# coords

Calculates coordinate transformation, and metric elements and curvatures if required, using FFTs.

[called by: global, bnorml, lforce, dforce, curent, jo00aa, metrix, sc00aa.] [calls: .]

## contents



### 1.1 coordinates

1. We work in coordinates, $(s,\theta,\zeta)$, which are be defined *inversely* via a transformation *to* Cartesian coordinates, $(x,y,z)$.

2. The toroidal angle, $\zeta$, is identical to the cylindrical angle, $\zeta \equiv \phi$.

3. The radial coordinate, $s$, is *not* a global variable: it only needs to be defined in each volume, and in each volume $s \in [-1,1]$.

4. The choice of poloidal angle, $\theta$, does not affect the following.

### 1.2 geometry

1. The geometry of the "ideal"-interfaces, $\mathbf{x}_v(\theta,\zeta)$, is given by $R(\theta,\zeta)$ and $Z(\theta,\zeta)$ as follows:

   - `Igeometry=1` : Cartesian
   $$\mathbf{x} \equiv \theta\,\hat{\mathbf{i}} + \zeta\,\hat{\mathbf{j}} + R\,\hat{\mathbf{k}} \tag{1}$$
   - `Igeometry=2` : Cylindrical
   $$\mathbf{x} = R\,\cos\theta\,\hat{\mathbf{i}} + R\,\sin\theta\,\hat{\mathbf{j}} + \zeta\,\hat{\mathbf{k}} \tag{2}$$
   - `Igeometry=3` : Toroidal
   $$\mathbf{x} \equiv R\,\hat{\mathbf{r}} + Z\,\hat{\mathbf{k}} \tag{3}$$
   where $\hat{\mathbf{r}} \equiv \cos\phi\,\hat{\mathbf{i}} + \sin\phi\,\hat{\mathbf{j}}$ and $\hat{\phi} \equiv -\sin\phi\,\hat{\mathbf{i}} + \cos\phi\,\hat{\mathbf{j}}$.

2. The geometry of the ideal interfaces is given as Fourier summation: e.g., for stellarator-symmetry
$$R_v(\theta,\zeta) \equiv \sum_j R_{j,v}\cos\alpha_j, \tag{4}$$
$$Z_v(\theta,\zeta) \equiv \sum_j Z_{j,v}\sin\alpha_j, \tag{5}$$
where $\alpha_j \equiv m_j\theta - n_j\zeta$.

### 1.3 interpolation between interfaces

1. The "coordinate" functions, $R(s,\theta,\zeta)$ and $Z(s,\theta,\zeta)$, are constructed by radially interpolating the Fourier representations of the ideal-interfaces.

2. The $v$-th volume is bounded by $\mathbf{x}_{v-1}$ and $\mathbf{x}_v$.

3. In each *annular* volume, the coordinates are constructed by linear interpolation:
$$\begin{aligned} R(s,\theta,\zeta) &\equiv \sum_j \left[\frac{(1-s)}{2}\,R_{j,v-1} + \frac{(1+s)}{2}\,R_{j,v}\right]\cos\alpha_j, \\ Z(s,\theta,\zeta) &\equiv \sum_j \left[\frac{(1-s)}{2}\,Z_{j,v-1} + \frac{(1+s)}{2}\,Z_{j,v}\right]\sin\alpha_j, \end{aligned} \tag{6}$$

### 1.3.1 coordinate singularity: regularized extrapolation

1. For cylindrical or toroidal geometry, in the innermost, "simple-torus" volume, the coordinates are constructed by an interpolation that "encourages" the interpolated coordinate surfaces to not intersect.

