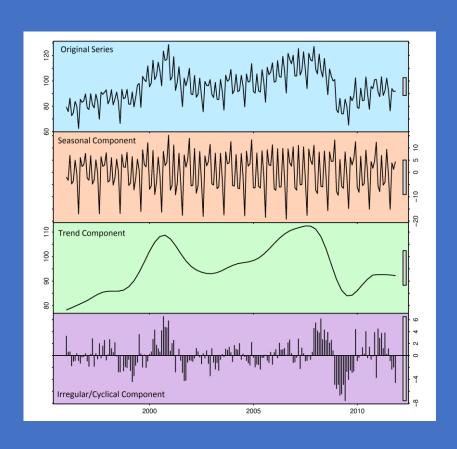


FORECASTING INDIVIDUAL ASSIGNMENT



Submitted By:

PRINEET KAUR BHURJI

EXERCISE 1

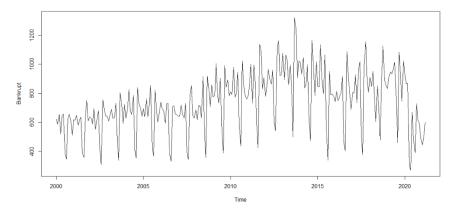
The purpose of this document is to describe the application of various Forecasting Models and the corresponding evaluation of them with an aim to come up with the expected Best Forecasts. For the first task, I used the "Bankrupt" dataset which contains the number of bankruptcies in Belgium (per month) for all economic activities, from January 2000 to March 2021. As a data scientist would need to analyse data to check for relevant trends and forecast the number of bankruptcies for Belgium until December 2022.

The Forecasting Process will consist of five main phases -

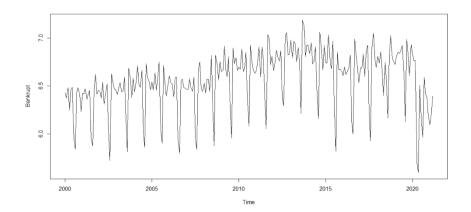
- Exploratory Data Analysis (Understanding and Decomposing the Time Series Data)
- Model Fitting (experimenting using different Methods)
- Evaluation of each of the Models
- Comparing All Models (picking the Best for Final Forecast)
- Final Forecast

EXPLORATORY DATA ANALYSIS

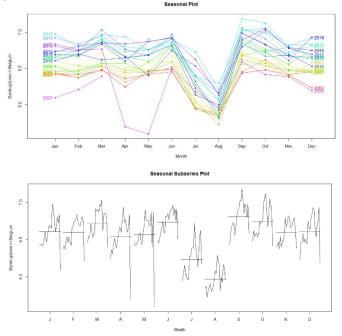
To explore the given data, I read the file and obtained the following graph by applying the plot() function.



From the above plot we can observe that there is no stationarity, instead, there is a clear increasing trend and a strong seasonal pattern. It can also be seen that the seasonality variation increases slightly as the level of the series increases (with an exception at the end), therefore it was a good idea to log transform the time series data. Below plot shows the data behaviour after the log transformation was done using log() function—



After the application of log transformation in the previous step, the time series variations seemed to be more compact now. To further explore the seasonality component, I went ahead with plotting the Seasonal and Seasonal subseries graphs



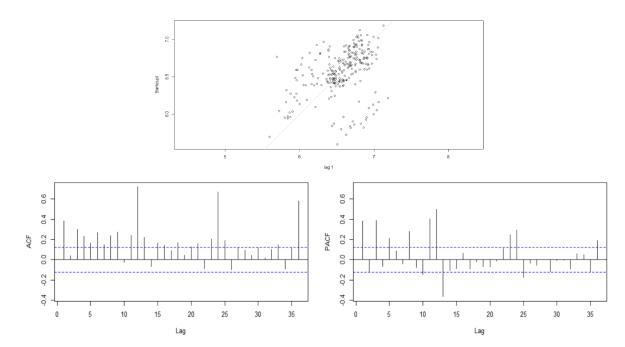
Both the graphs above show that there is Seasonality as well as a Trend in the data under consideration.

Further, the need was to check is there is any autocorrelation or partial autocorrelation if at all it exists.

Let us understand what we mean by ACF and PACF first -

- ACF is an (complete) auto-correlation function which gives us values of autocorrelation of any series with
 its lagged values. In simple terms, it describes how well the present value of the series is related with its
 past values.
- **PACF** is a partial autocorrelation function. Thus, instead of finding correlations of present with lags like ACF, it finds correlation of the residuals with the next lag value hence 'partial' and not 'complete' as we remove already found variations before we find the next correlation.

The graphs below show some points of high autocorrelation values occurring at lags 12, 24 and 36 which are above of 0.60 and even some component of seasonality. From this information, one may wonder about the lack of white noise. But if we see the PACF graph we see that there is not much white noise component.



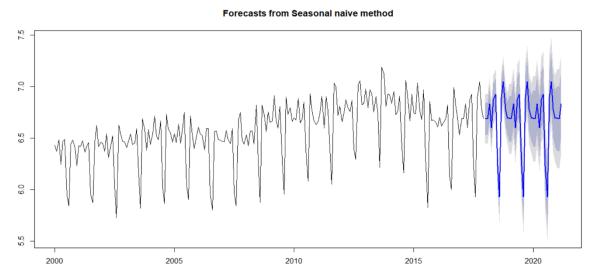
2

MODEL FITTING

To begin with the data was first splitted into the Train and Test sets. The Train set includes data from **January 2000 to December 2017** and the Test set includes data from **January 2018 to March 2021**.

