Performance metrics for time-varying drift and other diffusion based models for decision making

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Motivation

• Diffusion based models remain some of the most popular models used to explain choice and response time (RT) data across species. These models, derived from sequential sampling, have been used in a variety of learning and decision making tasks, and have also appeared as components of larger neural network models of cognition [1, 2, 3].

• This proliferation has led to the development of efficient codes for computing properties of the drift diffusion model (DDM) [4, 5]. These codes largely rely on the assumption that the signal to noise ratio of the decision evidence is accumulated at a constant rate:

$$dx(t) = a dt + \sigma dW(t), \qquad x(t_0) = x_0$$

where $x(t)$ represents the accumulated evidence (decision variable), a is a constant drift rate, and $\sigma dW(t)$ are independent white noise increments with variance $\sigma^2 dt$. A decision is made when $x(t)$ crosses one of the two boundaries at $\pm z$.

• Unfortunately, this model is unable to capture changes in drift rate during the decision process. Often the experiment or model is better suited for a time-varying a(t) [3, 6, 7], representing a change in signal (or evidence) during the decision process (e.g., from dynamic changes in "bottom-up" sensory information or in "top-down" allocation of attention).

Approach

• We derived analytic expressions for an n-stage DDM, whereby:

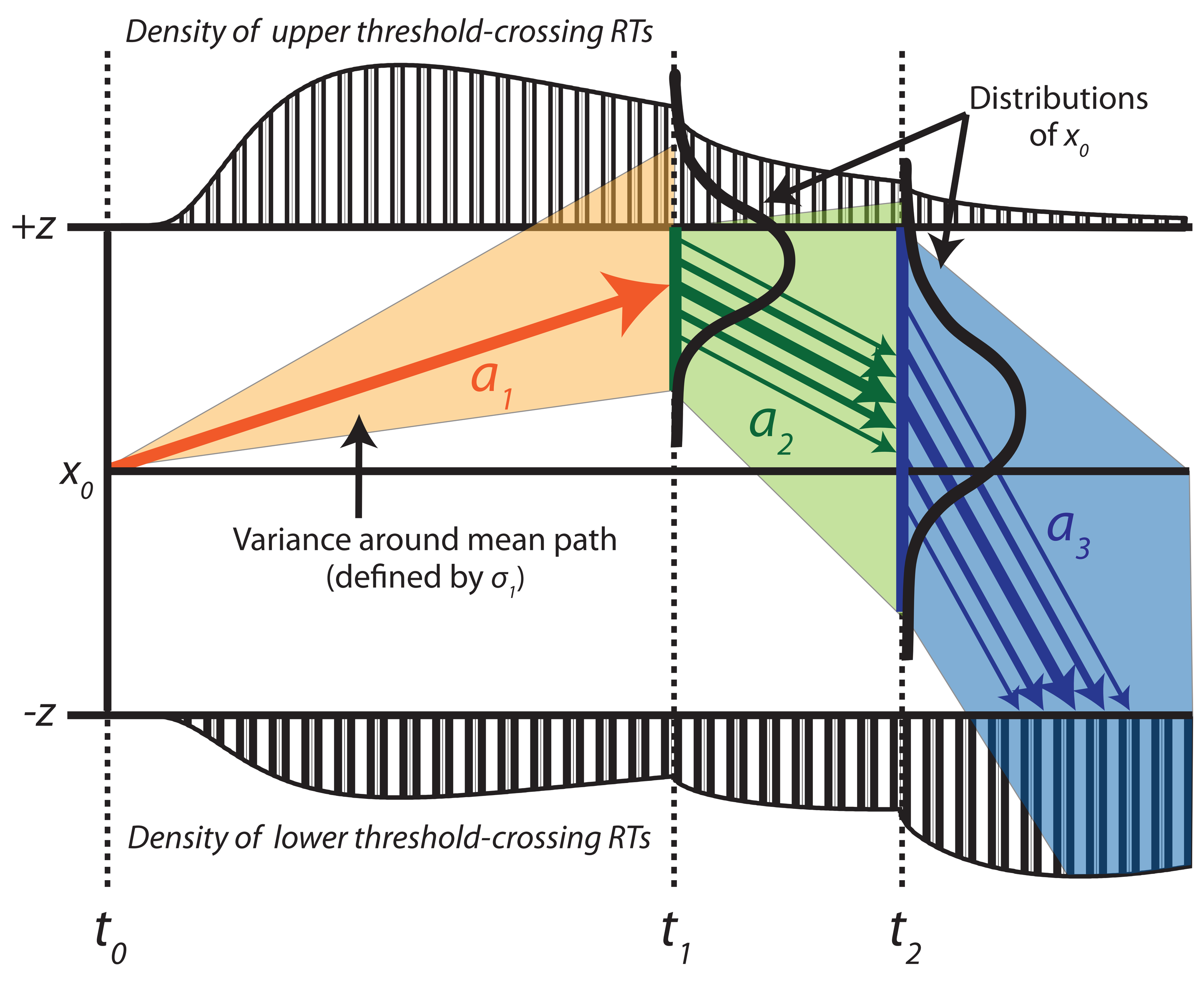
- the decision process begins at time t_0 from a starting point (x_0), or a distribution thereof, with a drift rate a_1 that noisily approaches one of two absorbing boundaries ($\pm z$), with Weiner noise coefficient σ_1
- drift rate (a_i), noise coefficient (σ_i), and/or threshold (z_i) change at each subsequent stage beginning at t_i

