CONTENTS

Pre	eface		XV
4c	Acknowledgments		
1	The	challenges of learning	1
•	1110	chancinges of learning	•
	1.1	Learning the best path	2
	1.2	Areas of application	4
	1.3	Major problem classes	12
	1.4	The different types of learning	13
	1.5	Learning from different communities	16
	1.6	Information collection using decision trees	18
		1.6.1 A basic decision tree	18
		1.6.2 Decision tree for offline learning	20
		1.6.3 Decision tree for online learning	21
		1.6.4 Discussion	25
	1.7	Website and downloadable software	26
	1.8	Goals of this book	26
		Problems	28
			vii

2	Ada	ptive learning	31
	2.1	The frequentist view	32
	2.2	The Bayesian view	33
		2.2.1 The updating equations for independent beliefs	34
		2.2.2 The expected value of information	36
		2.2.3 Updating for correlated normal priors	38
		2.2.4 Bayesian updating with an uninformative prior	41
	2.3	Updating for non-Gaussian priors	42
		2.3.1 The gamma-exponential model	43
		2.3.2 The gamma-Poisson model	44
		2.3.3 The Pareto-uniform model	45
		2.3.4 Models for learning probabilities*	46
		2.3.5 Learning an unknown variance*	49
	2.4	Monte Carlo simulation	51
	2.5	Why does it work?*	54
		2.5.1 Derivation of $\tilde{\sigma}$	54
		2.5.2 Derivation of Bayesian updating equations for independent	
		beliefs	55
	2.6	Bibliographic notes	57
		Problems	57
3	The	economics of information	61
	3.1	An elementary information problem	61
	3.2	The marginal value of information	65
	3.3	An information acquisition problem	68
	3.4	Bibliographic notes	70
		Problems	70
4	Ran	king and selection	71
	4.1	The model	72
	4.2	Measurement policies	75
		4.2.1 Deterministic vs. sequential policies	75
		4.2.2 Optimal sequential policies	76
		4.2.3 Heuristic policies	77
	4.3	Evaluating policies	81
	4.4	More advanced topics*	83
		4.4.1 An alternative representation of the probability space	83
		4.4.2 Equivalence of using true means and sample estimates	84

		CONTENTS	ix
	4.5	Bibliographic notes	85
		Problems	85
5	The	knowledge gradient	89
	5.1	The knowledge gradient for independent beliefs	90
		5.1.1 Computation	91
		5.1.2 Some properties of the knowledge gradient	93
		5.1.3 The four distributions of learning	94
	5.2	The value of information and the S-curve effect	95
	5.3	Knowledge gradient for correlated beliefs	98
	5.4	The knowledge gradient for some non-Gaussian distributions	103
		5.4.1 The gamma-exponential model	104
		5.4.2 The gamma-Poisson model	107
		5.4.3 The Pareto-uniform model	108
		5.4.4 The beta-Bernoulli model	109
		5.4.5 Discussion	111
	5.5	Relatives of the knowledge gradient	112
		5.5.1 Expected improvement	113
		5.5.2 Linear loss*	114
	5.6	Other issues	116
		5.6.1 Anticipatory vs. experiential learning	117
		5.6.2 The problem of priors	118
		5.6.3 Discussion	120
	5.7	Why does it work?*	121
		5.7.1 Derivation of the knowledge gradient formula	121
	5.8	Bibliographic notes	125
		Problems	126
6	Ban	dit problems	139
	6.1	The theory and practice of Gittins indices	141
		6.1.1 Gittins indices in the beta-Bernoulli model	142
		6.1.2 Gittins indices in the normal-normal model	145
		6.1.3 Approximating Gittins indices	147
	6.2	Variations of bandit problems	148
	6.3	Upper confidence bounding	149
	6.4	The knowledge gradient for bandit problems	151
		6.4.1 The basic idea	151
		6.4.2 Some experimental comparisons	153

