jo00aa

Measures error in Beltrami field, $||\nabla \times \mathbf{B} - \mu \mathbf{B}||$.

[called by: xspech.] [calls: coords.]

contents

1	jo00	aa :	1
	1.1	overview	1
	1.2	construction of current, $\mathbf{j} \equiv \nabla \times \nabla \times \mathbf{A}$	1
	1.3	quantification of the error	1
	1.4	details of the numerics	2
	1.5	comments	2

1.1 overview

This routine is called by xspech as a post diagnostic and only if Lcheck = 1.

1.2 construction of current, $\mathbf{j} \equiv \nabla \times \nabla \times \mathbf{A}$

1. The components of the vector potential, $\mathbf{A} = A_{\theta} \nabla + A_{\zeta} \nabla \zeta$, are

$$A_{\theta}(s,\theta,\zeta) = \sum_{i,l} A_{\theta,e,i,l} \, \overline{T}_{l,i}(s) \cos \alpha_i + \sum_{i,l} A_{\theta,o,i,l} \, \overline{T}_{l,i}(s) \sin \alpha_i, \tag{1}$$

$$A_{\zeta}(s,\theta,\zeta) = \sum_{i,l} A_{\zeta,e,i,l} \, \overline{T}_{l,i}(s) \cos \alpha_i + \sum_{i,l} A_{\zeta,o,i,l} \, \overline{T}_{l,i}(s) \sin \alpha_i, \tag{2}$$

where $\overline{T}_{l,i}(s) \equiv \overline{s}^{m_i/2} T_l(s)$, $T_l(s)$ is the Chebyshev polynomial, and $\alpha_j \equiv m_j \theta - n_j \zeta$. The regularity factor, $\overline{s}^{m_i/2}$, where $\overline{s} \equiv (1+s)/2$, is only included if there is a coordinate singularity in the domain (i.e. only in the innermost volume, and only in cylindrical and toroidal geometry.)

2. The magnetic field, $\sqrt{g} \mathbf{B} = \sqrt{g} B^s \mathbf{e}_s + \sqrt{g} B^\theta \mathbf{e}_\theta + \sqrt{g} B^\zeta \mathbf{e}_\zeta$, is

$$\sqrt{g} \mathbf{B} = \mathbf{e}_{s} \sum_{i,l} \left[(-m_{i} A_{\zeta,e,i,l} - n_{i} A_{\theta,e,i,l}) \overline{T}_{l,i} \sin \alpha_{i} + (+m_{i} A_{\zeta,o,i,l} + n_{i} A_{\theta,o,i,l}) \overline{T}_{l,i} \cos \alpha_{i} \right]
+ \mathbf{e}_{\theta} \sum_{i,l} \left[(-m_{i} A_{\zeta,e,i,l}) \overline{T}'_{l,i} \cos \alpha_{i} + (-m_{i} A_{\zeta,o,i,l} + n_{i} A_{\theta,o,i,l}) \overline{T}'_{l,i} \sin \alpha_{i} \right]
+ \mathbf{e}_{\zeta} \sum_{i,l} \left[(-A_{\theta,e,i,l}) \overline{T}'_{l,i} \cos \alpha_{i} + (-A_{\theta,o,i,l}) \overline{T}'_{l,i} \sin \alpha_{i} \right]$$
(3)

3. The current is

$$\sqrt{g}\,\mathbf{j} = (\partial_{\theta}B_{\zeta} - \partial_{\zeta}B_{\theta})\,\mathbf{e}_{s} + (\partial_{\zeta}B_{s} - \partial_{s}B_{\zeta})\,\mathbf{e}_{\theta} + (\partial_{s}B_{\theta} - \partial_{\theta}B_{s})\,\mathbf{e}_{\zeta},\tag{4}$$

where (for computational convenience) the covariant components of ${\bf B}$ are computed as

$$B_s = (\sqrt{g}B^s) g_{ss} / \sqrt{g} + (\sqrt{g}B^\theta) g_{s\theta} / \sqrt{g} + (\sqrt{g}B^\zeta) g_{s\zeta} / \sqrt{g}, \tag{5}$$

