Lecture 11: Practical applications of Bernoulli's equation

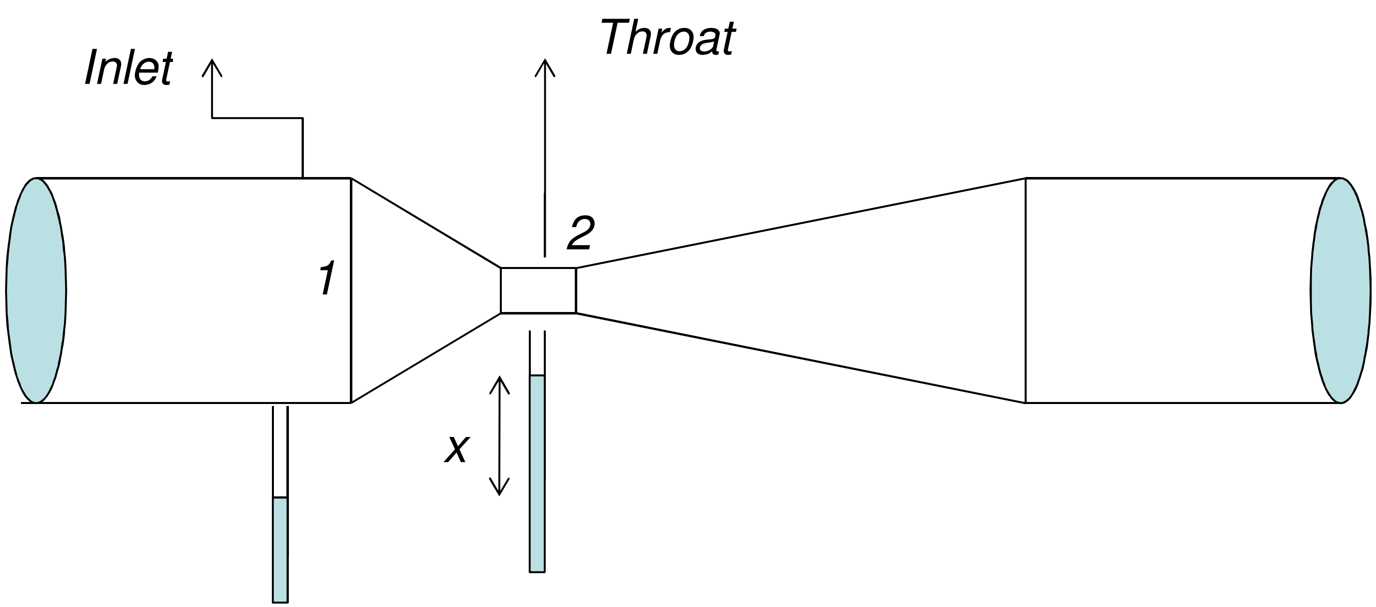
Objective: To study the applications of the Bernoulli's equation

- Venturimeter
- Orifice meter
- Pitot-tube

Venturimeter

Venturimeter: is a device used for measuring the rate of flow of a fluid flowing through a pipe. It consists of three parts:

- A short converging part
- Throat
- Diverging part



Let d_1 = diameter at the inlet (section 1)

p_1 = pressure at section 1

v_1 = velocity at section 1

A_1= area at section1

d_2, p_2, v_2, A_2 are the corresponding values at the throat (section 2)

Applying Bernoulli's equations at sections 1 and 2, we get

$$\frac{p_1}{\rho g}+\frac{w_1^{\ 2}}{2g}+z_1=\frac{p_2}{\rho g}+\frac{w_2^{\ 2}}{2g}+z_2.$$

As pipe is horizontal z_1=z_2

$$\Rightarrow \frac{p_1-p_2}{\rho g}=\frac{v_2^{\ 2}-v_1^{\ 2}}{2g}$$

$$\Rightarrow h=\frac{v_2^{\ 2}-v_1^{\ 2}}{2g}$$

Where $h \equiv \dfrac{p_1 - p_2}{\rho g}$, difference of pressure heads at sections 1 and 2.

From the continuity equation at sections 1 and 2, we obtain

$$A_1 v_1 = A_2 v_2 \Rightarrow v_1 = \frac{A_2 v_2}{A_1}$$

Hence $$h = \frac{v_2{}^2}{2g}\left[\frac{A_1^2 - A_2^2}{A_1^2}\right]$$

$$\Rightarrow v_2 = \frac{A_1}{\sqrt{A_1^2 - A_2^2}}\sqrt{2gh}$$

Discharge $$Q = A_1 v_1 = A_2 h$$

$$\Rightarrow Q = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2gh}$$

Note that the above expression is for ideal condition and is known as theoretical discharge.

Actual discharge will be less than theoretical discharge.

$$Q_{actual} = C_d \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gh}$$

C_d is the coefficient of venturimeter and its value is always less then 1.