Seasonal Naive Method

For the Forecasting purpose, I first chose the Seasonal Naive Method. Used to fit the Training set and evaluate the performance on both Train and Test sets. The results were as follows –

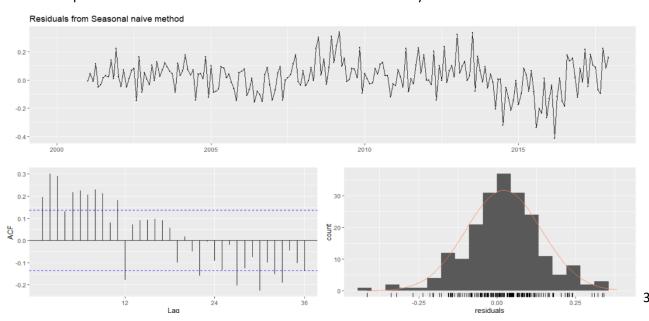


EVALUATION OF MODEL

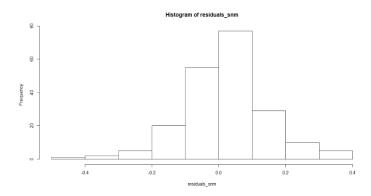
For evaluating the performance of the method used to build the models, I went ahead with checking the Residuals and applying the Ljung–Box test. Let us first understand what they signify –

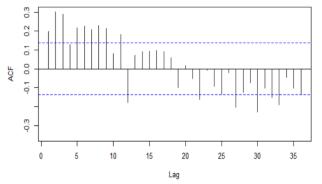
- **Residuals** in a time series model are what is left over after fitting a model. They show the difference between the observations and the corresponding fitted values. Residuals are useful in checking whether a model has adequately captured the information in the data and a good forecasting method will yield residuals with no or very less correlation.
- Ljung–Box test is a type of statistical test of whether any of a group of autocorrelations of a time series are different from zero. Instead of testing randomness at each distinct lag, it tests the "overall" randomness based on a number of lags and is therefore a portmanteau test.

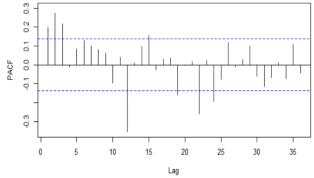
The below plots show the behaviour of the residuals and the accuracy of the model –



Went ahead with plotting the histogram of the residuals and the ACF and PACF plots to understand the autocorrelation and the partial autocorrelation if any.







```
Box-Ljung test
```

data: residuals_snm
X-squared = 134.86, df = 24, p-value < 2.2e-16</pre>

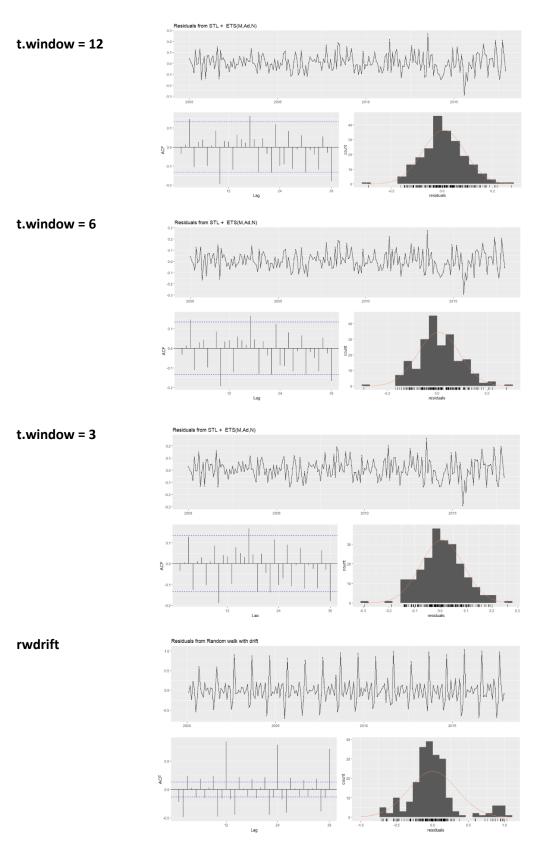
From the above Residual Plots visualizations and the Ljung Box tests performed for this model, we can see that the p-values (parameters in the tests) are less than .05 which means that they are significant. So, we can reject the null hypothesis of white noise and there is still something in the residuals that is not been captured by the model.

```
> LjungBox(residuals_snm[-c(1:12),
lags statistic df
                         p-value
       8.013276
                  1 4.643566e-03
                  2 1.449407e-06
   2
      26.888712
       44.466212
                  3 1.201425e-09
       48.008554
                  4 9.399166e-10
   5
       57.999535
                  5 3.146161e-11
      68.809770
                  6 7.170931e-13
       77.893785
                  7 3.697043e-14
   8
       89.175093
                  8 6.661338e-16
      98.920409
                  9 0.000000e+00
  10 100.324128 10 0.000000e+00
  11 107.441513 11 0.000000e+00
  12 114.387167
                12 0.000000e+00
  13 115.553955 13 0.000000e+00
  14 117.374331 14 0.000000e+00
  15 119.313308 15 0.000000e+00
  16 121.379064 16 0.000000e+00
  17 123.239848 17 0.000000e+00
  18 123.986039 18 0.000000e+00
  19 126.257970 19 0.000000e+00
  20 126.332952 20 0.000000e+00
  21 126.907991 21 0.000000e+00
  22 132.842555 22 0.000000e+00
  23 132.852609 23 0.000000e+00
  24 134.860759 24 0.000000e+00
```

In terms of accuracy, the following results we obtained. These will be used further as reference for model selection purposes.

