,i}\overline{T}'_{p,j}c_{i}c_{j}g_{s\theta} - \overline{T}'_{p,j}\overline{T}'_{l,i}c_{j}s_{i}g_{\theta\zeta})/\sqrt{2}$$

$$\frac{\partial}{\partial A_{\phi,o,i,l}} \int dv \ \mathbf{B} \cdot \mathbf{B} = 2 \int dv \ (+m_{j}n_{i}\overline{T}_{p,j}\overline{T}_{l,i}c_{j}c_{i}g_{ss} + m_{j}\overline{T}_{p,j}\overline{T}'_{l,i}c_{j}s_{i}g_{s\zeta} - n_{i}\overline{T}_{l,i}\overline{T}'_{p,j}c_{i}s_{j}g_{s\theta} - \overline{T}'_{p,j}\overline{T}'_{l,i}s_{j}s_{i}g_{\theta\zeta})/\sqrt{2}$$

$$\frac{\partial}{\partial A_{\phi,o,i,l}} \int dv \ \mathbf{B} \cdot \mathbf{B} = 2 \int dv \ (+n_{j}m_{i}\overline{T}_{p,j}\overline{T}_{l,i}s_{j}s_{i}g_{ss} + n_{j}\overline{T}_{p,j}\overline{T}'_{l,i}s_{j}c_{i}g_{s\theta} - m_{i}\overline{T}_{l,i}\overline{T}'_{p,j}s_{i}c_{j}g_{s\zeta} - \overline{T}'_{p,j}\overline{T}'_{l,i}s_{j}s_{i}g_{s\zeta})/\sqrt{2}$$

$$\frac{\partial}{\partial A_{\phi,o,i,l}} \int dv \ \mathbf{B} \cdot \mathbf{B} = 2 \int dv \ (-n_{j}m_{i}\overline{T}_{p,j}\overline{T}_{l,i}c_{j}s_{i}g_{ss} + n_{j}\overline{T}_{p,j}\overline{T}'_{l,i}s_{j}c_{i}g_{s\theta} - m_{i}\overline{T}_{l,i}\overline{T}'_{p,j}s_{i}c_{j}g_{s\zeta} - \overline{T}'_{p,j}\overline{T}'_{l,i}s_{j}c_{i}g_{\theta\zeta})/\sqrt{2}$$

$$\frac{\partial}{\partial A_{\phi,o,i,l}} \int dv \ \mathbf{B} \cdot \mathbf{B} = 2 \int dv \ (-n_{j}m_{i}\overline{T}_{p,j}\overline{T}_{l,i}c_{j}s_{i}g_{ss} - n_{j}\overline{T}_{p,j}\overline{T}'_{l,i}s_{j}c_{i}g_{s\theta} - m_{i}\overline{T}_{l,i}\overline{T}'_{p,j}s_{i}s_{j}g_{s\zeta} - \overline{T}'_{p,j}\overline{T}'_{l,i}s_{j}c_{i}g_{\theta\zeta})/\sqrt{2}$$

$$\frac{\partial}{\partial A_{\phi,o,i,l}} \int dv \ \mathbf{B} \cdot \mathbf{B} = 2 \int dv \ (-n_{j}m_{i}\overline{T}_{p,j}\overline{T}_{l,i}c_{j}s_{i}g_{ss} - n_{j}\overline{T}_{p,j}\overline{T}'_{l,i}s_{j}c_{i}g_{s\theta} - m_{i}\overline{T}_{l,i}\overline{T}'_{p,j}s_{i}s_{j}g_{s\zeta} - \overline{T}'_{p,j}\overline{T}'_{l,i}s_{j}c_{i}g_{\theta\zeta})/\sqrt{2}$$

$$\frac{\partial}{\partial A_{\phi,o,i,l}} \int dv \ \mathbf{B} \cdot \mathbf{B} = 2 \int dv \ (-n_{j}m_{i}\overline{T}_{p,j}\overline{T}_{l,i}s_{j}s_{i}g_{ss} -$$

$$\frac{\partial}{\partial A_{\theta,e,j,p}} \frac{\partial}{\partial A_{\zeta,o,i,l}} \int dv \, \mathbf{B} \cdot \mathbf{B} = 2 \int dv \, (-n_j m_i \overline{T}_{p,j} \overline{T}_{l,i} s_j c_i g_{ss} + n_j \overline{T}_{p,j} \overline{T}'_{l,i} s_j s_i g_{s\theta} + m_i \overline{T}_{l,i} \overline{T}'_{p,j} c_i c_j g_{s\zeta} - \overline{T}'_{p,j} \overline{T}'_{l,i} c_j s_i g_{\theta\zeta}) / \frac{\partial}{\partial A_{\theta,o,j,p}} \frac{\partial}{\partial A_{\zeta,o,i,l}} \int dv \, \mathbf{B} \cdot \mathbf{B} = 2 \int dv \, (+n_j m_i \overline{T}_{p,j} \overline{T}_{l,i} c_j c_i g_{ss} - n_j \overline{T}_{p,j} \overline{T}'_{l,i} c_j s_i g_{s\theta} + m_i \overline{T}_{l,i} \overline{T}'_{p,j} c_i s_j g_{s\zeta} - \overline{T}'_{p,j} \overline{T}'_{l,i} s_j s_i g_{\theta\zeta}) / \frac{\partial}{\partial A_{\zeta,o,i,l}} \frac{\partial}{\partial A_{\zeta,o,i,l}} \int dv \, \mathbf{B} \cdot \mathbf{B} = 2 \int dv \, (-m_j m_i \overline{T}_{p,j} \overline{T}_{l,i} s_j c_i g_{ss} + m_j \overline{T}_{p,j} \overline{T}'_{l,i} s_j s_i g_{s\theta} - m_i \overline{T}_{l,i} \overline{T}'_{p,j} c_i c_j g_{s\theta} + \overline{T}'_{p,j} \overline{T}'_{l,i} c_j s_i g_{\theta\theta}) / \frac{\partial}{\partial A_{\zeta,o,j,p}} \frac{\partial}{\partial A_{\zeta,o,i,l}} \int dv \, \mathbf{B} \cdot \mathbf{B} = 2 \int dv \, (+m_j m_i \overline{T}_{p,j} \overline{T}_{l,i} c_j c_i g_{ss} - m_j \overline{T}_{p,j} \overline{T}'_{l,i} c_j s_i g_{s\theta} - m_i \overline{T}_{l,i} \overline{T}'_{p,j} c_i s_j g_{s\theta} + \overline{T}'_{p,j} \overline{T}'_{l,i} s_j s_i g_{\theta\theta}) / \frac{\partial}{\partial A_{\zeta,o,j,p}} \frac{\partial}{\partial A_{\zeta,o,i,l}} \int dv \, \mathbf{B} \cdot \mathbf{B} = 2 \int dv \, (+m_j m_i \overline{T}_{p,j} \overline{T}_{l,i} c_j c_i g_{ss} - m_j \overline{T}_{p,j} \overline{T}'_{l,i} c_j s_i g_{s\theta} - m_i \overline{T}_{l,i} \overline{T}'_{p,j} c_i s_j g_{s\theta} + \overline{T}'_{p,j} \overline{T}'_{l,i} s_j s_i g_{\theta\theta}) / \frac{\partial}{\partial A_{\zeta,o,j,p}} \frac{\partial}{\partial A_{\zeta,o,j,p}}$$

## derivatives of helicity integrals

• The first derivatives of  $\int dv \ \mathbf{A} \cdot \mathbf{B}$  with respect to  $A_{\theta,e,i,l}, A_{\theta,o,i,l}, A_{\zeta,e,i,l}$  and  $A_{\zeta,o,i,l}$  are

$$\begin{split} &\frac{\partial}{\partial A_{\theta,e,i,l}}\int\!\!dv\;\mathbf{A}\cdot\mathbf{B} &= \int\!\!dv\;\left(\frac{\partial\mathbf{A}}{\partial A_{\theta,e,i,l}}\cdot\mathbf{B}+\mathbf{A}\cdot\frac{\partial\mathbf{B}}{\partial A_{\theta,e,i,l}}\right) = \int\!\!dv\;\left(\overline{T}_{l,i}\cos\alpha_{i}\nabla\theta\cdot\mathbf{B}+\mathbf{A}\cdot\overline{T}_{l,i}'\cos\alpha_{i}\,\mathbf{e}_{\zeta}/(59)\right) \\ &\frac{\partial}{\partial A_{\theta,o,i,l}}\int\!\!dv\;\mathbf{A}\cdot\mathbf{B} &= \int\!\!dv\;\left(\frac{\partial\mathbf{A}}{\partial A_{\theta,o,i,l}}\cdot\mathbf{B}+\mathbf{A}\cdot\frac{\partial\mathbf{B}}{\partial A_{\theta,o,i,l}}\right) = \int\!\!dv\;\left(\overline{T}_{l,i}\sin\alpha_{i}\nabla\theta\cdot\mathbf{B}+\mathbf{A}\cdot\overline{T}_{l,i}'\sin\alpha_{i}\,\mathbf{e}_{\zeta}/(59)\right) \\ &\frac{\partial}{\partial A_{\zeta,e,i,l}}\int\!\!dv\;\mathbf{A}\cdot\mathbf{B} &= \int\!\!dv\;\left(\frac{\partial\mathbf{A}}{\partial A_{\zeta,e,i,l}}\cdot\mathbf{B}+\mathbf{A}\cdot\frac{\partial\mathbf{B}}{\partial A_{\zeta,e,i,l}}\right) = \int\!\!dv\;\left(\overline{T}_{l,i}\cos\alpha_{i}\nabla\zeta\cdot\mathbf{B}-\mathbf{A}\cdot\overline{T}_{l,i}'\cos\alpha_{i}\,\mathbf{e}_{\theta}/(59)\right) \\ &\frac{\partial}{\partial A_{\zeta,o,i,l}}\int\!\!dv\;\mathbf{A}\cdot\mathbf{B} &= \int\!\!dv\;\left(\frac{\partial\mathbf{A}}{\partial A_{\zeta,o,i,l}}\cdot\mathbf{B}+\mathbf{A}\cdot\frac{\partial\mathbf{B}}{\partial A_{\zeta,o,i,l}}\right) = \int\!\!dv\;\left(\overline{T}_{l,i}\sin\alpha_{i}\nabla\zeta\cdot\mathbf{B}-\mathbf{A}\cdot\overline{T}_{l,i}'\sin\alpha_{i}\,\mathbf{e}_{\theta}/(59)\right) \end{split}$$

- Note that in the above expressions,  $\mathbf{A} \cdot \mathbf{e}_s = 0$  has been used.
- The second derivatives of  $\int dv \ \mathbf{A} \cdot \mathbf{B}$  with respect to  $A_{\theta,e,i,l}$ ,  $A_{\theta,o,i,l}$ ,  $A_{\zeta,e,i,l}$  and  $A_{\zeta,o,i,l}$  are

$$\begin{split} &\frac{\partial}{\partial A_{\theta,e,j,p}} \frac{\partial}{\partial A_{\theta,e,i,l}} \int\!\! dv \; \mathbf{A} \cdot \mathbf{B} &= \int\!\! dv \; \left[ + \overline{T}_{l,i} \cos \alpha_i \nabla \theta \cdot \overline{T}'_{p,j} \cos \alpha_j \mathbf{Q} \theta \cdot \overline{T}'_{l,i} \cos \alpha_i \; \mathbf{e}_{\zeta} \right] (163) \\ &\frac{\partial}{\partial A_{\theta,o,j,p}} \frac{\partial}{\partial A_{\theta,e,i,l}} \int\!\! dv \; \mathbf{A} \cdot \mathbf{B} &= \int\!\! dv \; \left[ + \overline{T}_{l,i} \cos \alpha_i \nabla \theta \cdot \overline{T}'_{p,j} \sin \alpha_j \; \mathbf{e}_{\zeta} + \overline{T}_{p,j} \sin \alpha_j \nabla \theta \cdot \overline{T}'_{l,i} \cos \alpha_i \; \mathbf{e}_{\zeta} \right] (163) \\ &\frac{\partial}{\partial A_{\zeta,e,j,p}} \frac{\partial}{\partial A_{\theta,e,i,l}} \int\!\! dv \; \mathbf{A} \cdot \mathbf{B} &= \int\!\! dv \; \left[ - \overline{T}_{l,i} \cos \alpha_i \nabla \theta \cdot \overline{T}'_{p,j} \cos \alpha_j \; \mathbf{e}_{\theta} + \overline{T}_{p,j} \cos \alpha_j \nabla \zeta \cdot \overline{T}'_{l,i} \cos \alpha_i \; \mathbf{e}_{\zeta} \right] (1649) \\ &\frac{\partial}{\partial A_{\zeta,o,j,p}} \frac{\partial}{\partial A_{\theta,e,i,l}} \int\!\! dv \; \mathbf{A} \cdot \mathbf{B} &= \int\!\! dv \; \left[ - \overline{T}_{l,i} \cos \alpha_i \nabla \theta \cdot \overline{T}'_{p,j} \sin \alpha_j \; \mathbf{e}_{\theta} + \overline{T}_{p,j} \sin \alpha_j \nabla \zeta \cdot \overline{T}'_{l,i} \cos \alpha_i \; \mathbf{e}_{\zeta} \right] (1649) \end{split}$$

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$$\frac{\partial}{\partial A_{\theta,e,j,p}} \frac{\partial}{\partial A_{\theta,o,i,l}} \int dv \, \mathbf{A} \cdot \mathbf{B} = \int dv \left[ +\overline{T_{l,i} \sin \alpha_i \nabla \theta} \cdot \overline{T'_{p,j} \cos \alpha_j \mathbf{e}_{\zeta}} + \overline{T_{p,j} \cos \alpha_j \nabla \theta} \cdot \overline{T'_{l,i} \sin \alpha_i \mathbf{e}_{\zeta}} \right] / \sqrt[4]{6}$$

$$\frac{\partial}{\partial A_{\theta,o,j,p}} \frac{\partial}{\partial A_{\theta,o,i,l}} \int dv \, \mathbf{A} \cdot \mathbf{B} = \int dv \left[ +\overline{T_{l,i} \sin \alpha_i \nabla \theta} \cdot \overline{T'_{p,j} \sin \alpha_j \mathbf{e}_{\zeta}} + \overline{T_{p,j} \sin \alpha_j \nabla \theta} \cdot \overline{T'_{l,i} \sin \alpha_i \mathbf{e}_{\zeta}} \right] / \sqrt[4]{6}$$

$$\frac{\partial}{\partial A_{\zeta,e,j,p}} \frac{\partial}{\partial A_{\theta,o,i,l}} \int dv \, \mathbf{A} \cdot \mathbf{B} = \int dv \left[ -\overline{T_{l,i} \sin \alpha_i \nabla \theta} \cdot \overline{T'_{p,j} \cos \alpha_j \mathbf{e}_{\theta}} + \overline{T_{p,j} \sin \alpha_j \nabla \zeta} \cdot \overline{T'_{l,i} \sin \alpha_i \mathbf{e}_{\zeta}} \right] / \sqrt[4]{6}$$

$$\frac{\partial}{\partial A_{\zeta,o,j,p}} \frac{\partial}{\partial A_{\zeta,o,i,l}} \int dv \, \mathbf{A} \cdot \mathbf{B} = \int dv \left[ -\overline{T_{l,i} \cos \alpha_i \nabla \zeta} \cdot \overline{T'_{p,j} \cos \alpha_j \mathbf{e}_{\theta}} + \overline{T_{p,j} \cos \alpha_j \nabla \zeta} \cdot \overline{T'_{l,i} \cos \alpha_i \mathbf{e}_{\zeta}} \right] / \sqrt[4]{6}$$

$$\frac{\partial}{\partial A_{\theta,o,j,p}} \frac{\partial}{\partial A_{\zeta,o,i,l}} \int dv \, \mathbf{A} \cdot \mathbf{B} = \int dv \left[ +\overline{T_{l,i} \cos \alpha_i \nabla \zeta} \cdot \overline{T'_{p,j} \cos \alpha_j \mathbf{e}_{\zeta}} - \overline{T_{p,j} \cos \alpha_j \nabla \delta} \cdot \overline{T'_{l,i} \cos \alpha_i \mathbf{e}_{\zeta}} \right] / \sqrt[4]{6}$$

$$\frac{\partial}{\partial A_{\zeta,o,j,p}} \frac{\partial}{\partial A_{\zeta,o,i,l}} \int dv \, \mathbf{A} \cdot \mathbf{B} = \int dv \left[ +\overline{T_{l,i} \cos \alpha_i \nabla \zeta} \cdot \overline{T'_{p,j} \cos \alpha_j \mathbf{e}_{\zeta}} - \overline{T_{p,j} \sin \alpha_j \nabla \delta} \cdot \overline{T'_{l,i} \cos \alpha_i \mathbf{e}_{\zeta}} \right] / \sqrt[4]{6}$$

$$\frac{\partial}{\partial A_{\zeta,o,j,p}} \frac{\partial}{\partial A_{\zeta,o,i,l}} \int dv \, \mathbf{A} \cdot \mathbf{B} = \int dv \left[ -\overline{T_{l,i} \cos \alpha_i \nabla \zeta} \cdot \overline{T'_{p,j} \cos \alpha_j \mathbf{e}_{\zeta}} - \overline{T_{p,j} \sin \alpha_j \nabla \zeta} \cdot \overline{T'_{l,i} \cos \alpha_i \mathbf{e}_{\zeta}} \right] / \sqrt[4]{6}$$

$$\frac{\partial}{\partial A_{\zeta,o,j,p}} \frac{\partial}{\partial A_{\zeta,o,i,l}} \int dv \, \mathbf{A} \cdot \mathbf{B} = \int dv \left[ -\overline{T_{l,i} \cos \alpha_i \nabla \zeta} \cdot \overline{T'_{p,j} \cos \alpha_j \mathbf{e}_{\zeta}} - \overline{T_{p,j} \sin \alpha_j \nabla \zeta} \cdot \overline{T'_{l,i} \cos \alpha_i \mathbf{e}_{\zeta}} \right] / \sqrt[4]{6}$$

$$\frac{\partial}{\partial A_{\zeta,o,j,p}} \frac{\partial}{\partial A_{\zeta,o,i,l}} \int dv \, \mathbf{A} \cdot \mathbf{B} = \int dv \left[ -\overline{T_{l,i} \sin \alpha_i \nabla \zeta} \cdot \overline{T'_{p,j} \sin \alpha_j \mathbf{e}_{\zeta}} - \overline{T_{p,j} \sin \alpha_j \nabla \zeta} \cdot \overline{T'_{l,i} \sin \alpha_i \mathbf{e}_{\zeta}} \right] / \sqrt[4]{6}$$

$$\frac{\partial}{\partial A_{\zeta,o,j,p}} \frac{\partial}{\partial A_{\zeta,o,i,l}} \int dv \, \mathbf{A} \cdot \mathbf{B} = \int dv \left[ -\overline{T_{l,i} \sin \alpha_i \nabla \zeta} \cdot \overline{T'_{p,j} \cos \alpha_j \mathbf{e}_{\zeta}} - \overline{T_{p,j} \cos \alpha_j \nabla \zeta} \cdot \overline{T'_{l,i} \sin \alpha_i \mathbf{e}_{\zeta}} \right] / \sqrt[4]{6}$$

$$\frac{\partial}{\partial A_{\zeta,o,j,p}} \frac{\partial}{\partial A_{\zeta,o,i,l}} \int dv \, \mathbf{A} \cdot \mathbf{B} = \int dv \left[ -\overline{T_{l,i} \sin \alpha_i \nabla \zeta} \cdot \overline{T'_{p,j} \cos \alpha_j \mathbf{e}_{\zeta}} - \overline{T_{p,j} \cos \alpha_j \nabla \zeta} \cdot \overline{T'_{l,$$

• In these expressions the terms  $\nabla \theta \cdot \mathbf{e}_{\theta} = \nabla \zeta \cdot \mathbf{e}_{\zeta} = 1$ , and  $\nabla \theta \cdot \mathbf{e}_{\zeta} = \nabla \zeta \cdot \mathbf{e}_{\theta} = 0$  have been included to show the structure of the derivation.

## derivatives of kinetic energy integrals

• The first derivatives of  $\int dv \ v^2$  with respect to  $v_{s.e.i.l}$  etc. are

$$\frac{\partial}{\partial v_{s,e,i,l}} \int dv \, \mathbf{v} \cdot \mathbf{v} = 2 \int dv \, \mathbf{v} \cdot \overline{T}_{l,i} \cos \alpha_i \nabla s \tag{178}$$

$$\frac{\partial}{\partial v_{s,o,i,l}} \int dv \, \mathbf{v} \cdot \mathbf{v} = 2 \int dv \, \mathbf{v} \cdot \overline{T}_{l,i} \sin \alpha_i \nabla s \tag{179}$$

$$\frac{\partial}{\partial v_{\theta,e,i,l}} \int dv \, \mathbf{v} \cdot \mathbf{v} = 2 \int dv \, \mathbf{v} \cdot \overline{T}_{l,i} \cos \alpha_i \nabla \theta \tag{180}$$

$$\frac{\partial}{\partial v_{\theta,o,i,l}} \int dv \, \mathbf{v} \cdot \mathbf{v} = 2 \int dv \, \mathbf{v} \cdot \overline{T}_{l,i} \sin \alpha_i \nabla \theta$$
 (181)

$$\frac{\partial}{\partial v_{\zeta, q, i, l}} \int dv \, \mathbf{v} \cdot \mathbf{v} = 2 \int dv \, \mathbf{v} \cdot \overline{T}_{l, i} \cos \alpha_i \nabla \zeta \tag{182}$$

$$\frac{\partial}{\partial v_{\zeta,o,i,l}} \int dv \, \mathbf{v} \cdot \mathbf{v} = 2 \int dv \, \mathbf{v} \cdot \overline{T}_{l,i} \sin \alpha_i \nabla \zeta$$
 (183)

(184)

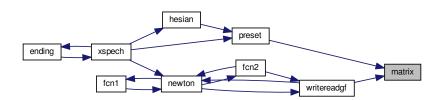
# calculation of volume-integrated basis-function-weighted metric information

• The required geometric information is calculated in ma00aa().

#### **Parameters**

in	Ivol	
in	mn	
in	Irad	

Referenced by preset(), and writereadgf().



# 8.11 Metric quantities

## **Functions/Subroutines**

• subroutine metrix (Iquad, Ivol)

Calculates the metric quantities,  $\sqrt{g} g^{\mu\nu}$ , which are required for the energy and helicity integrals.

# 8.11.1 Detailed Description

#### 8.11.2 Function/Subroutine Documentation

Calculates the metric quantities,  $\sqrt{g}\,g^{\mu\nu}$ , which are required for the energy and helicity integrals.

## metrics

• The Jacobian,  $\sqrt{g}$ , and the "lower" metric elements,  $g_{\mu\nu}$ , are calculated by coords(), and are provided on a regular grid in "real-space", i.e.  $(\theta, \zeta)$ , at a given radial location, i.e. where s is input.

# plasma region

• In the plasma region, the required terms are  $\bar{g}_{\mu\nu} \equiv g_{\mu\nu}/\sqrt{g}$ .

$$\sqrt{g} g^{ss} = (g_{\theta\theta}g_{\zeta\zeta} - g_{\theta\zeta}g_{\theta\zeta})/\sqrt{g}$$

$$\sqrt{g} g^{s\theta} = (g_{\theta\zeta}g_{s\zeta} - g_{s\theta}g_{\zeta\zeta})/\sqrt{g}$$

$$\sqrt{g} g^{s\zeta} = (g_{s\theta}g_{\theta\zeta} - g_{\theta\theta}g_{s\zeta})/\sqrt{g}$$

$$\sqrt{g} g^{\theta\theta} = (g_{\zeta\zeta}g_{ss} - g_{s\zeta}g_{s\zeta})/\sqrt{g}$$

$$\sqrt{g} g^{\theta\zeta} = (g_{s\zeta}g_{s\theta} - g_{\theta\zeta}g_{ss})/\sqrt{g}$$

$$\sqrt{g} g^{\xi\zeta} = (g_{ss}g_{\theta\theta} - g_{s\theta}g_{s\theta})/\sqrt{g}$$
(185)

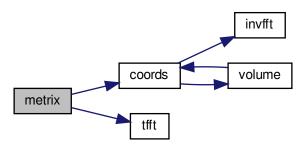
# **FFTs**

After constructing the required quantities in real space, FFTs provided the required Fourier harmonics, which
are returned through global.f90. (The "extended" Fourier resolution is used.)

References allglobal::cfmn, coords(), allglobal::cpus, allglobal::dbdx, allglobal::efmn, allglobal::gaussianabscissae, allglobal::guvij, allglobal::guvijsave, allglobal::im, allglobal::ime, allglobal::in, allglobal::ine, allglobal::ine, allglobal::nep, allglobal::nep, allglobal::nep, allglobal::nep, allglobal::nep, allglobal::nep, allglobal::guvijsave, allglobal::nep, allglobal::nep, allglobal::nep, allglobal::nep, allglobal::nep, allglobal::nep, allglobal::nep, allglobal::guvijsave, allglobal::nep, allgl

Referenced by ma00aa().

Here is the call graph for this function:





# 8.12 Solver for Beltrami (linear) system

#### **Functions/Subroutines**

subroutine mp00ac (Ndof, Xdof, Fdof, Ddof, Ldfjac, iflag)
 Solves Beltrami/vacuum (linear) system, given matrices.
 unpacking fluxes, helicity multiplier

# 8.12.1 Detailed Description

## 8.12.2 Function/Subroutine Documentation

Solves Beltrami/vacuum (linear) system, given matrices.

# unpacking fluxes, helicity multiplier

• The vector of "parameters",  $\mu$ , is unpacked. (Recall that  $\mu$  was "packed" in ma02aa() .) In the following,  $\psi \equiv (\Delta \psi_t, \Delta \psi_p)^T$ .

# construction of linear system

• The equation  $\nabla \times \mathbf{B} = \mu \mathbf{B}$  is cast as a matrix equation,

$$\mathcal{M} \cdot \mathbf{a} = \mathcal{R},\tag{186}$$

where a represents the degrees-of-freedom in the magnetic vector potential,  $\mathbf{a} \equiv \{A_{\theta,e,i,l}, A_{\zeta,e,i,l}, \ldots\}$ .

• The matrix  $\mathcal{M}$  is constructed from  $\mathcal{A} \equiv \text{dMA}$  and  $\mathcal{D} \equiv \text{dMD}$ , which were constructed in matrix(), according to

$$\mathcal{M} \equiv \mathcal{A} - \mu \mathcal{D}. \tag{187}$$

Note that in the vacuum region,  $\mu = 0$ , so  $\mathcal{M}$  reduces to  $\mathcal{M} \equiv \mathcal{A}$ .

- The construction of the vector  $\ensuremath{\mathcal{R}}$  is as follows:
  - if Lcoordinatesingularity=T, then

$$\mathcal{R} \equiv -\left(\mathcal{B} - \mu \mathcal{E}\right) \cdot \boldsymbol{\psi} \tag{188}$$

- if Lcoordinatesingularity=F and Lplasmaregion=T, then

$$\mathcal{R} \equiv -\mathcal{B} \cdot \psi \tag{189}$$

- if Lcoordinatesingularity=F and Lvacuumregion=T, then

$$\mathcal{R} \equiv -\mathcal{G} - \mathcal{B} \cdot \psi \tag{190}$$

The quantities  $\mathcal{B}\equiv \mathtt{dMB}$ ,  $\mathcal{E}\equiv \mathtt{dME}$  and  $\mathcal{G}\equiv \mathtt{dMG}$  are constructed in matrix() .

## solving linear system

It is *not* assumed that the linear system is positive definite. The LAPACK routine DSYSVX is used to solve the linear system.

## unpacking, ...

- The magnetic degrees-of-freedom are unpacked by packab() .
- The error flag, ImagneticOK, is set that indicates if the Beltrami fields were successfully constructed.

#### construction of "constraint" function

• The construction of the function  $f(\mu)$  is required so that iterative methods can be used to construct the Beltrami field consistent with the required constraints (e.g. on the enclosed fluxes, helicity, rotational-transform, ...).

### See also

ma02aa() for additional details.

## plasma region

- For Lcoordinatesingularity=T, the returned function is:

$$\mathbf{f}(\mu, \Delta \psi_p) \equiv \begin{cases} ( & 0 & , & 0 )^T, & \text{if Lconstraint} = -1 \\ ( & 0 & , & 0 )^T, & \text{if Lconstraint} = 0 \\ ( & \pounds(+1) - \text{iota (lvol )} & , & 0 )^T, & \text{if Lconstraint} = 1 \\ ( & ? & , & ? )^T, & \text{if Lconstraint} = 2 \end{cases}$$
(191)

- For Lcoordinatesingularity=F, the returned function is:

$$\mathbf{f}(\mu,\Delta\psi_p) \equiv \left\{ \begin{array}{lll} (&0&,&0&\\ (&0&,&0&\\ (&\pm(-1)-\mathrm{oita}\,(\mathrm{lvol-1})&,&\pm(+1)-\mathrm{iota}\,(\mathrm{lvol})&)^T,&\mathrm{if}\,\,\mathrm{Lconstraint}&=&-1\\ (&?&,&?&)^T,&\mathrm{if}\,\,\mathrm{Lconstraint}&=&1\\ (&?&,&?&)^T,&\mathrm{if}\,\,\mathrm{Lconstraint}&=&2 \end{array} \right.$$

### vacuum region

- For the vacuum region, the returned function is:

$$\mathbf{f}(\Delta\psi_t,\Delta\psi_p) \equiv \left\{ \begin{array}{lll} (&0&,&0&)^T, & \text{if Lconstraint} &=&-1\\ (&I-\text{curtor}&,&G-\text{curpol}&)^T, & \text{if Lconstraint} &=&0\\ (&{\scriptstyle t}(-1)-\text{oita(lvol-1)}&,&G-\text{curpol}&)^T, & \text{if Lconstraint} &=&1\\ (&?&,&?&)^T, & \text{if Lconstraint} &=&2 \end{array} \right. \tag{193}$$

• The rotational-transform,  $\pm$ , is computed by tr00ab(); and the enclosed currents, I and G, are computed by curent().

# early termination

• If  $|\mathbf{f}| < \text{mupftol}$ , then early termination is enforced (i.e., iflag is set to a negative integer). (See ma02aa() for details of how mp00ac() is called iteratively.)

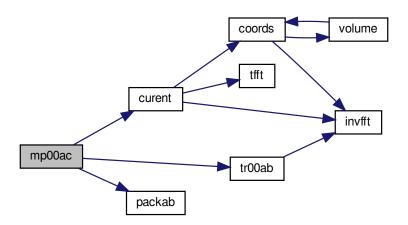
### **Parameters**

in	Ndof	
in	Xdof	
	Fdof	
	Ddof	
in	Ldfjac	
	iflag	indicates whether (i) iflag=1: "function" values are required; or (ii) iflag=2: "derivative" values are required

References allglobal::cpus, curent(), inputlist::curpol, inputlist::curtor, allglobal::dmd, inputlist::epsgmres, constants::half, inputlist::helicity, allglobal::im, allglobal::in, inputlist::iota, allglobal::iquad, allglobal::ivol, allglobal::loordinatesingularity, allglobal::liluprecond, allglobal::lplasmaregion, inputlist::lrad, allglobal::lvacuumregion, numerical::machprec, allglobal::mns, inputlist::mu, allglobal::myid, allglobal::ncpu, inputlist::nitergmres, allglobal::notmatrixfree, allglobal::notstellsym, allglobal::nt, inputlist::ntor, allglobal::nz, inputlist::oita, constants::one, fileunits::ounit, packab(), numerical::small, tr00ab(), inputlist::wmacros, allglobal::yesstellsym, and constants::zero.

Referenced by ma02aa().

Here is the call graph for this function:





## 8.13 Force-driver

#### **Functions/Subroutines**

```
• subroutine newton (NGdof, position, ihybrd)  \textit{Employs Newton method to find } \mathbf{F}(\mathbf{x}) = 0, \textit{where } \mathbf{x} \equiv \{ \text{geometry} \} \textit{ and } \mathbf{F} \textit{ is defined in dforce()} \ .
```

• subroutine writereadgf (readorwrite, NGdof, ireadhessian)

read or write force-derivative matrix

• subroutine fcn1 (NGdof, xx, fvec, irevcm)

subroutine fcn2 (NGdof, xx, fvec, fjac, Ldfjac, irevcm)

fcn2

### 8.13.1 Detailed Description

#### 8.13.2 Function/Subroutine Documentation

Employs Newton method to find F(x) = 0, where  $x \equiv \{\text{geometry}\}\$  and F is defined in dforce().

```
Solves \mathbf{F}(\xi) = 0, where \mathbf{F} \equiv \{[[p + B^2/2]]_{i,l}, I_{i,l}\} and \xi \equiv \{R_{i,l}, Z_{i,l}\}.
```

# iterative, reverse communication loop

- The iterative, Newton search to find  $\mathbf{x} \equiv \{\text{geometry}\} \equiv \{R_{i,l}, Z_{i,l}\}$  such that  $\mathbf{F}(\mathbf{x}) = 0$ , where  $\mathbf{F}$  and its derivatives,  $\nabla_{\mathbf{x}}\mathbf{F}$ , are calculated by dforce(), is provided by either
  - C05NDF if Lfindzero=1, which only uses function values; or
  - C05PDF if Lfindzero=2, which uses user-provided derivatives.
- The iterative search will terminate when the solution is within c05xtol of the true solution (see NAG documentation).
- The input variable c05factor is provided to determine the initial step bound (see NAG documentation).

## logic, writing/reading from file

- Before proceeding with iterative search, dforce() is called to determine the magnitude of the initial force imbalance, and if this is less than forcetol then the iterative search will not be performed.
- As the iterations proceed, wrtend() will be called to save itermediate information (also see xspech()).
- If the derivative matrix,  $\nabla_{\mathbf{x}} \mathbf{F}$ , is required, i.e. if Lfindzero=2, and if LreadGF=T then the derivative matrix will initially be read from .ext.sp.DF, if it exists, or from .sp.DF.
- As the iterations proceed, the derivative matrix will be written to .ext.sp.DF.

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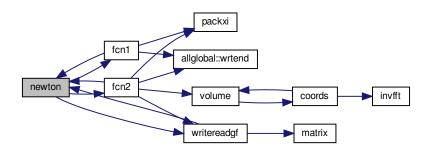
#### **Parameters**

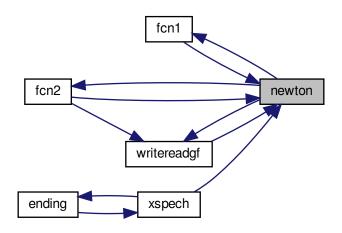
in	NGdof	
in,out	position	
out	ihybrd	

References allglobal::bbe, allglobal::bbo, allglobal::cpus, allglobal::dbbdmp, allglobal::dessian, allglobal:-dfdrz, allglobal::dessian, allglobal::energy, inputlist::ext, fcn1(), fcn2(), allglobal::forceerr, allglobal:-irbc, allglobal::irbc, a

Referenced by fcn1(), fcn2(), writereadgf(), and xspech().

Here is the call graph for this function:





read or write force-derivative matrix

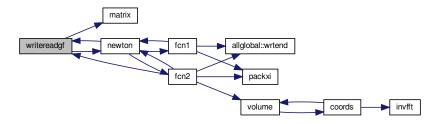
#### **Parameters**

in	readorwrite	
in	NGdof	
out	ireadhessian	

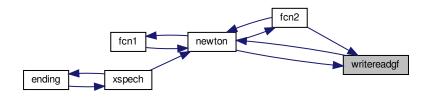
References allglobal::cpus, fileunits::dunit, inputlist::ext, allglobal::hessian, inputlist::igeometry, allglobal::im, allglobal::in, inputlist::istellsym, inputlist::lfreebound, allglobal::lhessianallocated, matrix(), inputlist::mpol, allglobal::myid, newton(), inputlist::ntor, inputlist::nvol, fileunits::ounit, and constants::zero.

Referenced by fcn2(), and newton().

Here is the call graph for this function:



Here is the caller graph for this function:



fcn1

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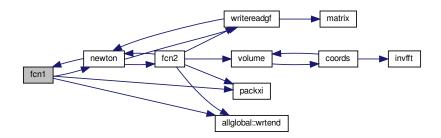
#### **Parameters**

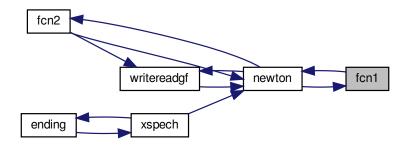
in	NGdof	
in	XX	
out	fvec	
in	irevcm	

References allglobal::bbe, allglobal::bbo, allglobal::cpus, allglobal::dbbdmp, allglobal::dessian, allglobal::dffdrz, allglobal::dmupfdx, allglobal::energy, inputlist::ext, allglobal::forceerr, allglobal::hessian, inputlist::igeometry, allglobal::iie, allglobal::iio, allglobal::im, allglobal::irbc, allglobal::irbs, allglobal::izbc, allglobal::izbc, allglobal::izbc, allglobal::irbs, allglobal::irbs, allglobal::irbs, allglobal::irbs, allglobal::irbs, allglobal::irps, allglobal::irbs, allglobal::i

Referenced by newton().

Here is the call graph for this function:





```
8.13.2.4 fcn2() subroutine fcn2 (
    integer, intent(in) NGdof,
    real, dimension(1:ngdof), intent(in) xx,
    real, dimension(1:ngdof), intent(out) fvec,
    real, dimension(1:ldfjac,1:ngdof), intent(out) fjac,
    integer, intent(in) Ldfjac,
    integer, intent(in) irevcm )
```

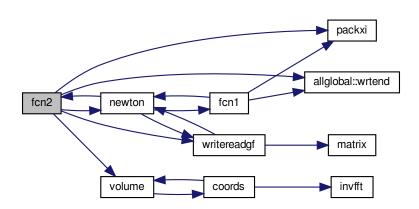
#### fcn2

#### **Parameters**

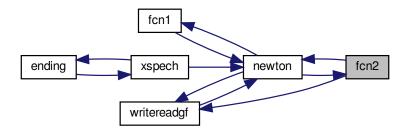
in	NGdof	
in	XX	
out	fvec	
out	fjac	
in	Ldfjac	
in	irevcm	

References allglobal::bbe, allglobal::bbo, allglobal::cpus, allglobal::dbdmp, allglobal::dessian, allglobal::dffdrz, allglobal::dmupfdx, allglobal::energy, inputlist::ext, allglobal::forceerr, allglobal::hessian, inputlist::igeometry, allglobal::iie, allglobal::iio, allglobal::im, allglobal::irbc, allglobal::irbs, allglobal::izbc, allglobal::izbc, allglobal::izbc, allglobal::irbs, allglobal::irbs, allglobal::irbs, allglobal::irbs, allglobal::irbs, allglobal::irbs, allglobal::ircpu, newtontime::ndcalls, newton(), newtontime::nfcalls, allglobal::nfreeboundaryiterations, allglobal::notstellsym, constants::one, fileunits::ounit, packxi(), numerical::sqrtmachprec, constants::ten, constants::two, volume(), inputlist::wmacros, writereadgf(), allglobal::wrtend(), and constants::zero.

