coords

Calculates coordinate transformation, and metric elements and curvatures if required, using FFTs.

[called by: global, bnorml, lforce, dforce, curent, jo00aa, metrix, sc00aa.] [calls: .]

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1.1 coordinates

1. We work in coordinates, (s, θ, ζ), which are be defined inversely via a transformation to Cartesian coordinates, (x, y, z).
2. The toroidal angle, ζ, is identical to the cylindrical angle, $\zeta \equiv \phi$.
3. The radial coordinate, s, is not a global variable: it only needs to be defined in each volume, and in each volume $s \in [-1, 1]$.
4. The choice of poloidal angle, θ, does not affect the following.

1.2 geometry

1. The geometry of the "ideal"-interfaces, $\mathbf{x}_v(\theta, \zeta)$, is given by $R(\theta, \zeta)$ and $Z(\theta, \zeta)$ as follows:
   - `Igeometry=1` : Cartesian
   $$\mathbf{x} \equiv \theta\, \hat{\mathbf{i}} + \zeta\, \hat{\mathbf{j}} + R\, \hat{\mathbf{k}} \tag{1}$$
   - `Igeometry=2` : Cylindrical
   $$\mathbf{x} = R\, \cos\theta\, \hat{\mathbf{i}} + R\, \sin\theta\, \hat{\mathbf{j}} + \zeta\, \hat{\mathbf{k}} \tag{2}$$
   - `Igeometry=3` : Toroidal
   $$\mathbf{x} \equiv R\, \hat{\mathbf{r}} + Z\, \hat{\mathbf{k}} \tag{3}$$
   where $\hat{\mathbf{r}} \equiv \cos\phi\, \hat{\mathbf{i}} + \sin\phi\, \hat{\mathbf{j}}$ and $\hat{\phi} \equiv -\sin\phi\, \hat{\mathbf{i}} + \cos\phi\, \hat{\mathbf{j}}$.
2. The geometry of the ideal interfaces is given as Fourier summation: e.g., for stellarator-symmetry
$$R_v(\theta, \zeta) \equiv \sum_j R_{j,v} \cos\alpha_j, \tag{4}$$
$$Z_v(\theta, \zeta) \equiv \sum_j Z_{j,v} \sin\alpha_j, \tag{5}$$
where $\alpha_j \equiv m_j\theta - n_j\zeta$.

1.3 interpolation between interfaces

1. The "coordinate" functions, $R(s, \theta, \zeta)$ and $Z(s, \theta, \zeta)$, are constructed by radially interpolating the Fourier representations of the ideal-interfaces.
2. The v-th volume is bounded by $\mathbf{x}_{v-1}$ and $\mathbf{x}_v$.
3. In each annular volume, the coordinates are constructed by linear interpolation:
$$\begin{aligned} R(s, \theta, \zeta) &\equiv \sum_j \left[\frac{(1-s)}{2}\, R_{j,v-1} + \frac{(1+s)}{2}\, R_{j,v} \right] \cos\alpha_j, \\ Z(s, \theta, \zeta) &\equiv \sum_j \left[\frac{(1-s)}{2}\, Z_{j,v-1} + \frac{(1+s)}{2}\, Z_{j,v} \right] \sin\alpha_j, \end{aligned} \tag{6}$$

1.3.1 coordinate singularity: regularized extrapolation

1. For cylindrical or toroidal geometry, in the innermost, "simple-torus" volume, the coordinates are constructed by an interpolation that "encourages" the interpolated coordinate surfaces to not intersect.

2. Introduce $\bar{s} \equiv (s+1)/2$, so that in each volume $\bar{s} \in [0,1]$, then

$$R_j(s) = R_{j,0} + (R_{j,1} - R_{j,0}) f_j, \tag{7}$$
$$Z_j(s) = Z_{j,0} + (Z_{j,1} - Z_{j,0}) f_j, \tag{8}$$

where, in toroidal geometry,

$$f_j \equiv \begin{cases} \bar{s} & , \quad \text{for } m_j = 0, \\ \bar{s}^{m_j/2} & , \quad \text{otherwise.} \end{cases} \tag{9}$$

3. Note: The location of the coordinate axis, i.e. the $R_{j,0}$ and $Z_{j,0}$, is set in the coordinate "packing" and "unpacking" routine, packxi.

1.4 Jacobian

1. The coordinate Jacobian (and some other metric information) is given by

- `Igeometry=1` : Cartesian

$$\mathbf{e}_\theta \times \mathbf{e}_\zeta = -R_\theta\, \hat{\mathbf{i}} - R_\zeta\, \hat{\mathbf{j}} + \hat{\mathbf{k}} \tag{10}$$
$$\boldsymbol{\xi} \cdot \mathbf{e}_\theta \times \mathbf{e}_\zeta = \delta R \tag{11}$$
$$\sqrt{g} = R_s \tag{12}$$

- `Igeometry=2` : Cylindrical

$$\mathbf{e}_\theta \times \mathbf{e}_\zeta = (R_\theta \sin\theta + R\cos\theta)\, \hat{\mathbf{i}} + (R\sin\theta - R_\theta \cos\theta)\, \hat{\mathbf{j}} - R R_\zeta\, \hat{\mathbf{k}} \tag{13}$$
$$\boldsymbol{\xi} \cdot \mathbf{e}_\theta \times \mathbf{e}_\zeta = \delta R\, R \tag{14}$$
$$\sqrt{g} = R_s\, R \tag{15}$$

- `Igeometry=3` : Toroidal

$$\mathbf{e}_\theta \times \mathbf{e}_\zeta = -R\, Z_\theta\, \hat{r} + (Z_\theta\, R_\zeta - R_\theta\, Z_\zeta)\hat{\phi} + R\, R_\theta\, \hat{z} \tag{16}$$
$$\boldsymbol{\xi} \cdot \mathbf{e}_\theta \times \mathbf{e}_\zeta = R(\delta Z\, R_\theta - \delta R\, Z_\theta) \tag{17}$$
$$\sqrt{g} = R(Z_s\, R_\theta - R_s\, Z_\theta) \tag{18}$$

1.4.1 cylindrical metrics

1. The cylindrical metrics and Jacobian are

$$\sqrt{g} = R_s R, \quad g_{ss} = R_s R_s, \quad g_{s\theta} = R_s R_\theta, \quad g_{s\zeta} = R_s R_\zeta, \quad g_{\theta\theta} = R_\theta R_\theta + R^2, \quad g_{\theta\zeta} = R_\theta R_\zeta, \quad g_{\zeta\zeta} = R_\zeta R_\zeta + 1 \tag{19}$$

1.5 logical control

1. The logical control is provided by `Lcurvature` as follows:

`Lcurvature=0` : only the coordinate transformation is computed, i.e. only R and Z are calculated
e.g. global

`Lcurvature=1` : the Jacobian, $\sqrt{g}$, and "lower" metrics, $g_{\mu,\nu}$, are calculated
e.g. bnorml, lforce, curent, metrix, sc00aa

`Lcurvature=2` : the "curvature" terms are calculated, by which I mean the second derivatives of the position vector; this information is required for computing the current, $\mathbf{j} = \nabla \times \nabla \times \mathbf{A}$
e.g. jo00aa

`Lcurvature=3` : the derivative of the $g_{\mu,\nu}/\sqrt{g}$ w.r.t. the interface boundary geometry is calculated
e.g. metrix, curent

`Lcurvature=4` : the derivative of the $g_{\mu,\nu}$ w.r.t. the interface boundary geometry is calculated
e.g. dforce