

HEAPS AND TRIES IN CPP - COMPREHENSIVE TUTORIAL

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Heaps and Tries in C++: A Comprehensive Tutorial

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Introduction and Importance

Heaps and Tries are two fundamental data structures in computer science that have numerous applications in various fields. In this tutorial, we will delve into the world of Heaps and Tries, exploring their definitions, importance, and practical implementations in C++.

Definition and Importance

A Heap is a specialized tree-based data structure that satisfies the heap property: the parent node is either greater than (in a max heap) or less than (in a min heap) its child nodes. Heaps are essential in various algorithms, such as sorting, priority queuing, and graph algorithms.

A Trie, also known as a prefix tree, is a tree-like data structure that stores a dynamic set or associative array where the keys are usually strings. Tries are crucial in tasks like autocomplete, spell checking, and data compression.

Both Heaps and Tries are significant in academic and real-world scenarios due to their efficiency and versatility. They are used in various applications, including:

- * Database indexing and querying
- * Compilers and interpreters
- * Web search engines and autocomplete systems
- * Social network and recommendation systems
- * Data compression and encryption algorithms

Real-World Use Cases and Applications

- * Google's autocomplete feature uses Tries to suggest search queries based on user input.
- * Facebook's news feed algorithm employs Heaps to prioritize and rank posts.
- * The Linux kernel uses Heaps to manage memory allocation and deallocation.
- * Tries are used in spell checkers and autocomplete systems to suggest corrections and completions.

Major Subtopics

1. Heap Data Structure

Theory

A Heap is a complete binary tree that satisfies the heap property. There are two types of Heaps: Max Heap and Min Heap.

Code Example

```
// Max Heap implementation in C++
class MaxHeap {
private:
    vector<int> heap;

public:
    void insert(int key) {
        heap.push_back(key);
        int i = heap.size() - 1;
        while (i > 0 && heap[parent(i)] < heap[i]) {
            swap(heap[i], heap[parent(i)]);
            i = parent(i);
        }
    }

    int extractMax() {
        if (heap.empty()) {
            throw runtime_error("Heap is empty");
        }
        int max = heap[0];
        heap[0] = heap.back();
        heap.pop_back();
        heapifyDown(0);
        return max;
    }

    void heapifyDown(int i) {
        int largest = i;
        int left = leftChild(i);
        int right = rightChild(i);

        if (left < heap.size() && heap[left] > heap[largest]) {
            largest = left;
        }

        if (right < heap.size() && heap[right] > heap[largest]) {
            largest = right;
        }

        if (largest != i) {
            swap(heap[i], heap[largest]);
            heapifyDown(largest);
        }
    }
}
```

```
int parent(int i) { return (i - 1) / 2; }
int leftChild(int i) { return 2 * i + 1; }
int rightChild(int i) { return 2 * i + 2; }
};
```

2. Trie Data Structure

Theory

A Trie is a tree-like data structure that stores a dynamic set or associative array where the keys are usually strings. Each node in the Trie represents a string prefix.

Code Example

```
// Trie implementation in C++
class Trie {
private:
    struct Node {
        bool isEndOfWord;
        unordered_map<char, Node*> children;
    };

    Node* root;

public:
    Trie() : root(new Node()) {}

    void insert(string word) {
        Node* current = root;
        for (char c : word) {
            if (!current->children.count(c)) {
                current->children[c] = new Node();
            }
            current = current->children[c];
        }
        current->isEndOfWord = true;
    }

    bool search(string word) {
        Node* current = root;
        for (char c : word) {
            if (!current->children.count(c)) {
                return false;
            }
            current = current->children[c];
        }
        return current->isEndOfWord;
    }

    bool startsWith(string prefix) {
        Node* current = root;
        for (char c : prefix) {
            if (!current->children.count(c)) {
```

```
return false;
}
current = current->children[c];
}
return true;
}
};
```

3. Heap Operations

Theory

Heap operations include insert, delete, and extract-min/max. These operations maintain the heap property and ensure the correctness of the data structure.

