volume

Computes volume of each region; and, if required, the derivatives of the volume with respect to the interface geometry.

[called by: dforce and xspech.] [calls: .]

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1.1 volume integral

1. The volume enclosed by the v-th interface is given by the integral

$$V = \int_{\mathcal{V}} dv = \frac{1}{3} \int_{\mathcal{V}} \nabla \cdot \mathbf{x} \, dv = \frac{1}{3} \int_{\mathcal{S}} \mathbf{x} \cdot d\mathbf{s} = \frac{1}{3} \int_{0}^{2\pi} d\theta \int_{0}^{2\pi/N} d\zeta \quad \mathbf{x} \cdot \mathbf{x}_{\theta} \times \mathbf{x}_{\zeta}|^{s}$$
(1)

where we have used $\nabla \cdot \mathbf{x} = 3$, and have assumed that the domain is periodic in the angles.

1.2 representation of surfaces

1. The coordinate functions are

$$R(\theta,\zeta) = \sum_{i} R_{e,i} \cos \alpha_i + \sum_{i} R_{o,i} \sin \alpha_i$$
 (2)

$$Z(\theta,\zeta) = \sum_{i} Z_{e,i} \cos \alpha_i + \sum_{i} Z_{o,i} \sin \alpha_i, \tag{3}$$

where $\alpha_i \equiv m_i \theta - n_i \zeta$.

1.3 geometry

- 1. The geometry is controlled by the input parameter Igeometry as follows:
- 2. Igeometry.eq.1 : Cartesian : $\sqrt{g} = R_s$

$$V = \int_0^{2\pi} d\theta \int_0^{2\pi/N} d\zeta R$$

= $2\pi \frac{2\pi}{N} R_{e,1}$ (4)

3. Igeometry.eq.2 : cylindrical : $\sqrt{g}=RR_s=\frac{1}{2}\partial_s(R^2)$

$$V = \frac{1}{2} \int_{0}^{2\pi} d\theta \int_{0}^{2\pi/N} d\zeta R^{2}$$

$$= \frac{1}{2} 2\pi \frac{2\pi}{N} \frac{1}{2} \sum_{i} \sum_{j} R_{e,i} R_{e,j} \left[\cos(\alpha_{i} - \alpha_{j}) + \cos(\alpha_{i} + \alpha_{j}) \right]$$

$$+ \frac{1}{2} 2\pi \frac{2\pi}{N} \frac{1}{2} \sum_{i} \sum_{j} R_{o,i} R_{o,j} \left[\cos(\alpha_{i} - \alpha_{j}) - \cos(\alpha_{i} + \alpha_{j}) \right]$$
(5)

4. Igeometry.eq.3: toroidal: $\mathbf{x} \cdot \mathbf{e}_{\theta} \times \mathbf{e}_{\zeta} = R(ZR_{\theta} - RZ_{\theta})$

$$V = \frac{1}{3} \int_{0}^{2\pi/N} d\theta \int_{0}^{2\pi/N} d\zeta R(ZR_{\theta} - RZ_{\theta})$$

$$= \frac{1}{3} \sum_{i} \sum_{j} \sum_{k} R_{e,i} (Z_{e,j}R_{o,k} - R_{e,j}Z_{o,k}) (+m_{k}) \iint d\theta d\zeta \cos \alpha_{i} \cos \alpha_{j} \cos \alpha_{k}$$

$$+ \frac{1}{3} \sum_{i} \sum_{j} \sum_{k} R_{e,i} (Z_{o,j}R_{e,k} - R_{o,j}Z_{e,k}) (-m_{k}) \iint d\theta d\zeta \cos \alpha_{i} \sin \alpha_{j} \sin \alpha_{k}$$

$$+ \frac{1}{3} \sum_{i} \sum_{j} \sum_{k} R_{o,i} (Z_{e,j}R_{e,k} - R_{e,j}Z_{e,k}) (-m_{k}) \iint d\theta d\zeta \sin \alpha_{i} \cos \alpha_{j} \sin \alpha_{k}$$