Referenced by newton().



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# 8.14 Some miscellaneous numerical routines

### **Functions/Subroutines**

- subroutine gi00ab (Mpol, Ntor, Nfp, mn, im, in)
   Assign Fourier mode labels.
- subroutine tfft (Nt, Nz, ijreal, ijimag, mn, im, in, efmn, ofmn, cfmn, sfmn, ifail)

Forward Fourier transform (fftw wrapper)

• subroutine invfft (mn, im, in, efmn, ofmn, cfmn, sfmn, Nt, Nz, ijreal, ijimag)

Inverse Fourier transform (fftw wrapper)

• subroutine gauleg (n, weight, abscis, ifail)

Gauss-Legendre weights and abscissae.

# 8.14.1 Detailed Description

#### 8.14.2 Function/Subroutine Documentation

```
8.14.2.1 gi00ab() subroutine gi00ab (
    integer, intent(in) Mpol,
    integer, intent(in) Ntor,
    integer, intent(in) Nfp,
    integer, intent(in) mn,
    integer, dimension(mn), intent(out) im,
    integer, dimension(mn), intent(out) in)
```

Assign Fourier mode labels.

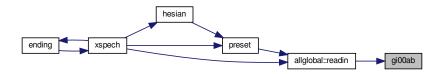
• This routine assigns the Fourier mode labels that converts a double-sum into a single sum; i.e., the  $m_j$  and  $n_i$  are assigned where

$$f(\theta,\zeta) = \sum_{n=0}^{N} f_{0,n} \cos(-n N_P \zeta) + \sum_{m=1}^{M} \sum_{n=-N}^{N} f_{m,n} \cos(m\theta - n N_P \zeta)$$

$$= \sum_{j} f_j \cos(m_j \theta - n_j \zeta),$$
(194)

where  $N\equiv {
m Ntor}$  and  $M\equiv {
m Mpol}$  are given on input, and  $N_P\equiv {
m Nfp}$  is the field periodicity.

Referenced by allglobal::readin().



```
8.14.2.2 tfft() subroutine tfft (
    integer Nt,
    integer Nz,
    real, dimension(1:nt*nz) ijreal,
    real, dimension(1:nt*nz) ijimag,
    integer mn,
    integer, dimension(1:mn) im,
    integer, dimension(1:mn) in,
    real, dimension(1:mn) efmn,
    real, dimension(1:mn) ofmn,
    real, dimension(1:mn) ofmn,
    real, dimension(1:mn) sfmn,
    real, dimension(1:mn) sfmn,
    integer ifail)
```

Forward Fourier transform (fftw wrapper)

- · This constructs the "forward" Fourier transform.
- Given a set of data,  $(f_i,g_i)$  for  $i=1,\ldots N_\theta N_\zeta$ , on a regular two-dimensional angle grid, where  $\theta_j=2\pi j/N_\theta$  for  $j=0,N_\theta-1$ , and  $\zeta_k=2\pi k/N_\zeta$  for  $k=0,N_\zeta-1$ . The "packing" is governed by  $i=1+j+kN_\theta$ . The "discrete" resolution is  $N_\theta\equiv {\rm Nt},\,N_\zeta\equiv {\rm Nz}$  and  ${\rm Ntz}={\rm Nt}\times {\rm Nz}$ , which are set in preset().
- The Fourier harmonics consistent with Eqn. (195) are constructed. The mode identification labels appearing in Eqn. (195) are  $m_j \equiv \text{im}(j)$  and  $n_j \equiv \text{in}(j)$ , which are set in readin() via a call to gi00ab().

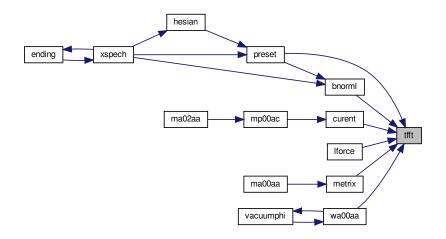
#### **Parameters**

Nt	
Nz	
ijreal	
ijimag	
mn	
im	
in	
efmn	
ofmn	
cfmn	
sfmn	
ifail	

References fftw\_interface::cplxin, fftw\_interface::cplxout, constants::half, inputlist::nfp, fileunits::ounit, constants ::pi2, allglobal::pi2nfp, fftw\_interface::planf, and constants::zero.

Referenced by bnorml(), curent(), lforce(), metrix(), preset(), and wa00aa().

Here is the caller graph for this function:



Inverse Fourier transform (fftw wrapper)

- · Given the Fourier harmonics, the data on a regular angular grid are constructed.
- This is the inverse routine to tfft().

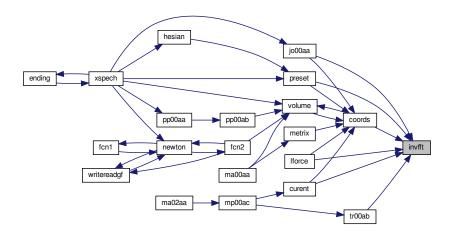
### **Parameters**

in	mn
in	im
in	in
in	efmn
in	ofmn
in	cfmn
in	sfmn
in	Nt
in	Nz
out	ijreal
out	ijimag

References fftw\_interface::cplxin, fftw\_interface::cplxout, constants::half, inputlist::nfp, fftw\_interface::planb, constants::two, and constants::zero.

Referenced by coords(), curent(), jo00aa(), lforce(), preset(), and tr00ab().

Here is the caller graph for this function:



Gauss-Legendre weights and abscissae.

- Compute Gaussian integration weights and abscissae.
- · From Numerical Recipes.

## **Parameters**

in	n	
out	weight	
out	abscis	
out	ifail	

References constants::one, constants::pi, constants::two, and constants::zero.

Referenced by preset().



# 8.15 "packing" of Beltrami field solution vector

#### **Functions/Subroutines**

- subroutine packab (packorunpack, Ivol, NN, solution, ideriv)
  - Packs and unpacks Beltrami field solution vector.
- subroutine packxi (NGdof, position, Mvol, mn, iRbc, iZbs, iRbs, iZbc, packorunpack, LComputeDerivatives, LComputeAxis)

Packs, and unpacks, geometrical degrees of freedom; and sets coordinate axis.

# 8.15.1 Detailed Description

## 8.15.2 Function/Subroutine Documentation

Packs and unpacks Beltrami field solution vector.

# construction of "vector" of independent degrees of freedom

- Numerical routines for solving linear equations typically require the unknown, independent degrees of freedom to be "packed" into a vector,  $\mathbf{x}$ .
- The magnetic field is defined by the independent degrees of freedom in the Chebyshev-Fourier representation of the vector potential,  $A_{\theta,e,i,l}$  and  $A_{\zeta,e,i,l}$ ; and the non-stellarator-symmetric terms if relevant,  $A_{\theta,o,i,l}$  and  $A_{\zeta,o,i,l}$ ; and the Lagrange multipliers,  $a_i,b_i,c_i,d_i,e_i$ , etc. as required to enforce the constraints:

$$\mathbf{x} \equiv \{ A_{\theta,e,i,l}, A_{\zeta,e,i,l}, A_{\theta,o,i,l}, A_{\zeta,o,i,l}, a_i, b_i, c_i, d_i, e_i, f_i, g_1, h_1 \}.$$
(196)

• The "packing" index is assigned in preset() .

# **Parameters**

packorunpack	
Ivol	
NN	
solution	
ideriv	

References allglobal::ate, allglobal::ate, allglobal::aze, allglobal::aze, allglobal::aze, allglobal::im, allglobal::im, allglobal::lmb, allglobal::lmb, allglobal::lmb, allglobal::lmc, allglobal::lmc, allglobal::lmc, allglobal::lmc, allglobal::lmg, allgl

allglobal::lmgvalue, allglobal::lmh, allglobal::lmhvalue, inputlist::lrad, allglobal::myid, allglobal::ncpu, allglobal::ncpu, allglobal::ncpu, allglobal::yesstellsym, and constants::zero.

Referenced by ma02aa(), and mp00ac().

Here is the caller graph for this function:



```
8.15.2.2 packxi() subroutine packxi (
    integer, intent(in) NGdof,
    real, dimension(0:ngdof) position,
    integer, intent(in) Mvol,
    integer, intent(in) mn,
    real, dimension(1:mn,0:mvol) iRbc,
    real, dimension(1:mn,0:mvol) iZbs,
    real, dimension(1:mn,0:mvol) iZbs,
    real, dimension(1:mn,0:mvol) iZbc,
    character packorunpack,
    logical, intent(in) LComputeDerivatives,
    logical, intent(in) LComputeAxis)
```

Packs, and unpacks, geometrical degrees of freedom; and sets coordinate axis.

## geometrical degrees of freedom

- The geometrical degrees-of-freedom, namely the  $R_{j,v}$  and  $Z_{j,v}$  where v labels the interface and j labels the Fourier harmonic, must be "packxi", and "unpackxi", into a single vector,  $\boldsymbol{\xi}$ , so that standard numerical routines can be called to find solutions to force-balance, i.e.  $\mathbf{F}[\boldsymbol{\xi}] = 0$ .
- · A coordinate "pre-conditioning" factor is included:

$$\boldsymbol{\xi}_k \equiv \frac{R_{j,v}}{\Psi_{j,v}},\tag{197}$$

where  $\Psi_{j,v} \equiv exttt{psifactor}( exttt{j,v})$  , which is defined in global.f90 .

# coordinate axis

- The coordinate axis is not an independent degree-of-freedom of the geometry. It is constructed by extrapolating the geometry of the innermost interface down to a line.
- Note that if the coordinate axis depends only on the geometry of the innermost interface then the block tridiagonal structure of the the force-derivative matrix is preserved.

· Define the arc-length weighted averages,

$$R_0(\zeta) \equiv \frac{\int_0^{2\pi} R_1(\theta, \zeta) dl}{L(\zeta)}, \qquad Z_0(\zeta) \equiv \frac{\int_0^{2\pi} Z_1(\theta, \zeta) dl}{L(\zeta)},$$
 (198)

where  $L(\zeta) \equiv \int_0^{2\pi} dl$  and  $dl \equiv \sqrt{\partial_\theta R_1(\theta,\zeta)^2 + \partial_\theta Z_1(\theta,\zeta)^2} \, d\theta$ .

- Note that if dl does not depend on  $\theta$ , i.e. if  $\theta$  is the equal arc-length angle, then the expressions simplify.
- Note that the geometry of the coordinate axis thus constructed only depends on the geometry of the innermost interface, by which I mean that the geometry of the coordinate axis is independent of the angle parameterization.

#### some numerical comments

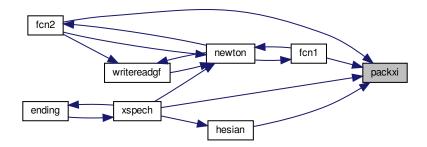
- First, the differential poloidal length,  $dl \equiv \sqrt{R_{\theta}^2 + Z_{\theta}^2}$ , is computed in real space using an inverse FFT from the Fourier harmonics of R and Z.
- Second, the Fourier harmonics of the dl are computed using an FFT. The integration over  $\theta$  to construct  $L \equiv \int dl$  is now trivial: just multiply the m=0 harmonics of dl by  $2\pi$ . The ajk (1:mn) variable is used.
- Next, the weighted  $R\,dl$  and  $Z\,dl$  are computed in real space, and the poloidal integral is similarly taken.
- · Lastly, the Fourier harmonics are constructed using an FFT after dividing in real space.

#### **Parameters**

NGdof	
position	
Mvol	
mn	
iRbc	
iZbs	
iRbs	
iZbc	
packorunpack	
LComputeDerivatives	
LComputeAxis	
	position  Mvol  mn  iRbc  iZbs  iRbs  iZbc  packorunpack  LComputeDerivatives

References allglobal::ajk, allglobal::cfmn, allglobal::comn, allglobal::cpus, allglobal::efmn, allglobal::evmn, inputlist::igeometry, allglobal::ijimag, allglobal::ijireal, allglobal::im, allglobal::in, allglobal::irij, allglobal::izij, allglobal::ijimag, allglobal::ijimag, allglobal::myid, allglobal::ncpu, allglobal::notstellsym, allglobal::nt, inputlist::ntor, inputlist::nvol, allglobal::nz, allglobal::odmn, allglobal::ofmn, fileunits::ounit, allglobal::psifactor, allglobal::rscale, allglobal::sfmn, allglobal::simn, allglobal::trij, allglobal::tzij, allglobal::yesstellsym, and constantscizero.

Referenced by fcn1(), fcn2(), hesian(), and xspech().



# 8.16 Conjugate-Gradient method

#### **Functions/Subroutines**

subroutine pc00aa (NGdof, position, Nvol, mn, ie04dgf)

Use preconditioned conjugate gradient method to find minimum of energy functional.

• subroutine pc00ab (mode, NGdof, Position, Energy, Gradient, nstate, iuser, ruser)

Returns the energy functional and it's derivatives with respect to geometry.

# 8.16.1 Detailed Description

#### 8.16.2 Function/Subroutine Documentation

```
8.16.2.1 pc00aa() subroutine pc00aa (
    integer, intent(in) NGdof,
    real, dimension(0:ngdof), intent(inout) position,
    integer, intent(in) Nvol,
    integer, intent(in) mn,
    integer ie04dgf)
```

Use preconditioned conjugate gradient method to find minimum of energy functional.

# energy functional

The energy functional is described in pc00ab().

## relevant input variables

- The following input variables control the operation of  ${\tt E04DGF}$  :
  - epsilon: weighting of "spectral energy"; see pc00ab()
  - maxstep: this is given to  ${\tt E04DGF}$  for the <code>Maximum Step Length</code>
  - maxiter: upper limit on derivative calculations used in the conjugate gradient iterations
  - verify: if verify=1, then E04DGF will confirm user supplied gradients (provided by pc00ab()) are correct;
- Todo Unfortunately,  $\verb"E04DGF"$  seems to require approximately 3N function evaluations before proceeding to minimize the energy functional, where there are N degrees of freedom. I don't know how to turn this off!

# **Parameters**

in	NGdof	
in,out	position	
in	Nvol	
in	mn	
	ie04dgf	

References allglobal::cpus, allglobal::energy, allglobal::forceerr, inputlist::forcetol, allglobal::myid, allglobal::ncpu, fileunits::ounit, pc00ab(), constants::ten, and constants::zero.

Here is the call graph for this function:



```
8.16.2.2 pc00ab() subroutine pc00ab (
    integer mode,
    integer NGdof,
    real, dimension(1:ngdof) Position,
    real Energy,
    real, dimension(1:ngdof) Gradient,
    integer nstate,
    integer, dimension(1:2) iuser,
    real, dimension(1:1) ruser)
```

Returns the energy functional and it's derivatives with respect to geometry.

## **Energy functional**

· The energy functional is

$$F \equiv \sum_{l=1}^{N} \int_{\mathcal{V}} \left( \frac{p}{\gamma - 1} + \frac{B^2}{2} \right) dv, \tag{199}$$

where  $N \equiv \mathtt{Nvol}$  is the number of interfaces.

- Assuming that the toroidal and poloidal fluxes,  $\psi_t$  and  $\psi_p$ , the helicity,  $\mathcal{K}$ , the helicity multiplier,  $\mu$ , and/or the interface rotational-transforms,  $\pm$ , are appropriately constrained, the Beltrami fields in each volume depend only the geometry of the adjacent interfaces. So, the energy functional is assumed to be a function of "position", i.e.  $F = F(R_{l,j}, Z_{l,j})$ .
- Introducing a ficitious time, t, the position may be advanced according to

$$\frac{\partial R_{j}}{\partial t} \equiv -\frac{\partial}{\partial R_{j}} \sum_{l=1}^{N} \int \left(\frac{p}{\gamma - 1} + \frac{B^{2}}{2}\right) dv, 
\frac{\partial Z_{j}}{\partial t} \equiv -\frac{\partial}{\partial Z_{j}} \sum_{l=1}^{N} \int \left(\frac{p}{\gamma - 1} + \frac{B^{2}}{2}\right) dv.$$
(200)

• There remain degrees of freedom in the angle representation of the interfaces.

# Spectral energy minimization

· Consider variations which do not affect the geometry of the surfaces,

$$\delta R = R_{\theta} u, \tag{201}$$

$$\delta Z = Z_{\theta} u, \tag{202}$$

where u is a angle variation.

· The corresponding variation in each of the Fourier harmonics is

$$\delta Z_j \equiv \oint \!\! \oint \! d\theta d\zeta \ Z_\theta \ u \ \sin \alpha_j, \tag{204}$$

· Following Hirshman et al., introducing the normalized spectral width

$$M \equiv \frac{\sum_{j} (m_{j}^{p} + n_{j}^{q}) (R_{l,j}^{2} + Z_{l,j}^{2})}{\sum_{j} (R_{l,j}^{2} + Z_{l,j}^{2})},$$
(205)

· Using the notation

$$N \equiv \sum_{j} \lambda_j (R_{l,j}^2 + Z_{l,j}^2), \tag{206}$$

$$D \equiv \sum_{i} (R_{l,j}^2 + Z_{l,j}^2), \tag{207}$$

where  $\lambda_j \equiv m_j^p + n_j^q$ , the variation in the normalized spectral width is

$$\delta M = (\delta N - M\delta D)/D. \tag{208}$$

· For tangential variations,

$$\delta N = 2 \oint \!\! \int \!\! d\theta d\zeta \ u \left( R_{\theta} \sum_{j} \lambda_{j} R_{j} \cos \alpha_{j} + Z_{\theta} \sum_{j} \lambda_{j} Z_{j} \sin \alpha_{j} \right), \tag{209}$$

$$\delta D = 2 \oint \!\! \int \!\! d\theta d\zeta \ u \left( R_{\theta} \sum_{j} R_{j} \cos \alpha_{j} + Z_{\theta} \sum_{j} Z_{j} \sin \alpha_{j} \right). \tag{210}$$

· The "tangential spectral-width descent direction" is thus

$$\frac{\partial u}{\partial t} = -\left[R_{\theta} \sum_{j} (\lambda_{j} - M) R_{j} \cos \alpha_{j} / D + Z_{\theta} \sum_{j} (\lambda_{j} - M) Z_{j} \sin \alpha_{j} / D\right]. \tag{211}$$

· This suggests that position should be advanced according to

$$\frac{\partial R_j}{\partial t} \equiv -\frac{\partial}{\partial R_j} \sum_{l=1}^N \int \left( \frac{p}{\gamma - 1} + \frac{B^2}{2} \right) dv - [R_\theta (R_\theta X + Z_\theta Y)]_j, \tag{212}$$

$$\frac{\partial Z_j}{\partial t} \equiv -\frac{\partial}{\partial Z_j} \sum_{l=1}^N \int \left( \frac{p}{\gamma - 1} + \frac{B^2}{2} \right) dv - [Z_\theta(R_\theta X + Z_\theta Y)]_j, \tag{213}$$

where  $X \equiv \sum_{j} (\lambda_j - M) R_j \cos \alpha_j / D$  and  $Y \equiv \sum_{j} (\lambda_j - M) Z_j \sin \alpha_j / D$ .

# numerical implementation

· The spectral condensation terms,

$$R_{\theta}(R_{\theta}X + Z_{\theta}Y) = \sum_{j,k,l} m_j m_k (\lambda_l - M) R_j (+R_k R_l \sin \alpha_j \sin \alpha_k \cos \alpha_l - Z_k Z_l \sin \alpha_j \cos \alpha_k \sin \alpha_l) \text{2DA}$$

$$Z_{\theta}(R_{\theta}X + Z_{\theta}Y) = \sum_{j,k,l} m_j m_k (\lambda_l - M) Z_j (-R_k R_l \cos \alpha_j \sin \alpha_k \cos \alpha_l + Z_k Z_l \cos \alpha_j \cos \alpha_k \sin \alpha_l) \text{2DA}$$

are calculated using triple angle expressions...

Todo IT IS VERY LIKELY THAT FFTs WOULD BE FASTER!!!

References allglobal::cpus, allglobal::dbbdrz, allglobal::diidrz, inputlist::epsilon, allglobal::forceerr, inputlist::forcetol, constants::half, inputlist::igeometry, allglobal::lbbintegral, allglobal::myid, inputlist::nvol, constants::one, fileunitsciounit, allglobal::yesstellsym, and constants::zero.

Referenced by pc00aa().



### 8.17 Initialization of the code

#### **Functions/Subroutines**

· subroutine preset

Allocates and initializes internal arrays.

### 8.17.1 Detailed Description

#### 8.17.2 Function/Subroutine Documentation

### 8.17.2.1 preset() subroutine preset

Allocates and initializes internal arrays.

### LGdof and NGdof: number of geometrical degrees-of-freedom

- LGdof = the number of degrees-of-freedom in the geometry (i.e. Fourier harmonics) of each interface
- $NGdof \equiv total number of degrees-of-freedom in geometry, i.e. of all interfaces$

### iota and oita: rotational transform on interfaces

- The input variables iota and oita are the rotational transform on "inner-side" and on the "outer-side" of each interface.
- · These quantities are formally inputs.
- Note that if  $q_l + \gamma q_r \neq 0$ , then iota is given by

$$t \equiv \frac{p_l + \gamma p_r}{q_l + \gamma q_r},\tag{216}$$

where  $p_l \equiv \mathtt{pl}$  ,  $q_l \equiv \mathtt{ql}$  , etc.; and similarly for oita .

# dtflux(1:Mvol) and dpflux(1:Mvol): enclosed fluxes

- dtflux  $\equiv \Delta \psi_{tor}/2\pi$  and dpflux  $\equiv \Delta \psi_{pol}/2\pi$  in each volume.
- Note that the total toroidal flux enclosed by the plasma boundary is  $\Phi_{edge} \equiv {\tt phiedge}$  .
- $\psi_{tor} \equiv \texttt{tflux}$  and  $\psi_{pol} \equiv \texttt{pflux}$  are immediately normalized (in readin() ) according to  $\psi_{tor,i} \to \psi_{tor,i}/\psi_0$  and  $\psi_{pol,i} \to \psi_{pol,i}/\psi_0$ , where  $\psi_0 \equiv \psi_{tor,N}$  on input.

# sweight(1:Mvol): star-like angle constraint weight

 the "star-like" poloidal angle constraint weights (only required for toroidal geometry, i.e. Igeometry=3) are given by

$$sweight_v \equiv upsilon \times (l_v/N_{vol})^w, \tag{217}$$

where  $l_v$  is the volume number, and  $w \equiv wpoloidal$ .

### TT(0:Mrad,0:1,0:1): Chebyshev polynomials at inner/outer interface

- TT (0:Lrad, 0:1, 0:1) gives the Chebyshev polynomials, and their first derivative, evaluated at s=-1
- Precisely, TT (1, i, d)  $\equiv T_i^{(d)}(s_i)$  for  $s_0 = -1$  and  $s_1 = +1$ .
- Note that  $T_l^{(0)}(s) = s^l$  and  $T_l^{(1)}(s) = s^{l+1}l^2$  for  $s = \pm 1$ .
- · Note that

$$T_l(-1) = \begin{cases} +1, & \text{if } l \text{ is even,} \\ -1, & \text{if } l \text{ is odd;} \end{cases} \qquad T_l(+1) = \begin{cases} +1, & \text{if } l \text{ is even,} \\ +1, & \text{if } l \text{ is odd;} \end{cases}$$
 (218)

$$T_{l}(-1) = \begin{cases} +1, & \text{if } l \text{ is even,} \\ -1, & \text{if } l \text{ is odd;} \end{cases} \qquad T_{l}(+1) = \begin{cases} +1, & \text{if } l \text{ is even,} \\ +1, & \text{if } l \text{ is odd;} \end{cases}$$

$$T'_{l}(-1) = \begin{cases} -l^{2}, & \text{if } l \text{ is even,} \\ +l^{2}, & \text{if } l \text{ is odd;} \end{cases} \qquad T'_{l}(+1) = \begin{cases} +l^{2}, & \text{if } l \text{ is even,} \\ +l^{2}, & \text{if } l \text{ is odd.} \end{cases}$$
(218)

- TT (0:Mrad, 0:1, 0:1) is used in routines that explicity require interface information, such as
  - the interface force-balance routine, Iforce()
  - the virtual casing routine, casing()
  - computing the rotational-transform on the interfaces, tr00ab()
  - computing the covariant components of the interface magnetic field, sc00aa()
  - enforcing the constraints on the Beltrami fields, matrix() and
  - computing the enclosed currents of the vacuum field, curent().

### ImagneticOK(1:Mvol): Beltrami/vacuum error flag

- · error flags that indicate if the magnetic field in each volume has been successfully constructed
- ImagneticOK is initialized to .false. in dforce() before the Beltrami solver routines are called. If the construction of the Beltrami field is successful (in either ma02aa() or mp00ac() ) then ImagneticOK is set to .true. .

## Lhessianallocated

• The internal logical variable, Lhessianallocated, indicates whether the 'Hessian' matrix of secondpartial derivatives (really, the first derivatives of the force-vector) has been allocated, or not!

## ki(1:mn,0:1): Fourier identification

· Consider the "abbreviated" representation for a double Fourier series,

$$\sum_{i} f_{i} \cos(m_{i}\theta - n_{i}\zeta) \equiv \sum_{n=0}^{N_{0}} f_{0,n} \cos(-n\zeta) + \sum_{m=1}^{M_{0}} \sum_{n=-N_{0}}^{N_{0}} f_{m,n} \cos(m\theta - n\zeta), \tag{220}$$

and the same representation but with enhanced resolution,

$$\sum_{k} \bar{f}_{k} \cos(\bar{m}_{k}\theta - \bar{n}_{k}\zeta) \equiv \sum_{n=0}^{N_{1}} f_{0,n} \cos(-n\zeta) + \sum_{m=1}^{M_{1}} \sum_{n=-N_{1}}^{N_{1}} f_{m,n} \cos(m\theta - n\zeta), \tag{221}$$

with  $M_1 \geq M_0$  and  $N_1 \geq N_0$ ; then  $k_i \equiv \text{ki}$  (i, 0) is defined such that  $\bar{m}_{k_i} = m_i$  and  $\bar{n}_{k_i} = n_i$ .

#### kija(1:mn,1:mn,0:1), kijs(1:mn,1:mn,0:1): Fourier identification

• Consider the following quantities, which are computed in ma00aa(), where  $\bar{g}^{\mu\nu} = \sum_k \bar{g}_k^{\mu\nu} \cos \alpha_k$  for  $\alpha_k \equiv m_k \theta - n_k \zeta$ ,

$$\oint \!\! \oint \! d\theta d\zeta \ \bar{g}^{\mu\nu} \cos \alpha_i \ \cos \alpha_j \ = \ \frac{1}{2} \oint \!\! \oint \! d\theta d\zeta \ \bar{g}^{\mu\nu} (+\cos \alpha_{k_{ij+}} + \cos \alpha_{k_{ij-}}), \tag{222}$$

$$\oint \!\! \oint d\theta d\zeta \ \bar{g}^{\mu\nu} \sin \alpha_i \cos \alpha_j = \frac{1}{2} \oint \!\! \oint d\theta d\zeta \ \bar{g}^{\mu\nu} (+\sin \alpha_{k_{ij+}} + \sin \alpha_{k_{ij-}}), \tag{224}$$

$$\oint \oint d\theta d\zeta \ \bar{g}^{\mu\nu} \sin \alpha_i \ \sin \alpha_j = \frac{1}{2} \oint \oint d\theta d\zeta \ \bar{g}^{\mu\nu} (-\cos \alpha_{k_{ij+}} + \cos \alpha_{k_{ij-}}), \tag{225}$$

where  $(m_{k_{ij+}},n_{k_{ij+}})=(m_i+m_j,n_i+n_j)$  and  $(m_{k_{ij-}},n_{k_{ij-}})=(m_i-m_j,n_i-n_j)$ ; then kija (i, j, 0)  $\equiv k_{ij+}$  and kijs (i, j, 0)  $\equiv k_{ij-}$ .

• Note that Eqn. (221) does not include m < 0; so, if  $m_i - m_j < 0$  then  $k_{ij-}$  is re-defined such that  $(m_{k_{ij-}}, n_{k_{ij-}}) = (m_j - m_i, n_j - n_i)$ ; and similarly for the case m = 0 and n < 0. Also, take care that the sign of the sine harmonics in the above expressions will change for these cases.

#### dikp

#### iotakki

#### cheby(0:Lrad,0:2): Chebyshev polynomial workspace

- cheby (0:Lrad, 0:2) is global workspace for computing the Chebyshev polynomials, and their derivatives, using the recurrence relations  $T_0(s) = 1$ ,  $T_1(s) = s$  and  $T_l(s) = 2sT_{l-1}(s) T_{l-2}(s)$ .
- These are computed as required, i.e. for arbitrary s, in bfield(), jo00aa() and ma00aa().
- Note that the quantities required for ma00aa() are for fixed s, and so these quantities should be precomputed.

## Iquad, gaussianweight, gaussianabscissae: Gauss-Legendre quadrature

- The volume integrals are computed using a "Fourier" integration over the angles and by Gauss-Legendre quadrature over the radial, i.e.  $\int f(s)ds = \sum_k \omega_k f(s_k)$ .
- The quadrature resolution in each volume is give by Iquad (1:Mvol) which is determined as follows:
  - if Nquad.gt.0, then Iquad(vvol)=Nquad
  - if Nquad.le.0 and .not.Lcoordinatesingularity, then Iquad(vvol)=2\*Lrad(vvol)-Nquad
  - if Nquad.le.0 and Lcoordinatesingularity , then Iquad(vvol)=2\*Lrad(vvol)-Nquad+Mpol
- The Gaussian weights and abscissae are given by gaussianweight (1:maxIquad, 1:Mvol) and gaussianabscissae (1:maxIquad, 1:Mvol), which are computed using modified Numerical Recipes routine gauleg().
- Iquad  $_v$  is passed through to ma00aa() to compute the volume integrals of the metric elements; also see jo00aa(), where Iquad  $_v$  is used to compute the volume integrals of  $||\nabla \times \mathbf{B} \mu \mathbf{B}||$ .

# LBsequad, LBnewton and LBlinear

• LBsequad, LBnewton and LBlinear depend simply on LBeltrami, which is described in global.f90

# BBweight(1:mn): weighting of force-imbalance harmonics

· weight on force-imbalance harmonics;

BBweight<sub>i</sub> 
$$\equiv$$
 opsilon  $\times \exp\left[-\text{escale} \times (m_i^2 + n_i^2)\right]$  (226)

· this is only used in dforce() in constructing the force-imbalance vector

# mmpp(1:mn): spectral condensation weight factors

· spectral condensation weight factors;

$$mmpp(i) \equiv m_i^p, \tag{227}$$

where  $p \equiv pcondense$ .

# NAdof, Ate, Aze, Ato and Azo: degrees-of-freedom in magnetic vector potential

- NAdof (1:Mvol) = total number of degrees-of-freedom in magnetic vector potential, including Lagrange multipliers, in each volume. This can de deduced from matrix().
- The components of the vector potential,  $\mathbf{A} = A_{\theta} \nabla + A_{\zeta} \nabla \zeta$ , are

$$A_{\theta}(s,\theta,\zeta) = \sum_{i,l} A_{\theta,e,i,l} \overline{T}_{l,i}(s) \cos \alpha_i + \sum_{i,l} A_{\theta,o,i,l} \overline{T}_{l,i}(s) \sin \alpha_i, \qquad (228)$$

$$A_{\zeta}(s,\theta,\zeta) = \sum_{i,l} A_{\zeta,e,i,l} \, \overline{T}_{l,i}(s) \cos \alpha_i + \sum_{i,l} A_{\zeta,o,i,l} \, \overline{T}_{l,i}(s) \sin \alpha_i, \tag{229}$$

where  $\overline{T}_{l,i}(s) \equiv \overline{s}^{m_i/2} T_l(s)$ ,  $T_l(s)$  is the Chebyshev polynomial, and  $\alpha_j \equiv m_j \theta - n_j \zeta$ . The regularity factor,  $\overline{s}^{m_i/2}$ , where  $\overline{s} \equiv (1+s)/2$ , is only included if there is a coordinate singularity in the domain (i.e. only in the innermost volume, and only in cylindrical and toroidal geometry.)

• The Chebyshev-Fourier harmonics of the covariant components of the magnetic vector potential are kept in

$$A_{\theta,e,i,l} \equiv \text{Ate}(v,0,j) \%s(1), \qquad (230)$$

$$A_{\zeta,e,i,l} \equiv \operatorname{Aze}(v,0,j) \%s(1), \qquad (231)$$

$$A_{\theta,o,i,l} \equiv \text{Ato}(v,0,j) \%s(1), \text{and}$$
 (232)

$$A_{\zeta,o,i,l} \equiv \text{Azo}(v,0,j)\%s(1); \tag{233}$$

where v=1, Mvol labels volume, j=1, mn labels Fourier harmonic, and l=0, Lrad(v) labels Chebyshev polynomial. (These arrays also contains derivative information.)

- If Linitguess=1, a guess for the initial state for the Beltrami fields is constructed. An initial state is required for iterative solvers of the Beltrami fields, see LBeltrami.
- If Linitguess=2, the initial state for the Beltrami fields is read from file (see ra00aa()). An initial state is required for iterative solvers of the Beltrami fields, see LBeltrami.

workspace

goomne, goomno: metric information These are defined in metrix(), and used in ma00aa(). gssmne, gssmno: metric information These are defined in metrix(), and used in ma00aa(). gstmne, gstmno: metric information These are defined in metrix(), and used in ma00aa(). gszmne, gszmno: metric information These are defined in metrix(), and used in ma00aa(). gttmne, gttmno: metric information These are defined in metrix(), and used in ma00aa(). gtzmne, gtzmno: metric information These are defined in metrix(), and used in ma00aa(). gzzmne, gzzmno: metric information These are defined in metrix(), and used in ma00aa(). cosi(1:Ntz,1:mn) and sini(1:Ntz,1:mn)

· Trigonometric factors used in various Fast Fourier transforms, where

$$\cos i_{j,i} = \cos(m_i \theta_j - n_i \zeta_j), \tag{234}$$

$$\sin_{i,i} = \sin(m_i \theta_i - n_i \zeta_i). \tag{235}$$

psifactor(1:mn,1:Mvol): coordinate "pre-conditioning" factor

· In toroidal geometry, the coordinate "pre-conditioning" factor is

$$f_{j,v} \equiv \begin{cases} \psi_{t,v}^0 &, & \text{for } m_j = 0, \\ \psi_{t,v}^{m_j/2} &, & \text{otherwise.} \end{cases}$$
 (236)

where  $\psi_{t,v} \equiv {\tt tflux}$  is the (normalized?) toroidal flux enclosed by the v-th interface.

- psifactor is used in packxi(), dforce() and hesian().
- · inifactor is similarly constructed, with

$$f_{j,v} \equiv \begin{cases} \psi_{t,v}^{1/2} &, & \text{for } m_j = 0, \\ \psi_{t,v}^{m_j/2} &, & \text{otherwise.} \end{cases}$$
 (237)

and used only for the initialization of the surfaces taking into account axis information if provided.

# **Bsupumn and Bsupvmn**

diotadxup and glambda: transformation to straight fieldline angle

- Given the Beltrami fields in any volume, the rotational-transform on the adjacent interfaces may be determined (in tr00ab()) by constructing the straight fieldline angle on the interfaces.
- The rotational transform on the inner or outer interface of a given volume depends on the magnetic field in that volume, i.e.  $\pm \pm (\mathbf{B}_{\pm})$ , so that

$$\delta \boldsymbol{t}_{\pm} = \frac{\partial \boldsymbol{t}_{\pm}}{\partial \mathbf{B}_{\pm}} \cdot \delta \mathbf{B}_{\pm}. \tag{238}$$

• The magnetic field depends on the Fourier harmonics of both the inner and outer interface geometry (represented here as  $x_j$ ), the helicity multiplier, and the enclosed poloidal flux, i.e.  $\mathbf{B}_{\pm} = \mathbf{B}_{\pm}(x_j, \mu, \Delta \psi_p)$ , so that

$$\delta \mathbf{B}_{\pm} = \frac{\partial \mathbf{B}_{\pm}}{\partial x_j} \delta x_j + \frac{\partial \mathbf{B}_{\pm}}{\partial \mu} \delta \mu + \frac{\partial \mathbf{B}_{\pm}}{\partial \Delta \psi_p} \delta \Delta \psi_p. \tag{239}$$

- The rotational-transforms, thus, can be considered to be functions of the geometry, the helicity-multiplier and the enclosed poloidal flux,  $\epsilon_{\pm} = \epsilon_{\pm}(x_j, \mu, \Delta \psi_p)$ .
- The rotational-transform, and its derivatives, on the inner and outer interfaces of each volume is stored in diotadxup(0:1,-1:2,1:Mvol). The indices label:
  - the first index labels the inner or outer interface,
  - the the second one labels derivative, with
    - \* -1 : indicating the derivative with respect to the interface geometry, i.e.  $\frac{\partial {m au}_{\pm}}{\partial x_{j}},$
    - $\star~$  0 : the rotational-transform itself,
    - \* 1,2 : the derivatives with respec to  $\mu$  and  $\Delta\psi_p$ , i.e.  $\frac{\partial\,\epsilon_{\,\pm}}{\partial\mu}$  and  $\frac{\partial\,\epsilon_{\,\pm}}{\partial\Delta\psi_p}$ ;
  - The third index labels volume.
- The values of diotadxup are assigned in mp00aa() after calling tr00ab().