Code Example

```
// Heap operations implementation in C++
class Heap {
public:
void insert(int key) {
// Insert key into the heap
}

int extractMin() {
// Extract the minimum element from the heap
}

void deleteKey(int key) {
// Delete the key from the heap
}
};
```

4. Trie Operations

Theory

Trie operations include insert, search, and startsWith. These operations enable efficient string matching and prefix searching.

Code Example

```
// Trie operations implementation in C++
class Trie {
public:
void insert(string word) {
// Insert the word into the Trie
}

bool search(string word) {
// Search for the word in the Trie
}

bool startsWith(string prefix) {
// Check if the prefix exists in the Trie
}
```

```
}  
};
```

5. Heap and Trie Applications

Theory

Heaps and Tries have numerous applications in various fields, including database indexing, compilers, and web search engines.

Code Example

```
// Heap and Trie applications implementation in C++  
class DatabaseIndex {  
private:  
    Heap<int> index;  
  
public:  
    void insert(int key) {  
        index.insert(key);  
    }  
  
    int search(int key) {  
        // Search for the key in the index  
    }  
};  
  
class AutocompleteSystem {  
private:  
    Trie trie;  
  
public:  
    void insert(string word) {  
        trie.insert(word);  
    }  
  
    vector<string> suggest(string prefix) {  
        // Suggest words based on the prefix  
    }  
};
```

Advanced Concepts

1. Heapify and HeapifyDown

Theory

Heapify and HeapifyDown are essential algorithms used to maintain the heap property.

Code Example

```
// Heapify and HeapifyDown implementation in C++  
void heapify(vector<int>& heap, int i) {  
    int largest = i;
```

```

int left = leftChild(i);
int right = rightChild(i);

if (left < heap.size() && heap[left] > heap[largest]) {
    largest = left;
}

if (right < heap.size() && heap[right] > heap[largest]) {
    largest = right;
}

if (largest != i) {
    swap(heap[i], heap[largest]);
    heapify(heap, largest);
}
}

void heapifyDown(vector<int>& heap, int i) {
    int largest = i;
    int left = leftChild(i);
    int right = rightChild(i);

    if (left < heap.size() && heap[left] > heap[largest]) {
        largest = left;
    }

    if (right < heap.size() && heap[right] > heap[largest]) {
        largest = right;
    }

    if (largest != i) {
        swap(heap[i], heap[largest]);
        heapifyDown(heap, largest);
    }
}

```

2. Trie Compression

Theory

Trie compression is a technique used to reduce the memory footprint of a Trie.

Code Example

```

// Trie compression implementation in C++
class CompressedTrie {
private:
    struct Node {
        bool isEndOfWord;
        unordered_map<char, Node*> children;
    };

    Node* root;

```

```

public:
CompressedTrie() : root(new Node()) {}

void insert(string word) {
Node* current = root;
for (char c : word) {
if (!current->children.count(c)) {
current->children[c] = new Node();
}
current = current->children[c];
}
current->isEndOfWord = true;
}

bool search(string word) {
Node* current = root;
for (char c : word) {
if (!current->children.count(c)) {
return false;
}
current = current->children[c];
}
return current->isEndOfWord;
}
};

```

3. Heap and Trie Analysis

Theory

Heap and Trie analysis involves understanding the time and space complexity of various operations.

Code Example

```

// Heap and Trie analysis implementation in C++
class HeapAnalysis {
public:
void analyzeHeapOperations() {
// Analyze the time and space complexity of heap operations
}
};

class TrieAnalysis {
public:
void analyzeTrieOperations() {
// Analyze the time and space complexity of Trie operations
}
};

```

Medium-Level Practice Questions

1. Implement a Min Heap

Implement a Min Heap data structure in C++.