Seasonal Naive Method with STL Decomposition

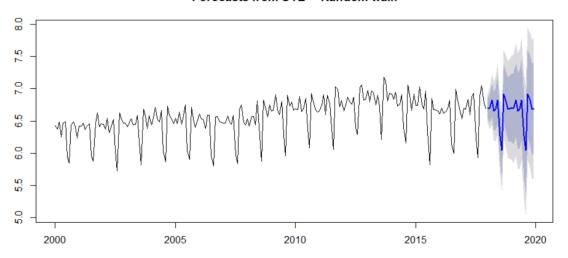
Three different cases with varying t.window values were taken (while keeping s.window as "periodic") along with rwdrift function. Residuals were checked for all cases and the results were as follows –



After analysing the above residual graphs, realized that the least residuals were seen in the case of t.window = 6. Also, the autocorrelation also seemed to be least. Thus, went ahead with choosing the same.

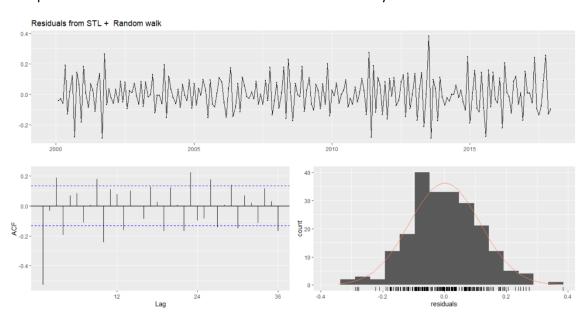
The seasonal naïve model with STL decomposition was fitted on the training set and the evaluation was done using both training and testing set. The Forecasting results were as follows –

Forecasts from STL + Random walk

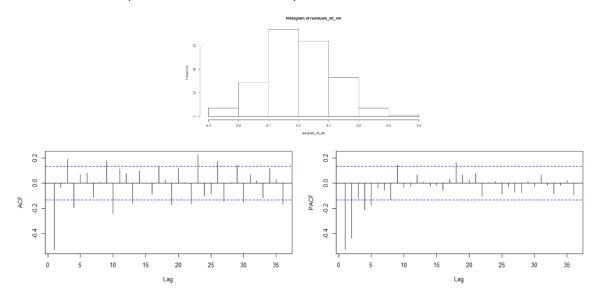


EVALUATION OF MODEL

The below plots show the behaviour of the residuals and the accuracy of the model.



Went ahead with plotting the histogram of the residuals and the ACF and PACF plots to understand the autocorrelation and the partial autocorrelation if any.



6

```
Box-Ljung test

data: residuals_stl_nm

X-squared = 153.63, df = 24, p-value < 2.2e-16
```

From the above Residual Plots visualizations and the Ljung Box tests performed for this model, we can see that the p-values (parameters in the tests) are less than .05 which means that they are significant. So, we can reject the null hypothesis of white noise and there is still something in the residuals that is not been captured by the model.

```
LiungBox(residuals_stl_nm[-1].
                       p-value
lags statistic df
    61.01598 1 5.662137e-15
               2 4.929390e-14
     61.28023
    69.12171
               3 6.550316e-15
     77.51116
               4 5.551115e-16
     78.57749
               5 1.665335e-15
  6
     80.07257
               6 3.441691e-15
     82.89794
               7 3.552714e-15
     82.90751
               8 1.265654e-14
     90.10455
               9 1.554312e-15
 10 103.57013 10 0.000000e+00
 11 106.39788 11 0.000000e+00
 12 107.78682 12 0.000000e+00
 13 113.86515 13 0.000000e+00
 14 116.19262 14 0.000000e+00
 15 116.19268 15 0.000000e+00
 16 117.99014 16 0.000000e+00
 17 121.87945 17 0.000000e+00
 18 122.04871 18 0.000000e+00
 19 128.75021 19 0.000000e+00
 20 132.28214 20 0.000000e+00
 21 132.28552 21 0.000000e+00
  22 139.02660 22 0.000000e+00
 23 151.24299 23 0.000000e+00
 24 153.62827 24 0.000000e+00
```

In terms of accuracy, the following results we obtained. These will be used further as reference for model selection purposes –

```
RMSE MAE MPE MAPE MASE ACF1
Training set 0.1143733 0.0904064 0.002379906 1.381122 0.9731942 -0.52902914
Test set 0.1519006 0.1211428 1.105732567 1.808763 1.3040611 0.06472759
```

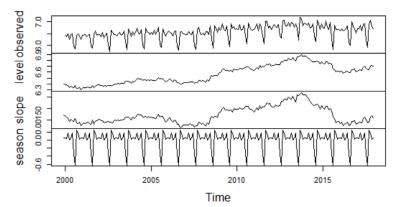
Exponential Smoothening Method (ETS)

For forecasting using the ETS method, I handpicked only 8 models that incorporate the seasonality and trend components from the 18 available models. The list of the models chosen includes –