### vvolume, IBBintegral and IABintegral

· volume integrals

vvolume(i) = 
$$\int_{\mathcal{V}_i} dv$$
 (240)

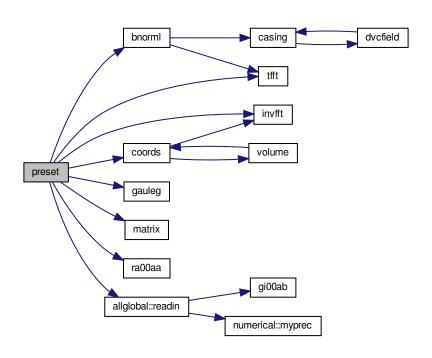
lBBintegral(i) = 
$$\int_{\mathcal{V}_i} \mathbf{B} \cdot \mathbf{B} \, dv$$
 (241)

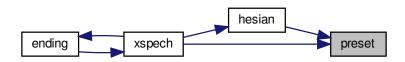
lABintegral(i) = 
$$\int_{\mathcal{V}} \mathbf{A} \cdot \mathbf{B} \, dv$$
 (242)

References allglobal::ate, allglobal::ato, allglobal::aze, allglobal::aze, allglobal::abe, allglobal::bbo, allglobal:-bbo, al ::bbweight, allglobal::bemn, allglobal::bloweremn, allglobal::bloweromn, bnorml(), allglobal::bomn, allglobal ::bsupumn, allglobal::bsupvmn, allglobal::btemn, allglobal::btomn, allglobal::bzemn, allglobal::bzomn, allglobal ::cfmn, allglobal::cheby, allglobal::comn, coords(), allglobal::cosi, fftw interface::cplxin, fftw interface::cplxin, allglobal::cpus, allglobal::diotadxup, allglobal::ditgpdxtp, allglobal::djkm, allglobal::djkp, allglobal::dpflux, allglobal ::dradr, allglobal::dradz, allglobal::drij, allglobal::drodr, allglobal::drodz, allglobal::dtflux, allglobal::dxyz, allglobal ::dzadr, allglobal::dzadz, allglobal::dzij, allglobal::dzodr, allglobal::dzodz, allglobal::efmn, inputlist::escale, allglobal::evmn, allglobal::fse, allglobal::fso, gauleg(), allglobal::gaussianabscissae, allglobal::gaussianweight, allglobal::glambda, allglobal::gmreslastsolution, allglobal::goomne, allglobal::goomno, allglobal::gssmne, allglobal ::gssmno, allglobal::gstmne, allglobal::gstmno, allglobal::gszmne, allglobal::gszmno, allglobal::gteta, allglobal← ::gttmne, allglobal::gttmno, allglobal::gtzmne, allglobal::gtzmno, allglobal::guvij, allglobal::gvuij, allglobal::gzeta, allglobal::gzzmne, allglobal::gzzmno, allglobal::halfmm, inputlist::helicity, allglobal::hnt, allglobal::hnz, allglobal ::iemn, inputlist::igeometry, allglobal::iie, allglobal::iio, allglobal::ijimag, allglobal::ijreal, allglobal::im, allglobal ::imagneticok, allglobal::ims, allglobal::in, allglobal::inifactor, allglobal::ins, invfft(), allglobal::iomn, inputlist ::iota. allqlobal::iotakadd. allqlobal::iotakkii. allqlobal::iotaksqn. allqlobal::iotaksub. allqlobal::iota ::ipdtdpf, allglobal::iquad, allglobal::irbc, allglobal::irbs, allglobal::irij, inputlist::ivolume, allglobal::izbc, allglobal ::izbs, allglobal::izij, allglobal::jiimag, allglobal::jireal, allglobal::jkimag, allglobal::jkreal, allglobal::jxyz, allglobal ::ki, allglobal::kija, allglobal::kijs, allglobal::kjimag, allglobal::kjreal, allglobal::labintegral, allglobal::lbbintegral, inputlist::lbeltrami, allglobal::lblinear, allglobal::lbnewton, allglobal::lbsequad, inputlist::lconstraint, allglobal ::lcoordinatesingularity, inputlist::lfindzero, inputlist::lfreebound, allglobal::lgdof, inputlist::lgmresprec, allglobal ← ::lhessianallocated, allglobal::liluprecond, inputlist::linitgues, inputlist::linitialize, allglobal::lma, inputlist::lmatsolver, allglobal::lmavalue, allglobal::lmb, allglobal::lmbvalue, allglobal::lmc, allglobal::lmcvalue, allglobal::lmd, allglobal:-lmd, allglobal:-lmcvalue, allglob ::Imdvalue, allglobal::Ime, allglobal::Imevalue, allglobal::Imf, allglobal::Imfvalue, allglobal::Img, allglobal::Imgvalue, allglobal::lmh, allglobal::lmhvalue, allglobal::lmns, allglobal::lmpol, allglobal::lntor, allglobal::localconstraint, inputlist::lp, inputlist::lq, inputlist::lrad, matrix(), inputlist::maxrndgues, allglobal::mmpp, allglobal::mne, allglobal ::mns, inputlist::mpol, inputlist::mu, constants::mu0, allglobal::mvol, allglobal::myid, allglobal::nadof, inputlist ← ::ndiscrete, allglobal::ndmas, allglobal::ndmasmax, allglobal::nfielddof, inputlist::nfp, allglobal::ngdof, allglobal ::notmatrixfree, allglobal::notstellsym, inputlist::nquad, allglobal::nt, inputlist::ntor, inputlist::nvol, allglobal::nxyz, allglobal::nz, allglobal::odmn, allglobal::ofmn, inputlist::oita, constants::one, inputlist::opsilon, fileunits::ounit, inputlist::pcondense, allglobal::pemn, inputlist::pflux, inputlist::phiedge, allglobal::pi2nfp, allglobal::pi2pi2nfp, allglobal::pi2pi2nfphalf, allglobal::pi2pi2nfpquart, inputlist::pl, fftw interface::planb, fftw interface::planf, allglobal ← ::pomn, inputlist::pr, allglobal::psifactor, inputlist::ql, inputlist::qr, ra00aa(), allglobal::readin(), allglobal::rij, inputlist. ::rp, inputlist::rq, allglobal::rscale, allglobal::rtm, allglobal::rtt, allglobal::semn, allglobal::sfmn, allglobal::sg, allglobal::simn, allglobal::sini, numerical::small, allglobal::smpol, allglobal::sntor, allglobal::somn, allglobal ← ::sontz, numerical::sgrtmachprec, allglobal::sweight, tfft(), inputlist::tflux, allglobal::trij, allglobal::tt, allglobal::tzij, inputlist::upsilon, numerical::vsmall, allglobal::vvolume, inputlist::wpoloidal, allglobal::yesstellsym, allglobal::zernike, constants::zero, and allglobal::zij.

Referenced by hesian(), and xspech().

Here is the call graph for this function:





# 8.18 Output file(s)

#### **Functions/Subroutines**

subroutine ra00aa (writeorread)
 Writes vector potential to .ext.sp.A .

### 8.18.1 Detailed Description

#### 8.18.2 Function/Subroutine Documentation

Writes vector potential to .ext.sp.A .

# representation of vector potential

• The components of the vector potential,  $\mathbf{A} = A_{\theta} \nabla + A_{\zeta} \nabla \zeta$ , are

$$A_{\theta}(s,\theta,\zeta) = \sum_{i,l} A_{\theta,e,i,l} \, \overline{T}_{l,i}(s) \cos \alpha_i + \sum_{i,l} A_{\theta,o,i,l} \, \overline{T}_{l,i}(s) \sin \alpha_i, \tag{243}$$

$$A_{\zeta}(s,\theta,\zeta) = \sum_{i,l} A_{\zeta,e,i,l} \, \overline{T}_{l,i}(s) \cos \alpha_i + \sum_{i,l} A_{\zeta,o,i,l} \, \overline{T}_{l,i}(s) \sin \alpha_i, \tag{244}$$

where  $\overline{T}_{l,i}(s) \equiv \overline{s}^{m_i/2} T_l(s)$ ,  $T_l(s)$  is the Chebyshev polynomial, and  $\alpha_j \equiv m_j \theta - n_j \zeta$ . The regularity factor,  $\overline{s}^{m_i/2}$ , where  $\overline{s} \equiv (1+s)/2$ , is only included if there is a coordinate singularity in the domain (i.e. only in the innermost volume, and only in cylindrical and toroidal geometry.)

# file format

• The format of the files containing the vector potential is as follows:

```
open(aunit, file="."/trim(ext)//".sp.A", status="replace", form="unformatted")
write(aunit) mvol, mpol, ntor, mn, nfp ! integers;
write(aunit) im(1:mn) ! integers; poloidal modes;
write(aunit) in(1:mn) ! integers; toroidal modes;
do vvol = 1, mvol ! integers; loop over volumes;
write(aunit) lrad(vvol) ! integers; the radial resolution in each volume may be different;
do ii = 1, mn
write(aunit) ate(vvol,ii)%s(0:lrad(vvol)) ! reals;
write(aunit) ate(vvol,ii)%s(0:lrad(vvol)) ! reals;
write(aunit) ato(vvol,ii)%s(0:lrad(vvol)) ! reals;
write(aunit) azo(vvol,ii)%s(0:lrad(vvol)) ! reals;
enddo ! end of do ii;
enddo ! end of do vvol;
close(aunit)
```

#### **Parameters**

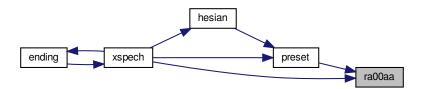
```
in writeorread 'W' to write the vector potential; 'R' to read it
```

References allglobal::ate, allglobal::ato, fileunits::aunit, allglobal::aze, allglobal::azo, allglobal::cpus, inputlist::ext,

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allglobal::im, allglobal::in, inputlist::lrad, inputlist::mpol, allglobal::mvol, allglobal::myid, allglobal::ncpu, inputlist::nfp, inputlist::ntor, fileunits::ounit, inputlist::wmacros, and constants::zero.

Referenced by preset(), and xspech().



# 8.19 Rotational Transform

#### **Functions/Subroutines**

subroutine tr00ab (Ivol, mn, NN, Nt, Nz, iflag, Idiota)
 Calculates rotational transform given an arbitrary tangential field.

#### 8.19.1 Detailed Description

### 8.19.2 Function/Subroutine Documentation

```
8.19.2.1 tr00ab() subroutine tr00ab (
    integer, intent(in) lvol,
    integer, intent(in) mn,
    integer, intent(in) NN,
    integer, intent(in) Nt,
    integer, intent(in) Nz,
    integer, intent(in) iflag,
    real, dimension(0:1,-1:2), intent(inout) ldiota )
```

Calculates rotational transform given an arbitrary tangential field.

Calculates transform,  $\iota = \dot{\theta}(1 + \lambda_{\theta}) + \lambda_{\zeta}$ , given  $\mathbf{B}|_{\mathcal{I}}$ .

# constructing straight field line angle on interfaces

• The algorithm stems from introducing a straight field line angle  $\theta_s = \theta + \lambda(\theta, \zeta)$ , where

$$\lambda = \sum_{j} \lambda_{o,j} \sin(m_j \theta - n_j \zeta) + \sum_{j} \lambda_{e,j} \cos(m_j \theta - n_j \zeta)$$
 (245)

and insisting that

$$\frac{\mathbf{B} \cdot \nabla \theta_s}{\mathbf{B} \cdot \nabla \zeta} = \dot{\theta} (1 + \lambda_{\theta}) + \lambda_{\zeta} = t, \tag{246}$$

where  $\pm$  is a constant that is to be determined.

• Writing  $\dot{\theta} = -\partial_s A_{\zeta}/\partial_s A_{\theta}$ , we have

$$\partial_s A_{\theta} + \partial_s A_{\zeta} \lambda_{\theta} - \partial_s A_{\theta} \lambda_{\zeta} = -\partial_s A_{\zeta} \tag{247}$$

· Expanding this equation we obtain

$$(A'_{\theta,e,k}\cos\alpha_k + A'_{\theta,o,k}\sin\alpha_k) +$$

$$+ (A'_{\zeta,e,k}\cos\alpha_k + A'_{\zeta,o,k}\sin\alpha_k) (+m_j\lambda_{o,j}\cos\alpha_j - m_j\lambda_{e,j}\sin\alpha_j)$$

$$- (A'_{\theta,e,k}\cos\alpha_k + A'_{\theta,o,k}\sin\alpha_k) (-n_j\lambda_{o,j}\cos\alpha_j + n_j\lambda_{e,j}\sin\alpha_j)$$

$$= - (A'_{\zeta,e,k}\cos\alpha_k + A'_{\zeta,o,k}\sin\alpha_k),$$
(248)

where summation over  $k=1, \min j=2, \min j$  is implied

· After applying double angle formulae,

$$(A'_{\theta,e,k}\cos\alpha_k + A'_{\theta,o,k}\sin\alpha_k) +$$

$$+ \lambda_{o,j} (+m_j A'_{\zeta,e,k} + n_j A'_{\theta,e,k}) [+\cos(\alpha_k + \alpha_j) + \cos(\alpha_k - \alpha_j)] / 2$$

$$+ \lambda_{e,j} (-m_j A'_{\zeta,e,k} - n_j A'_{\theta,e,k}) [+\sin(\alpha_k + \alpha_j) - \sin(\alpha_k - \alpha_j)] / 2$$

$$+ \lambda_{o,j} (+m_j A'_{\zeta,o,k} + n_j A'_{\theta,o,k}) [+\sin(\alpha_k + \alpha_j) + \sin(\alpha_k - \alpha_j)] / 2$$

$$+ \lambda_{e,j} (-m_j A'_{\zeta,o,k} - n_j A'_{\theta,o,k}) [-\cos(\alpha_k + \alpha_j) + \cos(\alpha_k - \alpha_j)] / 2$$

$$= - (A'_{\zeta,e,k}\cos\alpha_k + A'_{\zeta,o,k}\sin\alpha_k) ,$$
(249)

and equating coefficients, an equation of the form  $\mathbf{A} \cdot \mathbf{x} = \mathbf{b}$  is obtained, where

$$\mathbf{x} = (\underbrace{t}_{\mathbf{x}[1]}, \underbrace{\lambda_{o,2}, \lambda_{o,3}, \dots}_{\mathbf{x}[2:N]}, \underbrace{\lambda_{e,2}, \lambda_{e,3}, \dots}_{\mathbf{x}[N+1:2N-1]})^{T}.$$
(250)

#### alternative iterative method

• Consider the equation  $\dot{\theta}(1+\lambda_{\theta})+\lambda_{\zeta}=\pm$ , where  $\lambda=\sum_{j}\lambda_{j}\sin\alpha_{j}$ , given on a grid

$$\dot{\theta}_i + \dot{\theta}_i \sum_j m_j \cos \alpha_{i,j} \lambda_j - \sum_j n_j \cos \alpha_{i,j} \lambda_j = \pm, \tag{251}$$

where i labels the grid point.

· This is a matrix equation...

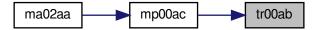
### **Parameters**

Ivol	
mn	
NN	
Nt	
Nz	
iflag	
Idiota	

References allglobal::cpus, allglobal::glambda, constants::goldenmean, constants::half, allglobal::im, inputlist::imethod, allglobal::ims, allglobal::in, allglobal::ins, invfft(), inputlist::iorder, inputlist::iotatol, inputlist::iprecon, inputlist::lrad, inputlist::lsparse, inputlist::lsvdiota, numerical::machprec, allglobal::mns, inputlist::mpol, allglobal-::mvol, allglobal::mvol, allglobal::ncpu, allglobal::notstellsym, inputlist::ntor, inputlist::nvol, constants::one, fileunits-::ounit, constants::pi2, allglobal::pi2nfp, numerical::small, numerical::sqrtmachprec, constants::third, constants::two, numerical::vsmall, inputlist::wmacros, allglobal::yesstellsym, and constants::zero.

Referenced by mp00ac().





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# 8.20 Plasma volume

### **Functions/Subroutines**

· subroutine volume (Ivol, vflag)

Computes volume of each region; and, if required, the derivatives of the volume with respect to the interface geometry.

### 8.20.1 Detailed Description

### 8.20.2 Function/Subroutine Documentation

```
8.20.2.1 volume() subroutine volume ( integer, intent(in) lvol, integer vflag)
```

Computes volume of each region; and, if required, the derivatives of the volume with respect to the interface geometry.

Calculates volume of each region;  $V_i \equiv \int dv$ .

## volume integral

• The volume enclosed by the v-th interface is given by the integral

$$V = \int_{\mathcal{V}} dv = \frac{1}{3} \int_{\mathcal{V}} \nabla \cdot \mathbf{x} \, dv = \frac{1}{3} \int_{\mathcal{S}} \mathbf{x} \cdot d\mathbf{s} = \frac{1}{3} \int_{0}^{2\pi} d\theta \int_{0}^{2\pi/N} d\zeta \quad \mathbf{x} \cdot \mathbf{x}_{\theta} \times \mathbf{x}_{\zeta}|^{s}$$
(252)

where we have used  $\nabla \cdot \mathbf{x} = 3$ , and have assumed that the domain is periodic in the angles.

# representation of surfaces

· The coordinate functions are

$$R(\theta,\zeta) = \sum_{i} R_{e,i} \cos \alpha_i + \sum_{i} R_{o,i} \sin \alpha_i$$
 (253)

$$Z(\theta,\zeta) = \sum_{i} Z_{e,i} \cos \alpha_i + \sum_{i} Z_{o,i} \sin \alpha_i, \qquad (254)$$

where  $\alpha_i \equiv m_i \theta - n_i \zeta$ .

#### geometry

- The geometry is controlled by the input parameter Igeometry as follows:
- Igeometry.eq.1: Cartesian:  $\sqrt{g}=R_s$

$$V = \int_0^{2\pi} d\theta \int_0^{2\pi/N} d\zeta R$$
$$= 2\pi \frac{2\pi}{N} R_{e,1}$$
 (255)

• Igeometry.eq.2: cylindrical:  $\sqrt{g}=RR_s=\frac{1}{2}\partial_s(R^2)$ 

$$V = \frac{1}{2} \int_{0}^{2\pi} d\theta \int_{0}^{2\pi/N} d\zeta R^{2}$$

$$= \frac{1}{2} 2\pi \frac{2\pi}{N} \frac{1}{2} \sum_{i} \sum_{j} R_{e,i} R_{e,j} \left[ \cos(\alpha_{i} - \alpha_{j}) + \cos(\alpha_{i} + \alpha_{j}) \right]$$

$$+ \frac{1}{2} 2\pi \frac{2\pi}{N} \frac{1}{2} \sum_{i} \sum_{j} R_{o,i} R_{o,j} \left[ \cos(\alpha_{i} - \alpha_{j}) - \cos(\alpha_{i} + \alpha_{j}) \right]$$
(256)

• Igeometry.eq.3: toroidal:  $\mathbf{x} \cdot \mathbf{e}_{\theta} \times \mathbf{e}_{\zeta} = R(ZR_{\theta} - RZ_{\theta})$  This is computed by fast Fourier transform:

$$V = \frac{1}{3} \int_{0}^{2\pi} d\theta \int_{0}^{2\pi/N} d\zeta \, R(ZR_{\theta} - RZ_{\theta})$$

$$= \frac{1}{3} \sum_{i} \sum_{j} \sum_{k} R_{e,i} (Z_{e,j}R_{o,k} - R_{e,j}Z_{o,k}) (+m_{k}) \iint d\theta d\zeta \cos \alpha_{i} \cos \alpha_{j} \cos \alpha_{k}$$

$$+ \frac{1}{3} \sum_{i} \sum_{j} \sum_{k} R_{e,i} (Z_{o,j}R_{e,k} - R_{o,j}Z_{e,k}) (-m_{k}) \iint d\theta d\zeta \cos \alpha_{i} \sin \alpha_{j} \sin \alpha_{k}$$

$$+ \frac{1}{3} \sum_{i} \sum_{j} \sum_{k} R_{o,i} (Z_{e,j}R_{e,k} - R_{e,j}Z_{e,k}) (-m_{k}) \iint d\theta d\zeta \sin \alpha_{i} \cos \alpha_{j} \sin \alpha_{k}$$

$$+ \frac{1}{3} \sum_{i} \sum_{j} \sum_{k} R_{o,i} (Z_{o,j}R_{o,k} - R_{o,j}Z_{o,k}) (+m_{k}) \iint d\theta d\zeta \sin \alpha_{i} \sin \alpha_{j} \cos \alpha_{k}$$

$$+ \frac{1}{3} \sum_{i} \sum_{j} \sum_{k} R_{o,i} (Z_{o,j}R_{o,k} - R_{o,j}Z_{o,k}) (+m_{k}) \iint d\theta d\zeta \sin \alpha_{i} \sin \alpha_{j} \cos \alpha_{k}$$

$$(257)$$

- (Recall that the integral over an odd function is zero, so various terms in the above expansion have been ignored.)
- · The trigonometric terms are

$$4 \cos \alpha_i \cos \alpha_j \cos \alpha_k = + \cos(\alpha_i + \alpha_j + \alpha_k) + \cos(\alpha_i + \alpha_j - \alpha_k) + \cos(\alpha_i - \alpha_j + \alpha_k) + \cos(\alpha_i - \alpha_j - \alpha_k)$$

$$4 \cos \alpha_i \sin \alpha_j \sin \alpha_k = - \cos(\alpha_i + \alpha_j + \alpha_k) + \cos(\alpha_i + \alpha_j - \alpha_k) + \cos(\alpha_i - \alpha_j + \alpha_k) - \cos(\alpha_i - \alpha_j - \alpha_k)$$

$$4 \sin \alpha_i \cos \alpha_j \sin \alpha_k = - \cos(\alpha_i + \alpha_j + \alpha_k) + \cos(\alpha_i + \alpha_j - \alpha_k) - \cos(\alpha_i - \alpha_j + \alpha_k) + \cos(\alpha_i - \alpha_j - \alpha_k)$$

$$4 \sin \alpha_i \sin \alpha_j \cos \alpha_k = - \cos(\alpha_i + \alpha_j + \alpha_k) - \cos(\alpha_i + \alpha_j - \alpha_k) + \cos(\alpha_i - \alpha_j + \alpha_k) + \cos(\alpha_i - \alpha_j - \alpha_k)$$

· The required derivatives are

$$3\frac{\partial V}{\partial R_{e,i}} = (+Z_{e,j}R_{o,k}m_k - R_{e,j}Z_{o,k}m_k - R_{e,j}Z_{o,k}m_k) \qquad \iint d\theta d\zeta \cos \alpha_i \cos \alpha_j \cos \alpha_k$$

$$+ (-Z_{o,j}R_{e,k}m_k + R_{o,j}Z_{e,k}m_k + R_{o,j}Z_{e,k}m_k) \qquad \iint d\theta d\zeta \cos \alpha_i \sin \alpha_j \sin \alpha_k$$

$$+ (-R_{o,k}Z_{e,j}m_i) \qquad \qquad \iint d\theta d\zeta \sin \alpha_i \cos \alpha_j \sin \alpha_k$$

$$+ (-R_{e,k}Z_{o,j}m_i) \qquad \qquad \iint d\theta d\zeta \sin \alpha_i \sin \alpha_j \cos \alpha_k$$

$$(259)$$

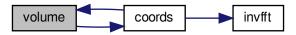
$$3\frac{\partial V}{\partial Z_{o,i}} = (-R_{e,k}R_{e,j}m_i) \quad \iint d\theta d\zeta \cos \alpha_i \cos \alpha_j \cos \alpha_k + (-R_{o,k}R_{o,j}m_i) \quad \iint d\theta d\zeta \cos \alpha_i \sin \alpha_j \sin \alpha_k + (-R_{e,j}R_{e,k}m_k) \quad \iint d\theta d\zeta \sin \alpha_i \cos \alpha_j \sin \alpha_k + (+R_{o,j}R_{o,k}m_k) \quad \iint d\theta d\zeta \sin \alpha_i \sin \alpha_j \cos \alpha_k$$
(260)

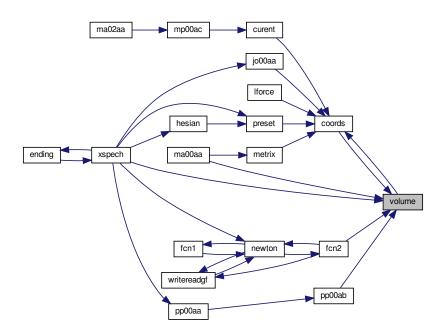
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References coords(), allglobal::cosi, allglobal::cpus, allglobal::dbdx, allglobal::djkm, allglobal::djkp, allglobal::djkp, allglobal::dvolume, constants::four, constants::half, inputlist::igeometry, allglobal::im, allglobal::in, allglobal::irbc, allglobal::irbc, allglobal::mvol, allglobal::myid, inputlist::nvol, constants::one, fileunits::ounit, constants::pi2, allglobal::pi2pi2nfp, allglobal::pi2pi2nfpquart, inputlist::pscale, constants::quart, allglobal::rij, allglobal::sini, numerical::small, constants::third, constants::two, numerical::vsmall, allglobal::vvolume, allglobal::yesstellsym, constants::zero, and allglobal::zij.

Referenced by coords(), fcn2(), ma00aa(), pp00ab(), and xspech().

Here is the call graph for this function:





# 8.21 Smooth boundary

### **Functions/Subroutines**

• subroutine wa00aa (iwa00aa)

Constructs smooth approximation to wall.

· subroutine vacuumphi (Nconstraints, rho, fvec, iflag)

Compute vacuum magnetic scalar potential (?)

### 8.21.1 Detailed Description

#### 8.21.2 Function/Subroutine Documentation

```
8.21.2.1 wa00aa() subroutine wa00aa ( integer iwa00aa )
```

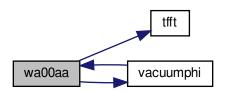
Constructs smooth approximation to wall.

### solution of Laplace's equation in two-dimensions

- The wall is given by a discrete set of points.
- · The points must go anti-clockwise.

References allglobal::cpus, fileunits::gunit, constants::half, allglobal::im, allglobal::in, allglobal::irbc, allglobal::irbc, allglobal::irbc, allglobal::irbc, allglobal::irbc, allglobal::irbc, allglobal::irbc, allglobal::mvol, allglobal::mvol, allglobal::mvol, allglobal::mvol, allglobal::nt, inputlist::ntor, inputlist::nvol, allglobal::nz, inputlist::odetol, constants::one, fileunits::ounit, constants::pi2, allglobal::rij, constants::ten, tfft(), vacuumphi(), numerical::vsmall, inputlist::wmacros, allglobal::yesstellsym, constants::zero, and allglobal::zij.

Referenced by vacuumphi().



Here is the caller graph for this function:



Compute vacuum magnetic scalar potential (?)

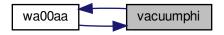
### **Parameters**

Nconstraints	
rho	
fvec	
iflag	

References allglobal::cpus, constants::half, allglobal::myid, allglobal::ncpu, constants::one, fileunits::ounit, constants::pi2, allglobal::rij, wa00aa(), inputlist::wmacros, constants::zero, and allglobal::zij.

Referenced by wa00aa().





8.22 physicslist 99

# 8.22 physicslist

The namelist physicslist controls the geometry, profiles, and numerical resolution.

Collaboration diagram for physicslist:

```
Input namelists and global variables physicslist
```

# **Variables**

```
• integer inputlist::igeometry = 3

selects Cartesian, cylindrical or toroidal geometry;
```

integer inputlist::istellsym = 1

stellarator symmetry is enforced if Istellsym==1

• integer inputlist::lfreebound = 0

compute vacuum field surrounding plasma

real inputlist::phiedge = 1.0

total enclosed toroidal magnetic flux;

• real inputlist::curtor = 0.0

total enclosed (toroidal) plasma current;

• real inputlist::curpol = 0.0

total enclosed (poloidal) linking current;

• real inputlist::gamma = 0.0

adiabatic index; cannot set  $|\gamma|=1$ 

integer inputlist::nfp = 1

field periodicity

• integer inputlist::nvol = 1

number of volumes

• integer inputlist::mpol = 0

number of poloidal Fourier harmonics

• integer inputlist::ntor = 0

number of toroidal Fourier harmonics

• integer, dimension(1:mnvol+1) inputlist::lrad = 4

Chebyshev resolution in each volume.

• integer inputlist::lconstraint = -1

selects constraints; primarily used in ma02aa() and mp00ac().

• real, dimension(1:mnvol+1) inputlist::tflux = 0.0

toroidal flux,  $\psi_t$ , enclosed by each interface

real, dimension(1:mnvol+1) inputlist::pflux = 0.0

poloidal flux,  $\psi_p$ , enclosed by each interface

real, dimension(1:mnvol) inputlist::helicity = 0.0

helicity, K, in each volume,  $V_i$ 

```
• real inputlist::pscale = 0.0
      pressure scale factor
• real, dimension(1:mnvol+1) inputlist::pressure = 0.0
      pressure in each volume
• integer inputlist::ladiabatic = 0
      logical flag

    real, dimension(1:mnvol+1) inputlist::adiabatic = 0.0

      adiabatic constants in each volume

    real, dimension(1:mnvol+1) inputlist::mu = 0.0

      helicity-multiplier, \mu, in each volume

    real, dimension(1:mnvol+1) inputlist::ivolume = 0.0

       Toroidal current constraint normalized by \mu_0 ( I_{volume} = \mu_0 \cdot [A]), in each volume. This is a cumulative quantity:
       I_{\mathcal{V},i} = \int_0^{\psi_{t,i}} \mathbf{J} \cdot \mathbf{dS}. Physically, it represents the sum of all non-pressure driven currents.

    real, dimension(1:mnvol) inputlist::isurf = 0.0

       Toroidal current normalized by \mu_0 at each interface (cumulative). This is the sum of all pressure driven currents.

    integer, dimension(0:mnvol) inputlist::pl = 0

       "inside" interface rotational-transform is \iota=(p_l+\gamma p_r)/(q_l+\gamma q_r), where \gamma is the golden mean, \gamma=(1+\sqrt{5})/2.

    integer, dimension(0:mnvol) inputlist::ql = 0

       "inside" interface rotational-transform is \iota=(p_l+\gamma p_r)/(q_l+\gamma q_r), where \gamma is the golden mean, \gamma=(1+\sqrt{5})/2.
• integer, dimension(0:mnvol) inputlist::pr = 0
       "inside" interface rotational-transform is \iota=(p_l+\gamma p_r)/(q_l+\gamma q_r), where \gamma is the golden mean, \gamma=(1+\sqrt{5})/2.
• integer, dimension(0:mnvol) inputlist::qr = 0
       "inside" interface rotational-transform is \iota=(p_l+\gamma p_r)/(q_l+\gamma q_r), where \gamma is the golden mean, \gamma=(1+\sqrt{5})/2.

    real, dimension(0:mnvol) inputlist::iota = 0.0

      rotational-transform, ε, on inner side of each interface
• integer, dimension(0:mnvol) inputlist::lp = 0
       "outer" interface rotational-transform is \iota=(p_l+\gamma p_r)/(q_l+\gamma q_r), where \gamma is the golden mean, \gamma=(1+\sqrt{5})/2.

    integer, dimension(0:mnvol) inputlist::lq = 0

       "outer" interface rotational-transform is \,\iota=(p_l+\gamma p_r)/(q_l+\gamma q_r), where \gamma is the golden mean, \gamma=(1+\sqrt{5})/2.
• integer, dimension(0:mnvol) inputlist::rp = 0
       "outer" interface rotational-transform is \,\iota=(p_l+\gamma p_r)/(q_l+\gamma q_r), where \gamma is the golden mean, \gamma=(1+\sqrt{5})/2.

    integer, dimension(0:mnvol) inputlist::rq = 0

       "outer" interface rotational-transform is \,\iota=(p_l+\gamma p_r)/(q_l+\gamma q_r), where \gamma is the golden mean, \gamma=(1+\sqrt{5})/2.
• real, dimension(0:mnvol) inputlist::oita = 0.0
      rotational-transform, ε, on outer side of each interface

 real inputlist::mupftol = 1.0e-14

      accuracy to which \mu and \Delta\psi_p are required
• integer inputlist::mupfits = 8
      an upper limit on the transform/helicity constraint iterations;
real inputlist::rpol = 1.0
      poloidal extent of slab (effective radius)
real inputlist::rtor = 1.0
      toroidal extent of slab (effective radius)
• integer inputlist::lreflect = 0
      =1 reflect the upper and lower bound in slab, =0 do not reflect

    real, dimension(0:mntor) inputlist::rac = 0.0

      stellarator symmetric coordinate axis;

    real, dimension(0:mntor) inputlist::zas = 0.0

      stellarator symmetric coordinate axis;

    real, dimension(0:mntor) inputlist::ras = 0.0
```

non-stellarator symmetric coordinate axis;

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- real, dimension( 0:mntor) inputlist::zac = 0.0
   non-stellarator symmetric coordinate axis;
- real, dimension(-mntor:mntor,-mmpol:mmpol) inputlist::rbc = 0.0 stellarator symmetric boundary components;
- real, dimension(-mntor:mntor,-mmpol:mmpol) inputlist::zbs = 0.0 stellarator symmetric boundary components;
- real, dimension(-mntor:mntor,-mmpol:mmpol) inputlist::rbs = 0.0
   non-stellarator symmetric boundary components;
- real, dimension(-mntor:mntor,-mmpol:mmpol) inputlist::zbc = 0.0 non-stellarator symmetric boundary components;
- real, dimension(-mntor:mntor,-mmpol:mmpol) inputlist::rwc = 0.0 stellarator symmetric boundary components of wall;
- real, dimension(-mntor:mntor,-mmpol:mmpol) inputlist::zws = 0.0 stellarator symmetric boundary components of wall;
- real, dimension(-mntor:mntor,-mmpol:mmpol) inputlist::rws = 0.0
   non-stellarator symmetric boundary components of wall;
- real, dimension(-mntor:mntor,-mmpol:mmpol) inputlist::zwc = 0.0
   non-stellarator symmetric boundary components of wall;
- real, dimension(-mntor:mntor,-mmpol:mmpol) inputlist::vns = 0.0 stellarator symmetric normal field at boundary; vacuum component;
- real, dimension(-mntor:mntor,-mmpol:mmpol) inputlist::bns = 0.0
   stellarator symmetric normal field at boundary; plasma component;
- real, dimension(-mntor:mntor,-mmpol:mmpol) inputlist::vnc = 0.0
   non-stellarator symmetric normal field at boundary; vacuum component;
- real, dimension(-mntor:mntor,-mmpol:mmpol) inputlist::bnc = 0.0
   non-stellarator symmetric normal field at boundary; plasma component;

## 8.22.1 Detailed Description

The namelist physicslist controls the geometry, profiles, and numerical resolution.

# 8.22.2 Variable Documentation

### **8.22.2.1** igeometry integer inputlist::igeometry = 3

selects Cartesian, cylindrical or toroidal geometry;

- Igeometry=1 : Cartesian; geometry determined by R;
- Igeometry=2: cylindrical; geometry determined by R;
- Igeometry=3 : toroidal; geometry determined by R and Z;

Referenced by bnorml(), coords(), dvcfield(), fcn1(), fcn2(), hesian(), jo00aa(), lforce(), newton(), packxi(), pc00ab(), pp00aa(), preset(), allglobal::readin(), stzxyz(), volume(), writereadgf(), allglobal::wrtend(), and xspech().

```
8.22.2.2 nfp integer inputlist::nfp = 1
```

field periodicity

- all Fourier representations are of the form  $\cos(m\theta nN\zeta)$ ,  $\sin(m\theta nN\zeta)$ , where  $N \equiv \text{Nfp}$
- constraint: Nfp >= 1

Referenced by invfft(), jo00aa(), preset(), ra00aa(), allglobal::readin(), tfft(), allglobal::wrtend(), and xspech().

**8.22.2.3 nvol** integer inputlist::nvol = 1

number of volumes

- each volume  $\mathcal{V}_l$  is bounded by the  $\mathcal{I}_{l-1}$  and  $\mathcal{I}_l$  interfaces
- note that in cylindrical or toroidal geometry,  $\mathcal{I}_0$  is the degenerate coordinate axis
- constraint: Nvol<=MNvol

Referenced by brcast(), df00ab(), dvcfield(), hesian(), jo00aa(), lforce(), packxi(), pc00ab(), pp00aa(), pp00ab(), preset(), allglobal::readin(), stzxyz(), tr00ab(), volume(), wa00aa(), writereadgf(), allglobal::wrtend(), and xspech().

**8.22.2.4 mpol** integer inputlist::mpol = 0

number of poloidal Fourier harmonics

· all Fourier representations of doubly-periodic functions are of the form

$$f(\theta,\zeta) = \sum_{n=0}^{\operatorname{Ntor}} f_{0,n} \cos(-n\operatorname{Nfp}\zeta) + \sum_{m=1}^{\operatorname{Mpol}} \sum_{n=-\operatorname{Ntor}}^{\operatorname{Ntor}} f_{m,n} \cos(m\theta - n\operatorname{Nfp}\zeta), \quad (261)$$

Internally these "double" summations are written as a "single" summation, e.g.  $f(\theta, \zeta) = \sum_j f_j \cos(m_j \theta - n_j \zeta)$ .

Referenced by bfield(), jo00aa(), ma00aa(), matrix(), preset(), ra00aa(), allglobal::readin(), tr00ab(), wa00aa(), writereadgf(), and allglobal::wrtend().

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**8.22.2.5 ntor** integer inputlist::ntor = 0

number of toroidal Fourier harmonics

· all Fourier representations of doubly-periodic functions are of the form

$$f(\theta,\zeta) = \sum_{n=0}^{\text{Ntor}} f_{0,n} \cos(-n \operatorname{Nfp} \zeta) + \sum_{m=1}^{\text{Mpol}} \sum_{n=-\text{Ntor}}^{\text{Ntor}} f_{m,n} \cos(m\theta - n \operatorname{Nfp} \zeta), \quad (262)$$

Internally these "double" summations are written as a "single" summation, e.g.  $f(\theta, \zeta) = \sum_j f_j \cos(m_j \theta - n_j \zeta)$ .

Referenced by coords(), mp00ac(), packxi(), preset(), ra00aa(), allglobal::readin(), stzxyz(), tr00ab(), wa00aa(), writereadgf(), and allglobal::wrtend().

**8.22.2.6** Irad integer, dimension(1:mnvol+1) inputlist::lrad = 4

Chebyshev resolution in each volume.

• constraint : Lrad(1:Mvol) >= 2

Referenced by bfield(), bnorml(), brcast(), curent(), dvcfield(), jo00aa(), lforce(), ma02aa(), mp00ac(), packab(), pp00aa(), preset(), ra00aa(), allglobal::readin(), tr00ab(), allglobal::wrtend(), and xspech().

#### **8.22.2.7 | Iconstraint** integer inputlist::lconstraint = -1

selects constraints; primarily used in ma02aa() and mp00ac().

- if Lconstraint==-1, then in the plasma regions  $\Delta \psi_t$ ,  $\mu$  and  $\Delta \psi_p$  are *not* varied and in the vacuum region (only for free-boundary)  $\Delta \psi_t$  and  $\Delta \psi_p$  are *not* varied, and  $\mu=0$ .
- if Lconstraint==0, then in the plasma regions  $\Delta\psi_t$ ,  $\mu$  and  $\Delta\psi_p$  are not varied and in the vacuum region (only for free-boundary)  $\Delta\psi_t$  and  $\Delta\psi_p$  are varied to match the prescribed plasma current, current, and the "linking" current, curpol, and  $\mu=0$
- if  ${\tt Lconstraint}==1$ , then in the plasma regions  $\mu$  and  $\Delta\psi_p$  are adjusted in order to satisfy the inner and outer interface transform constraints (except in the simple torus, where the enclosed poloidal flux is irrelevant, and only  $\mu$  is varied to satisfy the outer interface transform constraint); and in the vacuum region  $\Delta\psi_t$  and  $\Delta\psi_p$  are varied to match the transform constraint on the boundary and to obtain the prescribed linking current, curpol, and  $\mu=0$ .
- Todo if Lconstraint==2, under reconstruction.
- if Lconstraint.eq.3, then the  $\mu$  and  $\psi_p$  variables are adjusted in order to satisfy the volume and surface toroidal current computed with lbpol() (excepted in the inner most volume, where the volume current is irrelevant). Not implemented yet in free boundary.