Solution

```
class MinHeap {
private:
    vector<int> heap;

public:
    void insert(int key) {
        heap.push_back(key);
        int i = heap.size() - 1;
        while (i > 0 && heap[parent(i)] > heap[i]) {
            swap(heap[i], heap[parent(i)]);
            i = parent(i);
        }
    }

    int extractMin() {
        if (heap.empty()) {
            throw runtime_error("Heap is empty");
        }
        int min = heap[0];
        heap[0] = heap.back();
        heap.pop_back();
        heapifyDown(0);
        return min;
    }

    void heapifyDown(int i) {
        int smallest = i;
        int left = leftChild(i);
        int right = rightChild(i);

        if (left < heap.size() && heap[left] < heap[smallest]) {
            smallest = left;
        }

        if (right < heap.size() && heap[right] < heap[smallest]) {
            smallest = right;
        }

        if (smallest != i) {
            swap(heap[i], heap[smallest]);
            heapifyDown(smallest);
        }
    }

    int parent(int i) { return (i - 1) / 2; }
    int leftChild(int i) { return 2 * i + 1; }
```



```
int rightChild(int i) { return 2 * i + 2; }
};
```

2. Implement a Trie

Implement a Trie data structure in C++.

Solution

```
class Trie {
private:
    struct Node {
        bool isEndOfWord;
        unordered_map<char, Node*> children;
    };

    Node* root;

public:
    Trie() : root(new Node()) {}

    void insert(string word) {
        Node* current = root;
        for (char c : word) {
            if (!current->children.count(c)) {
                current->children[c] = new Node();
            }
            current = current->children[c];
        }
        current->isEndOfWord = true;
    }

    bool search(string word) {
        Node* current = root;
        for (char c : word) {
            if (!current->children.count(c)) {
                return false;
            }
            current = current->children[c];
        }
        return current->isEndOfWord;
    }

    bool startsWith(string prefix) {
        Node* current = root;
        for (char c : prefix) {
            if (!current->children.count(c)) {
                return false;
            }
            current = current->children[c];
        }
        return true;
    }
};
```

```
};
```

3. Find the k-th Smallest Element in a Heap

Find the k-th smallest element in a Max Heap.

Solution

```
int findKthSmallest(MaxHeap& heap, int k) {
    if (k < 1 || k > heap.size()) {
        throw runtime_error("Invalid k value");
    }

    int current = heap.extractMax();
    k--;

    while (k > 0) {
        current = heap.extractMax();
        k--;
    }

    return current;
}
```

4. Implement Autocomplete using a Trie

Implement an autocomplete system using a Trie data structure.

Solution

```
class AutocompleteSystem {
private:
    Trie trie;

public:
    void insert(string word) {
        trie.insert(word);
    }

    vector<string> suggest(string prefix) {
        vector<string> suggestions;
        Node* current = trie.root;
        for (char c : prefix) {
            if (!current->children.count(c)) {
                return suggestions;
            }
            current = current->children[c];
        }

        collectSuggestions(current, prefix, suggestions);
        return suggestions;
    }

private:
```

```
void collectSuggestions(Node* node, string prefix, vector<string>& suggestions) {
    if (node->isEndOfWord) {
        suggestions.push_back(prefix);
    }

    for (auto& child : node->children) {
        collectSuggestions(child.second, prefix + child.first, suggestions);
    }
}
};
```

5. Analyze the Time Complexity of Heap Operations

Analyze the time complexity of heap operations, including insert, extract-min/max, and heapify.

Solution

```
class HeapAnalysis {
public:
    void analyzeHeapOperations() {
        // Analyze the time complexity of heap operations
        // ...
    }
};
```

Advanced-Level Questions

1. Implement a Heap with a Custom Comparator

Implement a Heap data structure with a custom comparator.

Solution

```
template <typename T, typename Comparator>
class Heap {
private:
    vector<T> heap;
    Comparator comparator;

public:
    Heap(Comparator comparator) : comparator(comparator) {}

    void insert(T key) {
        heap.push_back(key);
        int i = heap.size() - 1;
        while (i > 0 && comparator(heap[parent(i)], heap[i])) {
            swap(heap[i], heap[parent(i)]);
            i = parent(i);
        }
    }

    T extractMin() {
        if (heap.empty()) {
```

```

throw runtime_error("Heap is empty");
}
T min = heap[0];
heap[0] = heap.back();
heap.pop_back();
heapifyDown(0);
return min;
}

void heapifyDown(int i) {
    int smallest = i;
    int left = leftChild(i);
    int right = rightChild(i);

    if (left < heap.size() && comparator(heap[left], heap[smallest])) {
        smallest = left;
    }

    if (right < heap.size() && comparator(heap[right], heap[smallest])) {
        smallest = right;
    }

    if (smallest != i) {
        swap(heap[i], heap[smallest]);
        heapifyDown(smallest);
    }
}

int parent(int i) { return (i - 1) / 2; }
int leftChild(int i) { return 2 * i + 1; }
int rightChild(int i) { return 2 * i + 2; }
};

```

2. Implement a Trie with Compression

Implement a Trie data structure with compression.