	Х	CONTENTS	S
--	---	----------	---

		6.4.3 Non-normal models	156
	6.5	Bibliographic notes	157
		Problems	157
7	Ele	ments of a learning problem	163
	7.1	The states of our system	164
	7.2	Types of decisions	166
	7.3	Exogenous information	167
	7.4	Transition functions	168
	7.5	Objective functions	168
		7.5.1 Designing versus controlling	168
		7.5.2 Measurement costs	170
		7.5.3 Objectives	170
	7.6	Evaluating policies	175
	7.7	Discussion	177
	7.8	Bibliographic notes	178
		Problems	178
8	Line	ear belief models	181
	8.1	Applications	182
	8.1	Applications 8.1.1 Maximizing ad clicks	182 182
	8.1		
	8.1	8.1.1 Maximizing ad clicks	182
	8.1	8.1.1 Maximizing ad clicks8.1.2 Dynamic pricing	182 184
	8.1	8.1.1 Maximizing ad clicks8.1.2 Dynamic pricing8.1.3 Housing loans	182 184 184
		8.1.1 Maximizing ad clicks8.1.2 Dynamic pricing8.1.3 Housing loans8.1.4 Optimizing dose response	182 184 184 185
		 8.1.1 Maximizing ad clicks 8.1.2 Dynamic pricing 8.1.3 Housing loans 8.1.4 Optimizing dose response A brief review of linear regression 	182 184 184 185 186
		 8.1.1 Maximizing ad clicks 8.1.2 Dynamic pricing 8.1.3 Housing loans 8.1.4 Optimizing dose response A brief review of linear regression 8.2.1 The normal equations 	182 184 184 185 186
		 8.1.1 Maximizing ad clicks 8.1.2 Dynamic pricing 8.1.3 Housing loans 8.1.4 Optimizing dose response A brief review of linear regression 8.2.1 The normal equations 8.2.2 Recursive least squares 	182 184 184 185 186 186
		 8.1.1 Maximizing ad clicks 8.1.2 Dynamic pricing 8.1.3 Housing loans 8.1.4 Optimizing dose response A brief review of linear regression 8.2.1 The normal equations 8.2.2 Recursive least squares 8.2.3 A Bayesian interpretation 	182 184 184 185 186 186 187
	8.2	 8.1.1 Maximizing ad clicks 8.1.2 Dynamic pricing 8.1.3 Housing loans 8.1.4 Optimizing dose response A brief review of linear regression 8.2.1 The normal equations 8.2.2 Recursive least squares 8.2.3 A Bayesian interpretation 8.2.4 Generating a prior 	182 184 184 185 186 186 187 188
	8.2 8.3	 8.1.1 Maximizing ad clicks 8.1.2 Dynamic pricing 8.1.3 Housing loans 8.1.4 Optimizing dose response A brief review of linear regression 8.2.1 The normal equations 8.2.2 Recursive least squares 8.2.3 A Bayesian interpretation 8.2.4 Generating a prior The knowledge gradient for a linear model 	182 184 184 185 186 187 188 189
	8.2 8.3 8.4	8.1.1 Maximizing ad clicks 8.1.2 Dynamic pricing 8.1.3 Housing loans 8.1.4 Optimizing dose response A brief review of linear regression 8.2.1 The normal equations 8.2.2 Recursive least squares 8.2.3 A Bayesian interpretation 8.2.4 Generating a prior The knowledge gradient for a linear model Application to drug discovery Application to dynamic pricing Bibliographic notes	182 184 184 185 186 186 187 188 189 191
	8.2 8.3 8.4 8.5	 8.1.1 Maximizing ad clicks 8.1.2 Dynamic pricing 8.1.3 Housing loans 8.1.4 Optimizing dose response A brief review of linear regression 8.2.1 The normal equations 8.2.2 Recursive least squares 8.2.3 A Bayesian interpretation 8.2.4 Generating a prior The knowledge gradient for a linear model Application to drug discovery Application to dynamic pricing 	182 184 184 185 186 186 187 188 189 191 192
9	8.2 8.3 8.4 8.5 8.6	8.1.1 Maximizing ad clicks 8.1.2 Dynamic pricing 8.1.3 Housing loans 8.1.4 Optimizing dose response A brief review of linear regression 8.2.1 The normal equations 8.2.2 Recursive least squares 8.2.3 A Bayesian interpretation 8.2.4 Generating a prior The knowledge gradient for a linear model Application to drug discovery Application to dynamic pricing Bibliographic notes	182 184 184 185 186 187 188 189 191 192 196 200
9	8.2 8.3 8.4 8.5 8.6	8.1.1 Maximizing ad clicks 8.1.2 Dynamic pricing 8.1.3 Housing loans 8.1.4 Optimizing dose response A brief review of linear regression 8.2.1 The normal equations 8.2.2 Recursive least squares 8.2.3 A Bayesian interpretation 8.2.4 Generating a prior The knowledge gradient for a linear model Application to drug discovery Application to dynamic pricing Bibliographic notes Problems	182 184 184 185 186 187 188 189 191 192 196 200