$$dx(t) = a(t)dt + \sigma(t)dW(t), \qquad x(t_0) = x_0$$

$$a(t) = a_i, \sigma(t) = \sigma_i, \text{ for } t_{i-1} \le t < t_i$$

• The final distribution of threshold crossing times is determined by compiling individual distributions for the single-stage DDMs given a distribution of starting point (x_0) values at each stage

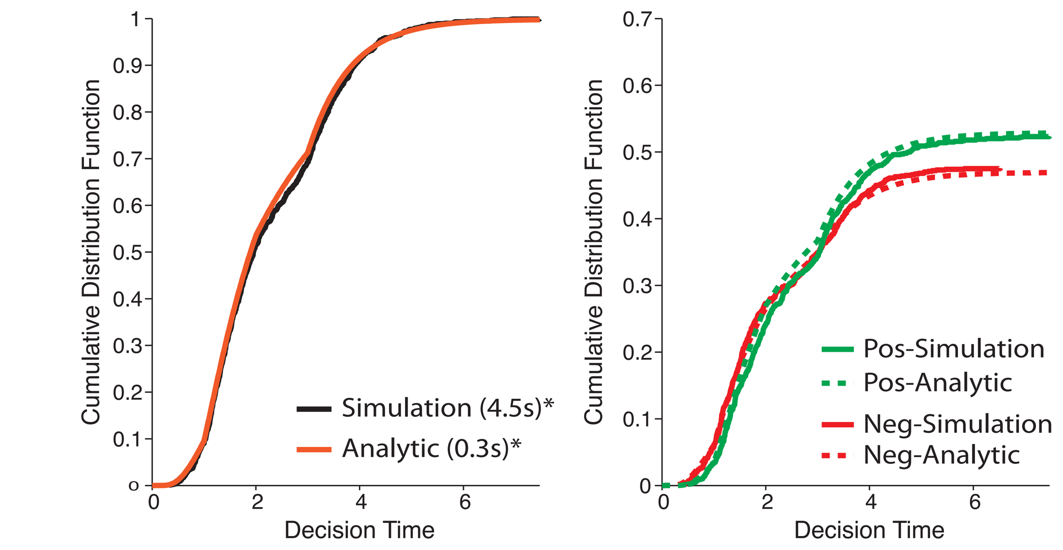
• The distribution of x_0 values on a given stage is calculated conditional on the DDM process in the prior stage by applying optional stopping theorem to carefully selected Martingales