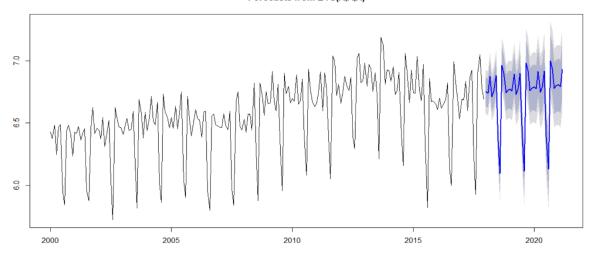
- ETS1 (A, N, A)
- ETS2 (A, N, Ad)
- ETS3 (A, A, A)
- ETS4 (A, A, Ad)
- ETS5 (M, N, A)
- ETS6 (M, N, Ad)
- ETS7 (M, A, A)
- ETS8 (M, A, Ad)

Checked the Residuals and Accuracy of each of the above models. (For all individual plots for these methods kindly refer to the code, for report purpose only included the results for the Auto ETS). As the results of all the models were quite similar, went ahead with applying the automated ETS method. It showed **ETS(A,A,A)** as the best performing method so went ahead with using the same for further analysis. The results of the auto ETS were as follows –

Decomposition by ETS(A,A,A) method

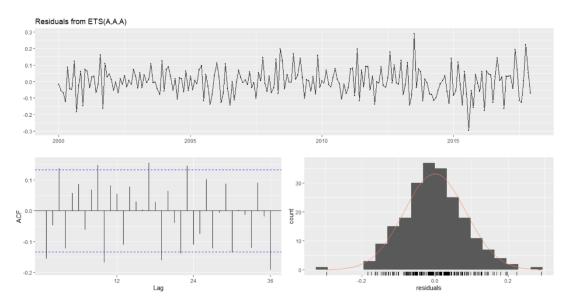


Forecasts from ETS(A,A,A)

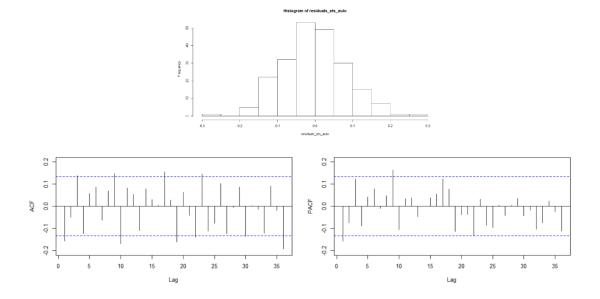


EVALUATION OF MODEL

For evaluating the performance of the method used to build the models, I went ahead with checking the Residuals and applying the Ljung–Box test. The below plots show the behaviour of the residuals and the accuracy of the model -



Went ahead with plotting the histogram of the residuals and the ACF and PACF plots to understand the autocorrelation and the partial autocorrelation if any.



```
Box-Ljung test

data: residuals_ets_auto
X-squared = 62.121, df = 24, p-value = 3.187e-05
```

From the above Residual Plots visualizations and the Ljung Box tests performed for this model, we can see that the p-values (parameters in the tests) are less than .05 which means that they are significant. So, we can reject the null hypothesis of white noise and there is still something in the residuals that is not been captured by the model.

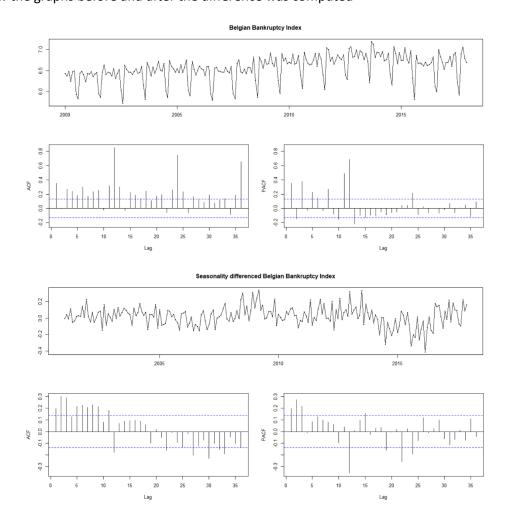
```
LjungBox(residuals_ets_auto, la
lags statistic df
                       p-value
      5.330841
               1 2.095127e-02
      5.826836
                2 5.428986e-02
    9.991152
                3 1.864150e-02
   4 13.290490
                4 9.940269e-03
  5 14.034880
                5
                  1.538936e-02
  6 15.717257
                6
                  1.535477e-02
  7 16.585828
                  2.027116e-02
  8 17.660034
                8
                 2.392479e-02
  9 22.690711
                9
                 6.929596e-03
 10 29.143578 10 1.181061e-03
 11 30.706043 11 1.226322e-03
 12 31.374487
                  1.726785e-03
              12
 13 34.180715 13
                  1.129769e-03
 14 35.620587 14
                 1.188920e-03
 15 35.833235 15 1.868387e-03
 16 35.836808 16
                  3.048243e-03
                 7.766388e-04
 17 41.557701 17
  18 41.740044 18
                  1.202162e-03
  19 47.908651 19 2.644342e-04
  20 48.897327 20 3.179206e-04
  21 49.280468 21 4.589753e-04
  22 53.952563 22
                 1.670369e-04
  23 59.136455 23 5.087817e-05
  24 62.120546 24 3.186914e-05
```

