# CHAPTER 1

The exercises in Chapter 1 are designed to provoke thought about issues that arise when numbers are used to communicate ideas. These questions do not have a single correct answer; therefore, solutions are not provided.

Similarly, solutions are not provided for the first few exercises in each chapter, which are intended to review concepts and definitions.

## CHAPTER 2

### Exercise 11

- a. The number of suicides is discrete.
- b. The response to treatment is ordinal.
- c. The concentration of lead is continuous.
- d. Political party affiliation is nominal.
- e. The presence or absence of hepatitis C is discrete.
- f. The length of time is continuous.
- g. The number of previous miscarriages is discrete.
- h. Satisfaction with care is ordinal.
- i. The age of a child is continuous.

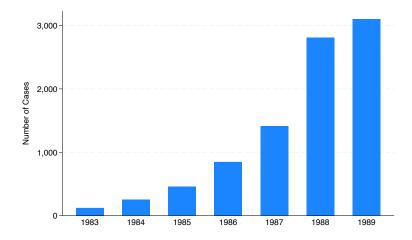
#### Exercise 12

The statement is not accurate. The intervals in the table are of unequal length; therefore, it does not make sense to compare the absolute frequencies within them. The interval 16–30 has length 15 minutes, for example, while the interval 11–15 is only 5 minutes long.

#### Exercise 13

A bar chart showing the numbers of cases of pediatric AIDS by year is displayed below. The graph indicates that the number of cases of AIDS increased each year between 1983 and 1989, with the biggest jump occurring between 1987 and 1988.

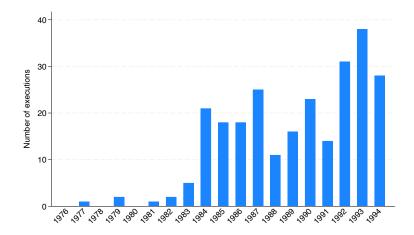
graph bar (asis) cases, over(Year)



## Exercise 14

A bar chart of the number of executions by year is shown on the following page. There were only a few executions in the eight years immediately following the 1976 Supreme Court decision. After that, the number of executions increased, and has continued to increase over time (although not steadily; there are periodic decreases).

graph bar (asis) executions, over(year, label(angle(forty\_five)))



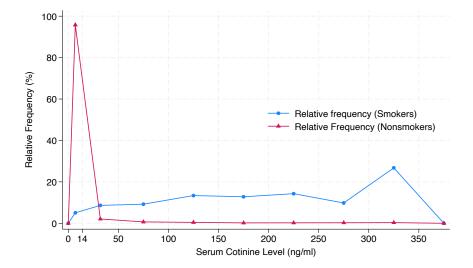
### Exercise 15

- a. Serum cotinine level is continuous.
- b. Because the total number of smokers is not equal to the total number of nonsmokers, it is not fair to compare the distributions of absolute frequencies for these two groups.
- c. The table of relative frequencies of serum cotinine levels appears below.

Cotinine Level	Smokers	Nonsmokers
(ng/ml)	(%)	(%)
0-13	5.1	95.8
14 - 49	8.6	2.1
50-99	9.2	0.7
100 - 149	13.4	0.4
150 - 199	12.8	0.2
200 – 249	14.3	0.2
250 – 299	9.8	0.3
300+	26.8	0.3

d. The frequency polygons are shown below. Note that the intervals are of unequal length. For the purposes of constructing the polygons, the last interval is assumed to be 300–349 ng/ml.

```
twoway (connected relfreq_smoke cotinine, msymbol(circle))
(connected relfreq_nonsmoke cotinine, msymbol(triangle)),
ytitle('"Relative Frequency (\%)"') ylabel(0 20 40 60 80 100)
xtitle('"Serum Cotinine Level (ng/ml)"') xlabel(0 14 50 100 150 200 250 300 350)
legend(position(3) ring(0))
```



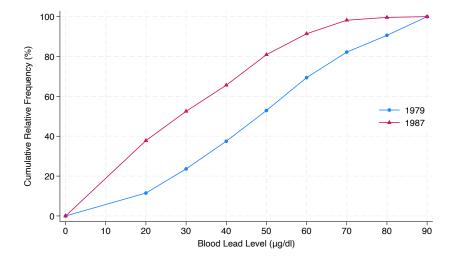
- e. The distribution of smokers is fairly uniform across cotinine levels. The relative frequency is smallest in the first interval (0–13 ng/ml). It then increases, and remains consistent (hovering around 10%) across subsequent intervals up to the last (300+ ng/ml), where the relative frequency increases. For nonsmokers, nearly everyone has a cotinine level below 13 ng/ml; the relative frequency in each of the other intervals is very small.
- f. Yes, it is possible that some of the subjects are misclassified. In particular, there are a number of self-reported "nonsmokers" with extremely high cotinine levels.

