curent

briefly

contents

1.1 enclosed currents

1. In the vacuum region, the enclosed currents are given by either surface integrals of the current density or line integrals of the magnetic field,

$$\int_{\mathcal{S}} \mathbf{j} \cdot d\mathbf{s} = \int_{\partial\mathcal{S}} \mathbf{B} \cdot d\mathbf{l}, \tag{1}$$

and line integrals are usually easier to compute than surface integrals . . .

2. The magnetic field is given by the curl of the magnetic vector potential, as described in e.g. bfield.

3. The toroidal, plasma current is obtained by taking a "poloidal" loop, $d\mathbf{l} = \mathbf{e}_\theta\, d\theta$, on the plasma boundary, where $B^s = 0$, to obtain

$$I \equiv \int_0^{2\pi} \mathbf{B} \cdot \mathbf{e}_\theta\, d\theta = \int_0^{2\pi} \left(-\partial_s A_\zeta\, \bar{g}_{\theta\theta} + \partial_s A_\theta\, \bar{g}_{\theta\zeta}\right) d\theta, \tag{2}$$

where $\bar{g}_{\mu\nu} \equiv g_{\mu\nu}/\sqrt{g}$.

4. The poloidal, "linking" current through the torus is obtained by taking a "toroidal" loop, $d\mathbf{l} = \mathbf{e}_\zeta\, d\zeta$, on the plasma boundary to obtain

$$G \equiv \int_0^{2\pi} \mathbf{B} \cdot \mathbf{e}_\zeta\, d\zeta = \int_0^{2\pi} \left(-\partial_s A_\zeta\, \bar{g}_{\theta\zeta} + \partial_s A_\theta\, \bar{g}_{\zeta\zeta}\right) d\zeta. \tag{3}$$

1.2 "Fourier integration"

1. Using $f \equiv -\partial_s A_\zeta\, \bar{g}_{\theta\theta} + \partial_s A_\theta\, \bar{g}_{\theta\zeta}$, the integral for the plasma current is

$$I = \sideset{}{'}\sum_i f_i \cos(n_i\zeta) 2\pi, \tag{4}$$

where $\sum'$ includes only the $m_i = 0$ harmonics.

2. Using $g \equiv -\partial_s A_\zeta\, \bar{g}_{\theta\zeta} + \partial_s A_\theta\, \bar{g}_{\zeta\zeta}$, the integral for the linking current is

$$G = \sideset{}{'}\sum_i g_i \cos(m_i\zeta) 2\pi, \tag{5}$$

where $\sum'$ includes only the $n_i = 0$ harmonics.

3. The plasma current, Eqn.(4), should be independent of ζ, and the linking current, Eqn.(5), should be independent of θ. (Perhaps this can be proved analytically; in any case it should be confirmed numerically.)