In terms of accuracy, the following results we obtained. These will be used further as reference for model selection purposes –

```
RMSE MAE MPE MAPE MASE ACF1
Training set 0.08448937 0.06532629 -0.02118966 0.997086 0.7032153 -0.1560134
Test set 0.36699750 0.23652551 -2.85637851 3.803810 2.5461168 0.6882844
```

Seasonal ARIMA Method

Before applying the auto ARIMA method, to add a component of stationarity and to help stabilize the mean, first computed the difference/variation in the observations and the output was plotted. The two graphs below show the graphs before and after the difference was computed —

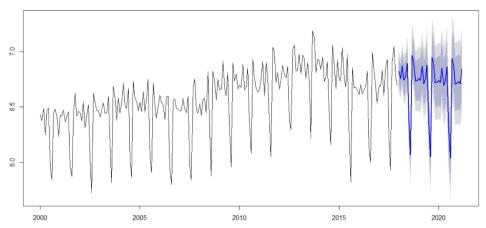


We can still observe some significant peaks in the PACF graph, in order to address this situation, some models were handpicked manually adjusting the elements in the trending and seasonal components (P, D, Q). The list of the models chosen includes –

- ARIMA (1, 0, 1) (2, 1, 1)[12]
- ARIMA (3, 0, 1) (0, 1, 1)[12]
- ARIMA (3, 0, 1) (3, 1, 1)[12]
- ARIMA (3, 1, 1) (3, 1, 1)[12]
- ARIMA (3, 0, 3) (3, 1, 1)[12]
- ARIMA (3, 0, 3) (3, 1, 3)[12]

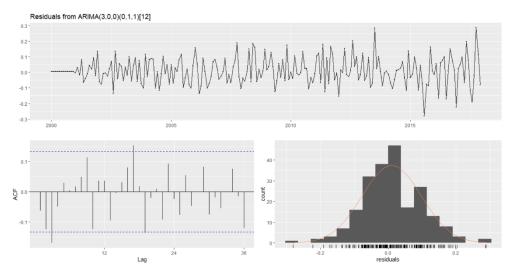
Checked the Residuals and Accuracy of each of the above models. (For all individual plots for these kindly refer to the code, for report purpose only included the results for the Auto ARIMA). As the results of all the models were not so great and to ensure that I choose the best model, went ahead with applying the automated ARIMA method. It showed **ARIMA(3,0,0)(0,1,1)[12]** as the best performing method so went ahead with using the same for further analysis. The results of the auto ARIMA were as follows —

Forecasts from ARIMA(3,0,0)(0,1,1)[12]

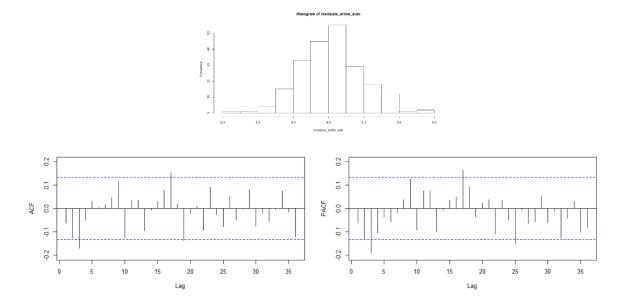


EVALUATION OF MODEL

For evaluating the performance of the method used to build the models, I went ahead with checking the Residuals and applying the Ljung–Box test. The below plots show the behaviour of the residuals and the accuracy of the model -



Went ahead with plotting the histogram of the residuals and the ACF and PACF plots to understand the autocorrelation and the partial autocorrelation if any.



Box-Ljung test

data: residuals_arima_auto
X-squared = 37.378, df = 24, p-value = 0.0401

From the above Residual Plots visualizations and the Ljung Box tests performed for this model, we can see that the p-values (parameters in the tests) are less than .05 which means that they are significant. So, we can reject the null hypothesis of white noise and there is still something in the residuals that is not been captured by the model.

LjungBox(residuals_arima_auto, statistic df p-value 0.8688761 1 0.35126664 4.2662580 2 0.11846603 3 0.01371084 3 10.6604848 4 11.2311411 4 0.02408552 11.4266844 5 0.04354651 11.4300093 6 0.07596156 11.4921839 7 0.11854457 12.0296619 8 0.14988534 14.9615970 9 0.09199626 10 18.4981166 10 0.04712034 11 18.8073988 11 0.06463960 12 19.1076709 12 0.08596296 13 21.1965484 13 0.06913100 21.2012219 14 0.09658603 14 15 21.4267584 15 0.12373939 16 22.9027475 16 0.11635913 28.4540931 17 0.03990401 17 28.5272651 18 0.05447154 32.9525226 19 0.02434502 20 33.0532189 20 0.03328978 21 33.0756907 21 0.04538453 22 35.1526952 22 0.03734495 23 37.2349358 23 0.03071778 24 37.3778769 24 0.04010404

In terms of accuracy, the following results we obtained –

RMSE MAE MPE MAPE MASE ACF1
Training set 0.08812702 0.06757031 0.1318679 1.028642 0.7273714 -0.06298586
Test set 0.33700432 0.21993784 -2.2999930 3.529802 2.3675561 0.65957101