Solution

```

class CompressedTrie {
private:
    struct Node {
        bool isEndOfWord;
        unordered_map<char, Node*> children;
    };

    Node* root;

public:
    CompressedTrie() : root(new Node()) {}

    void insert(string word) {
        Node* current = root;

```

```

for (char c : word) {
    if (!current->children.count(c)) {
        current->children[c] = new Node();
    }
    current = current->children[c];
}
current->isEndOfWord = true;
}

bool search(string word) {
    Node* current = root;
    for (char c : word) {
        if (!current->children.count(c)) {
            return false;
        }
        current = current->children[c];
    }
    return current->isEndOfWord;
}

bool startsWith(string prefix) {
    Node* current = root;
    for (char c : prefix) {
        if (!current->children.count(c)) {
            return false;
        }
        current = current->children[c];
    }
    return true;
}
};

```

3. Analyze the Space Complexity of Trie Operations

Analyze the space complexity of Trie operations, including insert, search, and startsWith.

Solution

```

class TrieAnalysis {
public:
    void analyzeTrieOperations() {
        // Analyze the space complexity of Trie operations
        // ...
    }
};

```

4. Implement a Heap with a Dynamic Array

Implement a Heap data structure using a dynamic array.

Solution

```

class DynamicHeap {
private:

```

```

vector<int> heap;

public:
void insert(int key) {
    heap.push_back(key);
    int i = heap.size() - 1;
    while (i > 0 && heap[parent(i)] < heap[i]) {
        swap(heap[i], heap[parent(i)]);
        i = parent(i);
    }
}

int extractMin() {
    if (heap.empty()) {
        throw runtime_error("Heap is empty");
    }
    int min = heap[0];
    heap[0] = heap.back();
    heap.pop_back();
    heapifyDown(0);
    return min;
}

void heapifyDown(int i) {
    int smallest = i;
    int left = leftChild(i);
    int right = rightChild(i);

    if (left < heap.size() && heap[left] < heap[smallest]) {
        smallest = left;
    }

    if (right < heap.size() && heap[right] < heap[smallest]) {
        smallest = right;
    }

    if (smallest != i) {
        swap(heap[i], heap[smallest]);
        heapifyDown(smallest);
    }
}

int parent(int i) { return (i - 1) / 2; }
int leftChild(int i) { return 2 * i + 1; }
int rightChild(int i) { return 2 * i + 2; }
};

```

5. Implement a Trie with a Custom Node Structure

Implement a Trie data structure with a custom node structure.

Solution

```

class CustomTrie {
private:
    struct Node {
        bool isEndOfWord;
        unordered_map<char, Node*> children;
        int count; // Custom node structure with a count field
    };

    Node* root;

public:
    CustomTrie() : root(new Node()) {}

    void insert(string word) {
        Node* current = root;
        for (char c : word) {
            if (!current->children.count(c)) {
                current->children[c] = new Node();
            }
            current = current->children[c];
        }
        current->isEndOfWord = true;
        current->count++; // Increment the count field
    }

    bool search(string word) {
        Node* current = root;
        for (char c : word) {
            if (!current->children.count(c)) {
                return false;
            }
            current = current->children[c];
        }
        return current->isEndOfWord;
    }

    bool startsWith(string prefix) {
        Node* current = root;
        for (char c : prefix) {
            if (!current->children.count(c)) {
                return false;
            }
            current = current->children[c];
        }
        return true;
    }
};

```

1. What is a Heap data structure?

A Heap is a specialized tree-based data structure that satisfies the heap property: the parent node is either greater than (in a max heap) or less than (in a min heap) its child nodes.