- a. The workers have lower blood lead levels in 1987.
- b. The cumulative relative frequencies for each group of workers are displayed below.

Blood Lead	1979	1987
$(\mu \mathrm{g}/\mathrm{dl})$	(%)	(%)
< 20	11.5	37.8
20 - 29	23.6	52.5
30 – 39	37.5	65.6
40 – 49	52.9	80.9
50 - 59	69.4	91.4
60 – 69	82.2	98.2
70 - 79	90.6	99.6
$\geq 80$	100.0	100.0

The cumulative frequency polygons are displayed on the next page. Note that, for the purposes of constructing the polygons, the last interval is assumed to be 80–89  $\mu$ g/dl.

twoway (connected year1 blood\_lead, msymbol(circle)) (connected year2
blood\_lead, msymbol(triangle)), ytitle('"Cumulative Relative Frequency (%)"')
ylabel(0 20 40 60 80 100) xtitle('"Blood Lead Level ({&mu}g/dl)"')
xlabel(0 10 20 30 40 50 60 70 80 90) legend(position(3) ring(0))



c. The distribution of blood lead levels is stochastically larger for the group of workers in 1979.

### Exercise 17

- a. The diagnosis of lung cancer is discrete.
- b. Smoking status is discrete.
- c. The graph tells us that the majority of men diagnosed with lung cancer were heavy, excessive, or chain smokers. For those who have not been diagnosed with lung cancer, the percentage of men in each category is more evenly distributed, except for heavy smokers which includes the highest percentage of men. There are almost an equal number of heavy smokers diagnosed with lung cancer as there are heavy smokers that were not diagnosed with lung cancer.
- d. The table is presented below. Note that the cumulative frequencies do not add to 100% because of rounding. The values used are from the original publication [68].

	Relative Frequency		Cumulative Frequency	
	Without		Without	
Smoking status	lung cancer	Lung cancer	lung cancer	Lung cancer
None	14.6	1.3	14.6	1.3
Light	11.5	2.3	26.1	3.6
Moderately heavy	19.0	10.1	45.1	13.7
Heavy	35.6	35.2	80.7	48.9
Excessive	11.5	30.9	92.2	70.8
Chain	7.6	20.3	99.8	100.1

## Exercise 18

- a. The graph tells us that South Asia had by far the highest number of smallpox cases. The graph also shows that South Asia was one of the last regions to eliminate smallpox.
- b. The graph tells us that the number of reported cases of smallpox was highest for Europe and Central Asia from 1920–1924, but decreased substantially after these years.
- c. Based on this graph we could say that for most regions of the world, the number of smallpox cases decreases over time; however, cases in South Asia continue to spike over time until the eradication of the disease.

a. The mean time to seizure is

$$\bar{x} = \frac{\sum_{i=1}^{13} x_i}{13}$$

$$= \frac{336.85}{13}$$

$$= 25.9 \text{ months.}$$

- b. The median is the middle value, or 24 months.
- c. Since two different measurements each occur twice, the data set has two modes 12 months and 24 months.
- d. The range is the largest value minus the smallest value, or

$$96 - 0.1 = 95.9 \text{ months.}$$

e. The interquartile range is the 75th percentile minus the 25th percentile, or

$$36-4 = 32 \text{ months.}$$

f. The standard deviation is

$$\sqrt{\frac{\sum_{i=1}^{13} (x_i - \bar{x})^2}{13 - 1}} = \sqrt{\frac{\sum_{i=1}^{13} (x_i - 25.9)^2}{12}}$$

$$= \sqrt{749.2}$$

$$= 27.4 \text{ months.}$$

Note that

$$\sum_{i=1}^{13} (x_i - \bar{x}) = (0.10 - 25.91) + (0.25 - 25.91) + (0.50 - 25.91)$$

$$+ (4 - 25.91) + (12 - 25.91) + (12 - 25.91)$$

$$+ (24 - 25.91) + (24 - 25.91) + (31 - 25.91)$$

$$+ (36 - 25.91) + (42 - 25.91) + (55 - 25.91)$$

$$+ (96 - 25.91)$$

$$= -25.81 - 25.66 - 25.41 - 21.91 - 13.91 - 13.91$$

$$- 1.91 - 1.91 + 5.09 + 10.09 + 16.09 + 29.09$$

$$+ 70.09$$

$$= 0.02$$

$$\approx 0.$$

Because we use the mean rounded to two decimal places in the calculations, the sum is not exactly equal to 0.