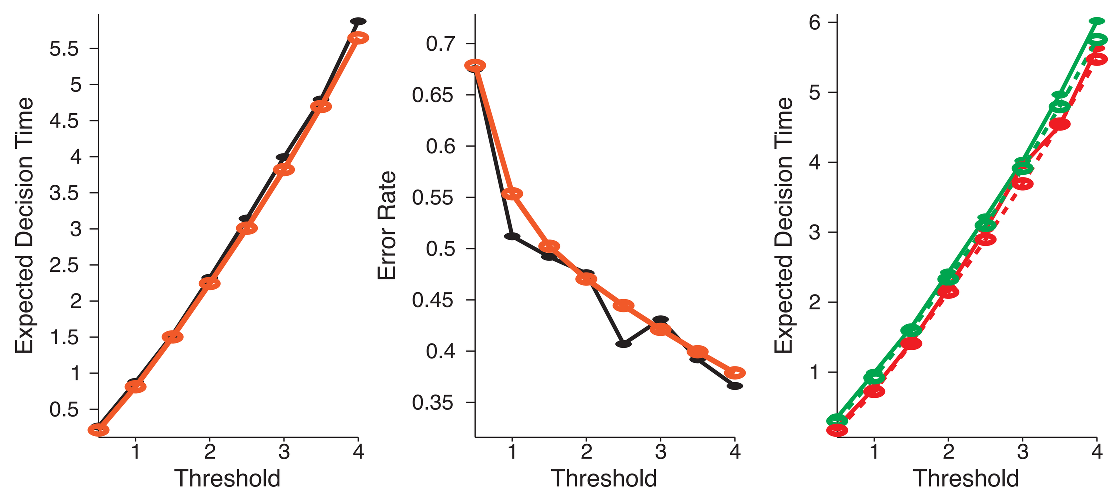
Results

Comparison to Numerical Simulation

Example: 4-stage DDM with biased starting point and different drift rates and noise coefficients at each stage



First passage time distributions for all responses (left) and conditional distributions for positive vs. negative threshold crossings (right) using analytic solution vs. 1000 simulated paths



Analytic vs. simulated expected error rate and response time across multiple (uniform) threshold settings

*Wall clock timing on Macbook Pro running Matlab 2012a, for reference. Code has not been optimized for efficiency.

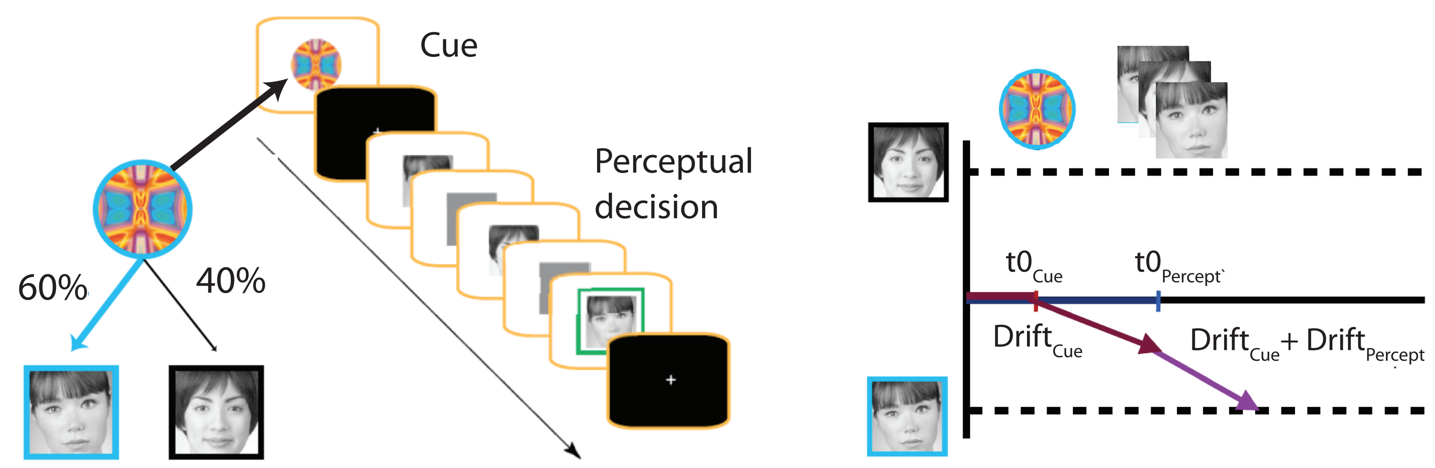
Model Fit to Empirical Data

(Bornstein, Aly, Feng, in prep)

