rzaxis

The coordinate axis is assigned via a poloidal average over an arbitrary surface.

[called by: packxi.]

contents

1.1 coordinate axis

1. The coordinate axis is not an independent degree-of-freedom of the geometry. It is constructed by extrapolating the geometry of a given interface down to a line.

2. Note that, if the coordinate axis depends only on the *geometry* of the interface and not the angle parameterization, then the block tri-diagonal structure of the the force-derivative matrix is preserved.

3. Define the arc-length-weighted averages,

$$R_0(\zeta) \equiv \frac{\displaystyle\int_0^{2\pi} R_1(\theta,\zeta)\,dl}{\displaystyle\int_0^{2\pi} dl}, \qquad Z_0(\zeta) \equiv \frac{\displaystyle\int_0^{2\pi} Z_1(\theta,\zeta)\,dl}{\displaystyle\int_0^{2\pi} dl}, \tag{1}$$

   where $dl \equiv \dot{l}\,d\theta = \sqrt{\partial_\theta R_1(\theta,\zeta)^2 + \partial_\theta Z_1(\theta,\zeta)^2}\,d\theta$.

4. (Note that if $\dot{l}$ does not depend on θ, i.e. if θ is the equal arc-length angle, then the expressions simplify. This constraint is *not* enforced.)

5. The geometry of the coordinate axis thus constructed only depends on the geometry of the interface, i.e. the angular parameterization of the interface is irrelevant.

1.2 coordinate axis: derivatives

1. The derivatives of the coordinate axis with respect to the Fourier harmonics of the given interface are given by

$$\frac{\partial R_0}{\partial R^c_{1,j}} = \int \left(\cos\alpha_j\,\dot{l} - \Delta R_1 R_{1,\theta}\,m_j \sin\alpha_j/\,\dot{l}\right) d\theta/L \tag{2}$$
$$\frac{\partial R_0}{\partial R^s_{1,j}} = \int \left(\sin\alpha_j\,\dot{l} + \Delta R_1 R_{1,\theta}\,m_j \cos\alpha_j/\,\dot{l}\right) d\theta/L \tag{3}$$
$$\frac{\partial R_0}{\partial Z^c_{1,j}} = \int \left(-\Delta R_1 Z_{1,\theta}\,m_j \sin\alpha_j/\,\dot{l}\right) d\theta/L \tag{4}$$
$$\frac{\partial R_0}{\partial Z^s_{1,j}} = \int \left(+\Delta R_1 Z_{1,\theta}\,m_j \cos\alpha_j/\,\dot{l}\right) d\theta/L \tag{5}$$
$$\frac{\partial Z_0}{\partial R^c_{1,j}} = \int \left(-\Delta Z_1 R_{1,\theta}\,m_j \sin\alpha_j/\,\dot{l}\right) d\theta/L \tag{6}$$
$$\frac{\partial Z_0}{\partial R^s_{1,j}} = \int \left(+\Delta Z_1 R_{1,\theta}\,m_j \cos\alpha_j/\,\dot{l}\right) d\theta/L \tag{7}$$
$$\frac{\partial Z_0}{\partial Z^c_{1,j}} = \int \left(\cos\alpha_j\,\dot{l} - \Delta Z_1 Z_{1,\theta}\,m_j \sin\alpha_j/\,\dot{l}\right) d\theta/L \tag{8}$$
$$\frac{\partial Z_0}{\partial Z^s_{1,j}} = \int \left(\sin\alpha_j\,\dot{l} + \Delta Z_1 Z_{1,\theta}\,m_j \cos\alpha_j/\,\dot{l}\right) d\theta/L \tag{9}$$

   where $L(\zeta) \equiv \displaystyle\int_0^{2\pi} dl$.

1.3 some numerical comments

1. First, the differential poloidal length, $\dot{l} \equiv \sqrt{R_\theta^2 + Z_\theta^2}$, is computed in real space using an inverse FFT from the Fourier harmonics of R and Z.
2. Second, the Fourier harmonics of dl are computed using an FFT. The integration over θ to construct $L \equiv \int dl$ is now trivial: just multiply the $m = 0$ harmonics of dl by 2π. The `ajk(1:mn)` variable is used, and this is assigned in global.
3. Next, the weighted $R\,dl$ and $Z\,dl$ are computed in real space, and the poloidal integral is similarly taken.
4. Lastly, the Fourier harmonics are constructed using an FFT after dividing in real space.

rzaxis.h last modified on 7-07-20 15:11:44.00; SPEC subroutines;