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1 Setup

1.1 Remap Escape

```
setxkbmap -option caps:swapescape # X11
gsettings set org.gnome.desktop.input-sources xkb-options
"['caps:swapescape']" # Wayland
```

1.2 vimrc

```
se nu rnu cin ts=4 sw=4 | sy on
inoremap {<CR> {<CR><Esc>0
nnoremap j gj
nnoremap k gk
colo evening
:bad input.txt
:let @# = 'input.txt'
```

1.3 default code

```
#include <bits/stdc++.h>
using namespace std;

#ifdef MIKU
string dbmc = "\033[1;38;2;57;197;187m", dbrs = "\033[0m";
#define debug(x...) cerr << dbmc << "[" << #x << "]" : ",
dout(x)
void dout() { cerr << dbrs << endl; }
template <typename T, typename ...U>
void dout(T t, U ...u) { cerr << t << (sizeof...(u) ? ", " :
""); dout(u...); }
```

```
#else
#define debug(...) 39
#endif

#define fs first
#define sc second
#define FOR(i, j, k) for (int i = j, Z = k; i < Z; i++)
using ll = long long;
typedef pair<int, int> pii;

void miku() {
    int a, b;
    cin >> a >> b;
    debug(a, b);
    cout << "DEC0*" << a * b << '\n';
}

int32_t main() {
    cin.tie(0) -> sync_with_stdio(false);
    cin.exceptions(cin.failbit);
    miku();
    return 0;
}
```

2 Graph

2.1 Block Cut Tree

```
// [0, n): round points, [n, tr.size()): square points
struct BlockCutTree { // 0 based
    vector<vector<int>> paths, tr;
    vector<int> idx, low, stk;
    int gid, dfn;
    int n;
    BlockCutTree(int _n) {
        n = _n;
        paths = tr = vector<vector<int>>(n);
        gid = 0;
        idx = low = vector<int>(n, -1);
        stk.clear();
    }
    void add_edge(int a, int b) {
        paths[a].push_back(b);
        paths[b].push_back(a);
    }
    void dfs(int now) {
        idx[now] = low[now] = ++dfn;
        stk.push_back(now);
        for(auto nxt:paths[now]) {
            if (idx[nxt] == -1) {
                dfs(nxt);
                low[now] = min(low[now], low[nxt]);
                if (low[nxt] == idx[now]) {
                    tr.push_back({});
                    int t = -1;
                    do {
                        t = stk.back();
                        stk.pop_back();
                        tr[gid+n].push_back(t);
                        tr[t].push_back(gid+n);
                    } while(t != nxt);
                    tr[now].push_back(gid+n);
                    tr[gid+n].push_back(now);
                    gid++;
                }
            }
            else low[now] = min(low[now], idx[nxt]);
        }
    }
    return;
}
```

```

vector<vector<int>>> solve() {
    dfn = 0;
    for(int i = 0; i < n; i++) {
        if (idx[i] == -1) dfs(i);
    }
    return tr;
}
};

```

2.2 Dinic

```

struct Dinic{//0-indexed
    struct E{
        int t,f,c;
        E(int tt = 0,int cc = 0,int ff = 0):t(tt),c(cc),f(ff){}
    };
    vector<vector<int>>> paths;
    vector<int> ptr,lv1;
    vector<E> e;
    queue<int> q;
    Dinic(int _n = 0){
        paths = vector<vector<int>>>(_n);
        ptr = lv1 = vector<int>(_n);
    }
    void add_edge(int a,int b,int c,int d = 0){
        paths[a].push_back(e.size());
        e.push_back(E(b,c));
        paths[b].push_back(e.size());
        e.push_back(E(a,d));
    }
    bool bfs(int s,int t){
        fill(lv1.begin(),lv1.end(),-1);
        q.push(s);
        lv1[s] = 0;
        while(!q.empty()){
            auto now = q.front();q.pop();
            for(auto &eid:paths[now]){
                if(e[eid].f == e[eid].c)continue;
                if(lv1[e[eid].t] == -1){
                    lv1[e[eid].t] = lv1[now]+1;
                    q.push(e[eid].t);
                }
            }
        }
        return lv1[t] != -1;
    }
    int dfs(int now,int t,int flow){
        if(now == t)return flow;
        for(int &i = ptr[now];i < paths[now].size();i++){
            int eid = paths[now][i];
            if(e[eid].f == e[eid].c || lv1[e[eid].t] != lv1[now]+1)continue;
            if(int re = dfs(e[eid].t,t,min(flow,e[eid].c-e[eid].f))){
                e[eid].f += re;
                e[eid^1].f -= re;
                return re;
            }
        }
        return 0;
    }
    int flow(int s,int t){
        int ans = 0;
        while(bfs(s,t)){
            fill(ptr.begin(),ptr.end(),0);
            while(auto re = dfs(s,t,INT_MAX)){
                ans += re;
            }
        }
        return ans;
    }
};