Referenced by brcast(), ma02aa(), pp00aa(), preset(), allglobal::readin(), allglobal::wrtend(), and xspech().

```
8.22.2.8 tflux real, dimension(1:mnvol+1) inputlist::tflux = 0.0
```

toroidal flux,  $\psi_t$ , enclosed by each interface

- For each of the plasma volumes, this is a constraint: tflux is not varied
- For the vacuum region (only if Lfreebound==1), tflux may be allowed to vary to match constraints
- Note that tflux will be normalized so that tflux (Nvol) = 1.0, so that tflux is arbitrary up to a scale factor

See also

phiedge

Referenced by preset(), allglobal::readin(), allglobal::wrtend(), and xspech().

```
8.22.2.9 helicity real, dimension(1:mnvol) inputlist::helicity = 0.0
```

helicity,  $\mathcal{K}$ , in each volume,  $\mathcal{V}_i$ 

• on exit, helicity is set to the computed values of  $\mathcal{K} \equiv \int \mathbf{A} \cdot \mathbf{B} \ dv$ 

Referenced by brcast(), df00ab(), hesian(), ma02aa(), mp00ac(), preset(), allglobal::readin(), allglobal::wrtend(), and xspech().

```
8.22.2.10 pscale real inputlist::pscale = 0.0
```

pressure scale factor

• the initial pressure profile is given by pscale \* pressure

Referenced by Iforce(), allglobal::readin(), volume(), allglobal::wrtend(), and xspech().

```
8.22.2.11 pressure real, dimension(1:mnvol+1) inputlist::pressure = 0.0
```

pressure in each volume

- The pressure is *not* held constant, but  $p_l V_l^{\gamma} = P_l$  is held constant, where  $P_l$  is determined by the initial pressures and the initial volumes,  $V_l$ .
- Note that if gamma==0.0, then  $p_l \equiv P_l$ .
- On output, the pressure is given by  $p_l = P_l/V_l^{\gamma}$ , where  $V_l$  is the final volume.
- pressure is only used in calculation of interface force-balance.

Referenced by allglobal::readin(), allglobal::wrtend(), and xspech().

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```
8.22.2.12 ladiabatic integer inputlist::ladiabatic = 0
```

logical flag

- If Ladiabatic==0, the adiabatic constants are determined by the initial pressure and volume.
- If Ladiabatic==1, the adiabatic constants are determined by the given input adiabatic.

Referenced by allglobal::readin(), allglobal::wrtend(), and xspech().

**8.22.2.13 adiabatic** real, dimension(1:mnvol+1) inputlist::adiabatic = 0.0

adiabatic constants in each volume

- The pressure is *not* held constant, but  $p_l V_l^{\gamma} = P_l \equiv {\tt adiabatic}$  is constant.
- Note that if gamma==0.0, then pressure==adiabatic.
- pressure is only used in calculation of interface force-balance.

Referenced by Iforce(), allglobal::readin(), allglobal::wrtend(), and xspech().

**8.22.2.14 pl** integer, dimension(0:mnvol) inputlist::pl = 0

"inside" interface rotational-transform is  $\ \iota=(p_l+\gamma p_r)/(q_l+\gamma q_r)$ , where  $\gamma$  is the golden mean,  $\gamma=(1+\sqrt{5})/2$ .

If both  $q_l=0$  and  $q_r=0$ , then the (inside) interface rotational-transform is defined by  ${\tt iota}$  .

Referenced by preset(), allglobal::readin(), and allglobal::wrtend().

**8.22.2.15** ql integer, dimension(0:mnvol) inputlist::ql = 0

"inside" interface rotational-transform is  $\iota = (p_l + \gamma p_r)/(q_l + \gamma q_r)$ , where  $\gamma$  is the golden mean,  $\gamma = (1 + \sqrt{5})/2$ .

If both  $q_l=0$  and  $q_r=0$ , then the (inside) interface rotational-transform is defined by  ${\tt iota}$  .

Referenced by preset(), allglobal::readin(), and allglobal::wrtend().

**8.22.2.16 pr** integer, dimension(0:mnvol) inputlist::pr = 0

"inside" interface rotational-transform is  $\ \iota=(p_l+\gamma p_r)/(q_l+\gamma q_r)$ , where  $\gamma$  is the golden mean,  $\gamma=(1+\sqrt{5})/2$ .

If both  $q_l=0$  and  $q_r=0$ , then the (inside) interface rotational-transform is defined by  ${\tt iota}$  .

Referenced by preset(), allglobal::readin(), and allglobal::wrtend().

**8.22.2.17 qr** integer, dimension(0:mnvol) inputlist::qr = 0

"inside" interface rotational-transform is  $\ \iota=(p_l+\gamma p_r)/(q_l+\gamma q_r)$ , where  $\gamma$  is the golden mean,  $\gamma=(1+\sqrt{5})/2$ .

If both  $q_l=0$  and  $q_r=0$ , then the (inside) interface rotational-transform is defined by  ${\tt iota}$  .

Referenced by preset(), allglobal::readin(), and allglobal::wrtend().

**8.22.2.18** iota real, dimension(0:mnvol) inputlist::iota = 0.0

rotational-transform, t, on inner side of each interface

• only relevant if illogical input for ql and qr are provided

Referenced by mp00ac(), pp00aa(), preset(), allglobal::readin(), and allglobal::wrtend().

**8.22.2.19** Ip integer, dimension(0:mnvol) inputlist::lp = 0

"outer" interface rotational-transform is  $t=(p_l+\gamma p_r)/(q_l+\gamma q_r)$ , where  $\gamma$  is the golden mean,  $\gamma=(1+\sqrt{5})/2$ .

If both  $q_l=0$  and  $q_r=0$ , then the (outer) interface rotational-transform is defined by oita .

Referenced by preset(), allglobal::readin(), and allglobal::wrtend().

**8.22.2.20** Iq integer, dimension(0:mnvol) inputlist::lq = 0

"outer" interface rotational-transform is  $t = (p_l + \gamma p_r)/(q_l + \gamma q_r)$ , where  $\gamma$  is the golden mean,  $\gamma = (1 + \sqrt{5})/2$ .

If both  $q_l=0$  and  $q_r=0$ , then the (outer) interface rotational-transform is defined by oita.

Referenced by preset(), allglobal::readin(), and allglobal::wrtend().

**8.22.2.21 rp** integer, dimension(0:mnvol) inputlist::rp = 0

"outer" interface rotational-transform is  $t=(p_l+\gamma p_r)/(q_l+\gamma q_r)$ , where  $\gamma$  is the golden mean,  $\gamma=(1+\sqrt{5})/2$ .

If both  $q_l=0$  and  $q_r=0$ , then the (outer) interface rotational-transform is defined by oita .

Referenced by preset(), allglobal::readin(), and allglobal::wrtend().

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```
8.22.2.22 rq integer, dimension(0:mnvol) inputlist::rq = 0
```

"outer" interface rotational-transform is  $t=(p_l+\gamma p_r)/(q_l+\gamma q_r)$ , where  $\gamma$  is the golden mean,  $\gamma=(1+\sqrt{5})/2$ . If both  $q_l=0$  and  $q_r=0$ , then the (outer) interface rotational-transform is defined by oita.

Referenced by preset(), allglobal::readin(), and allglobal::wrtend().

```
8.22.2.23 oita real, dimension(0:mnvol) inputlist::oita = 0.0
```

rotational-transform, t, on outer side of each interface

only relevant if illogical input for ql and qr are provided

Referenced by mp00ac(), pp00aa(), preset(), allglobal::readin(), and allglobal::wrtend().

```
8.22.2.24 mupftol real inputlist::mupftol = 1.0e-14
```

accuracy to which  $\mu$  and  $\Delta\psi_p$  are required

only relevant if constraints on transform, enclosed currents etc. are to be satisfied iteratively, see
 Lconstraint

Referenced by ma02aa(), allglobal::readin(), and allglobal::wrtend().

### **8.22.2.25** mupfits integer inputlist::mupfits = 8

an upper limit on the transform/helicity constraint iterations;

- only relevant if constraints on transform, enclosed currents etc. are to be satisfied iteratively, see Lconstraint
- constraint: mupfits > 0

Referenced by ma02aa(), allglobal::readin(), and allglobal::wrtend().

```
8.22.2.26 rpol real inputlist::rpol = 1.0
```

poloidal extent of slab (effective radius)

- only relevant if Igeometry==1
- poloidal size is  $L=2\pi*{\tt rpol}$

Referenced by coords(), allglobal::readin(), and allglobal::wrtend().

#### **8.22.2.27 rtor** real inputlist::rtor = 1.0

toroidal extent of slab (effective radius)

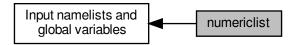
- only relevant if Igeometry==1
- toroidal size is  $L=2\pi*{\tt rtor}$

Referenced by coords(), allglobal::readin(), and allglobal::wrtend().

### 8.23 numericlist

The namelist numericlist controls internal resolution parameters that the user rarely needs to consider.

Collaboration diagram for numericlist:



#### **Variables**

• integer inputlist::linitialize = 0

Used to initialize geometry using a regularization / extrapolation method.

• integer inputlist::lautoinitbn = 1

Used to initialize  $B_{ns}$  using an initial fixed-boundary calculation.

• integer inputlist::lzerovac = 0

Used to adjust vacuum field to cancel plasma field on computational boundary.

• integer inputlist::ndiscrete = 2

resolution of the real space grid on which fast Fourier transforms are performed is given by Ndiscrete\*Mpol\*4

integer inputlist::nquad = -1

Resolution of the Gaussian quadrature.

• integer inputlist::impol = -4

Fourier resolution of straight-fieldline angle on interfaces.

integer inputlist::intor = -4

Fourier resolution of straight-fieldline angle on interfaces;.

• integer inputlist::lsparse = 0

controls method used to solve for rotational-transform on interfaces

• integer inputlist::lsvdiota = 0

controls method used to solve for rotational-transform on interfaces; only relevant if Lsparse = 0

• integer inputlist::imethod = 3

controls iterative solution to sparse matrix arising in real-space transformation to the straight-fieldline angle; only relevant if Lsparse.eq.2;

• integer inputlist::iorder = 2

controls real-space grid resolution for constructing the straight-fieldline angle; only relevant if Lsparse>0

integer inputlist::iprecon = 0

controls iterative solution to sparse matrix arising in real-space transformation to the straight-fieldline angle; only relevant if Lsparse.eq.2;

• real inputlist::iotatol = -1.0

tolerance required for iterative construction of straight-fieldline angle; only relevant if Lsparse.ge.2

integer inputlist::lextrap = 0

geometry of innermost interface is defined by extrapolation

• integer inputlist::mregular = -1

maximum regularization factor

• integer inputlist::lrzaxis = 1

controls the guess of geometry axis in the innermost volume or initialization of interfaces

integer inputlist::ntoraxis = 3

the number of n harmonics used in the Jacobian m=1 harmonic elimination method; only relevant if  $Lrzaxis. \leftarrow ge.1$ .

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## 8.23.1 Detailed Description

The namelist numericlist controls internal resolution parameters that the user rarely needs to consider.

#### 8.23.2 Variable Documentation

```
8.23.2.1 linitialize integer inputlist::linitialize = 0
```

Used to initialize geometry using a regularization / extrapolation method.

- if Linitialize = -I, where I is a positive integer, the geometry of the  $i=1,N_V-I$  surfaces constructed by an extrapolation
- if Linitialize = 0, the geometry of the interior surfaces is provided after the namelists in the input file
- if Linitialize = 1, the interior surfaces will be intialized as  $R_{l,m,n} = R_{N,m,n} \psi_{t,l}^{m/2}$ , where  $R_{N,m,n}$  is the plasma boundary and  $\psi_{t,l}$  is the given toroidal flux enclosed by the l-th interface, normalized to the total enclosed toroidal flux; a similar extrapolation is used for  $Z_{l,m,n}$
- Note that the Fourier harmonics of the boundary is always given by the Rbc and Zbs given in physicslist.
- if Linitialize = 2, the interior surfaces and the plasma boundary will be intialized as  $R_{l,m,n} = R_{W,m,n}\psi_{t,l}^{m/2}$ , where  $R_{W,m,n}$  is the computational boundary and  $\psi_{t,l}$  is the given toroidal flux enclosed by the l-th interface, normalized to the total enclosed toroidal flux; a similar extrapolation is used for  $Z_{l,m,n}$
- Note that, for free-boundary calculations, the Fourier harmonics of the computational boundary are *always* given by the Rwc and Zws given in physicslist.
- if Linitialize = 1, 2, it is not required to provide the geometry of the interfaces after the namelists

Referenced by preset(), allglobal::readin(), and allglobal::wrtend().

```
8.23.2.2 lautoinitbn integer inputlist::lautoinitbn = 1
```

Used to initialize  $B_{ns}$  using an initial fixed-boundary calculation.

- only relevant if Lfreebound = 1
- user-supplied Bns will only be considered if LautoinitBn = 0

Referenced by allglobal::readin(), allglobal::wrtend(), and xspech().

```
8.23.2.3 | Izerovac integer inputlist::lzerovac = 0
```

Used to adjust vacuum field to cancel plasma field on computational boundary.

• only relevant if Lfreebound = 1

Referenced by allglobal::readin(), allglobal::wrtend(), and xspech().

```
8.23.2.4 ndiscrete integer inputlist::ndiscrete = 2
```

resolution of the real space grid on which fast Fourier transforms are performed is given by Ndiscrete\*Mpol\*4

• constraint Ndiscrete>0

Referenced by preset(), allglobal::readin(), and allglobal::wrtend().

```
8.23.2.5 nquad integer inputlist::nquad = -1
```

Resolution of the Gaussian quadrature.

- The resolution of the Gaussian quadrature,  $\int\!\!f(s)ds=\sum_k\omega_kf(s_k)$ , in each volume is given by  $\operatorname{Iquad}_v$ ,
- Iquad v is set in preset()

Referenced by preset(), allglobal::readin(), and allglobal::wrtend().

```
8.23.2.6 impol integer inputlist::impol = -4
```

Fourier resolution of straight-fieldline angle on interfaces.

- the rotational-transform on the interfaces is determined by a transformation to the straight-fieldline angle, with poloidal resolution given by iMpol
- if iMpol<=0, then iMpol = Mpol iMpol

Referenced by allglobal::readin(), and allglobal::wrtend().

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```
8.23.2.7 intor integer inputlist::intor = -4
```

Fourier resolution of straight-fieldline angle on interfaces;.

• the rotational-transform on the interfaces is determined by a transformation to the straight-fieldline angle, with toroidal resolution given by iNtor

```
• if iNtor<=0 then iNtor = Ntor - iNtor
```

• if Ntor==0, then the toroidal resolution of the angle transformation is set lNtor=0

Referenced by allglobal::readin(), and allglobal::wrtend().

```
8.23.2.8 Isparse integer inputlist::lsparse = 0
```

controls method used to solve for rotational-transform on interfaces

- if Lsparse = 0, the transformation to the straight-fieldline angle is computed in Fourier space using a dense matrix solver, F04AAF
- if Lsparse = 1, the transformation to the straight-fieldline angle is computed in real space using a dense matrix solver, F04ATF
- if Lsparse = 2, the transformation to the straight-fieldline angle is computed in real space using a sparse matrix solver, F11DEF
- if Lsparse = 3, the different methods for constructing the straight-fieldline angle are compared

Referenced by allglobal::readin(), tr00ab(), and allglobal::wrtend().

```
8.23.2.9 Isvdiota integer inputlist::lsvdiota = 0
```

controls method used to solve for rotational-transform on interfaces; only relevant if  ${\tt Lsparse} = 0$ 

- if Lsvdiota = 0, use standard linear solver to construct straight fieldline angle transformation
- if Lsvdiota = 1, use SVD method to compute rotational-transform

Referenced by allglobal::readin(), tr00ab(), and allglobal::wrtend().

```
8.23.2.10 imethod integer inputlist::imethod = 3
```

controls iterative solution to sparse matrix arising in real-space transformation to the straight-fieldline angle; only relevant if Lsparse.eq.2;

See also

### tr00ab() for details

- if imethod = 1, the method is RGMRES
- if imethod = 2, the method is CGS
- if imethod = 3, the method is BICGSTAB

Referenced by allglobal::readin(), tr00ab(), and allglobal::wrtend().

```
8.23.2.11 iorder integer inputlist::iorder = 2
```

controls real-space grid resolution for constructing the straight-fieldline angle; only relevant if Lsparse>0

determines order of finite-difference approximation to the derivatives

- if iorder = 2,
- if iorder = 4,
- if iorder = 6,

Referenced by allglobal::readin(), tr00ab(), and allglobal::wrtend().

```
8.23.2.12 iprecon integer inputlist::iprecon = 0
```

controls iterative solution to sparse matrix arising in real-space transformation to the straight-fieldline angle; only relevant if Lsparse.eq.2;

See also

# tr00ab() for details

- if iprecon = 0, the preconditioner is 'N'
- if iprecon = 1, the preconditioner is 'J'
- if iprecon = 2, the preconditioner is 'S'

Referenced by allglobal::readin(), tr00ab(), and allglobal::wrtend().

8.23 numericlist 113

```
8.23.2.13 mregular integer inputlist::mregular = -1
```

maximum regularization factor

• if Mregular.ge.2, then regumm  $_i$  = Mregular /2 where m  $_i$  > Mregular

Referenced by allglobal::readin(), and allglobal::wrtend().

```
8.23.2.14 Irzaxis integer inputlist::lrzaxis = 1
```

controls the guess of geometry axis in the innermost volume or initialization of interfaces

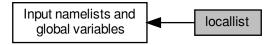
- if iprecon = 1, the centroid is used
- if iprecon = 2, the Jacobian m = 1 harmonic elimination method is used

Referenced by allglobal::readin(), and allglobal::wrtend().

# 8.24 locallist

The namelist locallist controls the construction of the Beltrami fields in each volume.

Collaboration diagram for locallist:



### **Variables**

• integer inputlist::lbeltrami = 4

Control flag for solution of Beltrami equation.

• integer inputlist::linitgues = 1

controls how initial guess for Beltrami field is constructed

• integer inputlist::lposdef = 0

redundant;

• real inputlist::maxrndgues = 1.0

the maximum random number of the Beltrami field if Linitgues = 3

• integer inputlist::lmatsolver = 3

1 for LU factorization, 2 for GMRES, 3 for GMRES matrix-free

• integer inputlist::nitergmres = 200

number of max iteration for GMRES

• real inputlist::epsgmres = 1e-14

the precision of GMRES

• integer inputlist::lgmresprec = 1

type of preconditioner for GMRES, 1 for ILU sparse matrix

• real inputlist::epsilu = 1e-12

the precision of incomplete LU factorization for preconditioning

### 8.24.1 Detailed Description

The namelist locallist controls the construction of the Beltrami fields in each volume.

The transformation to straight-fieldline coordinates is singular when the rotational-transform of the interfaces is rational; however, the rotational-transform is still well defined.

# 8.24.2 Variable Documentation

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#### **8.24.2.1 | Ibeltrami** integer inputlist::lbeltrami = 4

Control flag for solution of Beltrami equation.

• if LBeltrami = 1,3,5 or 7, (SQP) then the Beltrami field in each volume is constructed by minimizing the magnetic energy with the constraint of fixed helicity; this is achieved by using sequential quadratic programming as provided by E04UFF. This approach has the benefit (in theory) of robustly constructing minimum energy solutions when multiple, i.e. bifurcated, solutions exist.

- if LBeltrami = 2,3,6 or 7, (Newton) then the Beltrami fields are constructed by employing a standard Newton method for locating an extremum of  $F \equiv \int B^2 dv \mu (\int \mathbf{A} \cdot \mathbf{B} dv \mathcal{K})$ , where  $\mu$  is treated as an independent degree of freedom similar to the parameters describing the vector potential and  $\mathcal{K}$  is the required value of the helicity; this is the standard Lagrange multipler approach for locating the constrained minimum; this method cannot distinguish saddle-type extrema from minima, and which solution that will be obtained depends on the initial guess;
- if LBeltrami = 4,5,6 or 7, (linear) it is assumed that the Beltrami fields are parameterized by  $\mu$ ; in this case, it is only required to solve  $\nabla \times \mathbf{B} = \mu \mathbf{B}$  which reduces to a system of linear equations;  $\mu$  may or may not be adjusted iteratively, depending on Lconstraint, to satisfy either rotational-transform or helicity constraints;
- for flexibility and comparison, each of the above methods can be employed; for example:

```
- if LBeltrami = 1, only the SQP method will be employed;
```

- if LBeltrami = 2, only the Newton method will be employed;
- if LBeltrami = 4, only the linear method will be employed;
- if LBeltrami = 3, the SQP and the Newton method are used;
- if LBeltrami = 5, the SQP and the linear method are used;
- if LBeltrami = 6, the Newton and the linear method are used;
- if LBeltrami = 7, all three methods will be employed;

Referenced by preset(), allglobal::readin(), and allglobal::wrtend().

## **8.24.2.2 linitgues** integer inputlist::linitgues = 1

controls how initial guess for Beltrami field is constructed

- only relevant for routines that require an initial guess for the Beltrami fields, such as the SQP and Newton methods, or the sparse linear solver;
- if Linitgues = 0, the initial guess for the Beltrami field is trivial
- if Linitgues = 1, the initial guess for the Beltrami field is an integrable approximation
- if Linitgues = 2, the initial guess for the Beltrami field is read from file
- if Linitques = 3, the initial guess for the Beltrami field will be randomized with the maximum maxrndques

Referenced by preset(), allglobal::readin(), and allglobal::wrtend().

# 8.25 globallist

The namelist globallist controls the search for global force-balance.

Collaboration diagram for globallist:

```
Input namelists and global variables globallist
```

### **Variables**

```
    integer inputlist::lfindzero = 0
    use Newton methods to find zero of force-balance, which is computed by dforce()
```

• real inputlist::escale = 0.0

controls the weight factor, BBweight, in the force-imbalance harmonics

• real inputlist::opsilon = 1.0

weighting of force-imbalance

real inputlist::pcondense = 2.0

spectral condensation parameter

• real inputlist::epsilon = 0.0

weighting of spectral-width constraint

real inputlist::wpoloidal = 1.0

"star-like" poloidal angle constraint radial exponential factor used in preset() to construct sweight

• real inputlist::upsilon = 1.0

weighting of "star-like" poloidal angle constraint used in preset() to construct sweight

• real inputlist::forcetol = 1.0e-10

required tolerance in force-balance error; only used as an initial check

• real inputlist::c05xmax = 1.0e-06

required tolerance in position,  $\mathbf{x} \equiv \{R_{i,v}, Z_{i,v}\}$ 

• real inputlist::c05xtol = 1.0e-12

required tolerance in position,  $\mathbf{x} \equiv \{R_{i,v}, Z_{i,v}\}$ 

• real inputlist::c05factor = 1.0e-02

used to control initial step size in C05NDF and C05PDF

• logical inputlist::lreadgf = .true.

read  $\nabla_{\mathbf{x}}\mathbf{F}$  from file <code>ext.GF</code>

• integer inputlist::mfreeits = 0

maximum allowed free-boundary iterations

real inputlist::bnstol = 1.0e-06

redundant;

real inputlist::bnsblend = 0.666

redundant;

• real inputlist::gbntol = 1.0e-06

required tolerance in free-boundary iterations

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- real inputlist::gbnbld = 0.666
  - normal blend
- real inputlist::vcasingeps = 1.e-12

regularization of Biot-Savart; see bnorml(), casing()

• real inputlist::vcasingtol = 1.e-08

accuracy on virtual casing integral; see bnorml(), casing()

• integer inputlist::vcasingits = 8

minimum number of calls to adaptive virtual casing routine; see casing()

• integer inputlist::vcasingper = 1

periods of integragion in adaptive virtual casing routine; see casing()

integer inputlist::mcasingcal = 8

minimum number of calls to adaptive virtual casing routine; see casing(); redundant;

## 8.25.1 Detailed Description

The namelist globallist controls the search for global force-balance.

#### Comments:

• The "force" vector, **F**, which is constructed in dforce(), is a combination of pressure-imbalance Fourier harmonics,

$$F_{i,v} \equiv [[p+B^2/2]]_{i,v} \times \exp\left[-\operatorname{escale}(m_i^2 + n_i^2)\right] \times \operatorname{opsilon}, \tag{263}$$

and spectral-condensation constraints,  $I_{i,v}$ , and the "star-like" angle constraints,  $S_{i,v}$ , (see Iforce() for details)

$$F_{i,v} \equiv \operatorname{epsilon} \times I_{i,v} + \operatorname{upsilon} \times \left( \psi_v^{\omega} S_{i,v,1} - \psi_{v+1}^{\omega} S_{i,v+1,0} \right), \tag{264}$$

where  $\psi_v \equiv$  normalized toroidal flux, tflux, and  $\omega \equiv$  wpoloidal.

#### 8.25.2 Variable Documentation

# **8.25.2.1 Ifindzero** integer inputlist::lfindzero = 0

use Newton methods to find zero of force-balance, which is computed by dforce()

- if Lfindzero = 0, then dforce() is called once to compute the Beltrami fields consistent with the given geometry and constraints
- if Lfindzero = 1, then call C05NDF (uses function values only), which iteratively calls dforce()
- if Lfindzero = 2, then call CO5PDF (uses derivative information), which iteratively calls dforce()

Referenced by brcast(), hesian(), packxi(), preset(), allglobal::readin(), allglobal::wrtend(), and xspech().

```
8.25.2.2 escale real inputlist::escale = 0.0
```

controls the weight factor, BBweight, in the force-imbalance harmonics

- BBweight(i)  $\equiv$  opsilon  $\times \exp\left[-\mathrm{escale} \times (m_i^2 + n_i^2)\right]$
- defined in preset(); used in dforce()
- also see Eqn. (263)

Referenced by preset(), allglobal::readin(), and allglobal::wrtend().

```
8.25.2.3 opsilon real inputlist::opsilon = 1.0
```

weighting of force-imbalance

• used in dforce(); also see Eqn. (263)

Referenced by preset(), allglobal::readin(), and allglobal::wrtend().

## **8.25.2.4 pcondense** real inputlist::pcondense = 2.0

spectral condensation parameter

- used in preset() to define mmpp (i)  $\equiv m_i^p$ , where  $p \equiv {\tt pcondense}$
- the angle freedom is exploited to minimize  $\operatorname{epsilon} \sum_i m_i^p (R_i^2 + Z_i^2)$  with respect to tangential variations in the interface geometry
- also see Eqn. (264)

Referenced by preset(), allglobal::readin(), and allglobal::wrtend().

```
8.25.2.5 epsilon real inputlist::epsilon = 0.0
```

weighting of spectral-width constraint

• used in dforce(); also see Eqn. (264)

Referenced by pc00ab(), allglobal::readin(), and allglobal::wrtend().

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```
8.25.2.6 forcetol real inputlist::forcetol = 1.0e-10
```

required tolerance in force-balance error; only used as an initial check

 if the initially supplied interfaces are consistent with force-balance to within forcetol then the geometry of the interfaces is not altered

- if not, then the geometry of the interfaces is changed in order to bring the configuration into force balance so that the geometry of interfaces is within c05xtol, defined below, of the true solution
- to force execution of either C05NDF or C05PDF, regardless of the initial force imbalance, set forcetol <</li>

Referenced by pc00aa(), pc00ab(), allglobal::readin(), and allglobal::wrtend().

```
8.25.2.7 c05xtol real inputlist::c05xtol = 1.0e-12
```

required tolerance in position,  $\mathbf{x} \equiv \{R_{i,v}, Z_{i,v}\}$ 

- used by both C05NDF and C05PDF; see the NAG documents for further details on how the error is defined
- constraint c05xtol > 0.0

Referenced by allglobal::readin(), and allglobal::wrtend().

```
8.25.2.8 cO5factor real inputlist::c05factor = 1.0e-02
```

used to control initial step size in C05NDF and C05PDF

- constraint c05factor > 0.0
- only relevant if Lfindzero > 0

Referenced by allglobal::readin(), and allglobal::wrtend().

```
\textbf{8.25.2.9} \quad \textbf{lreadgf} \quad \texttt{logical inputlist::} \texttt{lreadgf = .true.}
```

 $\text{read } \nabla_{\mathbf{x}} F \text{ from file } \text{ext.GF}$ 

- only used if Lfindzero = 2
- only used in newton()

Referenced by allglobal::readin(), and allglobal::wrtend().

```
8.25.2.10 mfreeits integer inputlist::mfreeits = 0
```

maximum allowed free-boundary iterations

- only used if Lfreebound = 1
- only used in xspech()

Referenced by allglobal::readin(), allglobal::wrtend(), and xspech().

```
8.25.2.11 gbntol real inputlist::gbntol = 1.0e-06
```

required tolerance in free-boundary iterations

- only used if Lfreebound = 1
- only used in xspech()

Referenced by allglobal::readin(), allglobal::wrtend(), and xspech().

normal blend

• The "new" magnetic field at the computational boundary produced by the plasma currents is updated using a Picard scheme:

$$(\mathbf{B} \cdot \mathbf{n})^{j+1} = gBnbld \times (\mathbf{B} \cdot \mathbf{n})^j + (1 - gBnbld) \times (\mathbf{B} \cdot \mathbf{n})^*,$$
(265)

where j labels free-boundary iterations, and  $({f B}\cdot{f n})^*$  is computed by virtual casing.

- only used if Lfreebound = 1
- only used in xspech()

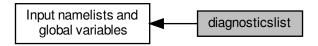
Referenced by allglobal::readin(), allglobal::wrtend(), and xspech().

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# 8.26 diagnosticslist

The namelist diagnosticslist controls post-processor diagnostics, such as Poincaré plot resolution, etc.

Collaboration diagram for diagnosticslist:



#### **Variables**

```
• real inputlist::odetol = 1.0e-07
```

o.d.e. integration tolerance for all field line tracing routines

real inputlist::absreq = 1.0e-08

redundant

real inputlist::relreq = 1.0e-08

redundant

real inputlist::absacc = 1.0e-04

redundant

• real inputlist::epsr = 1.0e-08

redundant

• integer inputlist::nppts = 0

number of toroidal transits used (per trajectory) in following field lines for constructing Poincaré plots; if nPpts < 1, no Poincaré plot is constructed;

• real inputlist::ppts = 0.0

stands for Poincare plot theta start. Chose at which angle (normalized over  $\pi$ ) the Poincare field-line tracing start.

• integer, dimension(1:mnvol+1) inputlist::nptrj = -1

number of trajectories in each annulus to be followed in constructing Poincaré plot

logical inputlist::lhevalues = .false.

to compute eigenvalues of  $abla \mathbf{F}$ 

• logical inputlist::lhevectors = .false.

to compute eigenvectors (and also eigenvalues) of  $abla \mathbf{F}$ 

logical inputlist::lhmatrix = .false.

to compute and write to file the elements of  $\nabla \mathbf{F}$ 

• integer inputlist::lperturbed = 0

to compute linear, perturbed equilibrium

integer inputlist::dpp = -1

perturbed harmonic

• integer inputlist::dqq = -1

perturbed harmonic

• integer inputlist::lerrortype = 0

the type of error output for Lcheck=1

integer inputlist::ngrid = -1

the number of points to output in the grid, -1 for Lrad(vvol)

```
• real inputlist::drz = 1E-5
```

difference in geometry for finite difference estimate (debug only)

• integer inputlist::lcheck = 0

implement various checks

logical inputlist::ltiming = .false.

to check timing

• real inputlist::fudge = 1.0e-00

redundant

• real inputlist::scaling = 1.0e-00

redundant

### 8.26.1 Detailed Description

The namelist diagnosticslist controls post-processor diagnostics, such as Poincaré plot resolution, etc.

### 8.26.2 Variable Documentation

```
8.26.2.1 nptrj integer, dimension(1:mnvol+1) inputlist::nptrj = -1
```

number of trajectories in each annulus to be followed in constructing Poincaré plot

• if nPtrj(1) <0, then nPtrj(1) = Ni(I), where Ni(1) is the grid resolution used to construct the Beltrami field in volume l

Referenced by pp00aa(), allglobal::readin(), allglobal::wrtend(), and xspech().

```
8.26.2.2 | Icheck integer inputlist::lcheck = 0
```

implement various checks

- if Lcheck = 0, no additional check on the calculation is performed
- if Lcheck = 1, the error in the current, i.e.  $\nabla \times \mathbf{B} \mu \mathbf{B}$  is computed as a post-diagnostic
- if Lcheck = 2, the analytic derivatives of the interface transform w.r.t. the helicity multiplier,  $\mu$ , and the enclosed poloidal flux,  $\Delta \psi_p$ , are compared to a finite-difference estimate
  - only if Lconstraint==1
  - only for dspec executable, i.e. must compile with DFLAGS = "-D DEBUG"
- if Lcheck = 3, the analytic derivatives of the volume w.r.t. interface Fourier harmonic is compared to a finite-difference estimate
  - must set Lfindzero = 2
  - set forcetol sufficiently small and set LreadGF = F, so that the matrix of second derivatives is calculated

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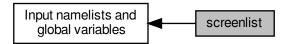
- only for dspec executable, i.e. must compile with DFLAGS = "-D DEBUG"
- if Lcheck = 4, the analytic calculation of the derivatives of the magnetic field,  $B^2$ , at the interfaces is compared to a finite-difference estimate
  - must set Lfindzero = 2
  - set forcetol sufficiently small
  - set LreadGF=F
  - only for dspec executable, i.e. must compile with DFLAGS = "-D DEBUG"
- if Lcheck = 5, the analytic calculation of the matrix of the derivatives of the force imbalance is compared to a finite-difference estimate
- if Lcheck = 6, the virtual casing calculation is compared to xdiagno (Lazerson 2013 [7])
  - the input file for xdiagno is written by bnorml()
  - this provides the Cartesian coordinates on the computational boundary where the virtual casing routine casing() computes the magnetic field, with the values of the magnetic field being written to the screen for comparison
  - must set Freebound=1, Lfindzero>0, mfreeits!=0
  - xdiagno must be executed manually

Referenced by bnorml(), hesian(), lforce(), ma02aa(), allglobal::readin(), allglobal::wrtend(), and xspech().

# 8.27 screenlist

The namelist screenlist controls screen output. Every subroutine, e.g. xy00aa.h, has its own write flag, Wxy00aa.h

Collaboration diagram for screenlist:



### **Variables**

- logical inputlist::wbuild\_vector\_potential = .false.
- logical inputlist::wreadin = .false.
   write screen output of readin()
- logical inputlist::wwrtend = .false.

write screen output of wrtend()

• logical inputlist::wmacros = .false.

write screen output from expanded macros

# 8.27.1 Detailed Description

The namelist screenlist controls screen output. Every subroutine, e.g. xy00aa.h, has its own write flag, Wxy00aa.h

## 8.27.2 Variable Documentation

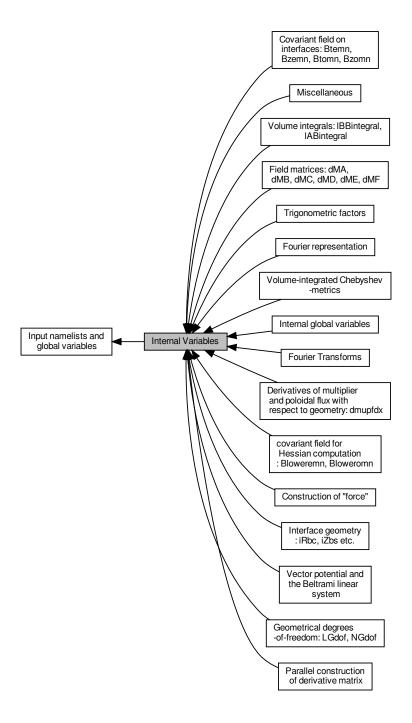
**8.27.2.1 wbuild\_vector\_potential** logical inputlist::wbuild\_vector\_potential = .false.

Todo: what is this?

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# 8.28 Internal Variables

Collaboration diagram for Internal Variables:



# Modules

- Fourier representation
- Interface geometry: iRbc, iZbs etc.

The Fourier harmonics of the interfaces are contained in iRbc(1:mn, 0:Mvol) and iZbs(1:mn, 0:Mvol), where iRbc(1, j), iZbs(1, j) contains the Fourier harmonics,  $R_j$ ,  $Z_j$ , of the l-th interface.

Fourier Transforms

The coordinate geometry and fields are mapped to/from Fourier space and real space using FFTW3. The resolution of the real space grid is given by Nt=Ndiscrete\*4\*Mpol and Nz=Ndiscrete\*4\*Ntor.

· Volume-integrated Chebyshev-metrics

These are allocated in dforce(), defined in ma00aa(), and are used in matrix() to construct the matrices.

- · Vector potential and the Beltrami linear system
- Field matrices: dMA, dMB, dMC, dMD, dME, dMF
- · Construction of "force"

The force vector is comprised of Bomn and Iomn.

· Covariant field on interfaces: Btemn, Bzemn, Btomn, Bzomn

The covariant field.

- · covariant field for Hessian computation: Bloweremn, Bloweromn
- · Geometrical degrees-of-freedom: LGdof, NGdof

The geometrical degrees-of-freedom.

- · Parallel construction of derivative matrix
- · Derivatives of multiplier and poloidal flux with respect to geometry: dmupfdx
- · Trigonometric factors
- · Volume integrals: IBBintegral, IABintegral
- · Internal global variables

internal global variables; internal logical variables; default values are provided here; these may be changed according to input values

Miscellaneous

The following are miscellaneous flags required for the virtual casing field, external (vacuum) field integration, ...

## **Data Types**

· type allglobal::derivative

 $d\mathbf{B}/d\mathbf{X}$  (?) More...

#### **Variables**

· logical allglobal::derivative::l

what is this?

• integer allglobal::derivative::vol

Used in coords(); required for global constraint force gradient evaluation.

integer allglobal::derivative::innout

what is this?

· integer allglobal::derivative::ii

what is this?

· integer allglobal::derivative::irz

what is this?

• integer allglobal::derivative::issym

what is this?

type(derivative) allglobal::dbdx

 $d\mathbf{B}/d\mathbf{X}$  (?)

## 8.28.1 Detailed Description

# 8.28.2 Data Type Documentation

### 8.28.2.1 type allglobal::derivative dB/dX (?)

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# **Class Members**

logical	1	what is this?
integer	vol	Used in coords(); required for global constraint force gradient evaluation.
integer	innout	what is this?
integer	ii	what is this?
integer	irz	what is this?
integer	issym	what is this?

# 8.28.3 Variable Documentation

**8.28.3.1** | logical allglobal::derivative::l

what is this?

**8.28.3.2 vol** integer allglobal::derivative::vol

Used in coords(); required for global constraint force gradient evaluation.

 $\textbf{8.28.3.3} \quad \textbf{innout} \quad \texttt{integer allglobal::derivative::innout}$ 

what is this?

 $\textbf{8.28.3.4} \quad \textbf{ii} \quad \texttt{integer allglobal::} \texttt{derivative::} \texttt{ii}$ 

what is this?