COMPARING ALL MODELS

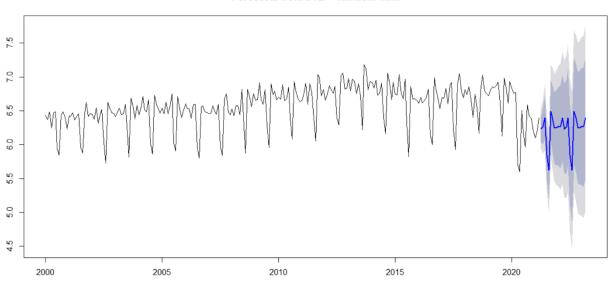
To choose which model performed better, let us compare the results of the final accuracy scores –

KPI	SNAIVE	SNAIVE STL	Auto ETS	Auto ARIMA
p-value	2.2E-16	2.2E-16	3.19E-05	0.0401
RMSE	0.3637954	0.1519006	0.3669975	0.33700432
MAE	0.24789251	0.1211428	0.23652551	0.21993784
MPE	-2.0490865	1.105732567	-2.85637851	-2.299993
MAPE	3.960417	1.808763	3.80381	3.529802
MASE	2.668479	1.3040611	2.5461168	2.3675561
ACF1	0.6127023	0.06472759	0.6882844	0.65957101

We can conclude that even though the models used did not perform well on the residuals test, but looking at the accuracy scores across various KPIs, we can say that the **Seasonal Naïve Method with STL Decomposition** performed overall the best. So, I went ahead with applying the same for coming up with the Final forecast.

FINAL FORECAST

To get the Final Forecast applied the evaluated Best Model in the previous step (i.e., Seasonal Naïve Method with STL) to the entire dataset and the results were as follows –



Forecasts from STL + Random walk

FINAL MODEL ACCURACY

RMSE MAE MPE MAPE MASE ACF1 0.1432197 0.1025948 -0.0373248 1.5752022 0.8435015 -0.2731486

With the above scores for some of the important KPI's like the MAPE, MAE and RMSE we can say that the Final Model accuracy is pretty good in terms of predictions and can be relied upon.

Thus, from the final forecast as seen from above graph, we can observe and conclude that the trend of Bankruptcies is downward in coming years and that the Bankruptcies are forecasted to decrease by December 2022.

EXERCISE 2

For this forecasting task, I used the "**Total Energy Monthly Data for the US**" since <u>January 1973 to January 2021</u> (which is the latest data released by the **U.S. Energy Information Administration** on <u>27th April 2021</u>).

Website Link - Total Energy Monthly Data - U.S. Energy Information Administration (EIA)

Dataset Link - http://www.eia.gov/totalenergy/data/monthly/#electricity (Source: Table 7.6.)

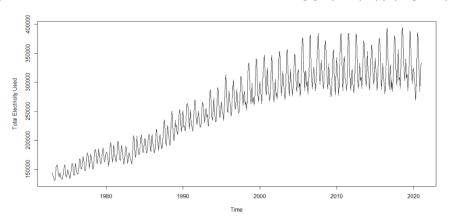
Further, a new sheet was created to just include the "Month" & "Total Electricity Used" columns and was saved as "Data_Formatted" in the main excel file.

The Forecasting Process will consist of five main phases -

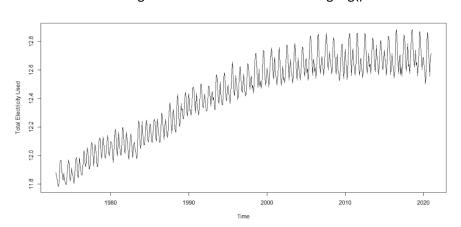
- Exploratory Data Analysis (Understanding and Decomposing the Time Series Data)
- Model Fitting (experimenting using different Methods)
- Evaluation of each of the Models
- Comparing All Models (picking the Best for Final Forecast)
- Final Forecast

EXPLORATORY DATA ANALYSIS

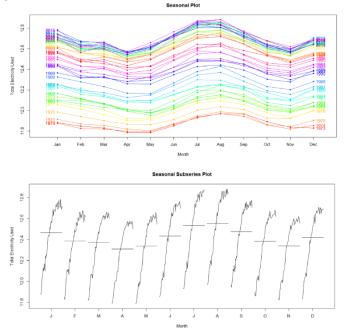
To explore the given data, I read the file and obtained the following graph by applying the plot() function.



From the above plot we can observe that there is no stationarity, instead, there is a clear increasing trend and a strong seasonal pattern. It can also be seen that the seasonality variation increases slightly as the level of the series increases, therefore it was a good idea to log transform the time series data. The below plot shows the data behaviour after the log transformation was done using log() function —



After the application of log transformation in the previous step, the time series variations seemed to be more compact now. To further explore the seasonality component, I went ahead with plotting the Seasonal and Seasonal subseries graphs

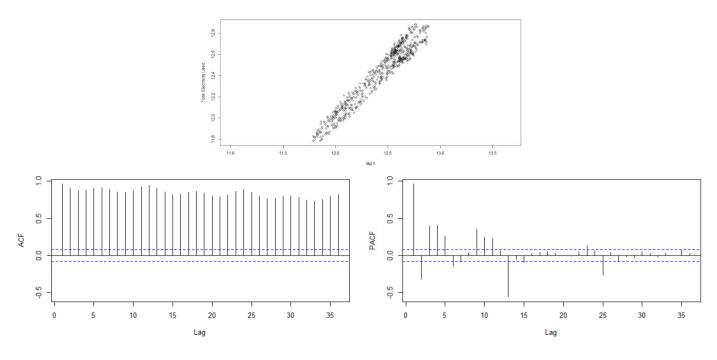


Both the graphs above show that there is Seasonality as well as a Trend in the data under consideration.

