### rzaxis

The coordinate axis is assigned via a poloidal average over an arbitrary surface.

[called by: preset, packxi.]

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## 1.1 coordinate axis

- 1. The coordinate axis is <u>not</u> an independent degree-of-freedom of the geometry. It is constructed by extrapolating the geometry of a given interface, as determined by  $i \equiv ivol$  which is given on input, down to a line.
- 2. If the coordinate axis depends only on the <u>geometry</u> of the interface and not the angle parameterization, then the block tridiagonal structure of the the force-derivative matrix is preserved.
- 3. Define the arc-length-weighted averages,

$$R_0(\zeta) \equiv \frac{\int_0^{2\pi} R_i(\theta, \zeta) \, dl}{\int_0^{2\pi} dl}, \qquad Z_0(\zeta) \equiv \frac{\int_0^{2\pi} Z_i(\theta, \zeta) \, dl}{\int_0^{2\pi} dl}, \tag{1}$$

where  $dl \equiv \dot{l} d\theta = \sqrt{\partial_{\theta} R_i(\theta, \zeta)^2 + \partial_{\theta} Z_i(\theta, \zeta)^2} d\theta$ .

- 4. (Note that if  $\dot{l}$  does not depend on  $\theta$ , i.e. if  $\theta$  is the equal arc-length angle, then the expressions simplify. This constraint is not enforced.)
- 5. The geometry of the coordinate axis thus constructed only depends on the geometry of the interface, i.e. the angular parameterization of the interface is irrelevant.

### 1.2 coordinate axis: derivatives

1. The derivatives of the coordinate axis with respect to the Fourier harmonics of the given interface are given by

$$\frac{\partial R_0}{\partial R_{i,j}^c} = \int \left(\cos \alpha_j \, \dot{l} - \Delta R_i R_{i,\theta} \, m_j \sin \alpha_j / \, \dot{l}\right) d\theta / L \tag{2}$$

$$\frac{\partial R_0}{\partial R_{i,j}^s} = \int \left( \sin \alpha_j \ \dot{l} + \Delta R_i R_{i,\theta} \ m_j \cos \alpha_j / \ \dot{l} \right) d\theta / L \tag{3}$$

$$\frac{\partial R_0}{\partial Z_{i,j}^c} = \int \left( -\Delta R_i Z_{i,\theta} \, m_j \sin \alpha_j / \, \dot{l} \right) d\theta / L \tag{4}$$

$$\frac{\partial R_0}{\partial Z_{i,j}^s} = \int \left( +\Delta R_i Z_{i,\theta} \, m_j \cos \alpha_j / \, \dot{l} \right) d\theta / L \tag{5}$$

$$\frac{\partial Z_0}{\partial R_{i,j}^c} = \int \left( -\Delta Z_i R_{i,\theta} \, m_j \sin \alpha_j / \, \dot{l} \right) d\theta / L \tag{6}$$

$$\frac{\partial Z_0}{\partial R_{i,j}^s} = \int \left( +\Delta Z_i R_{i,\theta} \, m_j \cos \alpha_j / \, \dot{l} \right) d\theta / L \tag{7}$$

$$\frac{\partial Z_0}{\partial Z_{i,j}^c} = \int \left(\cos \alpha_j \ \dot{l} - \Delta Z_i Z_{i,\theta} \ m_j \sin \alpha_j / \ \dot{l}\right) d\theta / L \tag{8}$$

$$\frac{\partial Z_0}{\partial Z_{i,j}^s} = \int \left( \sin \alpha_j \ \dot{l} + \Delta Z_i Z_{i,\theta} \ m_j \cos \alpha_j / \ \dot{l} \right) d\theta / L \tag{9}$$

where 
$$L(\zeta) \equiv \int_0^{2\pi} dl$$
.

# 1.3 some numerical comments

- 1. First, the differential poloidal length,  $\dot{l} \equiv \sqrt{R_{\theta}^2 + Z_{\theta}^2}$ , is computed in real space using an inverse FFT from the Fourier harmonics of R and Z.
- 2. Second, the Fourier harmonics of dl are computed using an FFT. The integration over  $\theta$  to construct  $L \equiv \int dl$  is now trivial: just multiply the m = 0 harmonics of dl by  $2\pi$ . The ajk(1:mn) variable is used, and this is assigned in global.
- 3. Next, the weighted R dl and Z dl are computed in real space, and the poloidal integral is similarly taken.
- 4. Last, the Fourier harmonics are constructed using an FFT after dividing in real space.

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 ${\bf SPEC\ subroutines};$