

**coords**

Calculates coordinate transformation, and metric elements and curvatures if required, using FFTs.

[called by: [global](#), [bnorml](#), [lforce](#), [dforce](#), [curent](#), [jo00aa](#), [metrix](#), [sc00aa](#).]

[calls: .]

**contents**

|   |          |
|---|----------|
| <b>1 coords</b>   | <b>1</b> |
| 1.1 coordinates . . . . .   | 1        |
| 1.2 geometry . . . . .  | 1        |
| 1.3 interpolation between interfaces . . . . .                    | 1        |
| 1.3.1 coordinate singularity: regularized extrapolation . . . . . | 2        |
| 1.4 Jacobian . . . . .  | 2        |
| 1.4.1 cylindrical metrics . . . . .                               | 2        |
| 1.5 logical control . . . . .                                     | 2        |

**1.1 coordinates**

1. We work in coordinates,  $(s, \theta, \zeta)$ , which are be defined inversely via a transformation to Cartesian coordinates,  $(x, y, z)$ .
2. The toroidal angle,  $\zeta$ , is identical to the cylindrical angle,  $\zeta \equiv \phi$ .
3. The radial coordinate,  $s$ , is not a global variable: it only needs to be defined in each volume, and in each volume  $s \in [-1, 1]$ .
4. The choice of poloidal angle,  $\theta$ , does not affect the following.

**1.2 geometry**

1. The geometry of the “ideal”-interfaces,  $\mathbf{x}_v(\theta, \zeta)$ , is given by  $R(\theta, \zeta)$  and  $Z(\theta, \zeta)$  as follows:

- Igeometry=1 : Cartesian

$$\mathbf{x} \equiv \theta \hat{\mathbf{i}} + \zeta \hat{\mathbf{j}} + R \hat{\mathbf{k}} \quad (1)$$

- Igeometry=2 : Cylindrical

$$\mathbf{x} = R \cos \theta \hat{\mathbf{i}} + R \sin \theta \hat{\mathbf{j}} + \zeta \hat{\mathbf{k}} \quad (2)$$

- Igeometry=3 : Toroidal

$$\mathbf{x} \equiv R \hat{\mathbf{r}} + Z \hat{\mathbf{k}} \quad (3)$$

where  $\hat{\mathbf{r}} \equiv \cos \phi \hat{\mathbf{i}} + \sin \phi \hat{\mathbf{j}}$  and  $\hat{\phi} \equiv -\sin \phi \hat{\mathbf{i}} + \cos \phi \hat{\mathbf{j}}$ .

2. The geometry of the ideal interfaces is given as Fourier summation: e.g., for stellarator-symmetry

$$R_v(\theta, \zeta) \equiv \sum_j R_{j,v} \cos \alpha_j, \quad (4)$$

$$Z_v(\theta, \zeta) \equiv \sum_j Z_{j,v} \sin \alpha_j, \quad (5)$$

where  $\alpha_j \equiv m_j \theta - n_j \zeta$ .

**1.3 interpolation between interfaces**

1. The “coordinate” functions,  $R(s, \theta, \zeta)$  and  $Z(s, \theta, \zeta)$ , are constructed by radially interpolating the Fourier representations of the ideal-interfaces.
2. The  $v$ -th volume is bounded by  $\mathbf{x}_{v-1}$  and  $\mathbf{x}_v$ .
3. In each annular volume, the coordinates are constructed by linear interpolation:

$$\begin{aligned} R(s, \theta, \zeta) &\equiv \sum_j \left[ \frac{(1-s)}{2} R_{j,v-1} + \frac{(1+s)}{2} R_{j,v} \right] \cos \alpha_j, \\ Z(s, \theta, \zeta) &\equiv \sum_j \left[ \frac{(1-s)}{2} Z_{j,v-1} + \frac{(1+s)}{2} Z_{j,v} \right] \sin \alpha_j, \end{aligned} \quad (6)$$