a. The mean calcium level is

$$\bar{x} = \frac{\sum_{i=1}^{8} x_i}{8}$$

$$= \frac{25.14}{8}$$

$$= 3.14 \text{ mmol/l.}$$

The median is the average of the 4th and 5th values, or

$$\frac{2.99 + 3.17}{2} = 3.08 \text{ mmol/l}.$$

The standard deviation is

$$s = \sqrt{\frac{\sum_{i=1}^{8} (x_i - \bar{x})^2}{8 - 1}}$$
$$= \sqrt{\frac{\sum_{i=1}^{8} (x_i - 3.14)^2}{7}}$$
$$= \sqrt{0.2608}$$
$$= 0.51 \text{ mmol/l.}$$

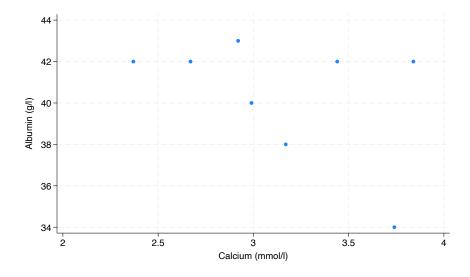
The range is the largest value minus the smallest value, or

$$3.84 - 2.37 = 1.47 \text{ mmol/l}.$$

b. The mean albumin level is  $40.4~\rm g/l.$  The median is  $42~\rm g/l.$  The standard deviation is  $3.0~\rm g/l.$  The range is  $9~\rm g/l.$ 

c. A scatter plot of albumin versus calcium is displayed below.

### twoway (scatter albumin calcium)

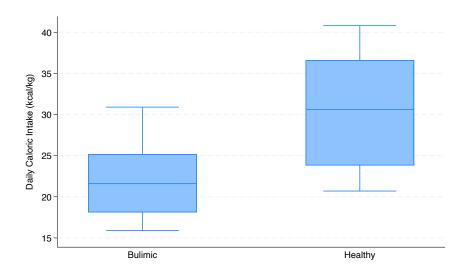


d. The patients suffering from vitamin D intoxication all have albumin levels within the normal range. However, they do not have normal blood levels of calcium. Both the mean and the median lie above the upper limit of the normal range; overall, 6 of the 8 patients have calcium levels that are above normal.

## Exercise 21

- a. The median daily caloric intake for the bulimic adolescents is 21.6 kcal/kg, and the median for the healthy adolescents is 30.6 kcal/kg.
- b. The interquartile range for the bulimic adolescents is the 75th percentile minus the 25th percentile, or 25.2 18.1 = 7.1 kcal/kg. The interquartile range for the healthy adolescents is 36.6 23.8 = 12.8 kcal/kg.
- c. A box plot of the daily caloric intake for bulimic and healthy adolescents is displayed below.

graph box intake, over(group)



d. Daily caloric intake tends to be higher for the healthy adolescents. This group also exhibits a greater amount of variability.

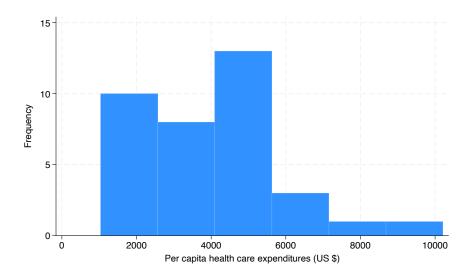
#### Exercise 22

- a. Europe has the smallest mean; its infant mortality rates are much lower than those of either Africa or Asia.
- b. The box plots indicate that Africa has the largest median; it has the greatest proportion of nations with relatively large infant mortality rates.
- c. Europe has the smallest standard deviation. Its infant mortality rates are more tightly clustered about the mean than those of either Africa or Asia.
- d. Since the distribution of values for Africa is unimodal and roughly symmetric, the mean and the median infant mortality rates should be fairly close.

We would not expect the mean and median to be equal for Asia, however; since the distribution is skewed to the right, the mean will be larger than the median.

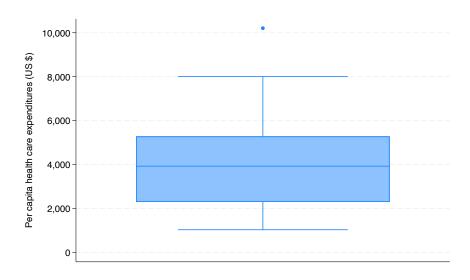
a. A histogram of per capita expenditure is displayed below.

## histogram per\_capita, frequency



b. A box plot of per capita expenditure is displayed below.

## graph box per\_capita



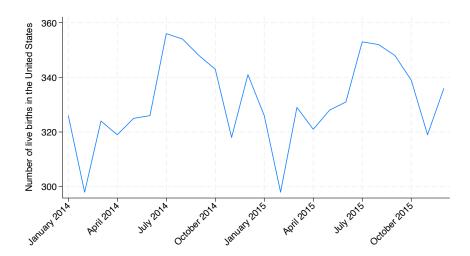
c. The figures show that the distribution of per capita health expenditure is right-skewed with a median value of approximately \$4,000. There is also an outlier shown in the box plot.

d. The histogram shows how many countries have per capita values in specific ranges, and this information is not provided in the box plot. The box plot shows there is an outlier, and this information is not seen in the histogram.