 $\textbf{8.28.3.5} \quad \textbf{irz} \quad \texttt{integer allglobal::derivative::irz}$ 

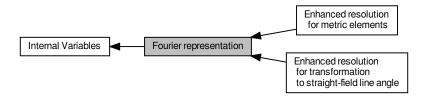
what is this?

**8.28.3.6 issym** integer allglobal::derivative::issym

what is this?

# 8.29 Fourier representation

Collaboration diagram for Fourier representation:



#### **Modules**

· Enhanced resolution for metric elements

Enhanced resolution is required for the metric elements,  $g_{ij}/\sqrt{g}$ , which is given by mne, ime, and ine. The Fourier resolution here is determined by 1 Mpol = 2\*Mpol and 1 Ntor = 2\*Ntor.

· Enhanced resolution for transformation to straight-field line angle

Enhanced resolution is required for the transformation to straight-field line angle on the interfaces, which is given by mns, ims and ins. The Fourier resolution here is determined by iMpol and iNtor.

# **Variables**

· integer allglobal::mn

total number of Fourier harmonics for coordinates/fields; calculated from Mpol, Ntor in readin()

• integer, dimension(:), allocatable allglobal::im

poloidal mode numbers for Fourier representation

• integer, dimension(:), allocatable allglobal::in

toroidal mode numbers for Fourier representation

• real, dimension(:), allocatable allglobal::halfmm

I saw this already somewhere...

• real, dimension(:), allocatable allglobal::regumm

I saw this already somewhere...

· real allglobal::rscale

no idea

real, dimension(:,:), allocatable allglobal::psifactor

no idea

real, dimension(:,:), allocatable allglobal::inifactor

no idea

real, dimension(:), allocatable allglobal::bbweight

weight on force-imbalance harmonics; used in dforce()

real, dimension(:), allocatable allglobal::mmpp

spectral condensation factors

· integer allglobal::Impol

what is this?

· integer allglobal::Intor

what is this?

• integer allglobal::smpol

what is this?

• integer allglobal::sntor

what is this?

• real allglobal::xoffset = 1.0

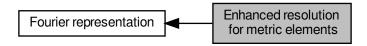
used to normalize NAG routines (which ones exacly where?)

# 8.29.1 Detailed Description

# 8.30 Enhanced resolution for metric elements

Enhanced resolution is required for the metric elements,  $g_{ij}/\sqrt{g}$ , which is given by mne, ime, and ine. The Fourier resolution here is determined by 1 Mpol = 2 \* Mpol and 1 Ntor = 2 \* Ntor.

Collaboration diagram for Enhanced resolution for metric elements:



### **Variables**

- integer allglobal::mne enhanced resolution for metric elements
- integer, dimension(:), allocatable allglobal::ime enhanced poloidal mode numbers for metric elements
- integer, dimension(:), allocatable allglobal::ine
   enhanced toroidal mode numbers for metric elements

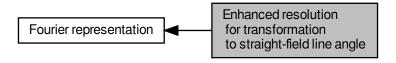
# 8.30.1 Detailed Description

Enhanced resolution is required for the metric elements,  $g_{ij}/\sqrt{g}$ , which is given by mne, ime, and ine. The Fourier resolution here is determined by 1 Mpol = 2 \* Mpol and 1 Ntor = 2 \* Ntor.

# 8.31 Enhanced resolution for transformation to straight-field line angle

Enhanced resolution is required for the transformation to straight-field line angle on the interfaces, which is given by mns, ims and ins. The Fourier resolution here is determined by iMpol and iNtor.

Collaboration diagram for Enhanced resolution for transformation to straight-field line angle:



#### **Variables**

- integer allglobal::mns enhanced resolution for straight field line transformation
- integer, dimension(:), allocatable allglobal::ims
   enhanced poloidal mode numbers for straight field line transformation
- integer, dimension(:), allocatable allglobal::ins
   enhanced toroidal mode numbers for straight field line transformation

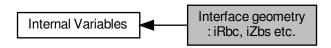
### 8.31.1 Detailed Description

Enhanced resolution is required for the transformation to straight-field line angle on the interfaces, which is given by mns, ims and ins. The Fourier resolution here is determined by iMpol and iNtor.

# 8.32 Interface geometry: iRbc, iZbs etc.

The Fourier harmonics of the interfaces are contained in iRbc(1:mn, 0:Mvol) and iZbs(1:mn, 0:Mvol), where iRbc(1,j), iZbs(1,j) contains the Fourier harmonics,  $R_i, Z_i$ , of the l-th interface.

Collaboration diagram for Interface geometry: iRbc, iZbs etc.:



#### **Variables**

- real, dimension(:,:), allocatable allglobal::irbc
- cosine R harmonics of interface surface geometry; stellarator symmetric
- real, dimension(:,:), allocatable allglobal::izbs
  - sine Z harmonics of interface surface geometry; stellarator symmetric
- real, dimension(:,:), allocatable allglobal::irbs
  - sine R harmonics of interface surface geometry; non-stellarator symmetric
- real, dimension(:,:), allocatable allglobal::izbc
  - cosine Z harmonics of interface surface geometry; non-stellarator symmetric
- real, dimension(:,:), allocatable allglobal::drbc
  - cosine R harmonics of interface surface geometry; stellarator symmetric; linear deformation
- real, dimension(:,:), allocatable allglobal::dzbs
  - sine Z harmonics of interface surface geometry; stellarator symmetric; linear deformation
- real, dimension(:,:), allocatable allglobal::drbs
  - sine R harmonics of interface surface geometry; non-stellarator symmetric; linear deformation
- real, dimension(:,:), allocatable allglobal::dzbc
  - cosine Z harmonics of interface surface geometry; non-stellarator symmetric; linear deformation
- real, dimension(:,:), allocatable allglobal::irij
  - interface surface geometry; real space
- real, dimension(:,:), allocatable allglobal::izij
  - interface surface geometry; real space
- real, dimension(:,:), allocatable allglobal::drij
  - interface surface geometry; real space
- real, dimension(:,:), allocatable allglobal::dzij
  - interface surface geometry; real space
- real, dimension(:,:), allocatable allglobal::trij
  - interface surface geometry; real space
- real, dimension(:,:), allocatable allglobal::tzij
  - interface surface geometry; real space
- real, dimension(:), allocatable allglobal::ivns
  - sine harmonics of vacuum normal magnetic field on interfaces; stellarator symmetric
- real, dimension(:), allocatable allglobal::ibns
  - sine harmonics of plasma normal magnetic field on interfaces; stellarator symmetric

- real, dimension(:), allocatable allglobal::ivnc
   cosine harmonics of vacuum normal magnetic field on interfaces; non-stellarator symmetric
- real, dimension(:), allocatable allglobal::ibnc
   cosine harmonics of plasma normal magnetic field on interfaces; non-stellarator symmetric
- real, dimension(:), allocatable allglobal::lrbc

local workspace

real, dimension(:), allocatable allglobal::lzbs

local workspace

- real, dimension(:), allocatable allglobal::lrbs local workspace
- real, dimension(:), allocatable allglobal::lzbc local workspace

# 8.32.1 Detailed Description

The Fourier harmonics of the interfaces are contained in iRbc(1:mn, 0:Mvol) and iZbs(1:mn, 0:Mvol), where iRbc(1,j), iZbs(1,j) contains the Fourier harmonics,  $R_j$ ,  $Z_j$ , of the l-th interface.

### 8.33 Fourier Transforms

The coordinate geometry and fields are mapped to/from Fourier space and real space using FFTW3. The resolution of the real space grid is given by Nt=Ndiscrete\*4\*Mpol and Nz=Ndiscrete\*4\*Ntor.

Collaboration diagram for Fourier Transforms:



#### **Variables**

· integer allglobal::nt

discrete resolution along  $\theta$  of grid in real space

integer allglobal::nz

discrete resolution along  $\zeta$  of grid in real space

· integer allglobal::ntz

discrete resolution; Ntz=Nt\*Nz shorthand

· integer allglobal::hnt

discrete resolution; Ntz=Nt\*Nz shorthand

integer allglobal::hnz

discrete resolution; Ntz=Nt\*Nz shorthand

· real allglobal::sontz

one / sqrt (one\*Ntz); shorthand

real, dimension(:,:,:), allocatable allglobal::rij

real-space grid; R

real, dimension(:,:,:), allocatable allglobal::zij

real-space grid; Z

• real, dimension(:,:,:), allocatable allglobal::xij

what is this?

real, dimension(:,:,:), allocatable allglobal::yij

what is this?

real, dimension(:,:), allocatable allglobal::sg

real-space grid; jacobian and its derivatives

real, dimension(:,:,:,:), allocatable allglobal::guvij

real-space grid; metric elements

• real, dimension(:,:,:), allocatable allglobal::gvuij

real-space grid; metric elements (?); 10 Dec 15;

• real, dimension(:,:,:), allocatable allglobal::guvijsave

what is this?

• integer, dimension(:,:), allocatable allglobal::ki

identification of Fourier modes

• integer, dimension(:,:,:), allocatable allglobal::kijs

identification of Fourier modes

- integer, dimension(:,:,:), allocatable allglobal::kija identification of Fourier modes
- integer, dimension(:), allocatable allglobal::iotakkii identification of Fourier modes
- integer, dimension(:,:), allocatable allglobal::iotaksub identification of Fourier modes
- integer, dimension(:,:), allocatable allglobal::iotakadd identification of Fourier modes
- integer, dimension(:,:), allocatable allglobal::iotaksgn identification of Fourier modes
- real, dimension(:), allocatable allglobal::efmn
   Fourier harmonics; dummy workspace.
- real, dimension(:), allocatable allglobal::ofmn Fourier harmonics; dummy workspace.
- real, dimension(:), allocatable allglobal::cfmn
  Fourier harmonics; dummy workspace.
- real, dimension(:), allocatable allglobal::sfmn
   Fourier harmonics; dummy workspace.
- real, dimension(:), allocatable allglobal::evmn Fourier harmonics; dummy workspace.
- real, dimension(:), allocatable allglobal::odmn
   Fourier harmonics; dummy workspace.
- real, dimension(:), allocatable allglobal::comn
   Fourier harmonics; dummy workspace.
- real, dimension(:), allocatable allglobal::simn Fourier harmonics; dummy workspace.
- real, dimension(:), allocatable allglobal::ijreal what is this?
- real, dimension(:), allocatable allglobal::ijimag what is this?
- real, dimension(:), allocatable allglobal::jireal what is this?
- real, dimension(:), allocatable allglobal::jiimag what is this?
- real, dimension(:), allocatable allglobal::jkreal what is this?
- real, dimension(:), allocatable allglobal::jkimag
   what is this?
- real, dimension(:), allocatable allglobal::kjreal what is this?
- real, dimension(:), allocatable allglobal::kjimag
   what is this?
- real, dimension(:,:,:), allocatable allglobal::bsupumn
- tangential field on interfaces;  $\theta$ -component; required for virtual casing construction of field; 11 Oct 12 real, dimension(:,:,:), allocatable allglobal::bsupvmn
- tangential field on interfaces;  $\zeta$  -component; required for virtual casing construction of field; 11 Oct 12 real, dimension(:,:), allocatable allglobal::goomne
- described in preset()real, dimension(:,:), allocatable allglobal::goomno
- real, dimension(:,:), allocatable allglobal::gssmne

described in preset()

described in preset()

- real, dimension(:,:), allocatable allglobal::gssmno described in preset()
- real, dimension(:,:), allocatable allglobal::gstmne described in preset()
- real, dimension(:,:), allocatable allglobal::gstmno described in preset()
- real, dimension(:,:), allocatable allglobal::gszmne described in preset()
- real, dimension(:,:), allocatable allglobal::gszmno described in preset()
- real, dimension(:,:), allocatable allglobal::gttmne described in preset()
- real, dimension(:,:), allocatable allglobal::gttmno described in preset()
- real, dimension(:,:), allocatable allglobal::gtzmne described in preset()
- real, dimension(:,:), allocatable allglobal::gtzmno described in preset()
- real, dimension(:,:), allocatable allglobal::gzzmne described in preset()
- real, dimension(:,:), allocatable allglobal::gzzmno described in preset()

### 8.33.1 Detailed Description

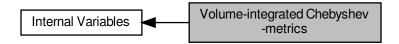
The coordinate geometry and fields are mapped to/from Fourier space and real space using FFTW3. The resolution of the real space grid is given by Nt=Ndiscrete\*4\*Mpol and Nz=Ndiscrete\*4\*Ntor.

Various workspace arrays are allocated. These include Rij(1:Ntz,0:3,0:3) and Zij(1:Ntz,0:3,0:3), which contain the coordinates in real space and their derivatives; sg(0:3,Ntz), which contains the Jacobian and its derivatives; and guv(0:6,0:3,1:Ntz), which contains the metric elements and their derivatives.

# 8.34 Volume-integrated Chebyshev-metrics

These are allocated in dforce(), defined in ma00aa(), and are used in matrix() to construct the matrices.

Collaboration diagram for Volume-integrated Chebyshev-metrics:



#### **Variables**

- real, dimension(:,:,:), allocatable allglobal::dtoocc
   volume-integrated Chebychev-metrics; see matrix()
- real, dimension(:,:,:,:), allocatable allglobal::dtoocs
   volume-integrated Chebychev-metrics; see matrix()
- real, dimension(:,:,:), allocatable allglobal::dtoosc volume-integrated Chebychev-metrics; see matrix()
- real, dimension(:,:,:), allocatable allglobal::dtooss
   volume-integrated Chebychev-metrics; see matrix()
- real, dimension(:,:,:,:), allocatable allglobal::ttsscc volume-integrated Chebychev-metrics; see matrix()
- real, dimension(:,:,:,:), allocatable allglobal::ttsscs
   volume-integrated Chebychev-metrics; see matrix()
- real, dimension(:,:,:,:), allocatable allglobal::ttsssc volume-integrated Chebychev-metrics; see matrix()
- real, dimension(:,:,:,:), allocatable allglobal::ttssss
   volume-integrated Chebychev-metrics; see matrix()
- real, dimension(:,:,:,:), allocatable allglobal::tdstcc
- volume-integrated Chebychev-metrics; see matrix()
   real, dimension(:,:,:,:), allocatable allglobal::tdstcs
- volume-integrated Chebychev-metrics; see matrix()real, dimension(:,:,:,:), allocatable allglobal::tdstsc
- volume-integrated Chebychev-metrics; see matrix()real, dimension(:,;;,;), allocatable allglobal::tdstss
- real, dimension(:,:,:,:), allocatable aligiobal::tdstss
   volume-integrated Chebychev-metrics; see matrix()
- real, dimension(:,:,:,:), allocatable allglobal::tdszcc volume-integrated Chebychev-metrics; see matrix()
- real, dimension(:,:,:,:), allocatable allglobal::tdszcs
   volume-integrated Chebychev-metrics; see matrix()
- real, dimension(:,:,:,:), allocatable allglobal::tdszsc volume-integrated Chebychev-metrics; see matrix()
- real, dimension(:,:,:,:), allocatable allglobal::tdszss
   volume-integrated Chebychev-metrics; see matrix()
- real, dimension(:,:,:,:), allocatable allglobal::ddttcc

- volume-integrated Chebychev-metrics; see matrix()
- real, dimension(:,:,:,:), allocatable allglobal::ddttcs
  - volume-integrated Chebychev-metrics; see matrix()
- real, dimension(:,:,:,:), allocatable allglobal::ddttsc volume-integrated Chebychev-metrics; see matrix()
- real, dimension(:,:,:), allocatable allglobal::ddttss
- $\label{eq:volume-integrated Chebychev-metrics} volume-integrated Chebychev-metrics; see \textit{matrix}() \\ \bullet \quad \text{real, dimension}(:,:,:,:), \ \text{allocatable allglobal::ddtzcc}$
- volume-integrated Chebychev-metrics; see matrix()
- real, dimension(:,:,:,:), allocatable allglobal::ddtzcs
   volume-integrated Chebychev-metrics; see matrix()
- real, dimension(:,:,:), allocatable allglobal::ddtzsc volume-integrated Chebychev-metrics; see matrix()
- real, dimension(:,:,:), allocatable allglobal::ddtzss
   volume-integrated Chebychev-metrics; see matrix()
- real, dimension(:,:,:,:), allocatable allglobal::ddzzcc
   volume-integrated Chebychev-metrics; see matrix()
- real, dimension(:,:,:,:), allocatable allglobal::ddzzcs
   volume-integrated Chebychev-metrics; see matrix()
- real, dimension(:,:,:,:), allocatable allglobal::ddzzsc volume-integrated Chebychev-metrics; see matrix()
- real, dimension(:,:,:,:), allocatable allglobal::ddzzss
   volume-integrated Chebychev-metrics; see matrix()
- real, dimension(:,:), allocatable allglobal::tsc what is this?
- real, dimension(:,:), allocatable allglobal::tss what is this?
- real, dimension(:,:), allocatable allglobal::dtc
   what is this?
- real, dimension(:,:), allocatable allglobal::dts
   what is this?
- real, dimension(:,:), allocatable allglobal::dzc

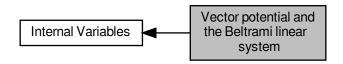
  what is this?
- real, dimension(:,:), allocatable allglobal::dzs what is this?
- real, dimension(:,:), allocatable allglobal::ttc what is this?
- real, dimension(:,:), allocatable allglobal::tzc what is this?
- real, dimension(:,:), allocatable allglobal::tts what is this?
- real, dimension(:,:), allocatable allglobal::tzs what is this?
- real, dimension(:), allocatable allglobal::dtflux  $\delta \psi_{toroidal} \ \textit{in each annulus}$
- real, dimension(:), allocatable allglobal::dpflux  $\delta\psi_{poloidal}$  in each annulus
- real, dimension(:), allocatable allglobal::sweight minimum poloidal length constraint weight

#### 8.34.1 Detailed Description

These are allocated in dforce(), defined in ma00aa(), and are used in matrix() to construct the matrices.

# 8.35 Vector potential and the Beltrami linear system

Collaboration diagram for Vector potential and the Beltrami linear system:



#### **Variables**

- integer, dimension(:), allocatable allglobal::nadof degrees of freedom in Beltrami fields in each annulus
- integer, dimension(:), allocatable allglobal::nfielddof

degrees of freedom in Beltrami fields in each annulus, field only, no Lagrange multipliers

 $\bullet \quad type( subgrid), \, dimension(:,:,:), \, allocatable \, \, all global :: ate$ 

magnetic vector potential cosine Fourier harmonics; stellarator-symmetric

- type(subgrid), dimension(:,:,:), allocatable allglobal::aze
  - magnetic vector potential cosine Fourier harmonics; stellarator-symmetric
- type(subgrid), dimension(:,:,:), allocatable allglobal::ato

magnetic vector potential sine Fourier harmonics; non-stellarator-symmetric

- type(subgrid), dimension(:,:,:), allocatable allglobal::azo
  - magnetic vector potential sine Fourier harmonics; non-stellarator-symmetric
- integer, dimension(:,:), allocatable allglobal::lma

Lagrange multipliers (?)

- integer, dimension(:,:), allocatable allglobal::Imb
  - Lagrange multipliers (?)
- integer, dimension(:,:), allocatable allglobal::Imc

Lagrange multipliers (?)

integer, dimension(:,:), allocatable allglobal::lmd

Lagrange multipliers (?)

integer, dimension(:,:), allocatable allglobal::lme

Lagrange multipliers (?)

integer, dimension(:,:), allocatable allglobal::lmf

Lagrange multipliers (?)

integer, dimension(:,:), allocatable allglobal::lmg

Lagrange multipliers (?)

integer, dimension(:,:), allocatable allglobal::lmh

Lagrange multipliers (?)

what is this?

- real, dimension(:,:), allocatable allglobal::lmavalue
- real, dimension(:,:), allocatable allglobal::Imbvalue
- real, dimension(:,:), allocatable allglobal::lmcvalue

what is this?

 real, dimension(:,:), allocatable allglobal::lmdvalue what is this?

- real, dimension(:,:), allocatable allglobal::lmevalue what is this?
- real, dimension(:,:), allocatable allglobal::Imfvalue what is this?
- real, dimension(:,:), allocatable allglobal::lmgvalue what is this?
- real, dimension(:,:), allocatable allglobal::lmhvalue what is this?
- integer, dimension(:,:), allocatable allglobal::fso what is this?
- integer, dimension(:,:), allocatable allglobal::fse what is this?
- · logical allglobal::lcoordinatesingularity

set by LREGION macro; true if inside the innermost volume

• logical allglobal::lplasmaregion

set by LREGION macro; true if inside the plasma region

· logical allglobal::lvacuumregion

set by LREGION macro; true if inside the vacuum region

· logical allglobal::lsavedguvij

flag used in matrix free

logical allglobal::localconstraint

what is this?

# 8.35.1 Detailed Description

- In each volume, the total degrees of freedom in the Beltrami linear system is NAdof(1:Nvol). This depends on Mpol, Ntor and Lrad(vvol).
- · The covariant components of the vector potential are written as

$$A_{\theta} = \sum_{i} \sum_{l=0}^{L} A_{\theta,e,i,l} T_{l}(s) \cos \alpha_{i} + \sum_{i} \sum_{l=0}^{L} A_{\theta,o,i,l} T_{l}(s) \sin \alpha_{i}$$
 (266)

$$A_{\zeta} = \sum_{i} \sum_{l=0}^{L} A_{\zeta,e,i,l} T_{l}(s) \cos \alpha_{i} + \sum_{i} \sum_{l=0}^{L} A_{\zeta,o,i,l} T_{l}(s) \sin \alpha_{i},$$
 (267)

where  $T_l(s)$  are the Chebyshev polynomials and  $\alpha_i \equiv m_i \theta - n_i \zeta$ .

• The following internal arrays are declared in preset():

$$\begin{aligned} &\mathrm{dAte}\left(0,\mathrm{i}\right) \$s(\mathrm{I}) \equiv A_{\theta,e,i,l} \\ &\mathrm{dAze}\left(0,\mathrm{i}\right) \$s(\mathrm{I}) \equiv A_{\zeta,e,i,l} \\ &\mathrm{dAto}\left(0,\mathrm{i}\right) \$s(\mathrm{I}) \equiv A_{\theta,o,i,l} \\ &\mathrm{dAzo}\left(0,\mathrm{i}\right) \$s(\mathrm{I}) \equiv A_{\zeta,o,i,l} \end{aligned}$$

# 8.36 Field matrices: dMA, dMB, dMC, dMD, dME, dMF

Collaboration diagram for Field matrices: dMA, dMB, dMC, dMD, dME, dMF:



#### **Variables**

- real, dimension(:,:), allocatable allglobal::dma energy and helicity matrices; quadratic forms
- real, dimension(:,:), allocatable allglobal::dmb energy and helicity matrices; quadratic forms
- real, dimension(:,:), allocatable allglobal::dmd energy and helicity matrices; quadratic forms
- real, dimension(:), allocatable allglobal::dmas sparse version of dMA, data
- real, dimension(:), allocatable allglobal::dmds sparse version of dMD, data
- integer, dimension(:), allocatable allglobal::idmas sparse version of dMA and dMD, indices
- integer, dimension(:), allocatable allglobal::jdmas sparse version of dMA and dMD, indices
- integer, dimension(:), allocatable allglobal::ndmasmax
   number of elements for sparse matrices
- integer, dimension(:), allocatable allglobal::ndmas
   number of elements for sparse matrices
- real, dimension(:), allocatable allglobal::dmg
- what is this?real, dimension(:,:), allocatable allglobal::solution
- this is allocated in dforce; used in mp00ac and ma02aa; and is passed to packab
- real, dimension(:,:,:), allocatable allglobal::gmreslastsolution
  - used to store the last solution for restarting GMRES
- real, dimension(:), allocatable allglobal::mbpsi
  - matrix vector products
- · logical allglobal::liluprecond
  - whether to use ILU preconditioner for GMRES
- real, dimension(:,:), allocatable allglobal::beltramiinverse
- real, dimension(:,:,:), allocatable allglobal::diotadxup
  - measured rotational transform on inner/outer interfaces for each volume; d(transform)/dx; (see dforce)
- real, dimension(:,:,:), allocatable allglobal::ditgpdxtp
  - measured toroidal and poloidal current on inner/outer interfaces for each volume; d(Itor,Gpol)/dx; (see dforce)
- real, dimension(:,:,:,:), allocatable allglobal::glambda
  - save initial guesses for iterative calculation of rotational-transform
- · integer allglobal::lmns
  - what is this?

what is this?

## 8.36.1 Detailed Description

• The energy,  $W\equiv\int\!dv~{f B}\cdot{f B}$ , and helicity,  $K\equiv\int\!dv~{f A}\cdot{f B}$ , functionals may be written

$$W = \frac{1}{2} a_i A_{i,j} a_j + a_i B_{i,j} \psi_j + \frac{1}{2} \psi_i C_{i,j} \psi_j$$

$$K = \frac{1}{2} a_i D_{i,j} a_j + a_i E_{i,j} \psi_j + \frac{1}{2} \psi_i F_{i,j} \psi_j$$
(268)

$$K = \frac{1}{2} a_i D_{i,j} a_j + a_i E_{i,j} \psi_j + \frac{1}{2} \psi_i F_{i,j} \psi_j$$
 (269)

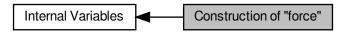
where  $\mathbf{a} \equiv \{A_{\theta,e,i,l}, A_{\zeta,e,i,l}, A_{\theta,o,i,l}, A_{\zeta,o,i,l}, f_{e,i}, f_{o,i}\}$  contains the independent degrees of freedom and  $\psi \equiv \{\Delta\psi_t, \Delta\psi_p\}$ .

• These are allocated and deallocated in dforce(), assigned in matrix(), and used in mp00ac() and (?) df00aa().

## 8.37 Construction of "force"

The force vector is comprised of Bomn and Iomn.

Collaboration diagram for Construction of "force":



#### **Variables**

- real, dimension(:,:,:), allocatable allglobal::bemn force vector; stellarator-symmetric (?)
- real, dimension(:,:), allocatable allglobal::iomn force vector; stellarator-symmetric (?)
- real, dimension(:,:,:), allocatable allglobal::somn force vector; non-stellarator-symmetric (?)
- real, dimension(:,:,:), allocatable allglobal::pomn force vector; non-stellarator-symmetric (?)
- real, dimension(:,:,:), allocatable allglobal::bomn force vector; stellarator-symmetric (?)
- real, dimension(:,:), allocatable allglobal::iemn force vector; stellarator-symmetric (?)
- real, dimension(:,:,:), allocatable allglobal::semn force vector; non-stellarator-symmetric (?)
- real, dimension(:,:,:), allocatable allglobal::pemn force vector; non-stellarator-symmetric (?)
- real, dimension(:), allocatable allglobal::bbe force vector (?); stellarator-symmetric (?)
- real, dimension(:), allocatable allglobal::iio
   force vector (?); stellarator-symmetric (?)
- real, dimension(:), allocatable allglobal::bbo force vector (?); non-stellarator-symmetric (?)
- real, dimension(:), allocatable allglobal::iie
   force vector (?); non-stellarator-symmetric (?)

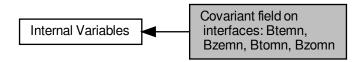
#### 8.37.1 Detailed Description

The force vector is comprised of Bomn and Iomn.

# 8.38 Covariant field on interfaces: Btemn, Bzemn, Btomn, Bzomn

The covariant field.

Collaboration diagram for Covariant field on interfaces: Btemn, Bzemn, Btomn, Bzomn:



#### **Variables**

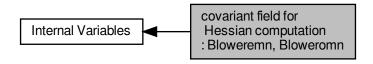
- real, dimension(:,:,:), allocatable allglobal::btemn
   covariant θ cosine component of the tangential field on interfaces; stellarator-symmetric
- real, dimension(:,:,:), allocatable allglobal::bzemn
   covariant ζ cosine component of the tangential field on interfaces; stellarator-symmetric
- real, dimension(:,;,:), allocatable allglobal::btomn
   covariant θ sine component of the tangential field on interfaces; non-stellarator-symmetric
- real, dimension(:,:,:), allocatable allglobal::bzomn
   covariant ζ sine component of the tangential field on interfaces; non-stellarator-symmetric

# 8.38.1 Detailed Description

The covariant field.

# 8.39 covariant field for Hessian computation: Bloweremn, Bloweromn

Collaboration diagram for covariant field for Hessian computation: Bloweremn, Bloweremn:



#### **Variables**

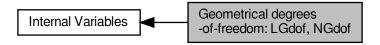
- real, dimension(:,:), allocatable allglobal::bloweremn covariant field for Hessian computation
- real, dimension(:,:), allocatable allglobal::bloweromn covariant field for Hessian computation

# 8.39.1 Detailed Description

# 8.40 Geometrical degrees-of-freedom: LGdof, NGdof

The geometrical degrees-of-freedom.

Collaboration diagram for Geometrical degrees-of-freedom: LGdof, NGdof:



## **Variables**

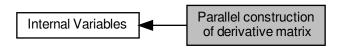
- integer allglobal::lgdof geometrical degrees of freedom associated with each interface
- integer allglobal::ngdof total geometrical degrees of freedom

# 8.40.1 Detailed Description

The geometrical degrees-of-freedom.

## 8.41 Parallel construction of derivative matrix

Collaboration diagram for Parallel construction of derivative matrix:



#### **Variables**

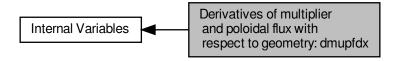
- real, dimension(:,;;), allocatable allglobal::dbbdrz
   derivative of magnetic field w.r.t. geometry (?)
- real, dimension(:,:), allocatable allglobal::diidrz derivative of spectral constraints w.r.t. geometry (?)
- real, dimension(:,:,:,:), allocatable allglobal::dffdrz derivatives of B<sup>2</sup> at the interfaces wrt geometry
- real, dimension(:,:,:), allocatable allglobal::dbbdmp derivatives of B<sup>2</sup> at the interfaces wrt mu and dpflux

## 8.41.1 Detailed Description

- The derivatives of force-balance,  $[[p+B^2/2]]$ , and the spectral constraints (see sw03aa()), with respect to the interface geometry is constructed in parallel by dforce().
- force-balance across the *l*-th interface depends on the fields in the adjacent interfaces.

# 8.42 Derivatives of multiplier and poloidal flux with respect to geometry: dmupfdx

Collaboration diagram for Derivatives of multiplier and poloidal flux with respect to geometry: dmupfdx:



#### **Variables**

- real, dimension(:,:,:,:), allocatable allglobal::dmupfdx
   derivatives of mu and dpflux wrt geometry at constant interface transform
- logical allglobal::lhessianallocated

flag to indicate that force gradient matrix is allocated (?)

- real, dimension(:,:), allocatable allglobal::hessian force gradient matrix (?)
- real, dimension(:,:), allocatable allglobal::dessian derivative of force gradient matrix (?)

## 8.42.1 Detailed Description

- The information in dmupfdx describes how the helicity multiplier,  $\mu$ , and the enclosed poloidal flux,  $\Delta\psi_p$ , must vary as the geometry is varied in order to satisfy the interface transform constraint.
- The internal variable dmupfdx (1:Mvol, 1:2, 1:LGdof, 0:1) is allocated/deallocated in newton(), and hesian() if selected.
- The magnetic field depends on the Fourier harmonics of both the inner and outer interface geometry (represented here as  $x_j$ ), the helicity multiplier, and the enclosed poloidal flux, i.e.  $\mathbf{B}_{\pm} = \mathbf{B}_{\pm}(x_j, \mu, \Delta \psi_p)$ , so that

$$\delta \mathbf{B}_{\pm} = \frac{\partial \mathbf{B}_{\pm}}{\partial x_j} \delta x_j + \frac{\partial \mathbf{B}_{\pm}}{\partial \mu} \delta \mu + \frac{\partial \mathbf{B}_{\pm}}{\partial \Delta \psi_p} \delta \Delta \psi_p. \tag{270}$$

• This information is used to adjust the calculation of how force-balance, i.e.  $B^2$  at the interfaces, varies with geometry at fixed interface rotational transform. Given

$$B_{\pm}^{2} = B_{\pm}^{2}(x_{j}, \mu, \Delta \psi_{p}), \tag{271}$$

we may derive

$$\frac{\partial B_{\pm}^{2}}{\partial x_{j}} = \frac{\partial B_{\pm}^{2}}{\partial x_{j}} + \frac{\partial B_{\pm}^{2}}{\partial \mu} \frac{\partial \mu}{\partial x_{j}} + \frac{\partial B_{\pm}^{2}}{\partial \Delta \psi_{p}} \frac{\partial \Delta \psi_{p}}{\partial x_{j}}$$
(272)

• The constraint to be enforced is that  $\mu$  and  $\Delta\psi_p$  must generally vary as the geometry is varied if the value of the rotational-transform constraint on the inner/outer interface is to be preserved, i.e.

$$\begin{pmatrix}
\frac{\partial \boldsymbol{t}_{-}}{\partial \mathbf{B}_{-}} \cdot \frac{\partial \mathbf{B}_{-}}{\partial \mu} & , & \frac{\partial \boldsymbol{t}_{-}}{\partial \mathbf{B}_{-}} \cdot \frac{\partial \mathbf{B}_{-}}{\partial \Delta \psi_{p}} \\
\frac{\partial \boldsymbol{t}_{+}}{\partial \mathbf{B}_{+}} \cdot \frac{\partial \mathbf{B}_{+}}{\partial \mu} & , & \frac{\partial \boldsymbol{t}_{+}}{\partial \mathbf{B}_{+}} \cdot \frac{\partial \mathbf{B}_{+}}{\partial \Delta \psi_{p}}
\end{pmatrix}
\begin{pmatrix}
\frac{\partial \mu}{\partial x_{j}} \\
\frac{\partial \Delta \psi_{p}}{\partial x_{j}}
\end{pmatrix} = - \begin{pmatrix}
\frac{\partial \boldsymbol{t}_{-}}{\partial \mathbf{B}_{-}} \cdot \frac{\partial \mathbf{B}_{-}}{\partial x_{j}} \\
\frac{\partial \boldsymbol{t}_{+}}{\partial \mathbf{B}_{+}} \cdot \frac{\partial \mathbf{B}_{+}}{\partial x_{j}}
\end{pmatrix}.$$
(273)

- This  $2 \times 2$  linear equation is solved in dforce() and the derivatives of the rotational-transform are given in diotadxup, see preset.f90 .
- A finite-difference estimate is computed if Lcheck==4.

# 8.43 Trigonometric factors

Collaboration diagram for Trigonometric factors:



#### **Variables**

- real, dimension(:,:), allocatable allglobal::cosi some precomputed cosines
- real, dimension(:,:), allocatable allglobal::sini some precomputed sines
- real, dimension(:), allocatable allglobal::gteta something related to  $\sqrt{g}$  and  $\theta$ ?
- real, dimension(:), allocatable allglobal::gzeta something related to  $\sqrt{g}$  and  $\zeta$  ?
- real, dimension(:), allocatable allglobal::ajk
   definition of coordinate axis
- real, dimension(:,:,:,:), allocatable allglobal::dradr derivatives of coordinate axis
- real, dimension(:,:,:,:), allocatable allglobal::dradz derivatives of coordinate axis
- real, dimension(:,:,:,:), allocatable allglobal::dzadr derivatives of coordinate axis
- real, dimension(:,:,:,:), allocatable allglobal::dzadz derivatives of coordinate axis
- real, dimension(:,:,:), allocatable allglobal::drodr derivatives of coordinate axis
- real, dimension(:,:,:), allocatable allglobal::drodz derivatives of coordinate axis
- real, dimension(:,:,:), allocatable allglobal::dzodr derivatives of coordinate axis
- real, dimension(:,:,:), allocatable allglobal::dzodz derivatives of coordinate axis
- integer, dimension(:,:), allocatable allglobal::djkp for calculating cylindrical volume
- integer, dimension(:,:), allocatable allglobal::djkm for calculating cylindrical volume

## 8.43.1 Detailed Description

- To facilitate construction of the metric integrals, various trigonometric identities are exploited.
- The following are used for volume integrals (see volume() ):

$$a_{i,j,k} = 4 m_k \oint d\theta d\zeta \cos(\alpha_i) \cos(\alpha_j) \cos(\alpha_k) / (2\pi)^2,$$
 (274)

$$b_{i,j,k} = 4 m_j \oint d\theta d\zeta \cos(\alpha_i) \sin(\alpha_j) \sin(\alpha_k) / (2\pi)^2, \tag{275}$$

# 8.44 Volume integrals: IBBintegral, IABintegral

Collaboration diagram for Volume integrals: IBBintegral, IABintegral:



#### **Variables**

- real, dimension(:), allocatable allglobal::lbbintegral
   B.B integral.
- real, dimension(:), allocatable allglobal::labintegral
   A.B integral.
- real, dimension(:), allocatable allglobal::vvolume volume integral of  $\sqrt{g}$ ; computed in volume
- · real allglobal::dvolume

derivative of volume w.r.t. interface geometry

## 8.44.1 Detailed Description

• The energy functional,  $F \equiv \sum_l F_l$ , where

$$F_{l} \equiv \left( \int_{\mathcal{V}_{l}} \frac{p_{l}}{\gamma - 1} + \frac{B_{l}^{2}}{2} dv \right) = \frac{P_{l}}{\gamma - 1} V_{l}^{1 - \gamma} + \int_{\mathcal{V}_{l}} \frac{B_{l}^{2}}{2} dv, \tag{276}$$

where the second expression is derived using  $p_l V_l^{\gamma} = P_l$ , where  $P_l$  is the adiabatic-constant. In Eqn. (276), it is implicit that  ${\bf B}$  satisfies (i) the toroidal and poloidal flux constraints; (ii) the interface constraint,  ${\bf B} \cdot \nabla s = 0$ ; and (iii) the helicity constraint (or the transform constraint).

• The derivatives of  $F_l$  with respect to the inner and outer adjacent interface geometry are stored in  $dFF(1 \leftarrow :Nvol,0:1,0:mn+mn-1)$ , where

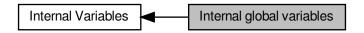
$$\begin{split} F_l &\equiv \text{dFF} \, (\text{1,0,0}) \\ \partial F_l / \partial R_{l-1,j} &\equiv \text{dFF} \, (\text{11,0,j}) \\ \partial F_l / \partial Z_{l-1,j} &\equiv \text{dFF} \, (\text{11,0,mnj}) \\ \partial F_l / \partial R_{l,j} &\equiv \text{dFF} \, (\text{11,1,j}) \\ \partial F_l / \partial Z_{l,j} &\equiv \text{dFF} \, (\text{11,1,mnj}) \end{split}$$

• The volume integrals  $\int dv$ ,  $\int B^2 dv$  and  $\int \mathbf{A} \cdot \mathbf{B} dv$  in each volume are computed and saved in volume (0  $\leftarrow$  :2,1:Nvol).