2. Introduce $\bar{s} \equiv (s+1)/2$, so that in each volume $\bar{s} \in [0,1]$, then

$$\begin{aligned} R_j(s) &= R_{j,0} + (R_{j,1} - R_{j,0}) f_j, && (7)\\ Z_j(s) &= Z_{j,0} + (Z_{j,1} - Z_{j,0}) f_j, && (8) \end{aligned}$$

   where, in toroidal geometry,

$$f_j \equiv \left\{ \begin{array}{lcl} \bar{s} & , & \text{for } m_j = 0, \\ \bar{s}^{m_j/2} & , & \text{otherwise.} \end{array} \right. \tag{9}$$

3. Note: The location of the coordinate axis, i.e. the $R_{j,0}$ and $Z_{j,0}$, is set in the coordinate "packing" and "unpacking" routine, packxi.

## 1.4 Jacobian

1. The coordinate Jacobian (and some other metric information) is given by

   - `Igeometry=1` : Cartesian

$$\begin{aligned} \mathbf{e}_\theta \times \mathbf{e}_\zeta &= -R_\theta \,\hat{\mathbf{i}} - R_\zeta \,\hat{\mathbf{j}} + \hat{\mathbf{k}} && (10)\\ \boldsymbol{\xi} \cdot \mathbf{e}_\theta \times \mathbf{e}_\zeta &= \delta R && (11)\\ \sqrt{g} &= R_s && (12) \end{aligned}$$

   - `Igeometry=2` : Cylindrical

$$\begin{aligned} \mathbf{e}_\theta \times \mathbf{e}_\zeta &= (R_\theta \sin\theta + R\cos\theta)\,\hat{\mathbf{i}} + (R\sin\theta - R_\theta\cos\theta)\,\hat{\mathbf{j}} - RR_\zeta \,\hat{\mathbf{k}} && (13)\\ \boldsymbol{\xi} \cdot \mathbf{e}_\theta \times \mathbf{e}_\zeta &= \delta R\, R && (14)\\ \sqrt{g} &= R_s\, R && (15) \end{aligned}$$

   - `Igeometry=3` : Toroidal

$$\begin{aligned} \mathbf{e}_\theta \times \mathbf{e}_\zeta &= -R\, Z_\theta \,\hat{r} + (Z_\theta \, R_\zeta - R_\theta \, Z_\zeta)\hat{\phi} + R\, R_\theta \,\hat{z} && (16)\\ \boldsymbol{\xi} \cdot \mathbf{e}_\theta \times \mathbf{e}_\zeta &= R(\delta Z \; R_\theta - \delta R \; Z_\theta) && (17)\\ \sqrt{g} &= R(Z_s \; R_\theta - R_s \; Z_\theta) && (18) \end{aligned}$$

### 1.4.1 cylindrical metrics

1. The cylindrical metrics and Jacobian are

$$\sqrt{g} = R_s R, \quad g_{ss} = R_s R_s, \quad g_{s\theta} = R_s R_\theta, \quad g_{s\zeta} = R_s R_\zeta, \quad g_{\theta\theta} = R_\theta R_\theta + R^2, \quad g_{\theta\zeta} = R_\theta R_\zeta, \quad g_{\zeta\zeta} = R_\zeta R_\zeta + 1 \tag{19}$$

## 1.5 logical control

1. The logical control is provided by `Lcurvature` as follows:

   `Lcurvature=0` : only the coordinate transformation is computed, i.e. only $R$ and $Z$ are calculated
   e.g. global

   `Lcurvature=1` : the Jacobian, $\sqrt{g}$, and "lower" metrics, $g_{\mu,\nu}$, are calculated
   e.g. bnorml, lforce, curent, metrix, sc00aa

   `Lcurvature=2` : the "curvature" terms are calculated, by which I mean the second derivatives of the position vector; this information is required for computing the current, $\mathbf{j} = \nabla \times \nabla \times \mathbf{A}$
   e.g. jo00aa

   `Lcurvature=3` : the derivative of the $g_{\mu,\nu}/\sqrt{g}$ w.r.t. the interface boundary geometry is calculated
   e.g. metrix, curent

   `Lcurvature=4` : the derivative of the $g_{\mu,\nu}$ w.r.t. the interface boundary geometry is calculated
   e.g. dforce