packxi

Packs, and unpacks, geometrical degrees of freedom; and sets coordinate axis.

[called by: dforce, global, hesian, newton and xspech.] [calls: rzaxis.]

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1.1 geometrical degrees of freedom

1. The geometrical degrees-of-freedom, namely the $R_{j,v}$ and $Z_{j,v}$ where v labels the interface and j labels the Fourier harmonic, must be "packxi", and "unpackxi", into a single vector, $\boldsymbol{\xi}$, so that standard numerical routines can be called to find solutions to force-balance, i.e. $\mathbf{F}[\boldsymbol{\xi}] = 0$.

2. A coordinate "pre-conditioning" factor is included:

$$\boldsymbol{\xi}_k \equiv \frac{R_{j,v}}{\Psi_{j,v}}, \tag{1}$$

where $\Psi_{j,v} \equiv$ `psifactor(j,v)`, which is defined in global.

1.2 coordinate axis

1. The coordinate axis is not an independent degree-of-freedom of the geometry. It is constructed by extrapolating the geometry of the innermost interface down to a line.

2. Note that if the coordinate axis depends only on the geometry of the innermost interface then the block tridiagonal structure of the the force-derivative matrix is preserved.

3. Define the arc-length weighted averages,

$$R_0(\zeta) \equiv \frac{\int_0^{2\pi} R_1(\theta,\zeta) dl}{L(\zeta)}, \qquad Z_0(\zeta) \equiv \frac{\int_0^{2\pi} Z_1(\theta,\zeta) dl}{L(\zeta)}, \tag{2}$$

where $L(\zeta) \equiv \int_0^{2\pi} dl$ and $dl \equiv \sqrt{\partial_\theta R_1(\theta,\zeta)^2 + \partial_\theta Z_1(\theta,\zeta)^2}\, d\theta$.

4. Note that if dl does not depend on θ, i.e. if θ is the equal arc-length angle, then the expressions simplify.

5. Note that the geometry of the coordinate axis thus constructed only depends on the geometry of the innermost interface, by which I mean that the geometry of the coordinate axis is independent of the angle parameterization.

1.3 some numerical comments

1. First, the differential poloidal length, $dl \equiv \sqrt{R_\theta^2 + Z_\theta^2}$, is computed in real space using an inverse FFT the from Fourier harmonics of R and Z.

2. Second, the Fourier harmonics of the dl are computed using an FFT. The integration over θ to construct $L \equiv \int dl$ is now trivial: just multiply the $m = 0$ harmonics of dl by 2π. The `ajk(1:mn)` variable is used.

3. Next, the weighted Rdl and Zdl are computed in real space, and the poloidal integral is similarly taken.

4. Lastly, the Fourier harmonics are constructed using an FFT after dividing in real space.