# 8.45 Internal global variables

internal global variables; internal logical variables; default values are provided here; these may be changed according to input values

Collaboration diagram for Internal global variables:



#### **Variables**

- · integer allglobal::ivol
  - labels volume; some subroutines (called by NAG) are fixed argument list but require the volume label
- · real allglobal::gbzeta
  - toroidal (contravariant) field; calculated in bfield; required to convert  $\dot{\theta}$  to  $B^{\theta}$ ,  $\dot{s}$  to  $B^{s}$
- integer, dimension(:), allocatable allglobal::iquad
  - internal copy of Nquad
- real, dimension(:,:), allocatable allglobal::gaussianweight
  - weights for Gaussian quadrature
- real, dimension(:,:), allocatable allglobal::gaussianabscissae
  - abscissae for Gaussian quadrature
- logical allglobal::lblinear
  - controls selection of Beltrami field solver; depends on LBeltrami
- · logical allglobal::lbnewton
  - controls selection of Beltrami field solver; depends on LBeltrami
- logical allglobal::lbsequad
  - controls selection of Beltrami field solver; depends on LBeltrami
- real, dimension(1:3) allglobal::orzp
  - used in mg00aa() to determine  $(s,\theta,\zeta)$  given  $(R,Z,\varphi)$

# 8.45.1 Detailed Description

internal global variables; internal logical variables; default values are provided here; these may be changed according to input values

8.46 Miscellaneous 153

## 8.46 Miscellaneous

The following are miscellaneous flags required for the virtual casing field, external (vacuum) field integration, ...

Collaboration diagram for Miscellaneous:



#### **Variables**

• integer allglobal::globaljk

labels position

real, dimension(:,:), allocatable allglobal::dxyz

computational boundary; position

real, dimension(:,:), allocatable allglobal::nxyz

computational boundary; normal

• real, dimension(:,:), allocatable allglobal::jxyz

plasma boundary; surface current

• real, dimension(1:2) allglobal::tetazeta

what is this?

• real allglobal::virtualcasingfactor = -one / (four\*pi)

this agrees with diagno

• integer allglobal::iberror

for computing error in magnetic field

· integer allglobal::nfreeboundaryiterations

number of free-boundary iterations already performed

• integer, parameter allglobal::node = 2

best to make this global for consistency between calling and called routines

• logical allglobal::first\_free\_bound = .false.

flag to indicate that this is the first free-boundary iteration

## 8.46.1 Detailed Description

The following are miscellaneous flags required for the virtual casing field, external (vacuum) field integration, ...

# 9 Module Documentation

# 9.1 allglobal Module Reference

global variable storage used as "workspace" throughout the code

## **Data Types**

· type derivative

 $\mathrm{d}\mathbf{B}/\mathrm{d}\mathbf{X}$  (?) More...

#### **Functions/Subroutines**

- subroutine build\_vector\_potential (Ivol, iocons, aderiv, tderiv)
- subroutine readin

The master node reads the input namelist and sets various internal variables. The relevant quantities are then broadcast.

· subroutine wrtend

The restart file is written.

• subroutine ismyvolume (vvol)

Check if volume vvol is associated to the corresponding MPI node.

subroutine whichcpuid (vvol, cpu\_id)

Returns which MPI node is associated to a given volume.

## **Variables**

· integer myid

MPI rank of current CPU.

• integer ncpu

number of MPI tasks

• integer ismyvolumevalue

flag to indicate if a CPU is operating on its assigned volume

· real cpus

initial time

• real pi2nfp

pi2/nfp; assigned in readin()

• real pi2pi2nfp

$$4\pi^2 \mathit{Nfp}$$

• real pi2pi2nfphalf

$$2\pi^2 Nfp$$

· real pi2pi2nfpquart

$$\pi^2 \mathit{Nfp}$$

real forceerr

total force-imbalance

· real energy

MHD energy.

real, dimension(:), allocatable ipdt

Toroidal pressure-driven current.

 real, dimension(:,:), allocatable ipdtdpf Toroidal pressure-driven current. · integer mvol number of total volumes; equal to Nvol for fixed-boundary; equal to Nvol+1 for free-boundary logical yesstellsym internal shorthand copies of Istellsym, which is an integer input; · logical notstellsym internal shorthand copies of Istellsym, which is an integer input; logical yesmatrixfree · logical notmatrixfree to use matrix-free method or not · real, dimension(:,:), allocatable cheby local workspace for evaluation of Chebychev polynomials • real, dimension(:,:,:), allocatable zernike local workspace for evaluation of Zernike polynomials • real, dimension(:,:,:), allocatable tt derivatives of Chebyshev polynomials at the inner and outer interfaces; • real, dimension(:,:,:,:), allocatable rtt derivatives of Zernike polynomials at the inner and outer interfaces; real, dimension(:,:), allocatable rtm  $r^m$  term of Zernike polynomials at the origin real, dimension(:), allocatable zernikedof Zernike degree of freedom for each m. logical, dimension(:), allocatable imagneticok · logical iconstraintok

used to indicate if Beltrami fields have been correctly constructed;

Used to break iteration loops of slaves in the global constraint minimization.

real, dimension(:,:), allocatable beltramierror

to store the integral of |curlB-mu\*B| computed by jo00aa;

integer mn

total number of Fourier harmonics for coordinates/fields; calculated from Mpol, Ntor in readin()

integer, dimension(:), allocatable im

poloidal mode numbers for Fourier representation

integer, dimension(:), allocatable in

toroidal mode numbers for Fourier representation

real, dimension(:), allocatable halfmm

I saw this already somewhere...

· real, dimension(:), allocatable regumm

I saw this already somewhere...

· real rscale

no idea

real, dimension(:,:), allocatable psifactor

no idea

real, dimension(:,:), allocatable inifactor

no idea

real, dimension(:), allocatable bbweight

weight on force-imbalance harmonics; used in dforce()

• real, dimension(:), allocatable mmpp

spectral condensation factors

· integer mne

enhanced resolution for metric elements

integer, dimension(:), allocatable ime

```
enhanced poloidal mode numbers for metric elements
· integer, dimension(:), allocatable ine
      enhanced toroidal mode numbers for metric elements
· integer mns
      enhanced resolution for straight field line transformation
• integer, dimension(:), allocatable ims
      enhanced poloidal mode numbers for straight field line transformation
• integer, dimension(:), allocatable ins
      enhanced toroidal mode numbers for straight field line transformation

    integer Impol

      what is this?

    integer Intor

      what is this?
· integer smpol
      what is this?
· integer sntor
      what is this?
• real xoffset = 1.0
      used to normalize NAG routines (which ones exacly where?)
• real, dimension(:,:), allocatable irbc
      cosine R harmonics of interface surface geometry; stellarator symmetric

    real, dimension(:,:), allocatable izbs

      sine Z harmonics of interface surface geometry; stellarator symmetric

    real, dimension(:,:), allocatable irbs

      sine R harmonics of interface surface geometry; non-stellarator symmetric

    real, dimension(:,:), allocatable izbc

      cosine Z harmonics of interface surface geometry; non-stellarator symmetric

    real, dimension(:,:), allocatable drbc

      cosine R harmonics of interface surface geometry; stellarator symmetric; linear deformation

    real, dimension(:,:), allocatable dzbs

      sine Z harmonics of interface surface geometry; stellarator symmetric; linear deformation
• real, dimension(:,:), allocatable drbs
      sine R harmonics of interface surface geometry; non-stellarator symmetric; linear deformation

    real, dimension(:,:), allocatable dzbc

      cosine Z harmonics of interface surface geometry; non-stellarator symmetric; linear deformation

    real, dimension(:,:), allocatable irij

      interface surface geometry; real space

    real, dimension(:,:), allocatable izij

      interface surface geometry; real space

    real, dimension(:,:), allocatable drij

      interface surface geometry; real space

    real, dimension(:,:), allocatable dzij

      interface surface geometry; real space
• real, dimension(:,:), allocatable trij
      interface surface geometry; real space

    real, dimension(:,:), allocatable tzij

      interface surface geometry; real space
· real, dimension(:), allocatable ivns
      sine harmonics of vacuum normal magnetic field on interfaces; stellarator symmetric

    real, dimension(:), allocatable ibns
```

```
sine harmonics of plasma normal magnetic field on interfaces; stellarator symmetric

    real, dimension(:), allocatable ivnc

      cosine harmonics of vacuum normal magnetic field on interfaces; non-stellarator symmetric

    real, dimension(:), allocatable ibnc

      cosine harmonics of plasma normal magnetic field on interfaces; non-stellarator symmetric

    real, dimension(:), allocatable Irbc

      local workspace

    real, dimension(:), allocatable lzbs

      local workspace
• real, dimension(:), allocatable Irbs
      local workspace
• real, dimension(:), allocatable lzbc
      local workspace
· integer nt
      discrete resolution along \theta of grid in real space
· integer nz
      discrete resolution along \zeta of grid in real space

    integer ntz

      discrete resolution; Ntz=Nt*Nz shorthand
· integer hnt
      discrete resolution; Ntz=Nt*Nz shorthand

    integer hnz

      discrete resolution; Ntz=Nt*Nz shorthand

    real sontz

      one / sqrt (one*Ntz); shorthand
• real, dimension(:,:,:), allocatable rij
      real-space grid; R

    real, dimension(:,:,:), allocatable zij

      real-space grid; Z

    real, dimension(:,:,:), allocatable xij

      what is this?

    real, dimension(:,:,:), allocatable yij

      what is this?
• real, dimension(:,:), allocatable sg
      real-space grid; jacobian and its derivatives

    real, dimension(:,:,:,:), allocatable guvij

      real-space grid; metric elements

    real, dimension(:,:,:), allocatable gvuij

      real-space grid; metric elements (?); 10 Dec 15;

    real, dimension(:,:,:,:), allocatable guvijsave

      what is this?

    integer, dimension(:,:), allocatable ki

      identification of Fourier modes

    integer, dimension(:,:,:), allocatable kijs

      identification of Fourier modes

    integer, dimension(:,:,:), allocatable kija

      identification of Fourier modes

    integer, dimension(:), allocatable iotakkii

      identification of Fourier modes

    integer, dimension(:,:), allocatable iotaksub
```

identification of Fourier modes

 integer, dimension(:,:), allocatable iotakadd identification of Fourier modes

```
    integer, dimension(:,:), allocatable iotaksgn

      identification of Fourier modes
• real, dimension(:), allocatable efmn
      Fourier harmonics; dummy workspace.
• real, dimension(:), allocatable ofmn
      Fourier harmonics; dummy workspace.

    real, dimension(:), allocatable cfmn

      Fourier harmonics; dummy workspace.
• real, dimension(:), allocatable sfmn
      Fourier harmonics; dummy workspace.

    real, dimension(:), allocatable evmn

      Fourier harmonics; dummy workspace.

    real, dimension(:), allocatable odmn

      Fourier harmonics; dummy workspace.
• real, dimension(:), allocatable comn
      Fourier harmonics; dummy workspace.
• real, dimension(:), allocatable simn
      Fourier harmonics; dummy workspace.
• real, dimension(:), allocatable ijreal
      what is this?
· real, dimension(:), allocatable ijimag
      what is this?

    real, dimension(:), allocatable jireal

      what is this?
· real, dimension(:), allocatable jiimag
      what is this?

    real, dimension(:), allocatable jkreal

      what is this?
· real, dimension(:), allocatable jkimag
      what is this?
• real, dimension(:), allocatable kireal
      what is this?
· real, dimension(:), allocatable kjimag
      what is this?

    real, dimension(:,:,:), allocatable bsupumn

      tangential field on interfaces; \theta-component; required for virtual casing construction of field; 11 Oct 12

    real, dimension(:,:,:), allocatable bsupvmn

      tangential field on interfaces; \zeta -component; required for virtual casing construction of field; 11 Oct 12

    real, dimension(:,:), allocatable goomne

      described in preset()
• real, dimension(:,:), allocatable goomno
      described in preset()
• real, dimension(:,:), allocatable gssmne
      described in preset()

    real, dimension(:,:), allocatable gssmno

      described in preset()

    real, dimension(:,:), allocatable gstmne

      described in preset()

    real, dimension(:,:), allocatable gstmno
```

```
described in preset()
• real, dimension(:,:), allocatable gszmne
      described in preset()

    real, dimension(:,:), allocatable gszmno

      described in preset()

    real, dimension(:,:), allocatable gttmne

      described in preset()

    real, dimension(:,:), allocatable gttmno

      described in preset()

    real, dimension(:,:), allocatable gtzmne

      described in preset()

    real, dimension(:,:), allocatable gtzmno

      described in preset()
• real, dimension(:,:), allocatable gzzmne
      described in preset()
• real, dimension(:,:), allocatable gzzmno
      described in preset()

    real, dimension(:,:,:,:), allocatable dtoocc

      volume-integrated Chebychev-metrics; see matrix()

    real, dimension(:,:,:,:), allocatable dtoocs

      volume-integrated Chebychev-metrics; see matrix()

    real, dimension(:,:,:,:), allocatable dtoosc

      volume-integrated Chebychev-metrics; see matrix()

    real, dimension(:,:,:,:), allocatable dtooss

      volume-integrated Chebychev-metrics; see matrix()
• real, dimension(:,:,:,:), allocatable ttsscc
      volume-integrated Chebychev-metrics; see matrix()

    real, dimension(:,:,:,:), allocatable ttsscs

      volume-integrated Chebychev-metrics; see matrix()

    real, dimension(:,:,:,:), allocatable ttsssc

      volume-integrated Chebychev-metrics; see matrix()

    real, dimension(:,:,:,:), allocatable ttssss

      volume-integrated Chebychev-metrics; see matrix()

    real, dimension(:,:,:,:), allocatable tdstcc

      volume-integrated Chebychev-metrics; see matrix()

    real, dimension(:,:,:,:), allocatable tdstcs

      volume-integrated Chebychev-metrics; see matrix()

    real, dimension(:,:,:,:), allocatable tdstsc

      volume-integrated Chebychev-metrics; see matrix()

    real, dimension(:,:,:,:), allocatable tdstss

      volume-integrated Chebychev-metrics; see matrix()

    real, dimension(:,:,:,:), allocatable tdszcc

      volume-integrated Chebychev-metrics; see matrix()

    real, dimension(:,:,:,:), allocatable tdszcs

      volume-integrated Chebychev-metrics; see matrix()

    real, dimension(:,:,:,:), allocatable tdszsc

      volume-integrated Chebychev-metrics; see matrix()

    real, dimension(:,:,:,:), allocatable tdszss

      volume-integrated Chebychev-metrics; see matrix()

    real, dimension(:,:,:,:), allocatable ddttcc
```

volume-integrated Chebychev-metrics; see matrix()

```
• real, dimension(:,:,:,:), allocatable ddttcs
      volume-integrated Chebychev-metrics; see matrix()

    real, dimension(:,:,:,:), allocatable ddttsc

      volume-integrated Chebychev-metrics; see matrix()

    real, dimension(:,:,:,:), allocatable ddttss

      volume-integrated Chebychev-metrics; see matrix()
• real, dimension(:,:,:), allocatable ddtzcc
      volume-integrated Chebychev-metrics; see matrix()

    real, dimension(:,:,:,:), allocatable ddtzcs

      volume-integrated Chebychev-metrics; see matrix()
• real, dimension(:,:,:,:), allocatable ddtzsc
      volume-integrated Chebychev-metrics; see matrix()

    real, dimension(:,:,:,:), allocatable ddtzss

      volume-integrated Chebychev-metrics; see matrix()

    real, dimension(:,:,:,:), allocatable ddzzcc

      volume-integrated Chebychev-metrics; see matrix()

    real, dimension(:,:,:,:), allocatable ddzzcs

      volume-integrated Chebychev-metrics; see matrix()

    real, dimension(:,:,:,:), allocatable ddzzsc

      volume-integrated Chebychev-metrics; see matrix()

    real, dimension(:,:,:,:), allocatable ddzzss

      volume-integrated Chebychev-metrics; see matrix()
• real, dimension(:,:), allocatable tsc
      what is this?

    real, dimension(:,:), allocatable tss

      what is this?
• real, dimension(:,:), allocatable dtc
      what is this?
• real, dimension(:,:), allocatable dts
      what is this?
• real, dimension(:,:), allocatable dzc
      what is this?
• real, dimension(:,:), allocatable dzs
      what is this?
• real, dimension(:,:), allocatable ttc
      what is this?

    real, dimension(:,:), allocatable tzc

      what is this?
• real, dimension(:,:), allocatable tts
      what is this?

    real, dimension(:,:), allocatable tzs

      what is this?
• real, dimension(:), allocatable dtflux
      \delta \psi_{toroidal} in each annulus
· real, dimension(:), allocatable dpflux
      \delta\psi_{poloidal} in each annulus

    real, dimension(:), allocatable sweight

      minimum poloidal length constraint weight

    integer, dimension(:), allocatable nadof

      degrees of freedom in Beltrami fields in each annulus
```

integer, dimension(:), allocatable nfielddof

degrees of freedom in Beltrami fields in each annulus, field only, no Lagrange multipliers type(subgrid), dimension(:,:,:), allocatable ate magnetic vector potential cosine Fourier harmonics; stellarator-symmetric type(subgrid), dimension(:,:,:), allocatable aze magnetic vector potential cosine Fourier harmonics; stellarator-symmetric • type(subgrid), dimension(:,:,:), allocatable ato magnetic vector potential sine Fourier harmonics; non-stellarator-symmetric type(subgrid), dimension(:,:,:), allocatable azo magnetic vector potential sine Fourier harmonics; non-stellarator-symmetric • integer, dimension(:,:), allocatable Ima Lagrange multipliers (?) • integer, dimension(:,:), allocatable Imb Lagrange multipliers (?) • integer, dimension(:,:), allocatable Imc Lagrange multipliers (?) integer, dimension(:,:), allocatable Imd Lagrange multipliers (?) integer, dimension(:,:), allocatable Ime Lagrange multipliers (?) integer, dimension(:,:), allocatable Imf Lagrange multipliers (?) integer, dimension(:,:), allocatable Img Lagrange multipliers (?) integer, dimension(:,:), allocatable lmh Lagrange multipliers (?) • real, dimension(:,:), allocatable Imavalue what is this? • real, dimension(:,:), allocatable Imbvalue what is this? real, dimension(:,:), allocatable Imcvalue what is this? • real, dimension(:,:), allocatable Imdvalue what is this? • real, dimension(:,:), allocatable Imevalue what is this? real, dimension(:,:), allocatable Imfvalue what is this? • real, dimension(:,:), allocatable Imgvalue what is this? real, dimension(:,:), allocatable Imhvalue what is this? integer, dimension(:,:), allocatable fso what is this? • integer, dimension(:,:), allocatable fse what is this? logical lcoordinatesingularity set by LREGION macro; true if inside the innermost volume logical lplasmaregion

logical lvacuumregion

set by LREGION macro; true if inside the plasma region

set by LREGION macro; true if inside the vacuum region

logical Isavedguvij

```
flag used in matrix free

    logical localconstraint

      what is this?
• real, dimension(:,:), allocatable dma
      energy and helicity matrices; quadratic forms
• real, dimension(:,:), allocatable dmb
      energy and helicity matrices; quadratic forms

    real, dimension(:,:), allocatable dmd

      energy and helicity matrices; quadratic forms
• real, dimension(:), allocatable dmas
      sparse version of dMA, data

    real, dimension(:), allocatable dmds

      sparse version of dMD, data
• integer, dimension(:), allocatable idmas
      sparse version of dMA and dMD, indices
• integer, dimension(:), allocatable jdmas
      sparse version of dMA and dMD, indices

    integer, dimension(:), allocatable ndmasmax

      number of elements for sparse matrices
• integer, dimension(:), allocatable ndmas
      number of elements for sparse matrices
· real, dimension(:), allocatable dmg
      what is this?
• real, dimension(:,:), allocatable solution
      this is allocated in dforce; used in mp00ac and ma02aa; and is passed to packab
• real, dimension(:,:,:), allocatable gmreslastsolution
      used to store the last solution for restarting GMRES
• real, dimension(:), allocatable mbpsi
      matrix vector products
· logical liluprecond
      whether to use ILU preconditioner for GMRES
• real, dimension(:,:), allocatable beltramiinverse
      what is this?
• real, dimension(:,:,:), allocatable diotadxup
      measured rotational transform on inner/outer interfaces for each volume; d(transform)/dx; (see dforce)

    real, dimension(:,:,:), allocatable ditgpdxtp

      measured toroidal and poloidal current on inner/outer interfaces for each volume; d(Itor, Gpol)/dx; (see dforce)

    real, dimension(:,:,:,:), allocatable glambda

      save initial guesses for iterative calculation of rotational-transform

    integer Imns

      what is this?

    real, dimension(:,:,:), allocatable bemn

      force vector; stellarator-symmetric (?)
• real, dimension(:,:), allocatable iomn
      force vector; stellarator-symmetric (?)

    real, dimension(:,:,:), allocatable somn

      force vector; non-stellarator-symmetric (?)

    real, dimension(:,:,:), allocatable pomn

      force vector; non-stellarator-symmetric (?)

    real, dimension(:,:,:), allocatable bomn
```

```
force vector; stellarator-symmetric (?)

    real, dimension(:,:), allocatable iemn

      force vector; stellarator-symmetric (?)
• real, dimension(:,:,:), allocatable semn
      force vector; non-stellarator-symmetric (?)

    real, dimension(:,:,:), allocatable pemn

      force vector; non-stellarator-symmetric (?)

    real, dimension(:), allocatable bbe

      force vector (?); stellarator-symmetric (?)

    real, dimension(:), allocatable iio

      force vector (?); stellarator-symmetric (?)

    real, dimension(:), allocatable bbo

      force vector (?); non-stellarator-symmetric (?)
• real, dimension(:), allocatable iie
      force vector (?); non-stellarator-symmetric (?)

    real, dimension(:,:,:), allocatable btemn

      covariant \theta cosine component of the tangential field on interfaces; stellarator-symmetric

    real, dimension(:,:,:), allocatable bzemn

      covariant \zeta cosine component of the tangential field on interfaces; stellarator-symmetric

    real, dimension(:,:,:), allocatable btomn

      covariant \theta sine component of the tangential field on interfaces; non-stellarator-symmetric

    real, dimension(:,:,:), allocatable bzomn

      covariant ( sine component of the tangential field on interfaces; non-stellarator-symmetric

    real, dimension(:,:), allocatable bloweremn

      covariant field for Hessian computation
• real, dimension(:,:), allocatable bloweromn
      covariant field for Hessian computation

    integer lgdof

      geometrical degrees of freedom associated with each interface

    integer ngdof

      total geometrical degrees of freedom

    real, dimension(:,:,:), allocatable dbbdrz

      derivative of magnetic field w.r.t. geometry (?)

    real, dimension(:,:), allocatable diidrz

      derivative of spectral constraints w.r.t. geometry (?)

    real, dimension(:,:,:,:), allocatable dffdrz

      derivatives of B^{\wedge}2 at the interfaces wrt geometry
• real, dimension(:,:,:,:), allocatable dbbdmp
      derivatives of B^{\wedge}2 at the interfaces wrt mu and dpflux

    real, dimension(:,:,:,:), allocatable dmupfdx

      derivatives of mu and dpflux wrt geometry at constant interface transform
· logical lhessianallocated
      flag to indicate that force gradient matrix is allocated (?)
• real, dimension(:,:), allocatable hessian
      force gradient matrix (?)

    real, dimension(:,:), allocatable dessian

      derivative of force gradient matrix (?)
• real, dimension(:,:), allocatable cosi
      some precomputed cosines

    real, dimension(:,:), allocatable sini

      some precomputed sines
```

real, dimension(1:3) orzp

```
    real, dimension(:), allocatable gteta

      something related to \sqrt{g} and \theta ?
· real, dimension(:), allocatable gzeta
      something related to \sqrt{g} and \zeta?

    real, dimension(:), allocatable ajk

      definition of coordinate axis
• real, dimension(:,:,:,:), allocatable dradr
      derivatives of coordinate axis

    real, dimension(:,:,:,:), allocatable dradz

      derivatives of coordinate axis

    real, dimension(:,:,:,:), allocatable dzadr

      derivatives of coordinate axis

    real, dimension(:,:,:,:), allocatable dzadz

      derivatives of coordinate axis

    real, dimension(:,:,:), allocatable drodr

      derivatives of coordinate axis

    real, dimension(:,:,:), allocatable drodz

      derivatives of coordinate axis

    real, dimension(:,:,:), allocatable dzodr

      derivatives of coordinate axis
• real, dimension(:,:,:), allocatable dzodz
      derivatives of coordinate axis
• integer, dimension(:,:), allocatable djkp
      for calculating cylindrical volume

    integer, dimension(:,:), allocatable djkm

      for calculating cylindrical volume

    real, dimension(:), allocatable lbbintegral

      B.B integral.
• real, dimension(:), allocatable labintegral
      A.B integral.
· real, dimension(:), allocatable vvolume
      volume integral of \sqrt{g}; computed in volume
· real dvolume
      derivative of volume w.r.t. interface geometry

    integer ivol

      labels volume; some subroutines (called by NAG) are fixed argument list but require the volume label
· real gbzeta
      toroidal (contravariant) field; calculated in bfield; required to convert \dot{\theta} to B^{\theta}, \dot{s} to B^{s}
• integer, dimension(:), allocatable iquad
      internal copy of Nquad

    real, dimension(:,:), allocatable gaussianweight

      weights for Gaussian quadrature
• real, dimension(:,:), allocatable gaussianabscissae
      abscissae for Gaussian quadrature
· logical Iblinear
      controls selection of Beltrami field solver; depends on LBeltrami

    logical Ibnewton

      controls selection of Beltrami field solver; depends on LBeltrami

    logical lbsequad

      controls selection of Beltrami field solver; depends on LBeltrami
```

```
used in mg00aa() to determine (s, \theta, \zeta) given (R, Z, \varphi)
```

• type(derivative) dbdx

 $d\mathbf{B}/d\mathbf{X}$  (?)

· integer globaljk

labels position

real, dimension(:,:), allocatable dxyz
 computational boundary; position

 real, dimension(:,:), allocatable nxyz computational boundary; normal

• real, dimension(:,:), allocatable jxyz

plasma boundary; surface current

• real, dimension(1:2) tetazeta

what is this?

• real virtualcasingfactor = -one / (four\*pi)

this agrees with diagno

integer iberror

for computing error in magnetic field

• integer nfreeboundaryiterations

number of free-boundary iterations already performed

• integer, parameter node = 2

best to make this global for consistency between calling and called routines

• logical first\_free\_bound = .false.

flag to indicate that this is the first free-boundary iteration

## 9.1.1 Detailed Description

global variable storage used as "workspace" throughout the code

# 9.1.2 Function/Subroutine Documentation

## 9.1.2.1 readin() subroutine allglobal::readin

The master node reads the input namelist and sets various internal variables. The relevant quantities are then broadcast.

#### machine precision

- The machine precision machprec is determined using myprec(), which is similar to corresponding the NAG routine
- The variables vsmall, small and sqrtmachprec are set.

#### input file extension (command line argument)

- The input file name, ext, is given as the first command line input, and the input file itself is  $ext \cdot sp$ 

- · Additional command line inputs recognized are:
  - help, -h: will give help information to user; under construction
  - readin will immediately set Wreadin=T; this may be over-ruled when the screenlist is read

#### reading of physicslist

- The internal variable, Mvol=Nvol+Lfreebound, gives the number of computational domains.
- The input value for the fluxes enclosed within each interface, tflux (1:Mvol) and tflux (1:Mvol), are immediately normalized:

```
tflux(1:Mvol) \rightarrow tflux(1:Mvol)/tflux(Nvol).
pflux(1:Mvol) \rightarrow pflux(1:Mvol)/tflux(Nvol).
```

The input  $\Phi_{edge} \equiv \text{phiedge}$  will provide the total toroidal flux; see preset().

• The input value for the toroidal current constraint (Isurf (1:Mvol) and Ivolume (1:Mvol) ) are also immediately normalized, using curtor .  $Ivolume \rightarrow Ivolume \cdot \frac{curtor}{\sum_i Isurf_i + Ivolume_i} Isurf \rightarrow Isurf \cdot \frac{curtor}{\sum_i Isurf_i + Ivolume_i}$ 

## **Current profiles normalization**

In case of a free boundary calculation (Lfreebound=1) and using a current constraint (Lconstraint=3), the current profiles are renormalized in order to match the linking current curtor. More specifically,

$$Isurf_i \rightarrow Isurf_i \cdot \frac{curtor}{\sum_{i=1}^{Mvol-1} Isurf_i + Ivol_i} Ivol_i \rightarrow Ivol_i \cdot \frac{curtor}{\sum_{i=1}^{Mvol-1} Isurf_i + Ivol_i}$$
(277)

Finally, the volume current in the vacuum region is set to 0.

reading of numericlist

reading of locallist

reading of globallist

reading of diagnosticslist

reading of screenlist

broadcast command line input

broadcast physicslist

broadcast numericlist

broadcast globallist

broadcast locallist

broadcast diagnosticslist

broadcast screenlist

set internal parameters that depend on physicslist

total number of volumes: Mvol

• The number of plasma volumes is Mvol = Nvol + Lfreebound.

## Fourier mode identification: mn, im(1:mn) and in(1:mn)

· The Fourier description of even periodic functions is

$$f(\theta,\zeta) = \sum_{n=0}^{N} f_{0,n} \cos(-n\zeta) + \sum_{m=1}^{M} \sum_{n=-N}^{N} f_{m,n} \cos(m\theta - n\zeta),$$
 (278)

where the resolution is given on input,  $M \equiv \text{Mpol}$  and  $N \equiv \text{Ntor}$ .

· For convenience, the Fourier summations are written as

$$f(s,\theta,\zeta) = \sum_{j} f_{j}(s) \cos(m_{j}\theta - n_{j}\zeta), \tag{279}$$

for j = 1, mn, where mn = N + 1 + M(2N + 1).

- The integer arrays im(1:mn) and in(1:mn) contain the  $m_j$  and  $n_j$ .
- The array in includes the Nfp factor.

### regularization factor: halfmm(1:mn), regumm(1:mn)

- The "regularization" factor, halfmm (1:mn) = im(1:mn) \* half, is real.
- This is used in Iforce(), bfield(), stzxyz(), coords(), jo00aa(), ma00aa(), sc00aa() and tr00ab().

## extended resolution Fourier mode identification: mne, ime and ine

• The "extended" Fourier resolution is defined by  ${\tt lMpol}=4\,{\tt Mpol},\,{\tt lNtor}=4{\tt Ntor}.$ 

Fourier mode identification for straight-fieldline angle: mns, ims and ins

set internal parameters that depend on numericlist

set internal parameters that depend on locallist

set internal parameters that depend on globallist

set internal parameters that depend on diagnosticslist

geometry: iRbc(1:mn,0:Mvol, iZbs(1:mn,0:Mvol), iRbs(1:mn,0:Mvol) and iZbc(1:mn,0:Mvol)

- iRbc, iZbs, iRbs and iZbc: Fourier harmonics of interface geometry
- iVns, iVnc, iBns and iBns: Fourier harmonics of normal field at computational boundary

## construction of coordinate axis: ajk

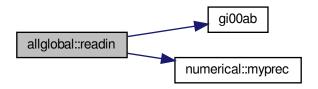
· This is only used in rzaxis() to perform the poloidal integration and is defined quite simply:

ajk
$$[i]\equiv 2\pi$$
 if  $m_i=0$ , and ajk $[i]\equiv 0$  if  $m_i 
eq 0$ .

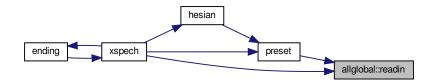
References inputlist::adiabatic, ajk, beltramierror, inputlist::bnc, inputlist::bns, inputlist::c05factor, inputlist::c05xmax, inputlist::c05xtol, cpus, inputlist::curpol, inputlist::curtor, inputlist::dpp, inputlist::dqq, drbc, drbs, inputlist::drz, dzbc, dzbs, inputlist::epsgmres, inputlist::epsilon, inputlist::epsilu, inputlist::escale, inputlist::ext, inputlist::forcetol, inputlist::gamma, inputlist::gbnbld, inputlist::gbntol, gi00ab(), constants::half, halfmm, inputlist::helicity, ibnc, ibns, inputlist::igeometry, im, ime, inputlist::imethod, inputlist::impol, ims, in, ine, ins, inputlist::intor, inputlist::iorder, inputlist::iota, inputlist::iotatol, inputlist::iprecon, irbc, irbs, inputlist::istellsym, inputlist::isurf, fileunits::iunit, ivnc, ivns, inputlist::ivolume, izbc, izbs, inputlist::ladiabatic, inputlist::lautoinitbn, inputlist::lbeltrami, inputlist::lcheck, inputlist. ::lconstraint, inputlist::lerrortype, inputlist::lextrap, inputlist::lfindzero, inputlist::lfreebound, inputlist::lgmresprec, inputlist::lhevalues, inputlist::lhevectors, inputlist::lhmatrix, inputlist::linitgues, inputlist::linitialize, inputlist::lmatsolver, Impol, Intor, inputlist::lp, inputlist::lperturbed, inputlist::lq, inputlist::lreadgf, inputlist::lreadgf ::lrzaxis, inputlist::lsparse, inputlist::lsvdiota, inputlist::ltiming, inputlist::lzerovac, numerical::machprec, inputlist ::maxrndgues, inputlist::mfreeits, inputlist::mmpol, mn, mne, mns, inputlist::mntor, inputlist::mnvol, inputlist: ::mpol, inputlist::mregular, inputlist::mu, inputlist::mupfits, inputlist::mupftol, mvol, myid, numerical::myprec(), inputlist::ndiscrete, inputlist::nfp, inputlist::ngrid, inputlist::nitergmres, notstellsym, inputlist::npts, inputlist::nptrj, inputlist::nquad, inputlist::ntor, inputlist::ntoraxis, inputlist::nvol, inputlist::odetol, inputlist::oita, constants::one, inputlist::opsilon, fileunits::ounit, inputlist::pcondense, inputlist::pflux, inputlist::phiedge, constants::pi2, inputlist::pl, inputlist::ppts, inputlist::pr, inputlist::pressure, inputlist::pscale, inputlist::ql, inputlist::qr, inputlist::rac, inputlist::ras, inputlist::rbc, inputlist::rbc, inputlist::rpc, inputlist::rpc, inputlist::rpc, inputlist::rpc, inputlist::rtor, inputlist::rvc, inputlist::rv inputlist::rws, numerical::small, smpol, sntor, numerical::sqrtmachprec, inputlist::tflux, inputlist::upsilon, inputlist ← ::vcasingeps, inputlist::vcasingits, inputlist::vcasingper, inputlist::vcasingtol, inputlist::vnc, inputlist::vns, numerical← ::vsmall, inputlist::wmacros, inputlist::wpoloidal, inputlist::wreadin, inputlist::wwrtend, yesstellsym, inputlist::zac, inputlist::zas, inputlist::zbs, inputlist::zbs, constants::zero, inputlist::zwc, and inputlist::zws.

Referenced by preset(), and xspech().

Here is the call graph for this function:



Here is the caller graph for this function:



## 9.2 constants Module Reference

some constants used throughout the code

#### **Variables**

```
• real, parameter zero = 0.0
• real, parameter one = 1.0
• real, parameter two = 2.0
• real, parameter three = 3.0
• real, parameter four = 4.0
• real, parameter five = 5.0
• real, parameter six = 6.0
• real, parameter seven = 7.0
• real, parameter eight = 8.0
• real, parameter nine = 9.0
• real, parameter ten = 10.0
• real, parameter eleven = 11.0
• real, parameter twelve = 12.0
• real, parameter hundred = 100.0
• real, parameter thousand = 1000.0
     1000
• real, parameter half = one / two
     1/2

 real, parameter third = one / three

• real, parameter quart = one / four

    real, parameter fifth = one / five

• real, parameter sixth = one / six

    real, parameter pi2 = 6.28318530717958623

• real, parameter pi = pi2 / two
• real, parameter mu0 = 2.0E-07 * pi2
• real, parameter goldenmean = 1.618033988749895
     golden mean = (1+\sqrt{5})/2;
• real, parameter version = 2.00
     version of SPEC
```

## 9.2.1 Detailed Description

some constants used throughout the code

# 9.3 cputiming Module Reference

timing variables

#### **Variables**

```
    real treadin = 0.0
        timing of readin()
    real twrtend = 0.0
        timing of wrtend()
```

## 9.3.1 Detailed Description

timing variables

# 9.4 fftw\_interface Module Reference

Interface to FFTW library.

#### **Variables**

```
    type(c_ptr) planf
        FFTW-related (?)
    type(c_ptr) planb
        FFTW-related (?)
    complex(c_double_complex), dimension(:,:,:), allocatable cplxin
        FFTW-related (?)
    complex(c_double_complex), dimension(:,:,:), allocatable cplxout
```

## 9.4.1 Detailed Description

FFTW-related (?)

Interface to FFTW library.

## 9.5 fileunits Module Reference

central definition of file units to avoid conflicts

#### **Variables**

```
• integer iunit = 10
```

input; used in global/readin:ext.sp, global/wrtend:ext.sp.end

• integer ounit = 6

screen output;

• integer gunit = 13

wall geometry; used in wa00aa

• integer aunit = 11

vector potential; used in ra00aa:.ext.AtAzmn;

• integer dunit = 12

derivative matrix; used in newton:.ext.GF;

• integer hunit = 14

eigenvalues of Hessian; under re-construction;

• integer munit = 14

matrix elements of Hessian;

integer lunit = 20

local unit; used in lunit+myid: pp00aa:.ext.poincare,.ext.transform;

• integer vunit = 15

for examination of adaptive quadrature; used in casing:.ext.vcint;

## 9.5.1 Detailed Description

central definition of file units to avoid conflicts

# 9.6 newtontime Module Reference

timing of Newton iterations

## **Variables**

• integer nfcalls

number of calls to get function values (?)

· integer ndcalls

number of calls to get derivative values (?)

real lastcpu

last CPU that called this (?)

## 9.6.1 Detailed Description

timing of Newton iterations

## 9.7 numerical Module Reference

platform-dependant numerical resolution

## **Functions/Subroutines**

real function myprec ()

Duplicates NAG routine X02AJF (machine precision)

#### **Variables**

· real machprec

machine precision according to NAG-like routine myprec()

real vsmall

very small number

· real small

small number

· real sqrtmachprec

square root of machine precision

• real, parameter logtolerance = 1.0e-32

this is used to avoid taking alog10(zero); see e.g. dforce()

### 9.7.1 Detailed Description

platform-dependant numerical resolution

## 9.7.2 Function/Subroutine Documentation

# 9.7.2.1 myprec() real function numerical::myprec

Duplicates NAG routine X02AJF (machine precision)

JAB; 27 Jul 17 I suggest that this be removed; SRH: 27 Feb 18;

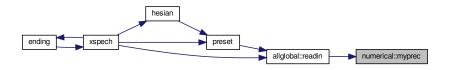
Returns

machine precision

References small.

Referenced by allglobal::readin().