```

```

bool inScut(int k){
    return lv1[k] != -1;
}
};

```

2.3 Dominator Tree

```

struct DominatorTree{
    //1-indexed
    //not reachable from s -> not on tree
    int n;
    vector<vector<int>>> G,rG;
    vector<int> pa,dfn,id;
    int dfnCnt;
    vector<int> semi,idom,best;
    vector<vector<int>>> ret;
    void init(int _n){
        n=_n;
        G = rG = ret = vector<vector<int>>>(n+1);
        pa = dfn = id = vector<int>(n+1,-1);
        dfnCnt = 0;
        semi = idom = best = vector<int>(n+1,-1);
    }
    void add_edge(int u,int v){
        G[u].push_back(v);
        rG[v].push_back(u);
    }
    void dfs(int u){
        id[dfn[u]++]=dfnCnt++;
        for(auto v:G[u]) if(!dfn[v]){
            dfs(v),pa[dfn[v]]=dfn[u];
        }
    }
    int find(int y,int x){
        if(y==x)return y;
        int tmp=find(pa[y],x);
        if(semi[best[y]]>semi[best[pa[y]]])
            best[y]=best[pa[y]];
        return pa[y]=tmp;
    }
    void tarjan(int root){
        dfnCnt=0;
        for(int i=1;i<=n;i++){
            dfn[i]=idom[i]=0;
            ret[i].clear();
            best[i]=semi[i]=i;
        }
        dfs(root);
        for(int i=dfnCnt;i>1;i--){
            int u=id[i];
            for(auto v:rG[u]) if(v=dfn[v]){
                find(v,i);
                semi[i]=min(semi[i],semi[best[v]]);
            }
            ret[semi[i]].push_back(i);
            for(auto v:ret[pa[i]]){
                find(v,pa[i]);
                idom[v] = semi[best[v]]==pa[i] ? pa[i] : best[v];
            }
            ret[pa[i]].clear();
        }
        for(int i=2;i<=dfnCnt;i++){
            if(idom[i]!=semi[i]) idom[i]=idom[idom[i]];
            ret[id[idom[i]]].push_back(id[i]);
        }
    }
    vector<vector<int>>> solve(int s){
        tarjan(s);
        return ret;
    }
};

```

2.4 Euler Tour

```
struct EulerTour{
    // undirected graph, 0-indexed, fails if doesn't exist
    // for directed graph, remove the g[b].push_back(pii(a,
    id)) line in add_edge
    // returns the order of edges
    vector<vector<pii>> g;
    vector<int> ptr;
    vector<bool> vis;
    vector<int> re;
    int n, ecnt;
    void init(int _n){
        n = _n;
        ecnt = 0;
        g = vector<vector<pii>>(n);
        ptr = vector<int>(n);
    }
    void add_edge(int a, int b, int id = -1){
        if(id == -1) id = ecnt;
        g[a].push_back(pii(b, id));
        g[b].push_back(pii(a, id));
        ecnt++;
    }
    void dfs(int now){
        for(int &i = ptr[now]; i < g[now].size(); i++){
            auto [to, eid] = g[now][i];
            if(vis[eid]) continue;
            vis[eid] = true;
            dfs(to);
            re.push_back(eid);
        }
        return;
    }
    vector<int> solve(int s){
        re.clear();
        vis = vector<bool>(ecnt, 0);
        dfs(s);
        reverse(re.begin(), re.end());
        return re;
    }
};
```

2.5 Gomory Hu

```
// needs dinic
struct GomoryHuTree{//0-indexed
#define pii pair<int,int>
#define tiii tuple<int,int,int>
    vector<tiii> edges;
    vector<vector<pii>> tr;
    vector<int> p;
    int n;
    GomoryHuTree(int _n = 0){
        n = _n;
        p = vector<int>(_n, 0);
        tr = vector<vector<pii>>(_n);
    }
    void add_edge(int a, int b, int c){
        edges.push_back(tiii(a, b, c));
    }
    vector<vector<pii>> make_tree(){
        fill(p.begin(), p.end(), 0);
        tr = vector<vector<pii>>(n);
        for(int i = 1; i < p.size(); i++){
            Dinic din(n);
            for(auto &[a, b, w]: edges){
                din.add_edge(a, b, w, w);
            }
        }
    }
};
```

```
    }
    int w = din.flow(i, p[i]);
    tr[i].push_back(pii(p[i], w));
    tr[p[i]].push_back(pii(i, w));
    for(int j = i+1; j < n; j++){
        if(p[j] == p[i] && din.inScut(j)) p[j] = i;
    }
    return tr;
}
#undef pii
#undef tiii
};
```