Further, the need was to check is there is any autocorrelation or partial autocorrelation if at all it exists. Let us understand what we mean by ACF and PACF first —

- ACF is an (complete) auto-correlation function which gives us values of autocorrelation of any series with
 its lagged values. In simple terms, it describes how well the present value of the series is related with its
 past values.
- PACF is a partial auto-correlation function. Thus, instead of finding correlations of present with lags like ACF, it finds correlation of the residuals with the next lag value hence 'partial' and not 'complete' as we remove already found variations before we find the next correlation.

The graphs below show that there is a strong autocorrelation with so many high values in the ACF graph and even a strong component of seasonality. From this information, one may wonder about the lack of white noise. But if we see the PACF graph we see that there is not much white noise component.

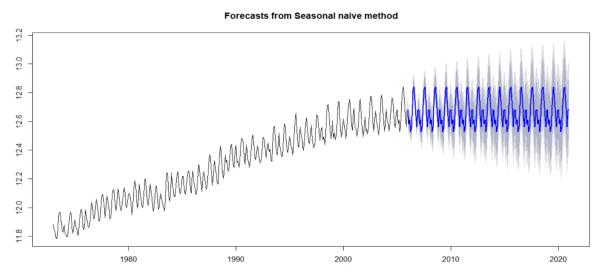


MODEL FITTING

To begin with the data was first splitted into the Train and Test sets. The Train set includes data from **January 1973 to December 2005** and the Test set includes data from **January 2006 to January 2021**.

Seasonal Naive Method

For the Forecasting purpose I first chose the Seasonal Naive Method. Used to fit the Training set and evaluate the performance on both Train & Test sets. The results were as follows -

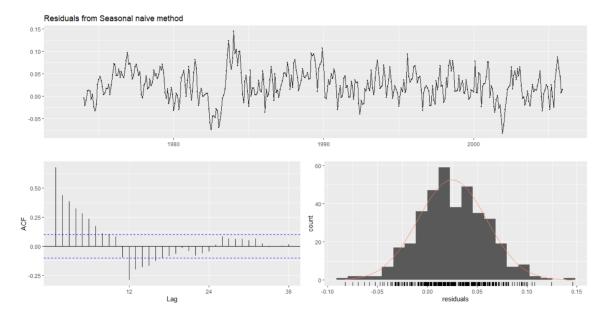


EVALUATION OF MODEL

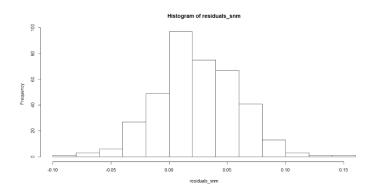
For evaluating the performance of the method used to build the models, I went ahead with checking the Residuals and applying the Ljung–Box test. Let us first understand what they signify –

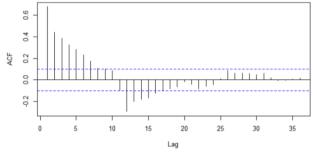
- **Residuals** in a time series model are what is left over after fitting a model. They show the difference between the observations and the corresponding fitted values. Residuals are useful in checking whether a model has adequately captured the information in the data and a good forecasting method will yield residuals with no or very less correlation.
- **Ljung–Box test** is a type of statistical test of whether any of a group of autocorrelations of a time series are different from zero. Instead of testing randomness at each distinct lag, it tests the "overall" randomness based on a number of lags and is therefore a portmanteau test.

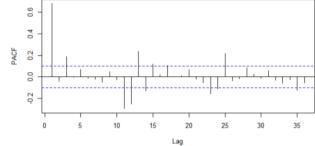
The below plots show the behaviour of the residuals and the accuracy of the model –



Went ahead with plotting the histogram of the residuals and the ACF and PACF plots to understand the autocorrelation and the partial autocorrelation if any.







Box-Ljung test

data: residuals_snm
X-squared = 526.83, df = 24, p-value < 2.2e-16</pre>

From the above Residual Plots visualizations and the Ljung Box tests performed for this model, we can see that the p-values (parameters in the tests) are less than .05 which means that they are significant. So, we can reject the null hypothesis of white noise and there is still something in the residuals that is not been captured by the model.

LjungBox(residuals_snm[-c(

In terms of accuracy, the following results we obtained. These will be used further as reference for model selection purposes.

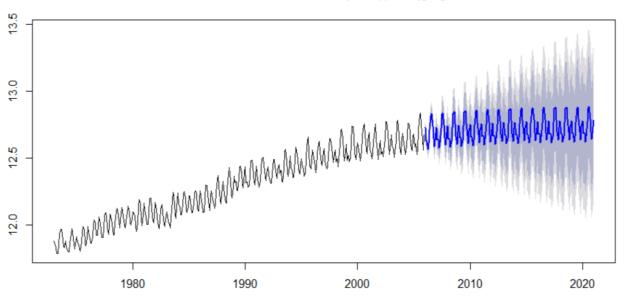
Seasonal ARIMA Method

Further, the auto ARIMA method was used to pick up the best model. The result of was **ARIMA (3, 0, 0) (0, 1, 1)[12]** with the following coefficients -

```
Coefficients:
         ar1
                   ar2
                           ar3
                                    sma1
                        0.2644
      0.8710
               -0.1525
                                 -0.7255
      0.0499
                0.0661
                        0.0494
                                  0.0353
                                   log likelihood=938.38
sigma^2 estimated as 0.0004389:
AIC=-1866.76
                AICc=-1866.61
                                 BIC=-1847.01
```