Here is the caller graph for this function:



# 9.8 typedefns Module Reference

type definitions for custom datatypes

# **Data Types**

- · type matrixlu
- · type subgrid

used for quantities which have different resolutions in different volumes, e.g. the vector potential More...

## 9.8.1 Detailed Description

type definitions for custom datatypes

#### 9.8.2 Data Type Documentation

#### **Class Members**

real, dimension(:,:), allocatable	mat	
integer, dimension(:), allocatable	ipivot	

## 9.8.2.1 type typedefns::matrixlu

**9.8.2.2 type typedefns::subgrid** used for quantities which have different resolutions in different volumes, e.g. the vector potential

#### **Class Members**

real, dimension(:), allocatable	s	coefficients
integer, dimension(:), allocatable	i	indices

# 10 Data Type Documentation

# 10.1 intghs\_module::intghs\_workspace Type Reference

## **Public Attributes**

- real, dimension(:,:), allocatable efmn
- real, dimension(:,:), allocatable ofmn
- real, dimension(:,:), allocatable cfmn
- real, dimension(:,:), allocatable sfmn
- real, dimension(:,:), allocatable evmn
- real, dimension(:,:), allocatable odmn
- real, dimension(:,:), allocatable ijreal

- real, dimension(:,:), allocatable jireal
- real, dimension(:,:), allocatable jkreal
- real, dimension(:,:), allocatable kjreal
- real, dimension(:,:,:), allocatable **bloweremn**
- real, dimension(:,:,:), allocatable **bloweromn**
- real, dimension(:,:,:), allocatable gbupper
- real, dimension(:,:,:), allocatable **blower**
- real, dimension(:,:,:,:), allocatable basis

#### 10.1.1 Member Data Documentation

10.1.1.1 efmn real, dimension(:,:), allocatable intghs\_module::intghs\_workspace::efmn  $\textbf{10.1.1.2} \quad \textbf{ofmn} \quad \texttt{real, dimension(:,:), allocatable intghs\_module::intghs\_workspace::ofmn}$ 10.1.1.3 cfmn real, dimension(:,:), allocatable intghs\_module::intghs\_workspace::cfmn 10.1.1.4 sfmn real, dimension(:,:), allocatable intghs\_module::intghs\_workspace::sfmn  $\textbf{10.1.1.5} \quad \textbf{evmn} \quad \texttt{real, dimension(:,:), allocatable intghs\_module::intghs\_workspace::evmn}$  $\textbf{10.1.1.6} \quad \textbf{odmn} \quad \texttt{real, dimension}(:,:), \ \texttt{allocatable intghs\_module::intghs\_workspace::odmn}$ 10.1.1.7 ijreal real, dimension(:,:), allocatable intghs\_module::intghs\_workspace::ijreal 10.1.1.8 jireal real, dimension(:,:), allocatable intghs\_module::intghs\_workspace::jireal 11 File Documentation 175

```
10.1.1.9 jkreal real, dimension(:,:), allocatable intghs_module::intghs_workspace::jkreal
```

```
\textbf{10.1.1.10} \quad \textbf{kjreal} \quad \texttt{real, dimension(:,:), allocatable intghs\_module::intghs\_workspace::kjreal}
```

```
10.1.1.11 bloweremn real, dimension(:,:,:), allocatable intghs_module::intghs_workspace↔ ::bloweremn
```

```
\textbf{10.1.1.13} \quad \textbf{gbupper} \quad \texttt{real, dimension} (:,:,:), \quad \texttt{allocatable intghs\_module::intghs\_workspace::gbupper}
```

10.1.1.14 blower real, dimension(:,:,:), allocatable intghs\_module::intghs\_workspace::blower

 $\textbf{10.1.1.15} \quad \textbf{basis} \quad \texttt{real, dimension} (:,:,:,:), \quad \texttt{allocatable intghs\_module} :: \texttt{intghs\_workspace} :: \texttt{basis}$ 

The documentation for this type was generated from the following file:

• intghs.f90

# 11 File Documentation

# 11.1 bfield.f90 File Reference

Returns  $\dot{s} \equiv B^s/B^{\zeta}$  and  $\dot{\theta} \equiv B^{\theta}/B^{\zeta}$ .

#### **Functions/Subroutines**

- subroutine bfield (zeta, st, Bst)
  - Compute the magnetic field.
- subroutine bfield\_tangent (zeta, st, Bst)

#### 11.1.1 Detailed Description

Returns  $\dot{s} \equiv B^s/B^\zeta$  and  $\dot{\theta} \equiv B^\theta/B^\zeta$ .

#### 11.2 bnorml.f90 File Reference

Computes  $\mathbf{B}_{Plasma} \cdot \mathbf{e}_{\theta} \times \mathbf{e}_{\zeta}$  on the computational boundary,  $\partial \mathcal{D}$ .

## **Functions/Subroutines**

• subroutine bnorml (mn, Ntz, efmn, ofmn)  $\textit{Computes $B_{Plasma} \cdot e_{\theta} \times e_{\zeta}$ on the computational boundary, $\partial \mathcal{D}$.}$ 

## 11.2.1 Detailed Description

Computes  $\mathbf{B}_{Plasma} \cdot \mathbf{e}_{\theta} \times \mathbf{e}_{\zeta}$  on the computational boundary,  $\partial \mathcal{D}$ .

# 11.3 brcast.f90 File Reference

Broadcasts Beltrami fields, profiles, . . .

#### **Functions/Subroutines**

• subroutine brcast (Ivol)

Broadcasts Beltrami fields, profiles, . . .

#### 11.3.1 Detailed Description

Broadcasts Beltrami fields, profiles, . . .

# 11.4 casing.f90 File Reference

Constructs the field created by the plasma currents, at an arbitrary, external location using virtual casing.

#### **Functions/Subroutines**

- subroutine casing (teta, zeta, gBn, icasing)
   Constructs the field created by the plasma currents, at an arbitrary, external location using virtual casing.
- subroutine dvcfield (Ndim, tz, Nfun, vcintegrand)

Differential virtual casing integrand.

## 11.4.1 Detailed Description

Constructs the field created by the plasma currents, at an arbitrary, external location using virtual casing.

#### 11.5 coords.f90 File Reference

Calculates coordinates,  $\mathbf{x}(s, \theta, \zeta) \equiv R \mathbf{e}_R + Z \mathbf{e}_Z$ , and metrics, using FFTs.

#### **Functions/Subroutines**

• subroutine coords (Ivol, Iss, Lcurvature, Ntz, mn) Calculates coordinates,  $\mathbf{x}(s, \theta, \zeta) \equiv R \, \mathbf{e}_R + Z \, \mathbf{e}_Z$ , and metrics, using FFTs.

#### 11.5.1 Detailed Description

Calculates coordinates,  $\mathbf{x}(s, \theta, \zeta) \equiv R \mathbf{e}_R + Z \mathbf{e}_Z$ , and metrics, using FFTs.

#### 11.6 curent.f90 File Reference

Computes the plasma current,  $I \equiv \int B_{\theta} d\theta$ , and the "linking" current,  $G \equiv \int B_{\zeta} d\zeta$ .

## **Functions/Subroutines**

• subroutine curent (Ivol, mn, Nt, Nz, iflag, IdItGp) Computes the plasma current,  $I \equiv \int B_{\theta} \, d\theta$ , and the "linking" current,  $G \equiv \int B_{\zeta} \, d\zeta$ .

#### 11.6.1 Detailed Description

Computes the plasma current,  $I \equiv \int B_{\theta} d\theta$ , and the "linking" current,  $G \equiv \int B_{\zeta} d\zeta$ .

# 11.7 df00ab.f90 File Reference

Evaluates volume integrals, and their derivatives w.r.t. interface geometry, using "packed" format.

#### **Functions/Subroutines**

• subroutine df00ab (pNN, xi, Fxi, DFxi, Ldfjac, iflag)

Evaluates volume integrals, and their derivatives w.r.t. interface geometry, using "packed" format.

#### 11.7.1 Detailed Description

Evaluates volume integrals, and their derivatives w.r.t. interface geometry, using "packed" format.

# 11.8 global.f90 File Reference

Defines input namelists and global variables, and opens some output files.

#### **Data Types**

· type typedefns::subgrid

used for quantities which have different resolutions in different volumes, e.g. the vector potential More...

- · type typedefns::matrixlu
- · type allglobal::derivative

 $\mathrm{d}\mathbf{B}/\mathrm{d}\mathbf{X}$  (?) More...

#### **Modules**

· module constants

some constants used throughout the code

· module numerical

platform-dependant numerical resolution

module fileunits

central definition of file units to avoid conflicts

· module cputiming

timing variables

· module typedefns

type definitions for custom datatypes

· module allglobal

global variable storage used as "workspace" throughout the code

• module fftw\_interface

Interface to FFTW library.

#### **Functions/Subroutines**

• real function numerical::myprec ()

Duplicates NAG routine X02AJF (machine precision)

- subroutine allglobal::build\_vector\_potential (Ivol, iocons, aderiv, tderiv)
- subroutine allglobal::readin

The master node reads the input namelist and sets various internal variables. The relevant quantities are then broadcast.

· subroutine allglobal::wrtend

The restart file is written.

• subroutine allglobal::ismyvolume (vvol)

Check if volume vvol is associated to the corresponding MPI node.

subroutine allglobal::whichcpuid (vvol, cpu\_id)

Returns which MPI node is associated to a given volume.

#### **Variables**

```
• real, parameter constants::zero = 0.0
• real, parameter constants::one = 1.0
• real, parameter constants::two = 2.0
• real, parameter constants::three = 3.0
• real, parameter constants::four = 4.0
• real, parameter constants::five = 5.0
  real, parameter constants::six = 6.0
• real, parameter constants::seven = 7.0
• real, parameter constants::eight = 8.0
• real, parameter constants::nine = 9.0
• real, parameter constants::ten = 10.0
• real, parameter constants::eleven = 11.0
• real, parameter constants::twelve = 12.0

    real, parameter constants::hundred = 100.0

    real, parameter constants::thousand = 1000.0

• real, parameter constants::half = one / two

    real, parameter constants::third = one / three

• real, parameter constants::quart = one / four
• real, parameter constants::fifth = one / five
• real, parameter constants::sixth = one / six
• real, parameter constants::pi2 = 6.28318530717958623
• real, parameter constants::pi = pi2 / two
real, parameter constants::mu0 = 2.0E-07 * pi2
     4\pi \cdot 10^{-7}
• real, parameter constants::goldenmean = 1.618033988749895
     golden mean = (1+\sqrt{5})/2;
• real, parameter constants::version = 2.00
```

```
version of SPEC
· real numerical::machprec
      machine precision according to NAG-like routine myprec()
· real numerical::vsmall
      very small number
  real numerical::small
     small number
  real numerical::sqrtmachprec
     square root of machine precision
  real, parameter numerical::logtolerance = 1.0e-32
      this is used to avoid taking alog10(zero); see e.g. dforce()
• integer fileunits::iunit = 10
      input; used in global/readin:ext.sp, global/wrtend:ext.sp.end
• integer fileunits::ounit = 6
     screen output;
• integer fileunits::gunit = 13
      wall geometry; used in wa00aa
• integer fileunits::aunit = 11
      vector potential; used in ra00aa:.ext.AtAzmn;
• integer fileunits::dunit = 12
      derivative matrix; used in newton:.ext.GF;
• integer fileunits::hunit = 14
      eigenvalues of Hessian; under re-construction;

    integer fileunits::munit = 14

      matrix elements of Hessian;
• integer fileunits::lunit = 20
      local unit; used in lunit+myid: pp00aa:.ext.poincare,.ext.transform;
• integer fileunits::vunit = 15
      for examination of adaptive quadrature; used in casing:.ext.vcint;
• real cputiming::treadin = 0.0
     timing of readin()
  real cputiming::twrtend = 0.0
     timing of wrtend()
· character inputlist::ext
      The input file is, ext.sp, where ext*100 or ext.sp*100 is given as command line input.

    integer, parameter inputlist::mnvol = 256

      The maximum value of Nvol is MNvol=256.
• integer, parameter inputlist::mmpol = 64
      The maximum value of Mpol is MNpol=64.
integer, parameter inputlist::mntor = 64
      The maximum value of Ntor is MNtor=64.

    integer inputlist::igeometry = 3

     selects Cartesian, cylindrical or toroidal geometry;
• integer inputlist::istellsym = 1
      stellarator symmetry is enforced if Istellsym==1

    integer inputlist::lfreebound = 0

      compute vacuum field surrounding plasma
• real inputlist::phiedge = 1.0
      total enclosed toroidal magnetic flux;
• real inputlist::curtor = 0.0
```

total enclosed (toroidal) plasma current;

```
 real inputlist::curpol = 0.0

      total enclosed (poloidal) linking current;
real inputlist::gamma = 0.0
      adiabatic index; cannot set |\gamma| = 1
• integer inputlist::nfp = 1
      field periodicity
integer inputlist::nvol = 1
      number of volumes
• integer inputlist::mpol = 0
      number of poloidal Fourier harmonics
• integer inputlist::ntor = 0
      number of toroidal Fourier harmonics

    integer, dimension(1:mnvol+1) inputlist::lrad = 4

       Chebyshev resolution in each volume.

    integer inputlist::lconstraint = -1

      selects constraints; primarily used in ma02aa() and mp00ac().

    real, dimension(1:mnvol+1) inputlist::tflux = 0.0

      toroidal flux, \psi_t, enclosed by each interface

    real, dimension(1:mnvol+1) inputlist::pflux = 0.0

      poloidal flux, \psi_p, enclosed by each interface
• real, dimension(1:mnvol) inputlist::helicity = 0.0
      helicity, K, in each volume, V_i
• real inputlist::pscale = 0.0
      pressure scale factor
• real, dimension(1:mnvol+1) inputlist::pressure = 0.0
      pressure in each volume

    integer inputlist::ladiabatic = 0

      logical flag
• real, dimension(1:mnvol+1) inputlist::adiabatic = 0.0
      adiabatic constants in each volume
• real, dimension(1:mnvol+1) inputlist::mu = 0.0
      helicity-multiplier, \mu, in each volume
• real, dimension(1:mnvol+1) inputlist::ivolume = 0.0
       Toroidal current constraint normalized by \mu_0 ( I_{volume} = \mu_0 \cdot [A]), in each volume. This is a cumulative quantity:
      I_{\mathcal{V},i} = \int_0^{\psi_{t,i}} \mathbf{J} \cdot \mathbf{dS}. Physically, it represents the sum of all non-pressure driven currents.

    real, dimension(1:mnvol) inputlist::isurf = 0.0

       Toroidal current normalized by \mu_0 at each interface (cumulative). This is the sum of all pressure driven currents.
• integer, dimension(0:mnvol) inputlist::pl = 0
       "inside" interface rotational-transform is \iota=(p_l+\gamma p_r)/(q_l+\gamma q_r), where \gamma is the golden mean, \gamma=(1+\sqrt{5})/2.

    integer, dimension(0:mnvol) inputlist::ql = 0

       "inside" interface rotational-transform is \iota=(p_l+\gamma p_r)/(q_l+\gamma q_r), where \gamma is the golden mean, \gamma=(1+\sqrt{5})/2.

    integer, dimension(0:mnvol) inputlist::pr = 0

       "inside" interface rotational-transform is \iota=(p_l+\gamma p_r)/(q_l+\gamma q_r), where \gamma is the golden mean, \gamma=(1+\sqrt{5})/2.

    integer, dimension(0:mnvol) inputlist::qr = 0

       "inside" interface rotational-transform is \iota=(p_l+\gamma p_r)/(q_l+\gamma q_r), where \gamma is the golden mean, \gamma=(1+\sqrt{5})/2.

    real, dimension(0:mnvol) inputlist::iota = 0.0

      rotational-transform, t, on inner side of each interface
• integer, dimension(0:mnvol) inputlist::lp = 0
       "outer" interface rotational-transform is \,\iota=(p_l+\gamma p_r)/(q_l+\gamma q_r), where \gamma is the golden mean, \gamma=(1+\sqrt{5})/2.

    integer, dimension(0:mnvol) inputlist::lq = 0

       "outer" interface rotational-transform is \iota=(p_l+\gamma p_r)/(q_l+\gamma q_r), where \gamma is the golden mean, \gamma=(1+\sqrt{5})/2.
```

integer inputlist::lautoinitbn = 1

```
    integer, dimension(0:mnvol) inputlist::rp = 0

      "outer" interface rotational-transform is \iota=(p_l+\gamma p_r)/(q_l+\gamma q_r), where \gamma is the golden mean, \gamma=(1+\sqrt{5})/2.

    integer, dimension(0:mnvol) inputlist::rq = 0

      "outer" interface rotational-transform is \,\iota=(p_l+\gamma p_r)/(q_l+\gamma q_r), where \gamma is the golden mean, \gamma=(1+\sqrt{5})/2.

    real, dimension(0:mnvol) inputlist::oita = 0.0

      rotational-transform, ε, on outer side of each interface
• real inputlist::mupftol = 1.0e-14
      accuracy to which \mu and \Delta\psi_p are required
integer inputlist::mupfits = 8
      an upper limit on the transform/helicity constraint iterations;
• real inputlist::rpol = 1.0
      poloidal extent of slab (effective radius)
real inputlist::rtor = 1.0
      toroidal extent of slab (effective radius)
• integer inputlist::lreflect = 0
      =1 reflect the upper and lower bound in slab, =0 do not reflect

    real, dimension(0:mntor) inputlist::rac = 0.0

      stellarator symmetric coordinate axis;

    real, dimension(0:mntor) inputlist::zas = 0.0

      stellarator symmetric coordinate axis;
• real, dimension( 0:mntor) inputlist::ras = 0.0
      non-stellarator symmetric coordinate axis;

    real, dimension(0:mntor) inputlist::zac = 0.0

      non-stellarator symmetric coordinate axis;

    real, dimension(-mntor:mntor,-mmpol:mmpol) inputlist::rbc = 0.0

      stellarator symmetric boundary components:

    real, dimension(-mntor:mntor,-mmpol:mmpol) inputlist::zbs = 0.0

      stellarator symmetric boundary components;

    real, dimension(-mntor:mntor,-mmpol:mmpol) inputlist::rbs = 0.0

      non-stellarator symmetric boundary components;
• real, dimension(-mntor:mntor,-mmpol:mmpol) inputlist::zbc = 0.0
      non-stellarator symmetric boundary components;
• real, dimension(-mntor:mntor,-mmpol:mmpol) inputlist::rwc = 0.0
      stellarator symmetric boundary components of wall;

    real, dimension(-mntor:mntor,-mmpol:mmpol) inputlist::zws = 0.0

      stellarator symmetric boundary components of wall;

    real, dimension(-mntor:mntor,-mmpol:mmpol) inputlist::rws = 0.0

      non-stellarator symmetric boundary components of wall:

    real, dimension(-mntor:mntor,-mmpol:mmpol) inputlist::zwc = 0.0

      non-stellarator symmetric boundary components of wall;

    real, dimension(-mntor:mntor,-mmpol:mmpol) inputlist::vns = 0.0

      stellarator symmetric normal field at boundary; vacuum component;

    real, dimension(-mntor:mntor,-mmpol:mmpol) inputlist::bns = 0.0

      stellarator symmetric normal field at boundary; plasma component;

    real, dimension(-mntor:mntor,-mmpol:mmpol) inputlist::vnc = 0.0

      non-stellarator symmetric normal field at boundary; vacuum component;

    real, dimension(-mntor:mntor,-mmpol:mmpol) inputlist::bnc = 0.0

      non-stellarator symmetric normal field at boundary; plasma component;
• integer inputlist::linitialize = 0
      Used to initialize geometry using a regularization / extrapolation method.
```

Used to initialize  $B_{ns}$  using an initial fixed-boundary calculation. • integer inputlist::lzerovac = 0

Used to adjust vacuum field to cancel plasma field on computational boundary.

• integer inputlist::ndiscrete = 2

resolution of the real space grid on which fast Fourier transforms are performed is given by Ndiscrete\*Mpol\*4

• integer inputlist::nquad = -1

Resolution of the Gaussian quadrature.

integer inputlist::impol = -4

Fourier resolution of straight-fieldline angle on interfaces.

• integer inputlist::intor = -4

Fourier resolution of straight-fieldline angle on interfaces;.

integer inputlist::lsparse = 0

controls method used to solve for rotational-transform on interfaces

• integer inputlist::lsvdiota = 0

controls method used to solve for rotational-transform on interfaces; only relevant if Lsparse = 0

integer inputlist::imethod = 3

controls iterative solution to sparse matrix arising in real-space transformation to the straight-fieldline angle; only relevant if Lsparse.eq.2;

• integer inputlist::iorder = 2

controls real-space grid resolution for constructing the straight-fieldline angle; only relevant if Lsparse>0

• integer inputlist::iprecon = 0

controls iterative solution to sparse matrix arising in real-space transformation to the straight-fieldline angle; only relevant if Lsparse.eq.2;

real inputlist::iotatol = -1.0

 $to lerance \ required \ for \ iterative \ construction \ of \ straight-field line \ angle; only \ relevant \ if \ Lsparse. ge. 2$ 

integer inputlist::lextrap = 0

geometry of innermost interface is defined by extrapolation

• integer inputlist::mregular = -1

maximum regularization factor

• integer inputlist::lrzaxis = 1

controls the guess of geometry axis in the innermost volume or initialization of interfaces

• integer inputlist::ntoraxis = 3

the number of n harmonics used in the Jacobian m=1 harmonic elimination method; only relevant if  $Lrzaxis. \leftarrow ge.1$ .

• integer inputlist::lbeltrami = 4

Control flag for solution of Beltrami equation.

integer inputlist::linitgues = 1

controls how initial guess for Beltrami field is constructed

• integer inputlist::lposdef = 0

redundant;

real inputlist::maxrndgues = 1.0

the maximum random number of the Beltrami field if Linitques = 3

• integer inputlist::lmatsolver = 3

1 for LU factorization, 2 for GMRES, 3 for GMRES matrix-free

integer inputlist::nitergmres = 200

number of max iteration for GMRES

• real inputlist::epsgmres = 1e-14

the precision of GMRES

integer inputlist::lgmresprec = 1

type of preconditioner for GMRES, 1 for ILU sparse matrix

• real inputlist::epsilu = 1e-12

redundant

```
the precision of incomplete LU factorization for preconditioning
• integer inputlist::lfindzero = 0
      use Newton methods to find zero of force-balance, which is computed by dforce()
• real inputlist::escale = 0.0
      controls the weight factor, {\tt BBweight}, in the force-imbalance harmonics

    real inputlist::opsilon = 1.0

      weighting of force-imbalance

    real inputlist::pcondense = 2.0

      spectral condensation parameter
• real inputlist::epsilon = 0.0
      weighting of spectral-width constraint
• real inputlist::wpoloidal = 1.0
      "star-like" poloidal angle constraint radial exponential factor used in preset() to construct sweight
• real inputlist::upsilon = 1.0
      weighting of "star-like" poloidal angle constraint used in preset() to construct sweight
• real inputlist::forcetol = 1.0e-10
      required tolerance in force-balance error; only used as an initial check
• real inputlist::c05xmax = 1.0e-06
      required tolerance in position, \mathbf{x} \equiv \{R_{i,v}, Z_{i,v}\}
real inputlist::c05xtol = 1.0e-12
      required tolerance in position, \mathbf{x} \equiv \{R_{i,v}, Z_{i,v}\}
• real inputlist::c05factor = 1.0e-02
      used to control initial step size in CO5NDF and CO5PDF

    logical inputlist::lreadgf = .true.

      \textit{read}\,\nabla_{\mathbf{x}}\mathbf{F} from file \textit{ext} . \textit{GF}
• integer inputlist::mfreeits = 0
      maximum allowed free-boundary iterations
real inputlist::bnstol = 1.0e-06
      redundant;
• real inputlist::bnsblend = 0.666
      redundant;
real inputlist::gbntol = 1.0e-06
      required tolerance in free-boundary iterations
• real inputlist::gbnbld = 0.666
      normal blend
• real inputlist::vcasingeps = 1.e-12
      regularization of Biot-Savart; see bnorml(), casing()
• real inputlist::vcasingtol = 1.e-08
      accuracy on virtual casing integral; see bnorml(), casing()
• integer inputlist::vcasingits = 8
      minimum number of calls to adaptive virtual casing routine; see casing()

    integer inputlist::vcasingper = 1

      periods of integragion in adaptive virtual casing routine; see casing()

    integer inputlist::mcasingcal = 8

      minimum number of calls to adaptive virtual casing routine; see casing(); redundant;
real inputlist::odetol = 1.0e-07
      o.d.e. integration tolerance for all field line tracing routines
• real inputlist::absreq = 1.0e-08
      redundant
• real inputlist::relreq = 1.0e-08
```

```
real inputlist::absacc = 1.0e-04
      redundant
• real inputlist::epsr = 1.0e-08
      redundant
integer inputlist::nppts = 0
      number of toroidal transits used (per trajectory) in following field lines for constructing Poincaré plots; if nPpts<1,
      no Poincaré plot is constructed;
• real inputlist::ppts = 0.0
      stands for Poincare plot theta start. Chose at which angle (normalized over \pi) the Poincare field-line tracing start.

    integer, dimension(1:mnvol+1) inputlist::nptrj = -1

      number of trajectories in each annulus to be followed in constructing Poincaré plot
• logical inputlist::lhevalues = .false.
      to compute eigenvalues of 
abla \mathbf{F}

    logical inputlist::lhevectors = .false.

      to compute eigenvectors (and also eigenvalues) of \nabla \mathbf{F}

    logical inputlist::lhmatrix = .false.

      to compute and write to file the elements of \nabla \mathbf{F}
• integer inputlist::lperturbed = 0
      to compute linear, perturbed equilibrium
integer inputlist::dpp = -1
      perturbed harmonic
integer inputlist::dqq = -1
      perturbed harmonic

    integer inputlist::lerrortype = 0

      the type of error output for Lcheck=1
• integer inputlist::ngrid = -1
      the number of points to output in the grid, -1 for Lrad(vvol)
• real inputlist::drz = 1E-5
      difference in geometry for finite difference estimate (debug only)
• integer inputlist::lcheck = 0
      implement various checks

    logical inputlist::ltiming = .false.

      to check timing
• real inputlist::fudge = 1.0e-00
      redundant

    real inputlist::scaling = 1.0e-00

      redundant

    logical inputlist::wbuild_vector_potential = .false.

• logical inputlist::wreadin = .false.
      write screen output of readin()
• logical inputlist::wwrtend = .false.
      write screen output of wrtend()

    logical inputlist::wmacros = .false.

      write screen output from expanded macros
· integer allglobal::myid
      MPI rank of current CPU.

    integer allglobal::ncpu

      number of MPI tasks

    integer allglobal::ismyvolumevalue

      flag to indicate if a CPU is operating on its assigned volume

    real allglobal::cpus
```

initial time

· real allglobal::pi2nfp

pi2/nfp; assigned in readin()

real allglobal::pi2pi2nfp

 $4\pi^2 Nfp$ 

· real allglobal::pi2pi2nfphalf

 $2\pi^2 Nfp$ 

· real allglobal::pi2pi2nfpquart

 $\pi^2 Nfp$ 

real allglobal::forceerr

total force-imbalance

· real allglobal::energy

MHD energy.

real, dimension(:), allocatable allglobal::ipdt

Toroidal pressure-driven current.

real, dimension(:,:), allocatable allglobal::ipdtdpf

Toroidal pressure-driven current.

· integer allglobal::mvol

number of total volumes; equal to Nvol for fixed-boundary; equal to Nvol+1 for free-boundary

· logical allglobal::yesstellsym

internal shorthand copies of Istellsym, which is an integer input;

· logical allglobal::notstellsym

internal shorthand copies of Istellsym, which is an integer input;

- logical allglobal::yesmatrixfree
- · logical allglobal::notmatrixfree

to use matrix-free method or not

• real, dimension(:,:), allocatable allglobal::cheby

local workspace for evaluation of Chebychev polynomials

• real, dimension(:,:,:), allocatable allglobal::zernike

local workspace for evaluation of Zernike polynomials

• real, dimension(:,:,:), allocatable allglobal::tt

derivatives of Chebyshev polynomials at the inner and outer interfaces;

• real, dimension(:,:,:,:), allocatable allglobal::rtt

derivatives of Zernike polynomials at the inner and outer interfaces;

• real, dimension(:,:), allocatable allglobal::rtm

 $r^m$  term of Zernike polynomials at the origin

• real, dimension(:), allocatable allglobal::zernikedof

Zernike degree of freedom for each m.

logical, dimension(:), allocatable allglobal::imagneticok

used to indicate if Beltrami fields have been correctly constructed;

· logical allglobal::iconstraintok

Used to break iteration loops of slaves in the global constraint minimization.

real, dimension(:,:), allocatable allglobal::beltramierror

to store the integral of |curlB-mu\*B| computed by jo00aa;

integer allglobal::mn

total number of Fourier harmonics for coordinates/fields; calculated from Mpol, Ntor in readin()

• integer, dimension(:), allocatable allglobal::im

poloidal mode numbers for Fourier representation

integer, dimension(:), allocatable allglobal::in

toroidal mode numbers for Fourier representation

• real, dimension(:), allocatable allglobal::halfmm

I saw this already somewhere... • real, dimension(:), allocatable allglobal::regumm I saw this already somewhere... · real allglobal::rscale no idea real, dimension(:,:), allocatable allglobal::psifactor real, dimension(:,:), allocatable allglobal::inifactor no idea • real, dimension(:), allocatable allglobal::bbweight weight on force-imbalance harmonics; used in dforce() real, dimension(:), allocatable allglobal::mmpp spectral condensation factors · integer allglobal::mne enhanced resolution for metric elements integer, dimension(:), allocatable allglobal::ime enhanced poloidal mode numbers for metric elements • integer, dimension(:), allocatable allglobal::ine enhanced toroidal mode numbers for metric elements integer allglobal::mns enhanced resolution for straight field line transformation • integer, dimension(:), allocatable allglobal::ims

enhanced poloidal mode numbers for straight field line transformation

 integer, dimension(:), allocatable allglobal::ins enhanced toroidal mode numbers for straight field line transformation

integer allglobal::Impol

what is this?

integer allglobal::Intor

what is this?

integer allglobal::smpol

what is this?

integer allglobal::sntor

what is this?

• real allglobal::xoffset = 1.0

used to normalize NAG routines (which ones exacly where?)

real, dimension(:,:), allocatable allglobal::irbc

cosine R harmonics of interface surface geometry; stellarator symmetric

• real, dimension(:,:), allocatable allglobal::izbs

sine Z harmonics of interface surface geometry; stellarator symmetric

real, dimension(:,:), allocatable allglobal::irbs

sine R harmonics of interface surface geometry; non-stellarator symmetric

real, dimension(:,:), allocatable allglobal::izbc

cosine Z harmonics of interface surface geometry; non-stellarator symmetric

real, dimension(:,:), allocatable allglobal::drbc

cosine R harmonics of interface surface geometry; stellarator symmetric; linear deformation

real, dimension(:,:), allocatable allglobal::dzbs

sine Z harmonics of interface surface geometry; stellarator symmetric; linear deformation

real, dimension(:,:), allocatable allglobal::drbs

sine R harmonics of interface surface geometry; non-stellarator symmetric; linear deformation

real, dimension(:,:), allocatable allglobal::dzbc

cosine Z harmonics of interface surface geometry; non-stellarator symmetric; linear deformation

```
    real, dimension(:,:), allocatable allglobal::irij

      interface surface geometry; real space

    real, dimension(:,:), allocatable allglobal::izij

      interface surface geometry; real space

    real, dimension(:,:), allocatable allglobal::drij

      interface surface geometry; real space
• real, dimension(:,:), allocatable allglobal::dzij
      interface surface geometry; real space

    real, dimension(:,:), allocatable allglobal::trij

      interface surface geometry; real space

    real, dimension(:,:), allocatable allglobal::tzij

      interface surface geometry; real space

    real, dimension(:), allocatable allglobal::ivns

      sine harmonics of vacuum normal magnetic field on interfaces; stellarator symmetric

    real, dimension(:), allocatable allglobal::ibns

      sine harmonics of plasma normal magnetic field on interfaces; stellarator symmetric

    real, dimension(:), allocatable allglobal::ivnc

      cosine harmonics of vacuum normal magnetic field on interfaces; non-stellarator symmetric
• real, dimension(:), allocatable allglobal::ibnc
      cosine harmonics of plasma normal magnetic field on interfaces; non-stellarator symmetric
· real, dimension(:), allocatable allglobal::lrbc
      local workspace

    real, dimension(:), allocatable allglobal::lzbs

      local workspace
• real, dimension(:), allocatable allglobal::lrbs
      local workspace

    real, dimension(:), allocatable allglobal::lzbc

      local workspace

    integer allglobal::nt

      discrete resolution along \theta of grid in real space
integer allglobal::nz
      discrete resolution along \zeta of grid in real space
· integer allglobal::ntz
      discrete resolution; Ntz=Nt*Nz shorthand
· integer allglobal::hnt
      discrete resolution; Ntz=Nt*Nz shorthand

    integer allglobal::hnz

      discrete resolution; Ntz=Nt*Nz shorthand
· real allglobal::sontz
      one / sqrt (one*Ntz); shorthand

    real, dimension(:,:,:), allocatable allglobal::rij

      real-space grid; R
• real, dimension(:,:,:), allocatable allglobal::zij
      real-space grid; Z

    real, dimension(:,:,:), allocatable allglobal::xij

      what is this?

    real, dimension(:,:,:), allocatable allglobal::yij

      what is this?

    real, dimension(:,:), allocatable allglobal::sg

      real-space grid; jacobian and its derivatives
```

real, dimension(:,:,:,:), allocatable allglobal::guvij

real-space grid; metric elements

- real, dimension(:,:,:), allocatable allglobal::gvuij
  - real-space grid; metric elements (?); 10 Dec 15;
- real, dimension(:,:,:), allocatable allglobal::guvijsave what is this?
- integer, dimension(:,:), allocatable allglobal::ki identification of Fourier modes
- integer, dimension(:,:,:), allocatable allglobal::kijs identification of Fourier modes
- integer, dimension(:,:,:), allocatable allglobal::kija identification of Fourier modes
- integer, dimension(:), allocatable allglobal::iotakkii identification of Fourier modes
- integer, dimension(:,:), allocatable allglobal::iotaksub identification of Fourier modes
- integer, dimension(:,:), allocatable allglobal::iotakadd identification of Fourier modes
- integer, dimension(:,:), allocatable allglobal::iotaksgn identification of Fourier modes
- real, dimension(:), allocatable allglobal::efmn
- Fourier harmonics; dummy workspace.

   real, dimension(:), allocatable allglobal::ofmn
  Fourier harmonics; dummy workspace.
- real, dimension(:), allocatable allglobal::cfmn
- Fourier harmonics; dummy workspace.
   real, dimension(:), allocatable allglobal::sfmn
- Fourier harmonics; dummy workspace.
   real, dimension(:), allocatable allglobal::evmn
  Fourier harmonics; dummy workspace.
- real, dimension(:), allocatable allglobal::odmn Fourier harmonics; dummy workspace.
- real, dimension(:), allocatable allglobal::comn Fourier harmonics; dummy workspace.
- real, dimension(:), allocatable allglobal::simn Fourier harmonics; dummy workspace.
- real, dimension(:), allocatable allglobal::ijreal what is this?
- real, dimension(:), allocatable allglobal::ijimag
   what is this?
- real, dimension(:), allocatable allglobal::jireal what is this?
- real, dimension(:), allocatable allglobal::jiimag
   what is this ?
- real, dimension(:), allocatable allglobal::jkreal what is this?
- real, dimension(:), allocatable allglobal::jkimag
   what is this?
- real, dimension(:), allocatable allglobal::kjreal what is this?
- real, dimension(:), allocatable allglobal::kjimag what is this?

```
    real, dimension(:,:,:), allocatable allglobal::bsupumn

      tangential field on interfaces; \theta-component; required for virtual casing construction of field; 11 Oct 12

    real, dimension(:,:,:), allocatable allglobal::bsupvmn

      tangential field on interfaces; \zeta -component; required for virtual casing construction of field; 11 Oct 12

    real, dimension(:,:), allocatable allglobal::goomne

      described in preset()

    real, dimension(:,:), allocatable allglobal::goomno

      described in preset()

    real, dimension(:,:), allocatable allglobal::gssmne

      described in preset()

    real, dimension(:,:), allocatable allglobal::gssmno

      described in preset()

    real, dimension(:,:), allocatable allglobal::gstmne

      described in preset()

    real, dimension(:,:), allocatable allglobal::gstmno

      described in preset()

    real, dimension(:,:), allocatable allglobal::gszmne

      described in preset()

    real, dimension(:,:), allocatable allglobal::gszmno

      described in preset()
• real, dimension(:,:), allocatable allglobal::gttmne
      described in preset()

    real, dimension(:,:), allocatable allglobal::gttmno

      described in preset()

    real, dimension(:,:), allocatable allglobal::gtzmne

      described in preset()
• real, dimension(:,:), allocatable allglobal::gtzmno
      described in preset()

    real, dimension(:,:), allocatable allglobal::gzzmne

      described in preset()
• real, dimension(:,:), allocatable allglobal::gzzmno
      described in preset()
• real, dimension(:,:,:,:), allocatable allglobal::dtoocc
      volume-integrated Chebychev-metrics; see matrix()

    real, dimension(:,:,:,:), allocatable allglobal::dtoocs

      volume-integrated Chebychev-metrics; see matrix()

    real, dimension(:,:,:,:), allocatable allglobal::dtoosc

      volume-integrated Chebychev-metrics; see matrix()

    real, dimension(:,:,:,:), allocatable allglobal::dtooss

      volume-integrated Chebychev-metrics; see matrix()

    real, dimension(:,:,:,:), allocatable allglobal::ttsscc

      volume-integrated Chebychev-metrics; see matrix()
• real, dimension(:,:,:), allocatable allglobal::ttsscs
      volume-integrated Chebychev-metrics; see matrix()
• real, dimension(:,:,:,:), allocatable allglobal::ttsssc
      volume-integrated Chebychev-metrics; see matrix()

    real, dimension(:,:,:,:), allocatable allglobal::ttssss

      volume-integrated Chebychev-metrics; see matrix()

    real, dimension(:,:,:,:), allocatable allglobal::tdstcc
```

volume-integrated Chebychev-metrics; see matrix()real, dimension(:,:,:,:), allocatable allglobal::tdstcs

volume-integrated Chebychev-metrics; see matrix() • real, dimension(:,:,:), allocatable allglobal::tdstsc volume-integrated Chebychev-metrics; see matrix() real, dimension(:,:,:,:), allocatable allglobal::tdstss volume-integrated Chebychev-metrics; see matrix() real, dimension(:,:,:,:), allocatable allglobal::tdszcc volume-integrated Chebychev-metrics; see matrix() real, dimension(:,:,:,:), allocatable allglobal::tdszcs volume-integrated Chebychev-metrics; see matrix() real, dimension(:,:,:,:), allocatable allglobal::tdszsc volume-integrated Chebychev-metrics; see matrix() real, dimension(:,:,:,:), allocatable allglobal::tdszss volume-integrated Chebychev-metrics; see matrix() • real, dimension(:,:,:,:), allocatable allglobal::ddttcc volume-integrated Chebychev-metrics; see matrix() real, dimension(:,:,:,:), allocatable allglobal::ddttcs volume-integrated Chebychev-metrics; see matrix() real, dimension(:,:,:,:), allocatable allglobal::ddttsc volume-integrated Chebychev-metrics; see matrix() real, dimension(:,:,:,:), allocatable allglobal::ddttss volume-integrated Chebychev-metrics; see matrix() real, dimension(:,:,:,:), allocatable allglobal::ddtzcc volume-integrated Chebychev-metrics; see matrix() real, dimension(:,:,:,:), allocatable allglobal::ddtzcs volume-integrated Chebychev-metrics; see matrix() real, dimension(:,:,:,:), allocatable allglobal::ddtzsc volume-integrated Chebychev-metrics; see matrix() real, dimension(:,:,:,:), allocatable allglobal::ddtzss volume-integrated Chebychev-metrics; see matrix() real, dimension(:,:,:,:), allocatable allglobal::ddzzcc volume-integrated Chebychev-metrics; see matrix() real, dimension(:,:,:,:), allocatable allglobal::ddzzcs volume-integrated Chebychev-metrics; see matrix() real, dimension(:,:,:,:), allocatable allglobal::ddzzsc volume-integrated Chebychev-metrics; see matrix() real, dimension(:,:,:,:), allocatable allglobal::ddzzss volume-integrated Chebychev-metrics; see matrix() real, dimension(:,:), allocatable allglobal::tsc what is this? • real, dimension(:,:), allocatable allglobal::tss what is this? real, dimension(:,:), allocatable allglobal::dtc what is this? real, dimension(:,:), allocatable allglobal::dts what is this? real, dimension(:,:), allocatable allglobal::dzc

what is this?