2.6 Incremental SCC

```
struct IncrementalSCC{
#define pii pair<int,int>
#define fs first
#define sc second
#define tiii tuple<int,int,int>
    //if u == v : ans[i] = -1
    //if not connected : ans[i] = m
    //all 0-indexed
    int n;
    vector<int> ans;
    int m;
    vector<tiii> all;
    vector<int> SCC(int n, vector<vector<int>>& paths){
        vector<int> scc_id(n, -1), idx(n, -1), low(n, -1), st;
        int cnt = 0, gcnt = 0;
        function<void(int)> dfs = [&](int now) -> void{
            low[now] = idx[now] = cnt++;
            st.push_back(now);
            for(auto nxt: paths[now]){
                if(scc_id[nxt] != -1) continue;
                if(idx[nxt] == -1){
                    dfs(nxt);
                    low[now] = min(low[now], low[nxt]);
                }
                else{
                    low[now] = min(low[now], idx[nxt]);
                }
            }
            if(low[now] == idx[now]){
                int id = -1;
                while(id != now){
                    id = st.back();
                    st.pop_back();
                    scc_id[id] = gcnt;
                }
                gcnt++;
            }
        };
        for(int i = 0; i < n; i++){
            if(scc_id[i] == -1) dfs(i);
        }
        return scc_id;
    }
    vector<int> mapping;
    void dc(int l, int r, vector<tiii> &edges){
        if(l == r){
            for(auto &[id, __, __]: edges) ans[id] = min(ans[id], l);
            return;
        }
        int mid = (l+r)>>1;
        int cnt = 0;
        for(auto &[t, u, v]: edges){
            if(mapping[u] == -1) mapping[u] = cnt++;
            if(mapping[v] == -1) mapping[v] = cnt++;
        }
    }
};
```

```

n = cnt;
vector<vector<int>> paths(n);
vector<int> vv;
for(auto &[t,u,v]:edges){
    vv.push_back(u);
    vv.push_back(v);
    u = mapping[u], v = mapping[v];
    if(t<=mid)paths[u].push_back(v);
}
for(auto &i:vv)mapping[i] = -1;

auto scc_id = SCC(n,paths);
vector<tiii> vl,vr;
for(auto &[t,u,v]:edges){
    if(scc_id[u] == scc_id[v]){
        ans[t] = min(ans[t],mid);
        vl.push_back(tiii(t,u,v));
    }
    else{
        u = scc_id[u], v = scc_id[v];
        vr.push_back(tiii(t,u,v));
    }
}
vector<tiii>().swap(edges);
dc(l,mid,vl);
dc(mid+1,r,vr);
return;
}

void add_edge(int u,int v){
    all.push_back(tiii(all.size(),u,v));
}

vector<tiii> solve(){//[time,u,v]
    m = all.size();
    vector<tiii> ret(m);
    for(auto [t,u,v]:all)ret[t] = tiii(m,u,v);
    for(auto [t,u,v]:all)n = max({n,u,v});
    n++;
    ans = vector<int>(m,m);
    for(auto [t,u,v]:all){
        if(u == v)ans[t] = -1;
    }
    mapping = vector<int>(n,-1);
    dc(0,m,all);
    for(int i = 0;i<m;i++)get<0>(ret[i]) = ans[i];
    return ret;
}

IncrementalSCC(){
    ans.clear();
    n = m = 0;
}

#undef tiii
#undef pii
#undef fs
#undef sc
};

```

2.7 KM

```

// Kuhn-Munkres : Bipartite matching with "maximum" weight
in O(n^3)
// NOTICE THAT match[y] = x
struct KM{
    const static int M = 500; // modify maximum number of
vertices
    int n;
    ll ans = 0;
    // 0-base
    vector<vector<ll>> w; // input weighted edges w[x][y]
    vector<int> match; // match[y] = x
    vector<ll> lx, ly, slack;
    bitset<M> visx, visy; // initialize with all zero

```

```

// abbr
#define forx for(int x=0; x<n; x++)
#define fory for(int y=0; y<n; y++)
#define z match[y]

bool dfs(int x){
    visx[x] = 1;
    fory{
        if(visy[y]) continue;
        ll d = lx[x]+ly[y]-w[x][y];
        if(!d){
            visy[y] = 1;
            if(z==-1 || (!visx[z] && dfs(z))){
                z = x;
                return 1;
            }
            else if(d<slack[y]) slack[y] = d;
        }
    }
    return 0;
}

bool augment(){
    fory if(!visy[y] && !slack[y]){
        visy[y] = 1;
        if(z==-1) return 1;
        else if(!visx[z] && dfs(z)){
            z = -1;
            return 1;
        }
    }
    return 0;
}

void relabel(){
    ll d = INT64_MAX;
    fory if(!visy[y]) d = min(d, slack[y]);
    forx if(visx[x]) lx[x] -= d;
    fory{
        if(visy[y]) ly[y] += d;
        else slack[y] -= d;
    }
}

KM(vector<vector<ll>> &W): n(W.size()), w(W) { // input
edges' weight
    //initialize
    slack.resize(n);
    match.assign(n, -1);
    lx.assign(n, INT64_MIN);
    ly.assign(n, 0);
    forx fory lx[x] = max(lx[x], w[x][y]);
    //matching
    forx{
        visx.reset();
        visy.reset();
        visx[x] = 1;
        fory slack[y] = lx[x]+ly[y]-w[x][y];
        while(!augment()) relabel();
        visx.reset();
        visy.reset();
        dfs(x);
    }
    //summing
    forx ans += lx[x];
    fory ans += ly[y];
}

# undef forx
# undef fory
# undef z
};