## Exercise 24

a. A line graph of the reported number of births over time is displayed below.

```
twoway (line births month), xtitle(, color(%0)) xlabel(1 "January 2014"
4 "April 2014" 7 "July 2014" 10 "October 2014" 13 "January 2015"
16 "April 2015" 19 "July 2015" 22 "October 2015", labels angle(forty_five))
```



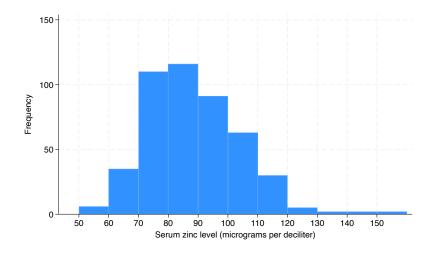
b. Yes, there does seem to be a seasonal pattern of live births in the United States. Based on this two-year period, there appears to be a tendency toward more births in the spring and summer months and fewer in fall and winter.

#### Exercise 25

- a. Serum zinc level is continuous.
- b. The table of relative frequencies appears on the following page. The serum zinc levels range from 50 to 159  $\mu g/dl$ ; however, most of the values lie between 70 and 109  $\mu g/dl$ . The intervals 70–79 and 80–89 contain the greatest numbers of observations.
- c. A histogram of the data appears on the next page.
- d. The histogram is unimodal and skewed to the right.
- e. The output from Stata used to calculate these values is presented below. The mean is 87.9  $\mu$ g/dl; the median is 6  $\mu$ g/dl; the range is 153-50 = 103  $\mu$ g/dl; the interquartile range is 98-76 = 22  $\mu$ g/dl; the standard deviation is 16  $\mu$ g/dl.
- f. Since the distribution of serum zinc level is skewed, the median is a better measure of central tendency to report.
- g. We would report the interquartile range and range.

Relative
Frequency
1.3%
7.6%
23.8%
25.1%
19.7%
13.6%
6.5%
1.1%
0.4%
0.4%
0.4%

histogram zinc, width(10) frequency xlabel(50 60 70 80 90 100 110 120 130 140 150, labels) (bin=11, start=50, width=10)



sum zinc, detail

Serum zinc level (micrograms per deciliter)

	Percentiles	Smallest		
1%	56	50		
5%	64	51		
10%	70	53	Obs	462
25%	76	55	Sum of wgt.	462
50%	86		Mean	87.93723
		Largest	Std. dev.	16.00469
75%	98	142		
90%	108	147	Variance	256.1501
95%	115	151	Skewness	.6211264
99%	135	153	Kurtosis	3.890067

a. A table of absolute and relative frequencies of age at time of death is displayed below. The Stata code and output are also shown below.

Age	Absolute	Relative
(Years)	Frequency	Frequency
10-19	76	36.02%
20 - 29	106	50.24%
30 – 39	20	9.479%
40 - 49	4	1.896%
50 - 59	2	0.9479%
60 – 69	3	1.42%
Total	211	100%

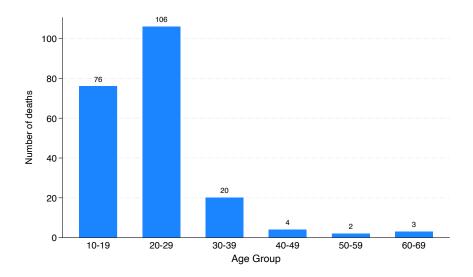
table ( age\_group ) () (), statistic(sumw) statistic(proportion)

	1	Sum of weights	Proportion
Age group at time of death			
10-19		76	.3602
20-29	1	106	.5024
30-39		20	.09479
40-49		4	.01896
50-59	1	2	.009479
60-69		3	.01422
Total	I	211	1

b. The table shows that most of the individuals who die while taking a selfie are between the ages of 10 and 29, with the decade 20-29 representing over 50% of deaths.

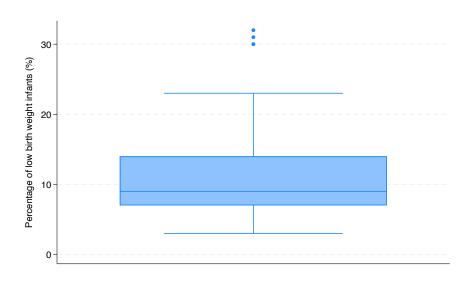
c. A bar chart of absolute number of deaths within each decade is displayed below.

graph bar (count), over(age\_group) blabel(bar) ytitle('"Number of deaths"')
b1title("Age Group")



Exercise 27
a. A box plot of the percentages of low birth weight infants is displayed below.