### 1.3.1 coordinate singularity: regularized extrapolation

1. For cylindrical or toroidal geometry, in the innermost, “simple-torus” volume, the coordinates are constructed by an interpolation that “encourages” the interpolated coordinate surfaces to not intersect.
2. Introduce  $\bar{s} \equiv (s + 1)/2$ , so that in each volume  $\bar{s} \in [0, 1]$ , then

$$R_j(s) = R_{j,0} + (R_{j,1} - R_{j,0})f_j, \quad (7)$$

$$Z_j(s) = Z_{j,0} + (Z_{j,1} - Z_{j,0})f_j, \quad (8)$$

where, in toroidal geometry,

$$f_j \equiv \begin{cases} \bar{s} & , \text{ for } m_j = 0, \\ \bar{s}^{m_j/2} & , \text{ otherwise.} \end{cases} \quad (9)$$

3. Note: The location of the coordinate axis, i.e. the  $R_{j,0}$  and  $Z_{j,0}$ , is set in the coordinate “packing” and “unpacking” routine, [packxi](#).

## 1.4 Jacobian

1. The coordinate Jacobian (and some other metric information) is given by

- Igeometry=1 : Cartesian

$$\mathbf{e}_\theta \times \mathbf{e}_\zeta = -R_\theta \hat{\mathbf{i}} - R_\zeta \hat{\mathbf{j}} + \hat{\mathbf{k}} \quad (10)$$

$$\boldsymbol{\xi} \cdot \mathbf{e}_\theta \times \mathbf{e}_\zeta = \delta R \quad (11)$$

$$\sqrt{g} = R_s \quad (12)$$

- Igeometry=2 : Cylindrical

$$\mathbf{e}_\theta \times \mathbf{e}_\zeta = (R_\theta \sin \theta + R \cos \theta) \hat{\mathbf{i}} + (R \sin \theta - R_\theta \cos \theta) \hat{\mathbf{j}} - R R_\zeta \hat{\mathbf{k}} \quad (13)$$

$$\boldsymbol{\xi} \cdot \mathbf{e}_\theta \times \mathbf{e}_\zeta = \delta R R \quad (14)$$

$$\sqrt{g} = R_s R \quad (15)$$

- Igeometry=3 : Toroidal

$$\mathbf{e}_\theta \times \mathbf{e}_\zeta = -R Z_\theta \hat{\mathbf{r}} + (Z_\theta R_\zeta - R_\theta Z_\zeta) \hat{\phi} + R R_\theta \hat{\mathbf{z}} \quad (16)$$

$$\boldsymbol{\xi} \cdot \mathbf{e}_\theta \times \mathbf{e}_\zeta = R(\delta Z R_\theta - \delta R Z_\theta) \quad (17)$$

$$\sqrt{g} = R(Z_s R_\theta - R_s Z_\theta) \quad (18)$$

### 1.4.1 cylindrical metrics

1. The cylindrical metrics and Jacobian are

$$\sqrt{g} = R_s R, \quad g_{ss} = R_s R_s, \quad g_{s\theta} = R_s R_\theta, \quad g_{s\zeta} = R_s R_\zeta, \quad g_{\theta\theta} = R_\theta R_\theta + R^2, \quad g_{\theta\zeta} = R_\theta R_\zeta, \quad g_{\zeta\zeta} = R_\zeta R_\zeta + 1 \quad (19)$$

## 1.5 logical control

1. The logical control is provided by **Lcurvature** as follows:

**Lcurvature=0** : only the coordinate transformation is computed, i.e. only  $R$  and  $Z$  are calculated

e.g. [global](#)

**Lcurvature=1** : the Jacobian,  $\sqrt{g}$ , and “lower” metrics,  $g_{\mu,\nu}$ , are calculated

e.g. [bnorml](#), [lforce](#), [curent](#), [metrix](#), [sc00aa](#)

**Lcurvature=2** : the “curvature” terms are calculated, by which I mean the second derivatives of the position vector; this information is required for computing the current,  $\mathbf{j} = \nabla \times \nabla \times \mathbf{A}$

e.g. [jo00aa](#)

**Lcurvature=3** : the derivative of the  $g_{\mu,\nu}/\sqrt{g}$  w.r.t. the interface boundary geometry is calculated

e.g. [metrix](#), [curent](#)

**Lcurvature=4** : the derivative of the  $g_{\mu,\nu}$  w.r.t. the interface boundary geometry is calculated

e.g. [dforce](#)