```

2.8 Max Clique

```
constexpr size_t KN = 150; using bits = bitset<KN>;
#define _all(T) T.begin(), T.end()
struct MaxClique {
    bits G[KN], cs[KN];
    int ans, sol[KN], q, cur[KN], d[KN], n;
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i) G[i].reset();
    }
    void add_edge(int u, int v) { G[u][v] = G[v][u] = 1; }
    void pre_dfs(vector<int> &v, int i, bits mask) {
        if (i < 4) {
            for (int x : v) d[x] = (int)(G[x] & mask).count();
            sort(_all(v), [&](int x, int y) {
                return d[x] > d[y]; });
        }
        vector<int> c(v.size());
        cs[1].reset(), cs[2].reset();
        int l = max(ans - q + 1, 1), r = 2, tp = 0, k;
        for (int p : v) {
            for (k = 1; (cs[k] & G[p]).any(); ++k);
            if (k >= r) cs[++r].reset();
            cs[k][p] = 1;
            if (k < l) v[tp++] = p;
        }
        for (k = l; k < r; ++k)
            for (auto p = cs[k]._Find_first();
                p < KN; p = cs[k]._Find_next(p))
                v[tp] = (int)p, c[tp] = k, ++tp;
        dfs(v, c, i + 1, mask);
    }
    void dfs(vector<int> &v, vector<int> &c,
            int i, bits mask) {
        while (!v.empty()) {
            int p = v.back(); v.pop_back(); mask[p] = 0;
            if (q + c.back() <= ans) return;
            cur[q++] = p;
            vector<int> nr;
            for (int x : v) if (G[p][x]) nr.push_back(x);
            if (!nr.empty()) pre_dfs(nr, i, mask & G[p]);
            else if (q > ans) ans = q, copy_n(cur, q, sol);
            c.pop_back(); --q;
        }
    }
    int solve() {
        vector<int> v(n); iota(_all(v), 0);
        ans = q = 0; pre_dfs(v, 0, bits(string(n, '1')));
        return ans; // sol[0 ~ ans-1]
    }
};
```

2.9 Minimum Cost Maximum Flow

```
#define T ll
const T inf = 1e12;
struct MCMF{//TC:O(VEF)
    struct E{
        int t,f;
        T c,w;
        E(int tt,T cap,T wei):t(tt),c(cap),w(wei),f(0){}
    };
    vector<E> e;
    vector<vector<int>> paths;
    vector<T> dis;
    vector<int> pre;
    vector<bool> inq;
```

```
queue<int> q;
int n;
MCMF(int _n = 0){
    n = _n;
    paths = vector<vector<int>>(n);
    e.clear();
    pre = vector<int>(n);
    dis = vector<T>(n);
    inq = vector<bool>(n);
}
void add_edge(int a,int b,int c,int d){//from,to,cap,wei
    paths[a].push_back(e.size());
    e.push_back(E(b,c,d));
    paths[b].push_back(e.size());
    e.push_back(E(a,0,-d));
}
bool SPFA(int s,int t){
    fill(dis.begin(),dis.end(),inf);
    fill(pre.begin(),pre.end(),-1);
    dis[s] = 0;
    q.push(s);inq[s] = true;
    while(!q.empty()){
        auto now = q.front();q.pop();
        inq[now] = false;
        //assert(dis[now]>=0);
        for(auto &eid:paths[now]){
            if(e[eid].f == e[eid].c)continue;
            int nxt = e[eid].t;
            if(dis[nxt]>dis[now]+e[eid].w){
                pre[nxt] = eid;
                dis[nxt] = dis[now]+e[eid].w;
                if(!inq[nxt]){
                    inq[nxt] = true;
                    q.push(nxt);
                }
            }
        }
    }
    return dis[t] != inf;
}
T flow(int s,int t,int cnt = INT_MAX){//cnt is the number
of flows
    T ans = 0;
    while(cnt--&&SPFA(s,t)){
        ans += dis[t];
        int now = t;
        while(pre[now] != -1){
            int eid = pre[now];
            e[eid].f++;
            e[eid^1].f--;
            now = e[eid^1].t;
        }
    }
    return ans;
}
};
#undef T
```