• real, dimension(:,:), allocatable allglobal::dzs

real, dimension(:,:), allocatable allglobal::ttc

```
what is this?

    real, dimension(:,:), allocatable allglobal::tzs

      what is this?
· real, dimension(:), allocatable allglobal::dtflux
      \delta \psi_{toroidal} in each annulus
• real, dimension(:), allocatable allglobal::dpflux
      \delta\psi_{poloidal} in each annulus

    real, dimension(:), allocatable allglobal::sweight

      minimum poloidal length constraint weight

    integer, dimension(:), allocatable allglobal::nadof

      degrees of freedom in Beltrami fields in each annulus

    integer, dimension(:), allocatable allglobal::nfielddof

      degrees of freedom in Beltrami fields in each annulus, field only, no Lagrange multipliers

    type(subgrid), dimension(:,:,:), allocatable allglobal::ate

      magnetic vector potential cosine Fourier harmonics; stellarator-symmetric
• type(subgrid), dimension(:,:,:), allocatable allglobal::aze
      magnetic vector potential cosine Fourier harmonics; stellarator-symmetric
• type(subgrid), dimension(:,:,:), allocatable allglobal::ato
      magnetic vector potential sine Fourier harmonics; non-stellarator-symmetric
• type(subgrid), dimension(:,:,:), allocatable allglobal::azo
      magnetic vector potential sine Fourier harmonics; non-stellarator-symmetric
• integer, dimension(:,:), allocatable allglobal::lma
      Lagrange multipliers (?)

    integer, dimension(:,:), allocatable allglobal::lmb

      Lagrange multipliers (?)

    integer, dimension(:,:), allocatable allglobal::lmc

      Lagrange multipliers (?)
• integer, dimension(:,:), allocatable allglobal::Imd
      Lagrange multipliers (?)
• integer, dimension(:,:), allocatable allglobal::lme
      Lagrange multipliers (?)
• integer, dimension(:,:), allocatable allglobal::Imf
      Lagrange multipliers (?)

    integer, dimension(:,:), allocatable allglobal::lmg

      Lagrange multipliers (?)
• integer, dimension(:,:), allocatable allglobal::Imh
      Lagrange multipliers (?)

    real, dimension(:,:), allocatable allglobal::lmavalue

      what is this?
• real, dimension(:,:), allocatable allglobal::Imbvalue
      what is this?
• real, dimension(:,:), allocatable allglobal::Imcvalue
      what is this?
• real, dimension(:,:), allocatable allglobal::Imdvalue
      what is this?

    real, dimension(:,:), allocatable allglobal::lmevalue

      what is this?

    real, dimension(:,:), allocatable allglobal::Imfvalue
```

real, dimension(:,:), allocatable allglobal::tzc

• real, dimension(:,:), allocatable allglobal::tts

what is this?

```
what is this?

    real, dimension(:,:), allocatable allglobal::lmgvalue

      what is this?

    real, dimension(:,:), allocatable allglobal::lmhvalue

      what is this?

    integer, dimension(:,:), allocatable allglobal::fso

      what is this?

    integer, dimension(:,:), allocatable allglobal::fse

      what is this?
· logical allglobal::lcoordinatesingularity
      set by LREGION macro; true if inside the innermost volume

    logical allglobal::lplasmaregion

      set by LREGION macro; true if inside the plasma region
• logical allglobal::lvacuumregion
      set by LREGION macro; true if inside the vacuum region

    logical allglobal::lsavedguvij

      flag used in matrix free
· logical allglobal::localconstraint
      what is this?

    real, dimension(:,:), allocatable allglobal::dma

      energy and helicity matrices; quadratic forms

    real, dimension(:,:), allocatable allglobal::dmb

      energy and helicity matrices; quadratic forms

    real, dimension(:,:), allocatable allglobal::dmd

      energy and helicity matrices; quadratic forms
• real, dimension(:), allocatable allglobal::dmas
      sparse version of dMA, data

    real, dimension(:), allocatable allglobal::dmds

      sparse version of dMD, data

    integer, dimension(:), allocatable allglobal::idmas

      sparse version of dMA and dMD, indices

    integer, dimension(:), allocatable allglobal::jdmas

      sparse version of dMA and dMD, indices

    integer, dimension(:), allocatable allglobal::ndmasmax

      number of elements for sparse matrices

    integer, dimension(:), allocatable allglobal::ndmas

      number of elements for sparse matrices
• real, dimension(:), allocatable allglobal::dmg
      what is this?
• real, dimension(:,:), allocatable allglobal::solution
      this is allocated in dforce; used in mp00ac and ma02aa; and is passed to packab

    real, dimension(:,:,:), allocatable allglobal::gmreslastsolution

      used to store the last solution for restarting GMRES

    real, dimension(:), allocatable allglobal::mbpsi

      matrix vector products

    logical allglobal::liluprecond

      whether to use ILU preconditioner for GMRES
• real, dimension(:,:), allocatable allglobal::beltramiinverse
```

measured rotational transform on inner/outer interfaces for each volume; d(transform)/dx; (see dforce)

real, dimension(:,:,:), allocatable allglobal::diotadxup

what is this?

```
    real, dimension(:,:,:), allocatable allglobal::ditgpdxtp

      measured toroidal and poloidal current on inner/outer interfaces for each volume; d(ltor, Gpol)/dx; (see dforce)

    real, dimension(:,:,:,:), allocatable allglobal::glambda

      save initial guesses for iterative calculation of rotational-transform

    integer allglobal::lmns

      what is this?
• real, dimension(:,:,:), allocatable allglobal::bemn
      force vector; stellarator-symmetric (?)

    real, dimension(:,:), allocatable allglobal::iomn

      force vector; stellarator-symmetric (?)

    real, dimension(:,:,:), allocatable allglobal::somn

      force vector; non-stellarator-symmetric (?)

    real, dimension(:,:,:), allocatable allglobal::pomn

      force vector; non-stellarator-symmetric (?)

    real, dimension(:,:,:), allocatable allglobal::bomn

      force vector; stellarator-symmetric (?)

    real, dimension(:,:), allocatable allglobal::iemn

      force vector; stellarator-symmetric (?)

    real, dimension(:,:,:), allocatable allglobal::semn

      force vector; non-stellarator-symmetric (?)
• real, dimension(:,:,:), allocatable allglobal::pemn
      force vector; non-stellarator-symmetric (?)

    real, dimension(:), allocatable allglobal::bbe

      force vector (?); stellarator-symmetric (?)
• real, dimension(:), allocatable allglobal::iio
      force vector (?); stellarator-symmetric (?)

    real, dimension(:), allocatable allglobal::bbo

      force vector (?); non-stellarator-symmetric (?)
• real, dimension(:), allocatable allglobal::iie
      force vector (?); non-stellarator-symmetric (?)
• real, dimension(:,:,:), allocatable allglobal::btemn
      covariant \theta cosine component of the tangential field on interfaces; stellarator-symmetric
• real, dimension(:,:,:), allocatable allglobal::bzemn
      covariant \zeta cosine component of the tangential field on interfaces; stellarator-symmetric
• real, dimension(:,:,:), allocatable allglobal::btomn
      covariant \theta sine component of the tangential field on interfaces; non-stellarator-symmetric

    real, dimension(:,:,:), allocatable allglobal::bzomn

      covariant ζ sine component of the tangential field on interfaces; non-stellarator-symmetric

    real, dimension(:,:), allocatable allglobal::bloweremn

      covariant field for Hessian computation

    real, dimension(:,:), allocatable allglobal::bloweromn

      covariant field for Hessian computation

    integer allglobal::lgdof

      geometrical degrees of freedom associated with each interface

    integer allglobal::ngdof

      total geometrical degrees of freedom

    real, dimension(:,:,:), allocatable allglobal::dbbdrz

      derivative of magnetic field w.r.t. geometry (?)

    real, dimension(:,:), allocatable allglobal::diidrz

      derivative of spectral constraints w.r.t. geometry (?)

    real, dimension(:,:,:,:,:), allocatable allglobal::dffdrz
```

```
derivatives of B^{\wedge}2 at the interfaces wrt geometry

    real, dimension(:,:,:,:), allocatable allglobal::dbbdmp

      derivatives of B^2 at the interfaces wrt mu and dpflux

    real, dimension(:,:,:,:), allocatable allglobal::dmupfdx

      derivatives of mu and dpflux wrt geometry at constant interface transform

    logical allglobal::lhessianallocated

      flag to indicate that force gradient matrix is allocated (?)

    real, dimension(:,:), allocatable allglobal::hessian

      force gradient matrix (?)

    real, dimension(:,:), allocatable allglobal::dessian

      derivative of force gradient matrix (?)

    real, dimension(:,:), allocatable allglobal::cosi

      some precomputed cosines
• real, dimension(:,:), allocatable allglobal::sini
      some precomputed sines

    real, dimension(:), allocatable allglobal::gteta

      something related to \sqrt{g} and \theta ?

    real, dimension(:), allocatable allglobal::gzeta

      something related to \sqrt{g} and \zeta ?

    real, dimension(:), allocatable allglobal::ajk

      definition of coordinate axis

    real, dimension(:,:,:,:), allocatable allglobal::dradr

      derivatives of coordinate axis

    real, dimension(:,:,:,:), allocatable allglobal::dradz

      derivatives of coordinate axis

    real, dimension(:,:,:,:), allocatable allglobal::dzadr

      derivatives of coordinate axis

    real, dimension(:,:,:,:), allocatable allglobal::dzadz

      derivatives of coordinate axis

    real, dimension(:,:,:), allocatable allglobal::drodr

      derivatives of coordinate axis

    real, dimension(:,:,:), allocatable allglobal::drodz

      derivatives of coordinate axis
• real, dimension(:,:,:), allocatable allglobal::dzodr
      derivatives of coordinate axis

    real, dimension(:,:,:), allocatable allglobal::dzodz

      derivatives of coordinate axis

    integer, dimension(:,:), allocatable allglobal::djkp

      for calculating cylindrical volume
• integer, dimension(:,:), allocatable allglobal::djkm
      for calculating cylindrical volume

    real, dimension(:), allocatable allglobal::lbbintegral

      B.B integral.

    real, dimension(:), allocatable allglobal::labintegral

      A.B integral.
• real, dimension(:), allocatable allglobal::vvolume
      volume integral of \sqrt{g}; computed in volume
· real allglobal::dvolume
      derivative of volume w.r.t. interface geometry
```

labels volume; some subroutines (called by NAG) are fixed argument list but require the volume label

integer allglobal::ivol

· real allglobal::gbzeta

```
• integer, dimension(:), allocatable allglobal::iquad
      internal copy of Nquad
• real, dimension(:,:), allocatable allglobal::gaussianweight
      weights for Gaussian quadrature

    real, dimension(:,:), allocatable allglobal::gaussianabscissae

      abscissae for Gaussian quadrature
· logical allglobal::lblinear
      controls selection of Beltrami field solver; depends on LBeltrami

    logical allglobal::lbnewton

      controls selection of Beltrami field solver; depends on LBeltrami
· logical allglobal::lbsequad
      controls selection of Beltrami field solver; depends on LBeltrami

    real, dimension(1:3) allglobal::orzp

      used in mg00aa() to determine (s, \theta, \zeta) given (R, Z, \varphi)
• type(derivative) allglobal::dbdx
      d\mathbf{B}/d\mathbf{X} (?)
· integer allglobal::globaljk
      labels position

    real, dimension(:,:), allocatable allglobal::dxyz

      computational boundary; position

    real, dimension(:,:), allocatable allglobal::nxyz

      computational boundary; normal

    real, dimension(:,:), allocatable allglobal::jxyz

      plasma boundary; surface current
• real, dimension(1:2) allglobal::tetazeta
      what is this?

    real allglobal::virtualcasingfactor = -one / (four*pi)

      this agrees with diagno
· integer allglobal::iberror
      for computing error in magnetic field
· integer allglobal::nfreeboundaryiterations
      number of free-boundary iterations already performed

    integer, parameter allglobal::node = 2

      best to make this global for consistency between calling and called routines
• logical allglobal::first_free_bound = .false.
      flag to indicate that this is the first free-boundary iteration
• type(c_ptr) fftw_interface::planf
      FFTW-related (?)
• type(c_ptr) fftw_interface::planb
      FFTW-related (?)
• complex(c_double_complex), dimension(:,:,:), allocatable fftw_interface::cplxin
      FFTW-related (?)

    complex(c_double_complex), dimension(:,:,:), allocatable fftw_interface::cplxout

      FFTW-related (?)
```

toroidal (contravariant) field; calculated in bfield; required to convert  $\dot{\theta}$  to  $B^{\theta}$ ,  $\dot{s}$  to  $B^{s}$ 

#### 11.8.1 Detailed Description

Defines input namelists and global variables, and opens some output files.

Note that all variables in namelist need to be broadcasted in readin.

#### Input geometry

• The geometry of the l-th interface, for l=0,N where  $N\equiv {\sf Nvol},$  is described by a set of Fourier harmonics, using an arbitrary poloidal angle,

$$R_l(\theta, \zeta) = \sum_j R_{j,l} \cos(m_j \theta - n_j \zeta),$$
 (280)

$$Z_l(\theta,\zeta) = \sum_j Z_{j,l} \sin(m_j \theta - n_j \zeta).$$
 (281)

• These harmonics are read from the ext.sp file and come directly after the namelists described above. The required format is as follows:

- The coordinate axis corresponds to j=0 and the outermost boundary corresponds to  $j=\mathsf{Nvol}.$
- An arbitrary selection of harmonics may be inluded in any order, but only those within the range specified by Mpol and Ntor will be used.
- The geometry of all the interfaces, i.e. l=0,N, including the degenerate "coordinate-axis" interface, must be given.

## 11.8.2 Data Type Documentation

**11.8.2.1 type typedefns::subgrid** used for quantities which have different resolutions in different volumes, e.g. the vector potential

#### **Class Members**

real, dimension	ı(:), allocatable	S	coefficients
integer, dimension	n(:), allocatable	i	indices

# Class Members

real, dimension(:,:), allocatable	mat	
integer, dimension(:), allocatable	ipivot	

#### 11.8.2.2 type typedefns::matrixlu

## 11.9 hesian.f90 File Reference

Computes eigenvalues and eigenvectors of derivative matrix,  $\nabla_{\xi} \mathbf{F}$ .

## **Functions/Subroutines**

• subroutine hesian (NGdof, position, Mvol, mn, LGdof) Computes eigenvalues and eigenvectors of derivative matrix,  $\nabla_{\xi} \mathbf{F}$ .

# 11.9.1 Detailed Description

Computes eigenvalues and eigenvectors of derivative matrix,  $\nabla_{\xi} \mathbf{F}$ .

# 11.10 jo00aa.f90 File Reference

Measures error in Beltrami equation,  $\nabla \times \mathbf{B} - \mu \mathbf{B}$ .

#### **Functions/Subroutines**

• subroutine jo00aa (Ivol, Ntz, Iquad, mn) Measures error in Beltrami equation,  $\nabla \times \mathbf{B} - \mu \mathbf{B}$ .

## 11.10.1 Detailed Description

Measures error in Beltrami equation,  $\nabla \times \mathbf{B} - \mu \mathbf{B}$ .

#### 11.11 Iforce.f90 File Reference

Computes  $B^2$ , and the spectral condensation constraints if required, on the interfaces,  $\mathcal{I}_i$ .

#### **Functions/Subroutines**

• subroutine Iforce (Ivol, iocons, ideriv, Ntz, dBB, XX, YY, length, DDI, MMI, iflag)

Computes  $B^2$ , and the spectral condensation constraints if required, on the interfaces,  $\mathcal{I}_i$ .

#### 11.11.1 Detailed Description

Computes  $B^2$ , and the spectral condensation constraints if required, on the interfaces,  $\mathcal{I}_i$ .

# 11.12 ma00aa.f90 File Reference

Calculates volume integrals of Chebyshev polynomials and metric element products.

#### **Functions/Subroutines**

• subroutine ma00aa (Iquad, mn, Ivol, Irad)

Calculates volume integrals of Chebyshev polynomials and metric element products.

## 11.12.1 Detailed Description

Calculates volume integrals of Chebyshev polynomials and metric element products.

## 11.13 ma02aa.f90 File Reference

Constructs Beltrami field in given volume consistent with flux, helicity, rotational-transform and/or parallel-current constraints.

#### **Functions/Subroutines**

• subroutine ma02aa (Ivol, NN)

Constructs Beltrami field in given volume consistent with flux, helicity, rotational-transform and/or parallel-current constraints.

## 11.13.1 Detailed Description

Constructs Beltrami field in given volume consistent with flux, helicity, rotational-transform and/or parallel-current constraints.

## 11.14 manual.f90 File Reference

Code development issues and future physics applications.

#### 11.14.1 Detailed Description

Code development issues and future physics applications.

See also

Manual / Documentation

## 11.15 matrix.f90 File Reference

Constructs energy and helicity matrices that represent the Beltrami linear system.

#### **Functions/Subroutines**

- subroutine matrix (Ivol, mn, Irad)
   Constructs energy and helicity matrices that represent the Beltrami linear system.
   gauge conditions
- · subroutine matrixbg (Ivol, mn, Irad)

#### 11.15.1 Detailed Description

Constructs energy and helicity matrices that represent the Beltrami linear system.

## 11.16 metrix.f90 File Reference

Calculates the metric quantities,  $\sqrt{g}\,g^{\mu\nu}$ , which are required for the energy and helicity integrals.

#### **Functions/Subroutines**

- subroutine metrix (Iquad, Ivol) Calculates the metric quantities,  $\sqrt{g} \ g^{\mu\nu}$ , which are required for the energy and helicity integrals.
- subroutine **compute\_guvijsave** (Iquad, vvol, ideriv, Lcurvature)

#### 11.16.1 Detailed Description

Calculates the metric quantities,  $\sqrt{g} g^{\mu\nu}$ , which are required for the energy and helicity integrals.

## 11.17 mp00ac.f90 File Reference

Solves Beltrami/vacuum (linear) system, given matrices.

## Functions/Subroutines

- subroutine mp00ac (Ndof, Xdof, Fdof, Ddof, Ldfjac, iflag)
   Solves Beltrami/vacuum (linear) system, given matrices.
   unpacking fluxes, helicity multiplier
- subroutine rungmres (n, nrestart, mu, vvol, rhs, sol, ipar, fpar, wk, nw, guess, a, au, jau, ju, iperm, ierr)
- subroutine **matvec** (n, x, ax, a, mu, vvol)
- subroutine prec\_solve (n, vecin, vecout, au, jau, ju, iperm)

#### 11.17.1 Detailed Description

Solves Beltrami/vacuum (linear) system, given matrices.

## 11.18 newton.f90 File Reference

Employs Newton method to find F(x) = 0, where  $x \equiv \{geometry\}$  and F is defined in dforce().

#### **Modules**

· module newtontime

timing of Newton iterations

#### **Functions/Subroutines**

• subroutine newton (NGdof, position, ihybrd)

Employs Newton method to find  ${f F}({f x})=0$ , where  ${f x}\equiv\{{
m geometry}\}$  and  ${f F}$  is defined in dforce() .

• subroutine writereadgf (readorwrite, NGdof, ireadhessian)

read or write force-derivative matrix

• subroutine fcn1 (NGdof, xx, fvec, irevcm)

fcn1

• subroutine fcn2 (NGdof, xx, fvec, fjac, Ldfjac, irevcm)

fcn2

#### **Variables**

· integer newtontime::nfcalls

number of calls to get function values (?)

· integer newtontime::ndcalls

number of calls to get derivative values (?)

· real newtontime::lastcpu

last CPU that called this (?)

#### 11.18.1 Detailed Description

Employs Newton method to find  $\mathbf{F}(\mathbf{x}) = 0$ , where  $\mathbf{x} \equiv \{\text{geometry}\}\$  and  $\mathbf{F}$  is defined in dforce().

#### 11.19 numrec.f90 File Reference

Various miscellaneous "numerical" routines.

#### **Functions/Subroutines**

• subroutine gi00ab (Mpol, Ntor, Nfp, mn, im, in)

Assign Fourier mode labels.

- subroutine **getimn** (Mpol, Ntor, Nfp, mi, ni, idx)
- · subroutine tfft (Nt, Nz, ijreal, ijimag, mn, im, in, efmn, ofmn, cfmn, sfmn, ifail)

Forward Fourier transform (fftw wrapper)

· subroutine invfft (mn, im, in, efmn, ofmn, cfmn, sfmn, Nt, Nz, ijreal, ijimag)

Inverse Fourier transform (fftw wrapper)

• subroutine gauleg (n, weight, abscis, ifail)

Gauss-Legendre weights and abscissae.

#### 11.19.1 Detailed Description

Various miscellaneous "numerical" routines.

# 11.20 packab.f90 File Reference

Packs, and unpacks, Beltrami field solution vector;  $\mathbf{a} \equiv \{A_{\theta,e,i,l}, A_{\zeta,e,i,l}, \text{etc.}\}.$ 

#### **Functions/Subroutines**

• subroutine packab (packorunpack, Ivol, NN, solution, ideriv)

Packs and unpacks Beltrami field solution vector.

#### 11.20.1 Detailed Description

Packs, and unpacks, Beltrami field solution vector;  $\mathbf{a} \equiv \{A_{\theta,e.i.l}, A_{\zeta,e.i.l}, \text{etc.}\}.$ 

## 11.21 packxi.f90 File Reference

Packs, and unpacks, geometrical degrees of freedom; and sets coordinate axis.

#### **Functions/Subroutines**

 subroutine packxi (NGdof, position, Mvol, mn, iRbc, iZbs, iRbs, iZbc, packorunpack, LComputeDerivatives, LComputeAxis)

Packs, and unpacks, geometrical degrees of freedom; and sets coordinate axis.

## 11.21.1 Detailed Description

Packs, and unpacks, geometrical degrees of freedom; and sets coordinate axis.

## 11.22 pc00aa.f90 File Reference

Use preconditioned conjugate gradient method to find minimum of energy functional.

#### **Functions/Subroutines**

subroutine pc00aa (NGdof, position, Nvol, mn, ie04dgf)
 Use preconditioned conjugate gradient method to find minimum of energy functional.

## 11.22.1 Detailed Description

Use preconditioned conjugate gradient method to find minimum of energy functional.

## 11.23 pc00ab.f90 File Reference

Returns the energy functional and it's derivatives with respect to geometry.

## **Functions/Subroutines**

• subroutine pc00ab (mode, NGdof, Position, Energy, Gradient, nstate, iuser, ruser)

Returns the energy functional and it's derivatives with respect to geometry.

#### 11.23.1 Detailed Description

Returns the energy functional and it's derivatives with respect to geometry.

# 11.24 pp00aa.f90 File Reference

Constructs Poincaré plot and "approximate" rotational-transform (driver).

#### **Functions/Subroutines**

subroutine pp00aa
 Constructs Poincaré plot and "approximate" rotational-transform (driver).

#### 11.24.1 Detailed Description

Constructs Poincaré plot and "approximate" rotational-transform (driver).

# 11.25 pp00ab.f90 File Reference

Follows magnetic fieldline using ode-integration routine from rksuite.f .

#### **Functions/Subroutines**

subroutine pp00ab (Ivol, sti, Nz, nPpts, poincaredata, fittedtransform, utflag)
 Constructs Poincaré plot and "approximate" rotational-transform (for single field line).

#### 11.25.1 Detailed Description

Follows magnetic fieldline using ode-integration routine from rksuite.f .

## 11.26 preset.f90 File Reference

Allocates and initializes internal arrays.

#### **Functions/Subroutines**

· subroutine preset

Allocates and initializes internal arrays.

#### 11.26.1 Detailed Description

Allocates and initializes internal arrays.

## 11.27 ra00aa.f90 File Reference

Writes vector potential to .ext.sp.A .

#### **Functions/Subroutines**

• subroutine ra00aa (writeorread)

Writes vector potential to .ext.sp.A .

## 11.27.1 Detailed Description

Writes vector potential to .ext.sp.A .

# 11.28 stzxyz.f90 File Reference

Calculates coordinates,  $\mathbf{x}(s, \theta, \zeta) \equiv R \mathbf{e}_R + Z \mathbf{e}_Z$ , and metrics, at given  $(s, \theta, \zeta)$ .

# Functions/Subroutines

• subroutine stzxyz (Ivol, stz, RpZ)  $\textit{Calculates coordinates, } \mathbf{x}(s,\theta,\zeta) \equiv R\,\mathbf{e}_R + Z\,\mathbf{e}_Z, \textit{and metrics, at given } (s,\theta,\zeta).$ 

#### 11.28.1 Detailed Description

Calculates coordinates,  $\mathbf{x}(s,\theta,\zeta) \equiv R \, \mathbf{e}_R + Z \, \mathbf{e}_Z$ , and metrics, at given  $(s,\theta,\zeta)$ .

# 11.29 tr00ab.f90 File Reference

Calculates rotational transform given an arbitrary tangential field.

## Functions/Subroutines

subroutine tr00ab (Ivol, mn, NN, Nt, Nz, iflag, Idiota)
 Calculates rotational transform given an arbitrary tangential field.

#### 11.29.1 Detailed Description

Calculates rotational transform given an arbitrary tangential field.

# 11.30 volume.f90 File Reference

Computes volume of each region; and, if required, the derivatives of the volume with respect to the interface geometry.

#### **Functions/Subroutines**

• subroutine volume (Ivol, vflag)

Computes volume of each region; and, if required, the derivatives of the volume with respect to the interface geometry.

## 11.30.1 Detailed Description

Computes volume of each region; and, if required, the derivatives of the volume with respect to the interface geometry.

## 11.31 wa00aa.f90 File Reference

Constructs smooth approximation to wall.

## **Functions/Subroutines**

• subroutine wa00aa (iwa00aa)

Constructs smooth approximation to wall.

• subroutine vacuumphi (Nconstraints, rho, fvec, iflag)

Compute vacuum magnetic scalar potential (?)

## **Variables**

· logical laplaces::stage1

what is this?

logical laplaces::exterior

what is this?

• logical laplaces::dorm

what is this?

integer laplaces::nintervals

what is this?

· integer laplaces::nsegments

what is this?

• integer laplaces::ic

what is this?

· integer laplaces::np4

what is this?

```
integer laplaces::np1
what is this?
```

• integer, dimension(:), allocatable laplaces::icint

what is this?

• real laplaces::originalalpha

what is this?

real, dimension(:), allocatable laplaces::xpoly

what is this?

real, dimension(:), allocatable laplaces::ypoly

what is this?

• real, dimension(:), allocatable laplaces::phi

what is this?

• real, dimension(:), allocatable laplaces::phid

what is this?

• real, dimension(:,:), allocatable laplaces::cc

what is this?

• integer laplaces::ilength

what is this?

· real laplaces::totallength

what is this?

• integer laplaces::niterations

counter; eventually redundant; 24 Oct 12;

• integer laplaces::iangle

angle; eventually redundant; 24 Oct 12;

· real laplaces::rmid

used to define local polar coordinate; eventually redundant; 24 Oct 12;

· real laplaces::zmid

used to define local polar coordinate; eventually redundant; 24 Oct 12;

· real laplaces::alpha

eventually redundant; 24 Oct 12;

## 11.31.1 Detailed Description

Constructs smooth approximation to wall.

## 11.32 xspech.f90 File Reference

Main program.

#### **Functions/Subroutines**

program xspech

Main program of SPEC.

· subroutine ending

todo remark

## 11.32.1 Detailed Description

Main program.

#### 11.32.2 Function/Subroutine Documentation

11.32.2.1 xspech() program xspech

Main program of SPEC.

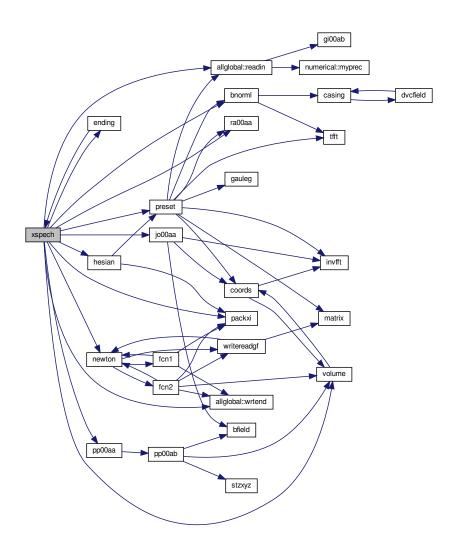
Returns

none

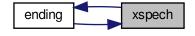
References inputlist::adiabatic, allglobal::ate, allglobal::ato, allglobal::aze, allglobal::aze, allglobal::bbe, allglobal: ::bbo, bnorml(), allglobal::btemn, allglobal::btomn, allglobal::bzemn, allglobal::bzemn, allglobal::cfmn, allglobal ::cpus, allglobal::dpflux, allglobal::dtflux, allglobal::efmn, ending(), inputlist::ext, allglobal::forceerr, inputlist::gamma, inputlist::gbnbld, inputlist::gbntol, inputlist::helicity, hesian(), allglobal::ibnc, allglobal::ibns, inputlist::igeometry, allglobal::iie, allglobal::iin, allglobal::imagneticok, allglobal::in, allglobal::iquad, allglobal::irbc, allglobal::irbs, inputlist::isurf, allglobal::ivnc, allglobal::ivns, inputlist::ivolume, allglobal::izbc, allglobal::izbs, jo00aa(), inputlist::ladiabatic, inputlist::lautoinitbn, inputlist::lcheck, inputlist::lconstraint, allglobal::lcoordinatesingularity, inputlist::lfindzero, inputlist::lfreebound, allglobal::lgdof, inputlist::lhevalues, inputlist::lhevectors, inputlist::lhmatrix, numerical::logtolerance, inputlist::lperturbed, allglobal::lplasmaregion, inputlist::lrad, fileunits::lunit, allglobal ↔ ::lvacuumregion, inputlist::lzerovac, inputlist::mfreeits, inputlist::mu, constants::mu0, allglobal::mvol, allglobal ::myid, allglobal::ncpu, newton(), inputlist::nfp, allglobal::ngdof, allglobal::notstellsym, inputlist::nppts, inputlist ← ::nptrj, inputlist::nvol, inputlist::odetol, allglobal::ofmn, constants::one, fileunits::ounit, packxi(), inputlist::pflux, inputlist::phiedge, constants::pi2, pp00aa(), preset(), inputlist::pressure, inputlist::pscale, ra00aa(), inputlist::rbc, inputlist::rbs, allglobal::readin(), allglobal::sfmn, inputlist::tflux, inputlist::vcasingtol, volume(), numerical::vsmall, allglobal::vvolume, inputlist::wmacros, allglobal::wrtend(), allglobal::yesstellsym, inputlist::zbc, inputlist::zbs, and constants::zero.

Referenced by ending().

Here is the call graph for this function:



Here is the caller graph for this function:



11.32.2.2 ending() subroutine ending

todo remark

**Todo** The following belongs to the docs of the program xspech, not to the ending() subroutine. If you know how to attach the docs to the program xspech, please fix this.

#### reading input, allocating global variables

- The input namelists and geometry are read in via a call to readin(). A full description of the required input is given in global.f90.
- Most internal variables, global memory etc., are allocated in preset() .
- All quantities in the input file are mirrored into the output file's group /input.

#### preparing output file group iterations

• The group /iterations is created in the output file. This group contains the interface geometry at each iteration, which is useful for constructing movies illustrating the convergence. The data structure in use is an unlimited array of the following compound datatype:

```
DATATYPE H5T_COMPOUND {
   H5T_NATIVE_INTEGER "nDcalls";
   H5T_NATIVE_DOUBLE "Energy";
   H5T_NATIVE_DOUBLE "ForceErr";
   H5T_ARRAY { [Mvol+1] [mn] H5T_NATIVE_DOUBLE } "iRbc";
   H5T_ARRAY { [Mvol+1] [mn] H5T_NATIVE_DOUBLE } "iZbs";
   H5T_ARRAY { [Mvol+1] [mn] H5T_NATIVE_DOUBLE } "iRbs";
   H5T_ARRAY { [Mvol+1] [mn] H5T_NATIVE_DOUBLE } "iZbc";
}
```

#### packing geometrical degrees-of-freedom into vector

• If NGdof.gt.0, where NGdof counts the geometrical degrees-of-freedom, i.e. the  $R_{bc}$ ,  $Z_{bs}$ , etc., then packxi() is called to "pack" the geometrical degrees-of-freedom into position (0:NGdof).

#### initialize adiabatic constants

• If Ladiabatic.eq.0 , then the "adiabatic constants" in each region,  $P_v$ , are calculated as

$$P_v \equiv p_v V_v^{\gamma},\tag{283}$$

where  $p_v \equiv \texttt{pressure}$  (vvol) , the volume  $V_v$  of each region is computed by volume() , and the adiabatic index  $\gamma \equiv \texttt{gamma}$  .

## solving force-balance

- If there are geometrical degress of freedom, i.e. if NGdof.gt.0, then
  - Todo If Lminimize.eq.1, call pc00aa() to find minimum of energy functional using quasi-Newton, preconditioned conjugate gradient method, E04DGF
  - If Lfindzero.gt.0, call newton() to find extremum of constrained energy functional using a Newton method, C05PDF.

#### post diagnostics

• The pressure is computed from the adiabatic constants from Eqn. (283), i.e.  $p = P/V^{\gamma}$ .

- The Beltrami/vacuum fields in each region are re-calculated using dforce() .
- If Lcheck.eq.5.or. LHevalues.or. LHevectors.or. Lperturbed.eq.1, then the force-gradient matrix is examined using hesian().

#### free-boundary: re-computing normal field

- If Lfreebound.eq.1 and Lfindzero.gt.0 and mfreeits.ne.0, then the magnetic field at the computational boundary produced by the plasma currents is computed using bnorml().
- The "new" magnetic field at the computational boundary produced by the plasma currents is updated using a Picard scheme:

$$\operatorname{Bns}_{i}^{j} = \lambda \operatorname{Bns}_{i}^{j-1} + (1 - \lambda) \operatorname{Bns}_{i}, \tag{284}$$

where j labels free-boundary iterations, the "blending parameter" is  $\lambda \equiv \mathtt{gBnbld}$ , and  $\mathtt{Bns}_i$  is computed by virtual casing. The subscript "\$i\$" labels Fourier harmonics.

• If the new (unblended) normal field is *not* sufficiently close to the old normal field, as quantified by <code>gBntol</code>, then the free-boundary iterations continue. This is quantified by

$$\sum_{i} |\mathrm{Bns}_{i}^{j-1} - \mathrm{Bns}_{i}|/N, \tag{285}$$

where N is the total number of Fourier harmonics.

- · There are several choices that are available:
  - if mfreeits=-2: the vacuum magnetic field (really, the normal component of the field produced by the external currents at the computational boundary) required to hold the given equlibrium is written to file. This information is required as input by FOCUS [9] for example. (This option probably needs to revised.)
  - if mfreeits=-1: after the plasma field is computed by virtual casing, the vacuum magnetic field is set to exactly balance the plasma field (again, we are really talking about the normal component at the computational boundary.) This will ensure that the computational boundary itself if a flux surface of the total magnetic field.
  - if mfreeits=0: the plasma field at the computational boundary is not updated; no "free-boundary" iterations take place.
  - if mfreeits>0: the plasma field at the computational boundary is updated according to the above blending Eqn. (284), and the free-boundary iterations will continue until either the tolerance condition is met (see gBntol and Eqn. (285)) or the maximum number of free-boundary iterations, namely mfreeits, is reached. For this case, Lzerovac is relevant: if Lzerovac=1, then the vacuum field is set equal to the normal field at every iteration, which results in the computational boundary being a flux surface. (I am not sure if this is identical to setting mfreeits=-1; the logic etc. needs to be revised.)

#### output files: vector potential

• The vector potential is written to file using ra00aa().

# final diagnostics

- sc00aa() is called to compute the covariant components of the magnetic field at the interfaces; these are related to the singular currents;
- if Lcheck=1, jo00aa() is called to compute the error in the Beltrami equation;

• pp00aa() is called to construct the Poincaré plot;

#### restart files

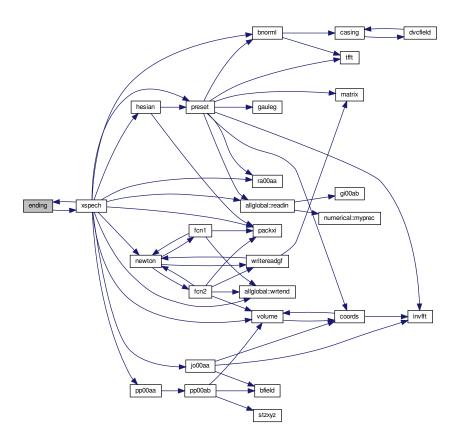
• wrtend() is called to write the restart files.

Closes output files, writes screen summary.

References allglobal::cpus, inputlist::ext, inputlist::ltiming, allglobal::myid, fileunits::ounit, cputiming::treadin, cputiming::twrtend, inputlist::wmacros, xspech(), and constants::zero.

Referenced by xspech().

Here is the call graph for this function:



Here is the caller graph for this function:



212 REFERENCES

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