Expression of 'h' given by differential U-tube manometer:

Case 1: The liquid in the manometer is heavier than the liquid flowing through the pipe

$$h = x\left[\frac{S_h}{S_0} - 1\right]$$

S_h: Specific gravity of the heavier liquid.

S_0: Specific gravity of the flowing liquid.

Case 2: The liquid in the manometer is lighter than the liquid flowing through the pipe

$$h = x\left[1 - \frac{S_L}{S_0}\right]$$

S_L: Specific gravity of the lighter liquid.

X: difference of the liquid columns in U-tube

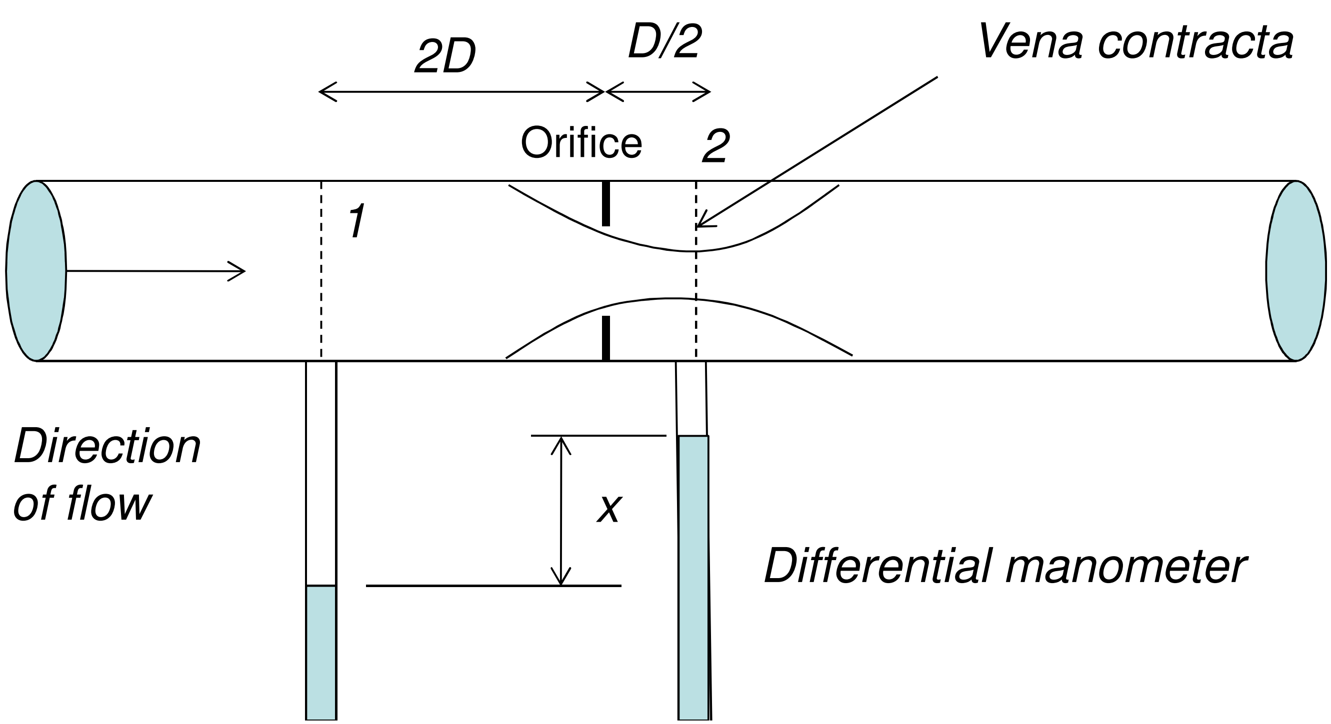
Orifice meter

• **Orifice meter:** is a device used for measuring the rate of flow of a fluid flowing through a pipe.

• It is a cheaper device as compared to venturimeter. This also work on the same principle as that of venturimeter.

• It consists of flat circular plate which has a circular hole, in concentric with the pipe. This is called orifice.

• The diameter of orifice is generally 0.5 times the diameter of the pipe (D), although it may vary from 0.4 to 0.8 times the pipe diameter.



Let d_1 = diameter at section 1

p_1 = pressure at section 1

v_1 = velocity at section 1

A_1= area at section1

d_2, p_2, v_2, A_2 are the corresponding values at section 2.

Applying Bernoulli's equations at sections 1 and 2, we get

$$\frac{p_1}{\rho g}+\frac{v_1^{\;2}}{2g}+z_1=\frac{p_2}{\rho g}+\frac{v_2^{\;2}}{2g}+z_2$$

$$\Rightarrow\left(\frac{p_1}{\rho g}+z_1\right)-\left(\frac{p_2}{\rho g}+z_2\right)=\frac{v_2^{\;2}-v_1^{\;2}}{2g}$$

$$\Rightarrow h=\frac{v_2^{\;2}-v_1^{\;2}}{2g}$$

$$\Rightarrow v_2=\sqrt{2gh+v_1^2}$$

where *h* is the differential head.