3 Data Structure

3.1 Dynamic Convex Hull

```
#define ll long long
// only works for integer coordinates!! maintain max

struct Line {
    mutable ll a, b, p;
    bool operator<(const Line &rhs) const { return a <
rhs.a; }
    bool operator<(ll x) const { return p < x; }
```



```

};
struct CHT : multiset<Line, less<>> {
    static const ll kInf = 1e18;
    ll Div(ll a, ll b) { return a / b - ((a ^ b) < 0 && a %
b); }
    bool isect(iterator x, iterator y) {
        if (y == end()) { x->p = kInf; return 0; }
        if (x->a == y->a) x->p = x->b > y->b ? kInf : -kInf;
        else x->p = Div(y->b - x->b, x->a - y->a);
        return x->p >= y->p;
    }
    void addline(ll a, ll b) {
        auto z = insert({a, b, 0}), y = z++, x = y;
        while (isect(y, z)) z = erase(z);
        if (x != begin() && isect(--x, y)) isect(x, y =
erase(y));
        while ((y = x) != begin() && (--x)->p >= y->p) isect(x,
erase(y));
    }
    ll query(ll x) {
        auto l = *lower_bound(x);
        return l.a * x + l.b;
    }
};

```

3.2 Link Cut Tree

```

#define ll long long
//needs splay
//vertex add paths sum link-cut
struct LCT{
    Splay sp;
    void access(int x){
        sp.splay(x);
        sp.ch[x][1] = 0;
        sp.pull(x);
        while(sp.fa[x]){
            int u = sp.fa[x];
            sp.splay(u);
            sp.push(u);
            sp.ch[u][1] = x;
            sp.pull(u);
            sp.splay(x);
        }
    }
    void makeroot(int x){
        access(x); sp.splay(x);
        sp.rev[x] ^= 1;
    }
    void link(int u, int v){
        makeroot(u);
        sp.splay(u);
        sp.fa[u] = v;
    }
    void cut(int u, int v){
        makeroot(u);
        access(v);
        sp.splay(v);
        int lc = sp.ch[v][0];
        sp.fa[lc] = 0;
        sp.ch[v][0] = 0;
        sp.pull(v);
    }
    ll path_sum(int u, int v){
        makeroot(u);
        access(v);
        sp.splay(v);
        return sp.sum[v];
    }
    void addval(int u, int val){
        sp.splay(u);

```

```

        sp.val[u] += val;
        sp.pull(u);
        return;
    }
    int find(int p){
        access(p);
        return sp.get_sz(p, 1);
    }
};

```

3.3 Li Chao

```

// range add line get min
// can even be used if modifies aren't range modify
#define ll long long
const ll SZ = 8e6+10;
const ll inf = 3e18;
vector<ll> all; // coordinates are stored here
struct Line{
    ll m, b;
    Line(ll mm = 0, ll bb = 0): m(mm), b(bb){}
    ll operator()(ll k){
        return m*k+b;
    }
};
struct LiChao{
#define ls now*2+1
#define rs now*2+2
#define mid ((l+r)>>1)
    Line seg[SZ];
    LiChao(){
        fill(seg, seg+SZ, Line(0, inf));
    }
    void modify(int now, int l, int r, int s, int e, Line v){
        if(l == r){
            if(seg[now](all[l]) > v(all[l])) swap(seg[now], v);
            return;
        }
        if(l >= s && e >= r){
            if(seg[now](all[mid]) > v(all[mid])) swap(seg[now], v);
            if(seg[now].m < v.m) modify(ls, l, mid, s, e, v);
            else modify(rs, mid+1, r, s, e, v);
        }
        else{
            if(mid >= s) modify(ls, l, mid, s, e, v);
            if(mid < e) modify(rs, mid+1, r, s, e, v);
        }
        return;
    }
    ll getval(int now, int l, int r, int p){
        if(l == r) return seg[now](all[p]);
        if(mid >= p) return min(seg[now],
(all[p]), getval(ls, l, mid, p));
        else return min(seg[now](all[p]), getval(rs, mid+1, r, p));
    }
    void add_line(int s, int e, Line v){
        modify(0, 0, all.size()-1, s, e, v);
        return;
    }
    ll getmin(int p){
        return getval(0, 0, all.size()-1, p);
    }
};
#undef ls
#undef rs
#undef mid
};
#undef ll long long