## graph box lowbwt



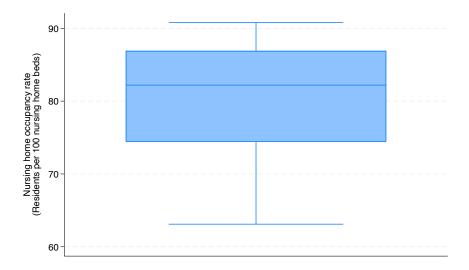
b. Albania has the lowest percentage of low birth weight infants (3%) and Yemen has the highest percentage of low birth weight infants (32%).

c. The distribution is skewed to the right; its tail extends in the direction of the higher values. d. According to the box plot, the data contain three outlying observations. In Bangladesh and India 30% of infants are low birth weight, in Sudan 31% are low birth weight, and in Yemen 32% are low birth weight. Note that there are 3 values that are outliers, but there are 4 countries total since Bangladesh and India have the same value.

- e. The mean is 10.85% and the median is 9%.
- f. The median is the preferred measure of central tendency since the distribution of percentage of low birth weight infants is skewed.

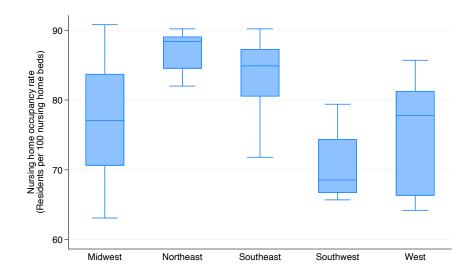
- a. Indiana has the smallest nursing home occupancy rate (63.1%), and South Dakota the highest number (90.8%). One factor that might influence the variability among states is the availability of support services for the elderly.
- b. The box plot is shown below.

graph box occupancy, ytitle("Nursing home occupancy rate" "(Residents per 100 nursing home beds)")



- c. The observations are skewed to the right. There is no state that can be categorized as an outlier.
- d. The box plots are displayed below.

graph box occupancy, ytitle("Nursing home occupancy rate" "(Residents per 100 nursing home beds)") over(region)

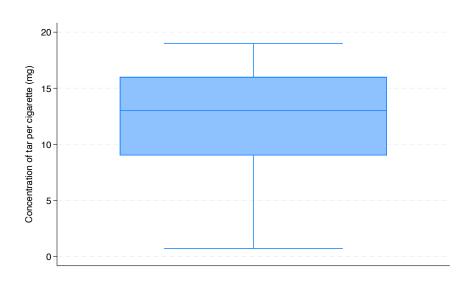


e. Yes, there are differences among the regions in terms of nursing home occupancy rate. The Northeast region has the highest median occupancy rate and the Southwest has the lowest median occupancy rate.

## Exercise 29

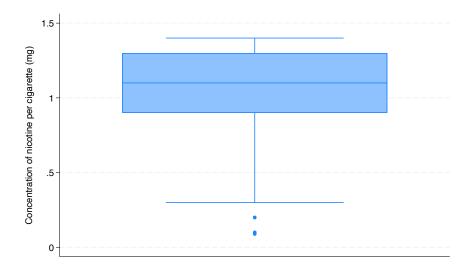
a. A box plot of the declared concentrations of tar per cigarette is displayed below.

## graph box tar



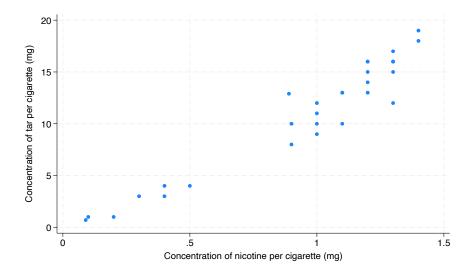
- b. The distribution of tar concentrations is right-skewed with a median of approximately  $13 \,\mathrm{mg}$ . There are no outliers.
- c. A box plot of the declared concentrations of nicotine per cigarette is displayed below.

## graph box nicotine



- d. The distribution of nicotine concentrations is right-skewed. However, there are three values that are low outliers.
- e. The one-way scatter plot appears below.

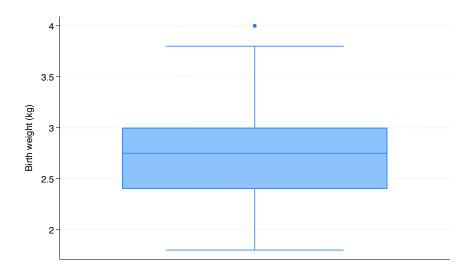
## twoway (scatter tar nicotine)



f. There does appear to be a relationship between these quantities; the concentration of tar increases as the concentration of nicotine increases.