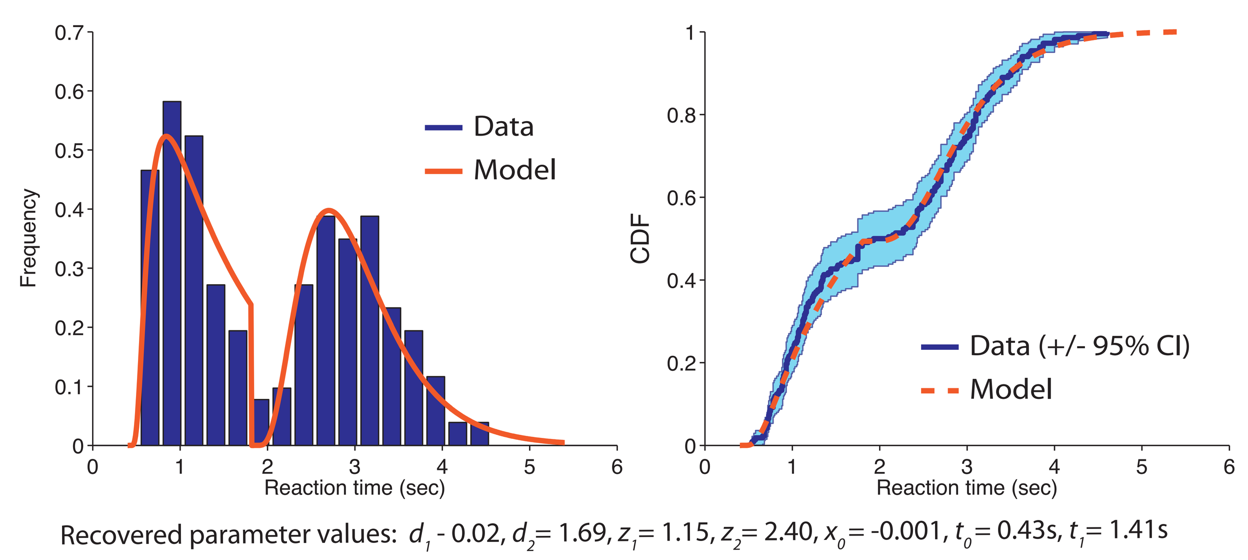
Participants in this study chose between two stimuli based on:

(1) Initial information about the probability of one stimulus or another being shown on that trial (based on past experience), followed by

(2) Noisy perceptual information reflecting one stimulus or the other



Model fits to trials with cued probability of 60% and perceptual coherence of 65% (n = 218 trials):



Recovered parameter values: d_1 = -0.02, d_2 = 1.69, z_1 = 1.15, z_2 = 2.40, x_0 = -0.001, t_0 = 0.43s, t_1 = 1.41s

Example Applications

Time-Varying Drift, Ex. 1: Shifting focus of attention

The n-stage DDM can describe situations in which valuation changes dynamically with the focus of attention (e.g., down-weighting unattended relative to attended items [3]):

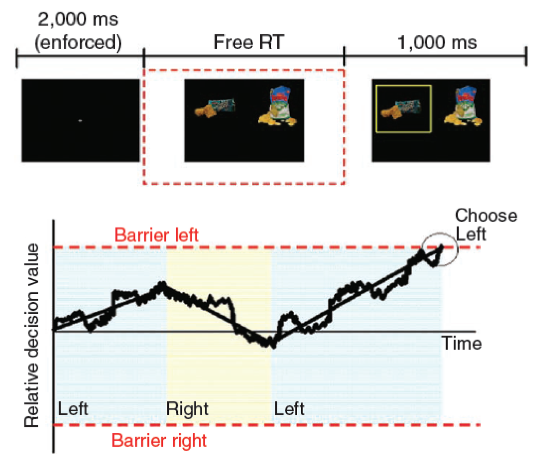
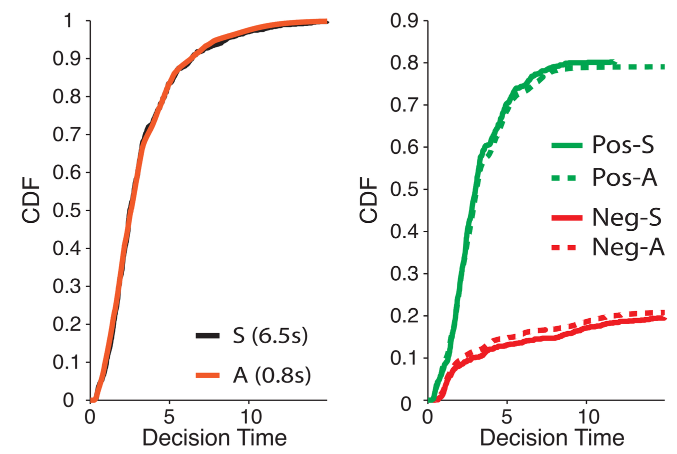