```

3.4 Quadrangle

```

struct QUADRANGLE {
    struct TUPLE {
        int l, r, id;
        TUPLE() {}
        TUPLE(int _l, int _r, int _id) : l(_l), r(_r),
id(_id) {}
    };
    int n, now;
    deque<TUPLE> dq;

    int calc_dp(int id, int i) {
        // ...
    }
    bool cmp(int cid, int pid, int i) {
        // ...
    }
    void init(int _n) {
        n = _n;
        now = 1;
        dq.clear();
    }
    void kill_head() {
        now++;
        if (dq.front().l == dq.front().r) dq.pop_front();
        else dq.front().l++;
    }
    void push(int id) {
        while (dq.size()) {
            TUPLE tl = dq.back();
            dq.pop_back();
            if (cmp(id, tl.id, tl.l)) {
                continue;
            }
            int l = tl.l, r = tl.r + 1;
            while (l + 1 < r) {
                int mid = (l + r) >> 1;
                (cmp(id, tl.id, mid) ? r : l) = mid;
            }
            dq.push_back(TUPLE(tl.l, l, tl.id));
            if (r <= n) dq.push_back(TUPLE(r, n, id));
            return;
        }
        dq.push_back(TUPLE(now, n, id));
    }
    int determine(int id) {
        return calc_dp(dq.front().id, id);
    }
};

```

3.5 Splay

```

#include <bits/stdc++.h>
using namespace std;

#define ll long long
const int SZ = 2e5+10;
//1-indexed,0 used for nullptr
//range reverse range sum
struct Splay{
#define ls ch[now][0]
#define rs ch[now][1]
    ll val[SZ];
    ll sum[SZ];
    int ch[SZ][2], fa[SZ], cnt, rev[SZ], sz[SZ];
    void pull(int now){
        if(!now) return;
        sum[now] = sum[ls]+sum[rs]+val[now];
        sz[now] = sz[ls]+sz[rs]+1;
        return;
    }
    void push(int now){

```

```

        if(!now) return;
        if(rev[now]){
            swap(ls,rs);
            rev[ls] ^= 1;
            rev[rs] ^= 1;
            rev[now] = 0;
        }
        pull(now);
        return;
    }
    Splay(){
        fill(sz+1,sz+SZ,1);
        return;
    }
    int newnode(){
        return ++cnt;
    }
    int dir(int now){//is ls or rs
        return ch[fa[now]][1] == now;
    }
    bool isroot(int k){//for LCT
        return !fa[k]||ch[fa[k]][dir(k)] != k;
    }
    void rot(int now){
        assert(now);
        assert(fa[now]);
        int p = fa[now];
        int g = fa[p];
        push(g);
        push(p);
        push(now);
        int d = dir(now);
        if(!isroot(p)) ch[g][dir(p)] = now;
        fa[ch[now][d^1]] = p;
        ch[p][d] = ch[now][d^1];
        fa[now] = g;
        fa[p] = now;
        ch[now][d^1] = p;
        pull(p);
        pull(now);
        return;
    }
    void splay(int now){
        if(!now) return;
        while(!isroot(now)){
            push(fa[fa[now]]);
            push(fa[now]);
            push(now);
            if(!isroot(fa[now])){
                if(dir(fa[now]) == dir(now)) rot(fa[now]);
                else rot(now);
            }
            rot(now);
        }
        push(now);
        return;
    }
    int get_sz(int now,int tar){
        push(now);
        while(now&&sz[ls]+1 != tar){
            if(sz[ls]>=tar) now = ls;
            else{
                tar -= sz[ls]+1;
                now = rs;
            }
            push(now);
        }
        return now;
    }
    void merge(int a,int b){
        if(!a||!b) return;
        splay(a);splay(b);
        a = get_sz(a,sz[a]);

```

```

    b = get_sz(b,1);
    splay(a);splay(b);
    ch[a][1] = b;
    fa[b] = a;
    pull(a);
    return;
}
pair<int,int> split(int a,int tar){
    splay(a);
    if(!tar)return make_pair(0,a);
    int b = get_sz(a,tar);
    splay(b);
    pair<int,int> re;
    re.first = b;
    re.second = ch[b][1];
    fa[ch[b][1]] = 0;
    ch[b][1] = 0;
    pull(b);
    return re;
}
#undef ls
#undef rs
};

```

3.6 Treap

```

#define ll long long
// range reverse range add range sum
// need to push before using the info on node
struct node{
    int pri;
    int pl,pr;
    ll sum,tag,val;
    int sz;
    int rev;
    node(){
        pl = pr = sum = tag = 0;
        sz = 0;
        rev = 0;
        pri = rand();
    }
};

const int SZ = 2e5+10;
struct Treap{
    node nd[SZ];
    int cnt = 0;
    Treap(){
        cnt = 0;
    }
    int newnode(){
        cnt++;
        nd[cnt].sz = 1;
        return cnt;
    }
    void pull(int now){
        if(!now)return;
        nd[now].sz = nd[nd[now].pr].sz+nd[nd[now].pl].sz+1;
        ll ls =
nd[nd[now].pl].sum+nd[nd[now].pl].tag*nd[nd[now].pl].sz;
        ll rs =
nd[nd[now].pr].sum+nd[nd[now].pr].tag*nd[nd[now].pr].sz;
        nd[now].sum = nd[now].val+ls+rs;
        return;
    }
    void push(int now){
        if(!now)return;
        if(nd[now].rev){
            swap(nd[now].pl,nd[now].pr);
            if(nd[now].pl)nd[nd[now].pl].rev ^= 1;
            if(nd[now].pr)nd[nd[now].pr].rev ^= 1;
        }
    }
};