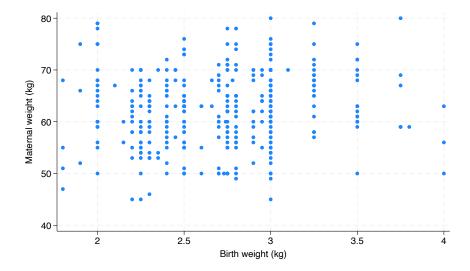
- a. Birth weight is continuous.
- b. The box plot of birth weight is displayed below.

## graph box bwt



- c. Looking at the box plot from part (b), the 25th percentile is approximately 2.4 kilograms, the 50th percentile is approximately 2.75 kilograms, and the 75th percentile is approximately 3 kilograms.
- d. Looking at the box plot from part (b), the minimum birth weight is approximately 1.75 kilograms and the maximum is approximately 4 kilograms. Yes, there is one outlying value at 4 kilograms.
- e. The two-way scatter plot of infant birth weight versus the weight of the mother is displayed below.

twoway (scatter m\_weight bwt)



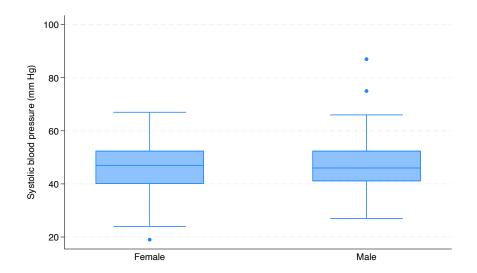
- f. No, there does not seem to be a relationship between the weight of the infant and its mother's weight. There is no discernible pattern to the points.
- g. Low birth weight status is discrete.
- h. The table of low birth weight is presented below. The relative frequency of low birth weight is 29.25%.

tab low\_bwt

Low birth   weight <   2500 g	Freq.	Percent	Cum.
No   Yes	283 117	70.75 29.25	70.75
Total	400	100.00	

- a. Systolic blood pressure is continuous.
- b. Sex is discrete.
- c. The box plots of systolic blood pressure measurements are shown below. The two distributions of values are quite similar; in particular, the 25th, 50th, and 75th percentiles are nearly the same. The males have two high outlying values, while the females have one low outlier.

graph box sbp, over(sex)



d. For girls, the mean systolic blood pressure is  $\bar{x}_f=46.5$  mm Hg and the standard deviation is  $s_f=11.1$  mm Hg; for boys, the mean is  $\bar{x}_m=47.9$  mm Hg and the standard deviation is  $s_m=11.8$  mm Hg. Males have a slightly larger mean and standard deviation.

sort sex
by sex: summarize sbp

-> sex= Variable	l Obs	Mean	Std. Dev.	Min	Max
	56		11.14526	19	67
-> sex= Variable	l Obs	Mean	Std. Dev.	Min	Max
sbp	l 44		11.80577	27	87

## CHAPTER 3

#### Exercise 7

The statement is misleading. Although the number of deaths has been increasing, the population base could be increasing as well.

#### Exercise 8

a. The crude birth rate is

$$\frac{70,704}{6,863,560} = 10.3 \text{ per } 1000 \text{ population.}$$

b. The crude death rate is

$$\frac{58,844}{6,863,560} = 8.6 \text{ per } 1000 \text{ population.}$$

c. The infant mortality rate is

$$\frac{263}{70,704}$$
 = 3.7 per 1000 live births.

#### Exercise 9

a. The infant mortality rates for each category of mother's body mass index (BMI) appear in the table below.

Mother's BMI	Infant Mortality Rate
$(kg/m^2)$	(per 1000 live births)
<18.5	5.8
18.5 – 24.9	4.6
25.0 – 29.9	5.2
$\geq 30.0$	7.1
Not stated	17.0

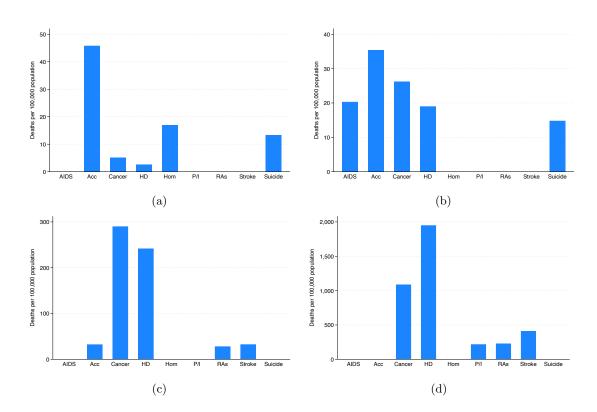
b. Infant mortality rate is the smallest for mother's in the Normal weight BMI category, and higher for those in the Underweight, Overweight, and Obese categories.

c. The infant mortality rate for children whose mother's BMI is in the Normal weight category is lower than the mortality rate for those whose mother's BMI is in a different category. Consequently, a large proportion of the infants whose mother's BMI is not stated are likely to have mothers in the Underweight, Overweight, and Obese categories, and perhaps more likely in the Obese category since this category has the highest infant mortality rate.