$$+ \frac{1}{3} \sum_{i} \sum_{j} \sum_{k} R_{o,i} (Z_{o,j}R_{o,k} - R_{o,j}Z_{o,k}) (+m_{k}) \iint d\theta d\zeta \sin \alpha_{i} \sin \alpha_{j} \cos \alpha_{k}$$

$$(6)$$

- 5. (Recall that the integral over an odd function is zero, so various terms in the above expansion have been ignored.)
- 6. The trigonometric terms are

$$4 \cos \alpha_{i} \cos \alpha_{j} \cos \alpha_{k} = + \cos(\alpha_{i} + \alpha_{j} + \alpha_{k}) + \cos(\alpha_{i} + \alpha_{j} - \alpha_{k}) + \cos(\alpha_{i} - \alpha_{j} + \alpha_{k}) + \cos(\alpha_{i} - \alpha_{j} - \alpha_{k})$$

$$4 \cos \alpha_{i} \sin \alpha_{j} \sin \alpha_{k} = - \cos(\alpha_{i} + \alpha_{j} + \alpha_{k}) + \cos(\alpha_{i} + \alpha_{j} - \alpha_{k}) + \cos(\alpha_{i} - \alpha_{j} + \alpha_{k}) - \cos(\alpha_{i} - \alpha_{j} - \alpha_{k})$$

$$4 \sin \alpha_{i} \cos \alpha_{j} \sin \alpha_{k} = - \cos(\alpha_{i} + \alpha_{j} + \alpha_{k}) + \cos(\alpha_{i} + \alpha_{j} - \alpha_{k}) - \cos(\alpha_{i} - \alpha_{j} + \alpha_{k}) + \cos(\alpha_{i} - \alpha_{j} - \alpha_{k})$$

$$4 \sin \alpha_{i} \cos \alpha_{k} = - \cos(\alpha_{i} + \alpha_{j} + \alpha_{k}) - \cos(\alpha_{i} + \alpha_{j} - \alpha_{k}) + \cos(\alpha_{i} - \alpha_{j} + \alpha_{k}) + \cos(\alpha_{i} - \alpha_{j} - \alpha_{k})$$

$$(7)$$

7. The required derivatives are

$$3\frac{\partial V}{\partial R_{e,i}} = (+Z_{e,j}R_{o,k}m_k - R_{e,j}Z_{o,k}m_k - R_{e,j}Z_{o,k}m_k) \qquad \iint d\theta d\zeta \cos \alpha_i \cos \alpha_j \cos \alpha_k$$

$$+ (-Z_{o,j}R_{e,k}m_k + R_{o,j}Z_{e,k}m_k) \qquad \iint d\theta d\zeta \cos \alpha_i \sin \alpha_j \sin \alpha_k$$

$$+ (-R_{o,k}Z_{e,j}m_i) \qquad \qquad \iint d\theta d\zeta \sin \alpha_i \cos \alpha_j \sin \alpha_k$$

$$+ (-R_{e,k}Z_{o,j}m_i) \qquad \qquad \iint d\theta d\zeta \sin \alpha_i \sin \alpha_j \cos \alpha_k$$

$$(8)$$

$$3\frac{\partial V}{\partial Z_{o,i}} = (-R_{e,k}R_{e,j}m_i) \quad \iint d\theta d\zeta \cos \alpha_i \cos \alpha_j \cos \alpha_k + (-R_{o,k}R_{o,j}m_i) \quad \iint d\theta d\zeta \cos \alpha_i \sin \alpha_j \sin \alpha_k + (-R_{e,j}R_{e,k}m_k) \quad \iint d\theta d\zeta \sin \alpha_i \cos \alpha_j \sin \alpha_k + (+R_{o,j}R_{o,k}m_k) \quad \iint d\theta d\zeta \sin \alpha_i \sin \alpha_j \cos \alpha_k$$

$$(9)$$

volume.h last modified on ; SPEC subroutines;