```

```

        nd[now].rev = 0;
    }
    int tl = nd[now].pl,tr = nd[now].pr;
    nd[now].val += nd[now].tag;
    if(tl)nd[tl].tag += nd[now].tag;
    if(tr)nd[tr].tag += nd[now].tag;
    nd[now].tag = 0;
    pull(now);
}
int merge(int a,int b){
    if(!a)return b;
    if(!b)return a;
    if(nd[a].pri>nd[b].pri){
        push(a);
        nd[a].pr = merge(nd[a].pr,b);
        pull(a);
        return a;
    }
    else{
        push(b);
        nd[b].pl = merge(a,nd[b].pl);
        pull(b);
        return b;
    }
}
void split(int now,int &a,int &b,int tar){
    if(!now){
        a = b = 0;
        return;
    }
    push(now);
    if(nd[nd[now].pl].sz+1<=tar){
        a = now;
        split(nd[now].pr,nd[a].pr,b,tar-
(nd[nd[now].pl].sz+1));
    }
    else{
        b = now;
        split(nd[now].pl,a,nd[b].pl,tar);
    }
    pull(a);
    pull(b);
    return;
}
};
Treap T;

```

4 Geometry

4.1 Point

```

template<typename T = int>
struct Pt{
    T x,y;
    Pt (T xx = (T)(0),T yy = (T)(0)):x(xx),y(yy){}
    Pt operator+(Pt b)const{return Pt(x+b.x,y+b.y);}
    Pt operator-(Pt b)const{return Pt(x-b.x,y-b.y);}
    T operator*(Pt b)const{return x*b.x+y*b.y;}
    T operator^(Pt b)const{return x*b.y-y*b.x;}
    T operator/(Pt b)const{return x*b.y-y*b.x;}
    bool operator<=>(const Pt& b) = default; // since C++20

    friend int dir(Pt a,Pt b){//returns sign(a ^ b)
        auto re = a ^ b;
        return re<0?-1:re>0?1:0;
    }
    friend bool onseg(Pt x,Pt s,Pt e){
        if(((e-x)^(s-x)) != 0)return false;
        else if((s-x)*(e-x)>0)return false;
        return true;
    }
};

```



```

    }
    friend int intersect(Pt s1,Pt e1,Pt s2,Pt e2){//returns 0
    if doesn't intersect,1 if intersect,2 if on line
        if(onseg(s1,s2,e2)||onseg(e1,s2,e2)||onseg(s2,s1,e1)||
onseg(e2,s1,e1))return 2;
        if(dir(s1-s2,e2-s2)*dir(e1-s2,e2-s2)<0&&dir(s2-s1,e1-
s1)*dir(e2-s1,e1-s1)<0)return 1;
        return 0;
    }
};

```

4.2 Convex Hull

```

// needs Point.cpp
template<typename T = int>
struct ConvexHull{
    //returns in clockwise direction
    // returns strictly on convex hull
    vector<Pt<T>> solve(vector<Pt<T>> v){
        sort(v.begin(),v.end());
        vector<Pt<T>> u,d;
        for(auto &i:v){
            if (!u.empty() && u.back() == i) continue;
            while(u.size()>1&&((i-u.end()[-1])^(u.end()[-2]-
u.end()[-1]))>=0)u.pop_back();
            while(d.size()>1&&((i-d.end()[-1])^(d.end()[-2]-
d.end()[-1]))<=0)d.pop_back();
            u.push_back(i);
            d.push_back(i);
        }
        for(int i = 1;i<d.size();i++)u.push_back(d.end()[-1-
i]);
        return u;
    }
};

```

4.3 Tangent of Convex Hull

```

int ori(Pt a, Pt b, Pt c) {
    ll tmp = (b-a) ^ (c-a);
    return tmp>0?1:tmp<0?-1:0;
}

int cyc_tsearch(int n, auto pred) {
    if (n == 1) return 0;
    int l = 0, r = n; bool rv = pred(1, 0);
    while(r-l > 1) {
        int m = (l+r) / 2;
        if (pred(0, m) ? rv : pred(m, (m+1) % n)) r = m;
        else l = m;
    }
    return pred(l, r % n) ? l : r % n;
}

pii get_tangent(const vector<Pt> &cvx, Pt p) {
    auto gao = [&](int s) {
        return cyc_tsearch(cvx.size(), [&](int x, int y)
        { return ori(p, cvx[x], cvx[y]) == s; });
    };
    return pii(gao(1), gao(-1));
}

```

4.4 Minkowski Sum

```

// needs Point template
template <typename T>
vector<Pt<T>> minkowski(vector<Pt<T>> va,vector<Pt<T>> vb){