		CONTENTS	хi
		9.2.1 Setting prior means and variances	207
		9.2.2 Two strategies for setting prior covariances	208
	9.3	Larger sets	209
		9.3.1 Using simulation to reduce the problem size	210
		9.3.2 Computational issues	212
		9.3.3 Experiments	213
	9.4	Very large sets	214
	9.5	Bibliographic notes	216
		Problems	216
10	Opti	mizing a scalar function	219
	10.1	Deterministic measurements	219
	10.2	Stochastic measurements	223
		10.2.1 The model	223
		10.2.2 Finding the posterior distribution	224
		10.2.3 Choosing the measurement	226
		10.2.4 Discussion	229
	10.3	Bibliographic notes	229
		Problems	229
11	Opti	imal bidding	231
	11.1	Modeling customer demand	233
		11.1.1 Some valuation models	233
		11.1.2 The logit model	234
	11.2	Bayesian modeling for dynamic pricing	237
		11.2.1 A conjugate prior for choosing between two demand curves	237
		11.2.2 Moment matching for non-conjugate problems	239
		11.2.3 An approximation for the logit model	242
	11.3	Bidding strategies	244
		11.3.1 An idea from multi-armed bandits	245
		11.3.2 Bayes-greedy bidding	245
		11.3.3 Numerical illustrations	247
	11.4	Why does it work?*	251
		11.4.1 Moment matching for Pareto prior	251
		11.4.2 Approximating the logistic expectation	252
	11.5	Bibliographic notes	253
		Problems	254

XII CONTENTS

12	Stop	pping problems	255
	12.1	Sequential probability ratio test	255
	12.2	The secretary problem	260
		12.2.1 Setup	261
		12.2.2 Solution	263
	12.3	Bibliographic notes	266
		Problems	266
13	Acti	ve learning in statistics	269
	13.1	Deterministic policies	270
	13.2	Sequential policies for classification	274
		13.2.1 Uncertainty sampling	274
		13.2.2 Query by committee	275
		13.2.3 Expected error reduction	276
	13.3	A variance minimizing policy	277
	13.4	Mixtures of Gaussians	279
		13.4.1 Estimating parameters	280
		13.4.2 Active learning	281
	13.5	Bibliographic notes	283
14	Sim	ulation optimization	285
	14.1	Indifference zone selection	287
		14.1.1 Batch procedures	288
		14.1.2 Sequential procedures	290
		14.1.3 The 0-1 procedure: connection to linear loss	291
	14.2	Optimal computing budget allocation	292
		14.2.1 Indifference-zone version	293
		14.2.2 Linear loss version	294
		14.2.3 When does it work?	295
	14.3	Model-based simulated annealing	296
		Other areas of simulation optimization	298
	14.5	Bibliographic notes	299
15	Lea	rning in mathematical programming	301
	15.1	Applications	303
		15.1.1 Piloting a hot air balloon	303
		15.1.2 Optimizing a portfolio	308

			CONTENTS	xiii
		15.1.3 Network problems		309
		15.1.4 Discussion		313
	15.2	Learning on graphs		313
		Alternative edge selection policies		316
		Learning costs for linear programs*		317
		Bibliographic notes		324
16	Opti	mizing over continuous measurements		325
	16.1	The belief model		327
		16.1.1 Updating equations		328
		16.1.2 Parameter estimation		330
	16.2	Sequential kriging optimization		332
	16.3	The knowledge gradient for continuous parameters*		334
		16.3.1 Maximizing the knowledge gradient		334
		16.3.2 Approximating the knowledge gradient		335
		16.3.3 The gradient of the knowledge gradient		336
		16.3.4 Maximizing the knowledge gradient		338
		16.3.5 The KGCP policy		339
	16.4	Efficient global optimization		340
	16.5	Experiments		341
	16.6	Extension to higher dimensional problems		342
	16.7	Bibliographic notes		343
17	Lea	rning with a physical state		345
	17.1	Introduction to dynamic programming		347
		17.1.1 Approximate dynamic programming		348
		17.1.2 The exploration vs. exploitation problem		350
		17.1.3 Discussion		351
	17.2	Some heuristic learning policies		352
	17.3	The local bandit approximation		353
		The knowledge gradient in dynamic programming		355
		17.4.1 Generalized learning using basis functions		355
		17.4.2 The knowledge gradient		358
		17.4.3 Experiments		361
	17.5	An expected improvement policy		363
		Bibliographic notes		364
Inde	ex			379