### Exercise 10

a. The bar charts of death rates per 100,000 population by age group are displayed on the following page. Note that "Acc" represents accidents, "HD" represents heart disease, "Hom" represents homicide, "P/I" represents pneumonia/influenza, and "RAs" represents respiratory ailments.

```
graph bar (asis) deaths if age == "15-24", over(cause) ytitle('"Deaths per 100,000 population"') graph bar (asis) deaths if age == "25-44", over(cause) ytitle('"Deaths per 100,000 population"') graph bar (asis) deaths if age == "45-64", over(cause) ytitle('"Deaths per 100,000 population"') graph bar (asis) deaths if age == "65+", over(cause) ytitle('"Deaths per 100,000 population"')
```



b. Cancer and heart disease are influential across all four age groups. Homicide and suicide affect primarily the younger groups; accidents also do not have a large impact on the elderly. Respiratory ailments, pneumonia/influenza and stroke affect the older groups. AIDS has a significant impact on those 25–44 years of age.

a. First, sum all province populations presented in Column 2 and call this x. Then, sum the number of hospitals in each province and call this y. The aggregate country number in the last row will be  $\frac{x}{y} = \frac{54,772,000}{544} = 1.00$ .

b. First, sum the number of public hospitals in each province and call this x. Then, calculate the number of uninsured individuals in each province using Columns 7 and 2 and call this z, and sum these numbers and call this y:

$$z_i = \#$$
 uninsured in province  $i = (100 - {\rm Column~7})/100 * {\rm Column~2}$  
$$y = \sum_{i=1}^9 z_i$$

The country number (0.71 per 100,000 uninsured population) in the last row of Column 8 is  $\frac{x}{\frac{y}{100,000}}$ .

- c. Similarly to answer (b) above, we can calculate the total number of private hospitals by summing the values in Column 4, and then use Columns 2 and 7 to calculate the total number of insured individuals across provinces and then divide this by 100,000. The country number (2.48 per 100,000 insured population) in the last row of Column 9 is then the total number of private hospitals divided by the total insured population divided by 100,000.
- d. There is much more variation across provinces for the number of public hospitals per 100,000 uninsured in the population (Column 8) and much less across provinces for the number of private hospitals per 100,000 insured in the population (Column 9). This means individuals with insurance will have approximately the same number of hospitals available to them in any province, while the availability of hospitals can differ greatly by province for uninsured individuals.

a. The rates of reported cases of polio per 100,000 children are 40.8 for the vaccine group and 80.5 for the placebo group.

The rates of true instances of polio per 100,000 children are 28.4 for the vaccine group and 70.6 for the placebo group.

The rates of incorrect diagnoses of polio per 100,000 children are 12.5 for the vaccine group and 9.9 for the placebo group.

The rates of paralytic disease per 100,000 children are 16.4 for the vaccine group and 57.1 for the placebo group.

The rates of nonparalytic disease per 100,000 children are 12.0 for the vaccine group and 13.4 for the placebo group.

b. The Salk vaccine appears to have help prevented cases of paralytic polio. It had no apparent effect on nonparalytic polio.

#### Exercise 13

Unlike the age-adjusted death rates, the crude death rates do not take into account the effect of age; therefore, the increasing crude rates reflect the fact that the population is growing older.

## Exercise 14

a. The crude mortality rate for Maine is

$$\frac{11,082}{796,832}$$
 = 13.9 per 1000 population,

and the crude rate for South Carolina is

$$\frac{22,401}{1,738,173}$$
 = 12.9 per 1000 population.

b. The proportions of the total population in each age group are displayed below.

	Proportion		
Age	Maine	South Carolina	
0-4	9.4%	11.8%	
5-9	9.3%	12.8%	
10 - 14	8.6%	12.2%	
15 - 19	7.6%	9.6%	
20 – 24	13.3%	12.6%	
25 - 34	12.7%	11.0%	
35 - 44	11.3%	8.3%	
45 - 54	10.0%	13.9%	
55 – 64	9.1%	4.6%	
65 - 74	5.8%	2.3%	
75 +	2.8%	1.0%	
Total	100.0%	100.0%	

The population in Maine is somewhat older than the population in South Carolina.

c. The age-specific mortality rates for each state are below.