$$B_{\theta} = (\sqrt{g}B^{s}) g_{s\theta} / \sqrt{g} + (\sqrt{g}B^{\theta}) g_{\theta\theta} / \sqrt{g} + (\sqrt{g}B^{\zeta}) g_{\theta\zeta} / \sqrt{g}, \tag{6}$$

$$B_{\zeta} = (\sqrt{g}B^{s}) g_{s\zeta} / \sqrt{g} + (\sqrt{g}B^{\theta}) g_{\theta\zeta} / \sqrt{g} + (\sqrt{g}B^{\zeta}) g_{\zeta\zeta} / \sqrt{g}.$$
 (7)

1.3 quantification of the error

1. The measures of the error are

$$||(\mathbf{j} - \mu \mathbf{B}) \cdot \nabla s|| \equiv \int ds \oint d\theta d\zeta ||\nabla g \mathbf{j} \cdot \nabla s - \mu \sqrt{g} \mathbf{B} \cdot \nabla s|, \qquad (8)$$

$$||(\mathbf{j} - \mu \mathbf{B}) \cdot \nabla \theta|| = \int ds \oint d\theta d\zeta ||\nabla g \mathbf{j} \cdot \nabla \theta - \mu \sqrt{g} \mathbf{B} \cdot \nabla \theta|, \qquad (9)$$

$$||(\mathbf{j} - \mu \mathbf{B}) \cdot \nabla \zeta|| \equiv \int ds \oint d\theta d\zeta ||\nabla g \mathbf{j} \cdot \nabla \zeta - \mu \sqrt{g} \mathbf{B} \cdot \nabla \zeta|.$$
(10)

1.4 details of the numerics

- 1. The integration over s is performed using Gaussian integration, e.g., $\int f(s)ds \approx \sum_k \omega_k f(s_k)$; with the abscissae, s_k , and the weights, ω_k , for k=1, Iquad_v, determined by CDGQF. The resolution, $\mathbb{N} \equiv \text{Iquad}_v$, is determined by Nquad (see global and preset). A fatal error is enforced by jo00aa if CDGQF returns an ifail $\neq 0$.
- 2. Inside the Gaussian quadrature loop, i.e. for each s_k ,
 - (a) The metric elements, $g_{\mu,\nu} \equiv \text{gij}(1:6,0,1:\text{Ntz})$, and the Jacobian, $\sqrt{g} \equiv \text{sg}(0,1:\text{Ntz})$, are calculated on a regular angular grid, (θ_i,ζ_j) , in coords. The derivatives $\partial_i g_{\mu,\nu} \equiv \text{gij}(1:6,i,1:\text{Ntz})$ and $\partial_i \sqrt{g} \equiv \text{sg}(i,1:\text{Ntz})$, with respect to $i \in \{s,\theta,\zeta\}$ are also returned.
 - (b) The Fourier components of the vector potential given in Eqn.(1) and Eqn.(2), and their first and second radial derivatives, are summed.
 - (c) The quantities $\sqrt{g}B^s$, $\sqrt{g}B^\theta$ and $\sqrt{g}B^\zeta$, and their first and second derivatives with respect to (s, θ, ζ) , are computed on the regular angular grid (using FFTs).
 - (d) The following quantities are then computed on the regular angular grid

$$\sqrt{g}j^{s} = \sum_{u} \left[\partial_{\theta}(\sqrt{g}B^{u}) g_{u,\zeta} + (\sqrt{g}B^{u}) \partial_{\theta}g_{u,\zeta} - (\sqrt{g}B^{u})g_{u,\zeta} \partial_{\theta}\sqrt{g}/\sqrt{g} \right] / \sqrt{g}$$

