Scientific Report on Modeling Tumor Growth in the Patient

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Problem and Data

On February 1st, the patient was found by MRI to have a malignant tumor of volume 650 cubic millimeters in the suprasellar cavity of the brain. Another MRI was performed a week later on February 8th which measured the tumor's volume to have grown to 810 cubic millimeters. The goal of this report is to give and explain a mathematical model for the tumor's growth which is both simple and accurate enough to advise the specialists at AOS, Inc. on the treatment of the patient.

Model

A Gompertz curve is the generally excepted mathematical model for the growth of tumors large enough to detect and typically subject to some constraint.[1] The constraint in this scenario is the volume of the suprasellar cavity, which is 3,200 cubic millimeters. We therefore apply it here as

$$\frac{dV(t)}{dt} = rV(t)\ln(\frac{k}{V(t)}) \tag{1}$$

where $V(t)$ is the volume of the tumor in cubic millimeters as a function of time, r and k are constants, and t is time, which is measured in days with $t = 0$ referring to February 1st. By analysis shown later, we find

$$V(t) = e^{-\frac{A}{e^{rt}}} \tag{2}$$

where A is some constant. Then, from the measurements on February 1st and 8th respectively,

$$V(0) = 650\text{mm}^3 \tag{3}$$

$$V(7) = 810\text{mm}^3 \tag{4}$$

Analyses

Returning to equation (1), seperating variables, and using properties of logs, we have

$$\frac{dV(t)}{V(t)(\ln(k) - \ln(V(t))} = r\,dt \tag{5}$$

We integrate both sides, using a u-substitution on the left with $u = \ln(V(t))$, $du = \frac{dV(t)}{V(t)}$, to find

$$-\ln(\ln(\frac{k}{V(t)})) = rt + C \tag{6}$$

Simplifying this we find equation (2)

$$V(t) = e^{-\frac{A}{e^{rt}}}$$

where $A = e^C$ which is some constant. Now to find the constant k we return to the original equation and observe that since the volume of the tumor is constrained to 3200 cubic millimeters, then the change in volume when $V(t) = 3200$ is 0. We also know that r must be nonzero, otherwise the tumor wouldn't have grown by this model, so k must equal 3200 so that $\ln(\frac{k}{V(t)}) = 0$ when $V(t) = 3200$. Then, to find A, notice that $V(0) = 650$ so

$$V(0) = 650 = \frac{3200}{e^{\frac{A}{e^0}}}$$

$$A = \ln(\frac{3200}{650}) \approx 1.5939$$

Then, to find r, notice that $V(7) = 810$ so

$$V(7) = 810 = \frac{3200}{e^{\frac{1.5939}{e^{7r}}}}$$

$$\frac{1.5939}{e^{7r}} = \ln(\frac{3200}{810})$$

$$r = \frac{\ln(\frac{1.5939}{\ln(\frac{3200}{810})})}{7} \approx 0.02122$$

Summary

Bibliography

[1] Fornalski, K. W., Reszczyńska, J., Dobrzyński, L., Wysocki, P., & Janiak, M. K. (2020). Possible Source of the Gompertz Law of Proliferating Cancer Cells: Mechanistic Modeling of Tumor Growth. Acta Physica Polonica A, 138(6), 854–862. https://doi.org/10.12693/aphyspola.138.854