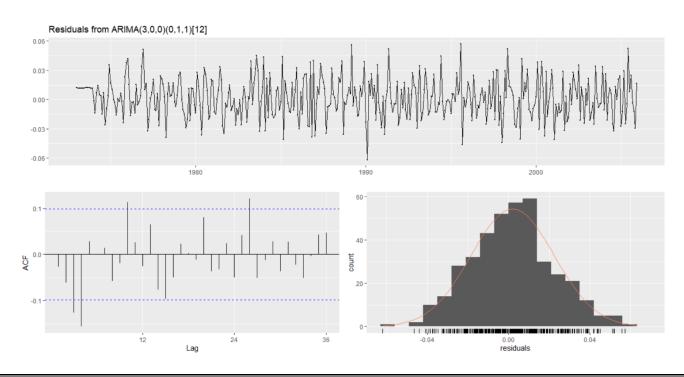
The Forecast from the auto ARIMA was as follows -

Forecasts from ARIMA(3,0,0)(0,1,1)[12]

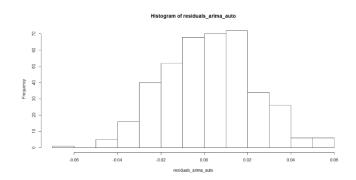


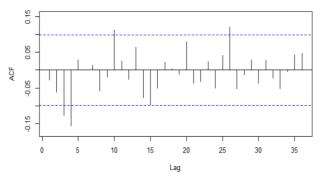
EVALUATION OF MODEL

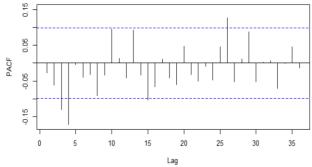
For evaluating the performance of the method used to build the models, I went ahead with checking the Residuals and applying the Ljung–Box test. The below plots show the behaviour of the residuals and the accuracy of the model –



Even went ahead with plotting the histogram of the residuals and the ACF and PACF plots to understand the autocorrelation and the partial autocorrelation if any.







Box-Ljung test

data: residuals_arima_auto

X-squared = 39.817, df = 24, p-value = 0.02237

From the above Residual Plots visualizations and the Ljung Box tests performed for this model, we can see that the p-values (parameters in the tests) are less than .05 which means that they are significant. So, we can reject the null hypothesis of white noise and there is still something in the residuals that is not been captured by the model.

```
lags
     statistic df
                       p-value
     0.3018367
                1 0.582733255
  1
     1.7942443
                2 0.407741387
  2
     8.1291335
                3 0.043416949
  4 17.9365136
                4 0.001269857
  5 18.2568784
                5 0.002641110
  6 18.2568786
                6 0.005621306
                7 0.010575094
  7 18.3278765
  8 19.6783492
                 8 0.011623932
  9 19.8389030
                9 0.018933039
 10 25.0341312 10 0.005281185
 11 25.3064677 11 0.008219783
 12 25.5787630 12 0.012306256
 13 27.2970835 13 0.011325598
 14 29.6849280 14 0.008431393
 15 33.5284353 15 0.003964094
 16 34.5856724 16 0.004526620
 17 34.7856426 17 0.006637006
 18 34.7898219 18 0.010044801
 19 34.8479865 19 0.014567194
 20 37.5008199 20 0.010184116
 21 38.0751944 21 0.012629590
 22 38.5217836 22 0.015999372
 23 38.7569241 23 0.021093758
 24 39.8171182 24 0.022373951
```

In terms of accuracy, the following results we obtained –

COMPARING BOTH THE MODELS

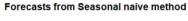
To choose which model performed better, let us compare the results of the final accuracy scores –

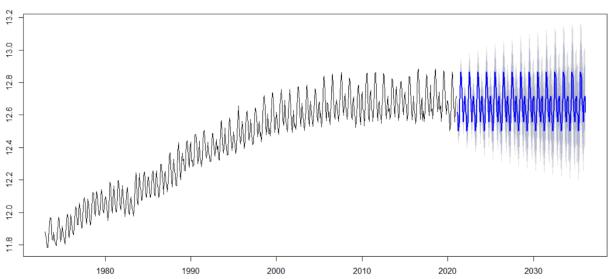
KPI	SNAIVE	ARIMA	
p-value	2.2E-16	0.02237	
RMSE	0.03602182	0.04662397	
MAE	0.02901740	0.03880271	
MPE	0.131627	-0.29312656	
MAPE	0.2287681	0.3066585	
MASE	0.8603794	1.1505183	
ACF1	0.5167383	0.64917552	

We can conclude that even though the models used did not perform really well on the residuals test, but looking at the accuracy scores across various KPIs, we can say that the **Seasonal Naïve Method** performed better than the Seasonal ARIMA (3, 0, 0) (0, 1, 1)[12].

FINAL FORECAST

To get the Final Forecast applied the evaluated Best Model (i.e. Seasonal Naïve Method) to the entire dataset and the results were as follows –





FINAL MODEL ACCURACY

With the above scores for some of the important KPI's like the MAPE, MAE and RMSE we can say that the Final Model accuracy is pretty good in terms of predictions and can be relied upon.

Thus, from the final forecast as seen from above graph, we can observe and conclude that the trend of Total Electricity usage in the US will remain comparable to the immediate previous years.

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