Let A_0 is the area of the orifice.

Coefficient of contraction, $C_c = \frac{A_2}{A_0}$

By continuity equation, we have

$$A_1 v_1 = A_2 v_2$$

$$\Rightarrow v_1 = \frac{A_0 C_c}{A_1} v_2$$

Hence,

$$v_2 = \sqrt{2gh + \frac{A_0^2 C_c^2 v_2^2}{A_1^2}}$$

$$\Rightarrow v_2 = \frac{\sqrt{2gh}}{\sqrt{1 - \frac{A_0^2}{A_1^2} C_c^2}}$$

Thus, discharge,

$$Q = A_2 v_2 = v_2 A_0 C_c = \frac{A_0 C_c \sqrt{2gh}}{\sqrt{1 - \frac{A_0^2}{A_1^2} C_c^2}}$$

If C_d is the co-efficient of discharge for orifice meter, which is defined as

$$C_d = C_c \frac{\sqrt{1 - \frac{A_0^2}{A_1^2}}}{\sqrt{1 - \frac{A_0^2}{A_1^2} C_c^2}}$$

$$\Rightarrow C_c = C_d \frac{\sqrt{1 - \frac{A_0^2}{A_1^2} C_c^2}}{\sqrt{1 - \frac{A_0^2}{A_1^2}}}$$

Hence,

$$Q = C_d \frac{A_0 A_1 \sqrt{2gh}}{\sqrt{A_1^2 - A_0^2}}$$

The coefficient of discharge of the orifice meter is much smaller than that of a venturimeter.

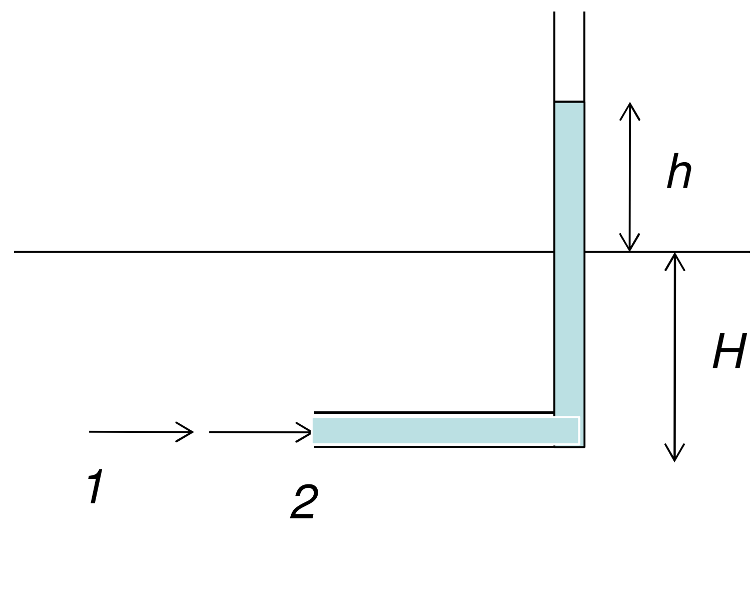
Pitot-tube

• **Orifice meter:** is a device used for measuring the velocity of flow at any point in a pipe or a channel.

• Principle: If the velocity at any point decreases, the pressure at that point increases due to the conservation of the kinetic energy into pressure energy.

• In simplest form, the pitot tube consists of a glass tube, bent at right angles.

Let p_1 = pressure at section 1
p_2 = pressure at section 2
v_1 = velocity at section 1
v_2 = velocity at section 2 = 0
H = depth of tube in the liquid
h = rise of liquid in the tube above the free surface



Point 2 is just at the inlet of the Pitot-tube
Point 1 is far away from the tube

Applying Bernoulli's equations at sections 1 and 2, we get

$$\frac{p_1}{\rho g}+\frac{v_1^{\ 2}}{2g}+z_1=\frac{p_2}{\rho g}+\frac{v_2^{\ 2}}{2g}+z_2$$

But $z_1 = z_2$, and $v_2 = 0.$

$\frac{p_1}{\rho g} =$ Pressure head at 1=H

$\frac{p_2}{\rho g} =$ Pressure head at 2=h+H

Substituting these values, we get

$$H+\frac{v_1^{\ 2}}{2g}=h+H$$

$$\Rightarrow v_1=\sqrt{2gh}$$

This is theoretical velocity. Actual velocity is given by

$$\left(v_1\right)_{act}=C_v\sqrt{2gh}$$

$C_v \equiv$ coefficient of pitot-tube