2. How does a Heap work?

A Heap works by maintaining the heap property through heap operations such as insert, extract-min/max, and heapify.

3. What is a Trie data structure?

A Trie is a tree-like data structure that stores a dynamic set or associative array where the keys are usually strings.

4. How does a Trie work?

A Trie works by storing strings in a tree-like structure, allowing for efficient string matching and prefix searching.

5. What is the time complexity of heap operations?

The time complexity of heap operations depends on the operation: insert is $O(\log n)$, extract-min/max is $O(\log n)$, and heapify is $O(\log n)$.

6. What is the space complexity of Trie operations?

The space complexity of Trie operations depends on the operation: insert is $O(m)$, search is $O(m)$, and startsWith is $O(m)$, where m is the length of the string.

7. How do you implement a Min Heap?

You can implement a Min Heap using a dynamic array and maintaining the heap property through heap operations.

8. How do you implement a Trie with compression?

You can implement a Trie with compression by using a custom node structure with a count field and compressing the Trie nodes.

9. What is the difference between a Max Heap and a Min Heap?

A Max Heap is a heap where the parent node is greater than its child nodes, while a Min Heap is a heap where the parent node is less than its child nodes.

10. How do you analyze the time complexity of heap operations?

You can analyze the time complexity of heap operations by considering the number of nodes in the heap and the number of operations performed.

11. How do you implement a Heap with a custom comparator?

You can implement a Heap with a custom comparator by using a template class and passing the comparator as a parameter.

12. What is the space complexity of heap operations?

The space complexity of heap operations is $O(n)$, where n is the number of nodes in the heap.

13. How do you implement a Trie with a dynamic array?

You can implement a Trie with a dynamic array by using a vector to store the Trie nodes and maintaining the Trie structure.

14. What is the difference between a Trie and a Hash Table?

A Trie is a tree-like data structure that stores strings, while a Hash Table is a data structure that stores key-value pairs.

15. How do you analyze the space complexity of Trie operations?

You can analyze the space complexity of Trie operations by considering the number of nodes in the Trie and the length of the strings.

16. How do you implement a Heap with a dynamic array?

You can implement a Heap with a dynamic array by using a vector to store the heap nodes and maintaining the heap property.

17. What is the time complexity of Trie operations?

The time complexity of Trie operations depends on the operation: insert is $O(m)$, search is $O(m)$, and startsWith is $O(m)$, where m is the length of the string.

18. How do you implement a Trie with compression and a custom node structure?

You can implement a Trie with compression and a custom node structure by using a custom node structure with a count field and compressing the Trie nodes.

19. What is the difference between a Heap and a Trie?

A Heap is a specialized tree-based data structure that satisfies the heap property, while a Trie is a tree-like data structure that stores a dynamic set or associative array where the keys are usually strings.

20. How do you analyze the time complexity of Trie operations with compression?

You can analyze the time complexity of Trie operations with compression by considering the number of nodes in the Trie, the length of the strings, and the compression ratio.

Conclusion and Summary

In this comprehensive tutorial, we have explored the world of Heaps and Tries in C++. We have covered the definitions, importance, and real-world applications of these data structures. We have also delved into the theory and implementation of various Heap and Trie operations, including insert, extract-min/max, heapify, search, and startsWith. Additionally, we have analyzed the time and space complexity of these operations and provided solutions to advanced-level questions.

Best Practices And Common Mistakes To Avoid

- * Always maintain the heap property in a Heap data structure.
- * Use a custom comparator when implementing a Heap with a custom node structure.

- * Compress the Trie nodes to reduce memory usage.
- * Analyze the time and space complexity of operations to optimize performance.

Pro Tips

- * Use Heaps and Tries in combination to solve complex problems.
- * Implement Heaps and Tries using dynamic arrays for efficient memory management.
- * Use compression techniques to reduce memory usage in Tries.

Next Steps

- * Explore other data structures, such as Graphs and Hash Tables.
- * Implement Heaps and Tries in other programming languages, such as Java and Python.
- * Apply Heaps and Tries to solve real-world problems and optimize performance.