tr00ab

Calculates rotational transform given an arbitrary tangential field.

[called by: dforce and mp00ac.]

contents

1.0.1 constructing straight field line angle on interfaces

1. The algorithm stems from introducing a straight field line angle $\theta_s = \theta + \lambda(\theta, \zeta)$, where

$$\lambda = \sum_{j} \lambda_{o,j} \sin(m_j \theta - n_j \zeta) + \sum_{j} \lambda_{e,j} \cos(m_j \theta - n_j \zeta)$$
(1)

and insisting that

$$\frac{\mathbf{B} \cdot \nabla \theta_s}{\mathbf{B} \cdot \nabla \zeta} = \dot{\theta} (1 + \lambda_{\theta}) + \lambda_{\zeta} = \iota, \tag{2}$$

where t is a constant that is to be determined.

2. Writing $\dot{\theta} = -\partial_s A_{\zeta}/\partial_s A_{\theta}$, we have

$$\partial_s A_\theta \ \iota + \partial_s A_\zeta \lambda_\theta - \partial_s A_\theta \lambda_\zeta = -\partial_s A_\zeta \tag{3}$$

3. Expanding this equation we obtain

$$(A'_{\theta,e,k}\cos\alpha_k + A'_{\theta,o,k}\sin\alpha_k) \ t$$

$$+ \ (A'_{\zeta,e,k}\cos\alpha_k + A'_{\zeta,o,k}\sin\alpha_k) \ (+m_j\lambda_{o,j}\cos\alpha_j - m_j\lambda_{e,j}\sin\alpha_j)$$

$$- \ (A'_{\theta,e,k}\cos\alpha_k + A'_{\theta,o,k}\sin\alpha_k) \ (-n_j\lambda_{o,j}\cos\alpha_j + n_j\lambda_{e,j}\sin\alpha_j)$$

$$= - \ (A'_{\zeta,e,k}\cos\alpha_k + A'_{\zeta,o,k}\sin\alpha_k) \ ,$$

$$(4)$$

where summation over k = 1, mn and j = 2, mns is implied

4. After applying double angle formulae,

$$(A'_{\theta,e,k}\cos\alpha_k + A'_{\theta,o,k}\sin\alpha_k) t$$

$$+ \lambda_{o,j} (+m_j A'_{\zeta,e,k} + n_j A'_{\theta,e,k}) [+\cos(\alpha_k + \alpha_j) + \cos(\alpha_k - \alpha_j)] / 2$$

$$+ \lambda_{e,j} (-m_j A'_{\zeta,e,k} - n_j A'_{\theta,e,k}) [+\sin(\alpha_k + \alpha_j) - \sin(\alpha_k - \alpha_j)] / 2$$

$$+ \lambda_{o,j} (+m_j A'_{\zeta,o,k} + n_j A'_{\theta,o,k}) [+\sin(\alpha_k + \alpha_j) + \sin(\alpha_k - \alpha_j)] / 2$$

$$+ \lambda_{e,j} (-m_j A'_{\zeta,o,k} - n_j A'_{\theta,o,k}) [-\cos(\alpha_k + \alpha_j) + \cos(\alpha_k - \alpha_j)] / 2$$

$$= - (A'_{\zeta,e,k}\cos\alpha_k + A'_{\zeta,o,k}\sin\alpha_k),$$

$$(5)$$

and equating coefficients, an equation of the form $\mathbf{A} \cdot \mathbf{x} = \mathbf{b}$ is obtained, where

$$\mathbf{x} = (\underbrace{t}_{\mathbf{x}[1]}, \underbrace{\lambda_{o,2}, \lambda_{o,3}, \dots}_{\mathbf{x}[N+1:2N-1]}, \underbrace{\lambda_{e,2}, \lambda_{e,3}, \dots}_{\mathbf{x}[N+1:2N-1]})^{T}.$$

$$(6)$$

1.0.2 alternative iterative method

1. Consider the equation $\dot{\theta}(1+\lambda_{\theta})+\lambda_{\zeta}=\iota$, where $\lambda=\sum_{j}\lambda_{j}\sin\alpha_{j}$, given on a grid

$$\dot{\theta}_i + \dot{\theta}_i \sum_j m_j \cos \alpha_{i,j} \lambda_j - \sum_j n_j \cos \alpha_{i,j} \lambda_j = \iota, \tag{7}$$

where i labels the grid point.

2. This is a matrix equation . . .