tr00ab

Calculates rotational transform given an arbitrary tangential field.

[called by: dforce and mp00ac.]

contents

1.0.1 constructing straight field line angle on interfaces

1. The algorithm stems from introducing a straight field line angle $\theta_s = \theta + \lambda(\theta, \zeta)$, where

$$\lambda = \sum_j \lambda_{o,j} \sin(m_j\theta - n_j\zeta) + \sum_j \lambda_{e,j} \cos(m_j\theta - n_j\zeta) \tag{1}$$

and insisting that

$$\frac{\mathbf{B} \cdot \nabla\theta_s}{\mathbf{B} \cdot \nabla\zeta} = \dot{\theta}(1 + \lambda_\theta) + \lambda_\zeta = \iota, \tag{2}$$

where ι is a constant that is to be determined.

2. Writing $\dot{\theta} = -\partial_s A_\zeta / \partial_s A_\theta$, we have

$$\partial_s A_\theta \, \iota + \partial_s A_\zeta \, \lambda_\theta - \partial_s A_\theta \, \lambda_\zeta = -\partial_s A_\zeta \tag{3}$$

3. Expanding this equation we obtain

$$
\begin{aligned}
& \left(A'_{\theta,e,k}\cos\alpha_k + A'_{\theta,o,k}\sin\alpha_k\right)\iota \\
+ \; & \left(A'_{\zeta,e,k}\cos\alpha_k + A'_{\zeta,o,k}\sin\alpha_k\right)\left(+m_j\lambda_{o,j}\cos\alpha_j - m_j\lambda_{e,j}\sin\alpha_j\right) \\
- \; & \left(A'_{\theta,e,k}\cos\alpha_k + A'_{\theta,o,k}\sin\alpha_k\right)\left(-n_j\lambda_{o,j}\cos\alpha_j + n_j\lambda_{e,j}\sin\alpha_j\right) \\
= \; - \; & \left(A'_{\zeta,e,k}\cos\alpha_k + A'_{\zeta,o,k}\sin\alpha_k\right),
\end{aligned}
\tag{4}
$$

where summation over $k = 1, \mathtt{mn}$ and $j = 2, \mathtt{mns}$ is implied

4. After applying double angle formulae,

$$
\begin{aligned}
& \left(A'_{\theta,e,k}\cos\alpha_k + A'_{\theta,o,k}\sin\alpha_k\right)\iota \\
+ \; & \lambda_{o,j}\left(+m_j A'_{\zeta,e,k} + n_j A'_{\theta,e,k}\right)\left[+\cos(\alpha_k + \alpha_j) + \cos(\alpha_k - \alpha_j)\right]/2 \\
+ \; & \lambda_{e,j}\left(-m_j A'_{\zeta,e,k} - n_j A'_{\theta,e,k}\right)\left[+\sin(\alpha_k + \alpha_j) - \sin(\alpha_k - \alpha_j)\right]/2 \\
+ \; & \lambda_{o,j}\left(+m_j A'_{\zeta,o,k} + n_j A'_{\theta,o,k}\right)\left[+\sin(\alpha_k + \alpha_j) + \sin(\alpha_k - \alpha_j)\right]/2 \\
+ \; & \lambda_{e,j}\left(-m_j A'_{\zeta,o,k} - n_j A'_{\theta,o,k}\right)\left[-\cos(\alpha_k + \alpha_j) + \cos(\alpha_k - \alpha_j)\right]/2 \\
= \; - \; & \left(A'_{\zeta,e,k}\cos\alpha_k + A'_{\zeta,o,k}\sin\alpha_k\right),
\end{aligned}
\tag{5}
$$

and equating coefficients, an equation of the form $\mathbf{A} \cdot \mathbf{x} = \mathbf{b}$ is obtained, where

$$\mathbf{x} = (\underbrace{\iota}_{\mathtt{x[1]}}, \underbrace{\lambda_{o,2}, \lambda_{o,3}, \dots}_{\mathtt{x[\ 2:\ N\ ]}}, \underbrace{\lambda_{e,2}, \lambda_{e,3}, \dots}_{\mathtt{x[N+1:2N-1]}})^T. \tag{6}$$

1.0.2 alternative iterative method

1. Consider the equation $\dot{\theta}(1 + \lambda_\theta) + \lambda_\zeta = \iota$, where $\lambda = \sum_j \lambda_j \sin\alpha_j$, given on a grid

$$\dot{\theta}_i + \dot{\theta}_i \sum_j m_j \cos\alpha_{i,j}\lambda_j - \sum_j n_j \cos\alpha_{i,j}\lambda_j = \iota, \tag{7}$$

where i labels the grid point.

2. This is a matrix equation . . .