	Death Rate		
	per 1000 Population		
Age	Maine	South Carolina	
0-4	20.6	23.9	
5 - 9	1.9	1.9	
10 – 14	1.4	1.8	
15 - 19	2.2	4.3	
20 – 24	3.7	6.5	
25 – 34	3.9	8.7	
35 – 44	5.5	12.4	
45 - 54	10.8	19.9	
55 – 64	20.4	33.1	
65 - 74	52.2	61.5	
75 +	136.5	141.4	

Disregarding the effect of infant mortality, mortality rate increases with age in each state.

- d. It is necessary to control for the effect of age when comparing mortality rates in Maine and South Carolina. Age is a confounder; it is associated with both the population distribution and mortality rate.
- e. The age-adjusted mortality rates are 12.0 per 1000 population for Maine and 17.2 per 1000 population for South Carolina.
- f. The rates are now flipped; the crude mortality rate for Maine (13.9 per 1000 population) is higher than for South Carolina (12.9 per 1000 population), but the age-adjusted mortality rate for Maine (12 per 1000 population) is smaller than the age-adjusted mortality rate for South Carolina (17.2 per 1000 population). This is due to Maine having more older individuals in its population.

## CHAPTER 4

#### Exercise 6

a. The proportion of individuals that will live to their 21st birthday is

$$\frac{l_{21}}{l_0} = \frac{98,927}{100,000} \\
= 0.989.$$

b. The proportion of 21-year-olds that will live to their 40th birthday is

$$\frac{l_{40}}{l_{21}} = \frac{96,537}{98,927} \\
= 0.976.$$

c. The proportion of 40-year-olds that will live to their 70th birthday is

$$\begin{array}{rcl} \frac{l_{70}}{l_{40}} & = & \frac{79,092}{96,537} \\ & = & 0.819. \end{array}$$

c. The proportion of 70-year-olds that will live to their 90th birthday is

$$\frac{l_{90}}{l_{70}} = \frac{28,285}{79,092} \\
= 0.358.$$

## Exercise 7

a. Say you sell the policies to 346 50-year-olds who will pay you the first year on the policy. Using Halley's life table in Figure 4.3, we can see that 11 people died before reaching age 51 since there are only 335 51-year-olds. Therefore, those 11 individuals' inheritors will receive \$1,000 in the first year, and the remaining 335 will pay the second year. Similarly, 11 individuals' inheritors will receive \$1,000 in the second year and the remaining 324 will pay the third year, 11 individuals' inheritors will receive \$1,000 in the third year and the remaining 313 will pay the fourth year, and 11 individuals' inheritors will receive \$1,000 in the fourth year and the remaining 302 individuals will pay a fifth year. In the fifth year, 10 will die. So, you will pay out \$1,000 to  $4 \times 11 + 10 = $54,000$  to the inheritors, after having collected \$X (payment for a policy for a year) each year from 346 + 335 + 324 + 313 + 302 = 1,620 payments. To break even,  $1620 \times $X = $54,000$ . Solving for X you will need to charge a 50-year-old \$33.34 for a 5-year policy in order to break even, and more than this to make a profit. b. If you extend the calculations from part a. for another 5 years, you will pay inheritors

\$104,000 while receiving 346+335+324+313+302+292+282+272+262+252=2,980 payments. To break even, you will need to charge a 50-year-old \$34.90 for a 10-year policy. c. Using Table 4.2 (the 2016 US life table), you would need to charge a 50-year-old \$4.50 for a 5-year policy and \$5.65 for a 10-year policy.

### Exercise 8

The total person-years lived outside institutions from age x on (Column 7) is obtained by summing column 6 from the bottom up. The average remaining years outside institutions (Column 8) is calculated by dividing column 7 by column 2.

(7)	(8)
Total Person-Years	Average Remaining
Outside Institutions	Years Outside
From Age $x$ On	of Institutions
7,005,116	70.1
5,532,705	56.5
4,562,720	47.3
2,671,609	28.9
982,595	13.6

Life expectancy at birth has been increasing steadily since 1970, and has always been longer for females than for males for both races. Whites individuals have longer life expectancies than black individuals of the same gender.

#### Exercise 10

Individuals in the highest income quintile have longer life expectancies compared to individuals in lower income quintiles for both 1930 and 1960. However, the gap in life expectancy between higher-income individuals and those lower on the socioeconomic distribution expanded considerably between 1930 and 1960.

### Exercise 11

- a. At the ages when males have a higher rate of accidental and violent death than females, we would see higher age-specific death rates among the males.
- b. Since  $e_x$  reflects the mortality rates for age x and beyond, we would expect the average life expectancies of males to be lower than those of females for these early age groups.

### Exercise 12

Life expectancies at birth and at age 60 increased consistently over the centuries for both males and females. Life expectancies at age 80 also increased for females; for males, the life expectancy first dropped and then increased. Females have a longer life expectancy than males at each age and in every time period. The difference between genders is largest in the most recent time period, 1971–1975.