Figure from Krajbich et al., 2010



Time-Varying Drift, Ex. 2: Narrowing attentional spotlight

The n-stage DDM can similarly describe situations in which a single focus of attention changes over time (e.g., "shrinking spotlight model" of Flanker task [6]):

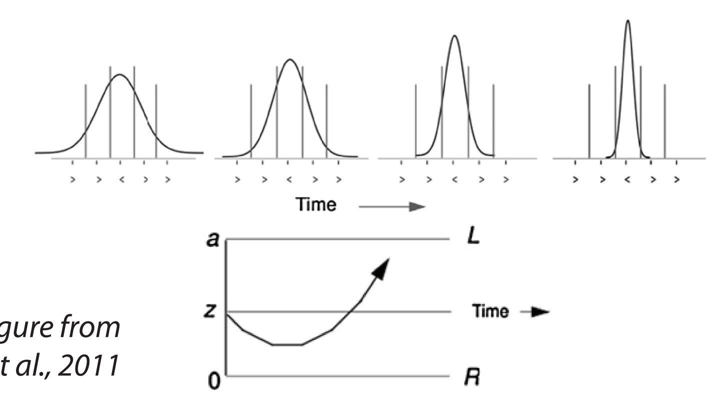
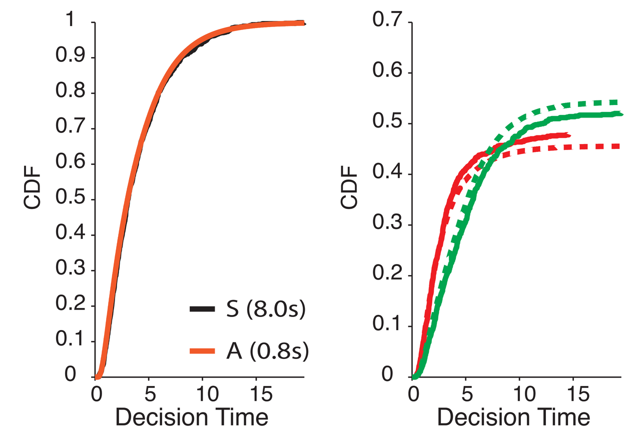
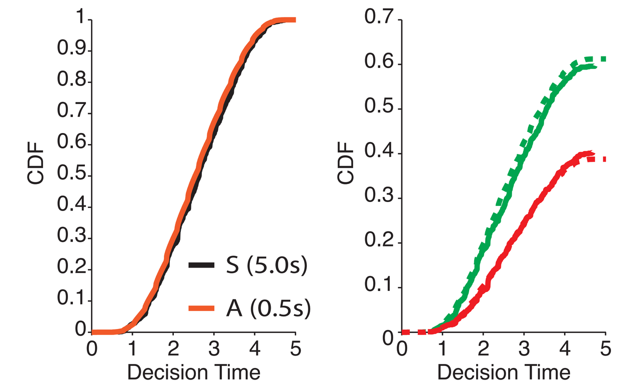


Figure from White et al., 2011



Time-Varying (e.g., Collapsing) Threshold

The n-stage DDM also allows modeling of dynamic rather than uniform thresholds. This can be used to model discrete changes in choice strategy or continuously changing thresholds, as in models involving collapsing boundaries as a deadline approaches [8, 9]:



Ornstein-Uhlenbeck (OU) Processes

Our approach can also be extended to an OU process, in which evidence accumulated early or late in the decision process exerts an outsize influence on the decision process (depending on whether the friction term λ in the following equation is negative or positive):

$$dx(t) = a(t)dt - \lambda x(t)dt + \sigma(t)dW(t), \qquad x(t_0) = x_0$$

Conclusions/Future Directions

• We derive analytic expressions for efficiently calculating expected error rate/RT, and first passage distributions (conditional on response) for a DDM with arbitrary numbers of changes in drift rate, diffusion rate, and threshold at specified time points.

• This work solves and generalizes previously described 2-stage versions of the DDM [e.g., 10, 11], using a Martingale approach. However, alternative approaches are available [11, 12, 13], most notably a Monte Carlo-based approximation to a diffusion process by Diederich and colleagues [7, 12].

• Ongoing work focuses on extending the code to implement expressions for OU processes, estimating the limitations on parameter fits to data (e.g., identifying uncertain timepoints at which a parameter change occurs), and increasing the computational efficiency of the code.

References

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Additional Information

Code:

https://github.com/PrincetonUniversity/msddm

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