$$- \sum_{u} \left[\partial_{\zeta}(\sqrt{g}B^{u}) g_{u,\theta} + (\sqrt{g}B^{u}) \partial_{\zeta}g_{u,\theta} - (\sqrt{g}B^{u})g_{u,\theta} \partial_{\zeta}\sqrt{g}/\sqrt{g} \right] / \sqrt{g}, \tag{11}$$

$$\sqrt{g}j^{\theta} = \sum_{u} \left[\partial_{\zeta}(\sqrt{g}B^{u}) g_{u,\beta} + (\sqrt{g}B^{u}) \partial_{\zeta}g_{u,\beta} - (\sqrt{g}B^{u})g_{u,\beta} \partial_{\zeta}\sqrt{g}/\sqrt{g} \right] / \sqrt{g}$$

$$\sqrt{g}j^{\theta} = \sum_{u} \left[\partial_{\zeta}(\sqrt{g}B^{u}) g_{u,s} + (\sqrt{g}B^{u}) \partial_{\zeta}g_{u,s} - (\sqrt{g}B^{u})g_{u,s} \partial_{\zeta}\sqrt{g}/\sqrt{g} \right] / \sqrt{g}
- \sum_{u} \left[\partial_{s}(\sqrt{g}B^{u}) g_{u,\zeta} + (\sqrt{g}B^{u}) \partial_{s}g_{u,\zeta} - (\sqrt{g}B^{u})g_{u,\zeta} \partial_{s}\sqrt{g}/\sqrt{g} \right] / \sqrt{g},$$
(12)

$$\sqrt{g}j^{\zeta} = \sum_{u} \left[\partial_{s}(\sqrt{g}B^{u}) g_{u,\theta} + (\sqrt{g}B^{u}) \partial_{s}g_{u,\theta} - (\sqrt{g}B^{u})g_{u,\theta} \partial_{s}\sqrt{g}/\sqrt{g} \right] / \sqrt{g}$$

$$- \sum_{u} \left[\partial_{\theta}(\sqrt{g}B^{u}) g_{u,s} + (\sqrt{g}B^{u}) \partial_{\theta}g_{u,s} - (\sqrt{g}B^{u})g_{u,s} \partial_{\theta}\sqrt{g}/\sqrt{g} \right] / \sqrt{g}. \tag{13}$$

3. The final calculation of the error, which is written to screen, is a sum over the angular grid:

$$E^{s} \equiv \frac{1}{N} \sum_{k} \omega_{k} \sum_{i,j} |\sqrt{g}j^{s} - \mu \sqrt{g}B^{s}|, \tag{14}$$

$$E^{\theta} \equiv \frac{1}{N} \sum_{k} \omega_{k} \sum_{i,j} |\sqrt{g}j^{\theta} - \mu \sqrt{g}B^{\theta}|, \tag{15}$$

$$E^{\zeta} \equiv \frac{1}{N} \sum_{k} \omega_{k} \sum_{i,j} |\sqrt{g}j^{\zeta} - \mu \sqrt{g}B^{\zeta}|, \tag{16}$$

where $N \equiv \sum_{i,j} 1$.

1.5 comments

- 1. Is there a better definition and quantification of the error? For example, should we employ an error measure that is dimensionless?
- 2. If the coordinate singularity is in the domain, then $|\nabla \theta| \to \infty$ at the coordinate origin. What then happens to $||(\mathbf{j} \mu \mathbf{B}) \cdot \nabla \theta||$ as defined in Eqn.(9)?
- 3. What is the predicted scaling of the error in the Chebyshev-Fourier representation scale with numerical resolution? Note that the predicted error scaling for E^s , E^θ and E^ζ may not be standard, as various radial derivatives are taken to compute the components of \mathbf{j} . (See for example the discussion in Sec.IV.C in [Hudson, Dewar et al., Phys. Plasmas 19, 112502 (2012)], where the expected scaling of the error for a finite-element implementation is confirmed numerically.)
- 4. Instead of using Gaussian integration to compute the integral over s, an adaptive quadrature algorithm may be preferable.

jo00aa.h last modified on 18-07-26 10:50:26.7;

SPEC subroutines;