```

```

deque<Pt<T>> a,b;
for(auto &i:va)a.push_back(i);
for(auto &i:vb)b.push_back(i);
Pt head = *min_element(a.begin(),a.end());
while(a[0].x != head.x||a[0].y != head.y){
    a.push_back(a[0]);
    a.pop_front();
}
head = *min_element(b.begin(),b.end());
while(b[0].x != head.x||b[0].y != head.y){
    b.push_back(b[0]);
    b.pop_front();
}
a.push_back(a[0]);
b.push_back(b[0]);
int p1 = 0,p2 = 0;
vector<Pt<T>> re;
while(p1 < a.size()&&p2 < b.size()){
    //cerr<<a.size()<<','<<b.size()<<":"<<p1<< ' '<<p2<<endl;
    int dir = 0;
    re.push_back(a[p1]+b[p2]);
    if(p1+1 == a.size())dir = 1;
    else if(p2+1 == b.size())dir = 0;
    else if(((a[p1+1]-a[p1])^(b[p2+1]-b[p2]))>0)dir = 0;
    else dir = 1;
    if(dir == 0)p1++;
    else p2++;
}
return re;
}

```

4.5 Half Plane Intersection

```

// please don't use with other geometry templates
#define iter(v) v.begin(), v.end()
#define SZ(v) int(v.size())
#define pb emplace_back
#define ff first
#define ss second

using ll = long long;
using pii = pair<int, int>;
using pll = pair<ll, ll>;

template<class A, class B>
ostream &operator<<(ostream &o, pair<A, B> p) {
    return o << '(' << p.ff << ',' << p.ss << ')';
}

#define temp template<class T>
#define ptt pair<T, T>
#define X ff
#define Y ss
using ld = long double;
using pdd = pair<ld, ld>;

temp ptt operator+(ptt a, ptt b) {
    return {a.X + b.X, a.Y + b.Y};
}
temp ptt operator-(ptt a, ptt b) {
    return {a.X - b.X, a.Y - b.Y};
}
temp ptt operator*(ptt v, T i) {
    return {v.X * i, v.Y * i};
}
temp ptt operator*(T i, ptt v) {
    return {v.X * i, v.Y * i};
}
temp ptt operator/(ptt v, T i) {
    return {v.X / i, v.Y / i};
}

```

```

temp T dot(ptt a, ptt b) {
    return a.X * b.X + a.Y * b.Y;
}
temp T cross(ptt a, ptt b) {
    return a.X * b.Y - a.Y * b.X;
}
temp T abs2(ptt a) {
    return dot(a, a);
}
temp ld abs(ptt a) {
    return sqrt(abs2(a));
}
temp int sgn(T v) {
    return v > 0 ? 1 : (v < 0 ? -1 : 0);
}
temp int ori(ptt a, ptt b, ptt c) {
    return sgn(cross(b - a, c - a));
}
// intersects Line(p1, p2), Line(p3, p4)
pdd intersect(pdd p1, pdd p2, pdd p3, pdd p4) {
    ld a123 = cross(p2 - p1, p3 - p1);
    ld a124 = cross(p2 - p1, p4 - p1);
    return (p4 * a123 - p3 * a124) / (a123 - a124);
}

int cmp(pll a, pll b, bool same = true) {
#define is_neg(k) (sgn(k.Y) < 0 || (sgn(k.Y) == 0 && sgn(k.X) < 0))
    int A = is_neg(a), B = is_neg(b);
    if (A != B) return A < B;
    if (sgn(cross(a, b)) == 0) return same ? abs2(a) < abs2(b) : -1;
    return sgn(cross(a, b)) > 0;
}

using Line = pair<pll, pll>;
// cross(p - line.X, line.Y-line.X) <= 0 <=> p in half plane
// LHS when going from line.X to line.Y
pll area_pair(Line a, Line b) {
    return pll(cross(a.Y - a.X, b.X - a.X), cross(a.Y - a.X, b.Y - a.X));
}
bool isin(Line l0, Line l1, Line l2) {
    auto [a02X, a02Y] = area_pair(l0, l2);
    auto [a12X, a12Y] = area_pair(l1, l2);
    if (a12X - a12Y < 0) a12X *= -1, a12Y *= -1;
    return ((__int128) a02Y * a12X - (__int128) a02X * a12Y > 0;
}
vector<Line> HalfPlaneInter(vector<Line> arr) {
    sort(iter(arr), [&](Line a, Line b) -> int {
        if (cmp(a.Y - a.X, b.Y - b.X, 0) != -1)
            return cmp(a.Y - a.X, b.Y - b.X, 0);
        return ori(a.X, a.Y, b.Y) < 0;
    });
    deque<Line> dq(1, arr[0]);
    for (auto p : arr) {
        if (cmp(dq.back().Y - dq.back().X, p.Y - p.X, 0) == -1)
            continue;
        while (SZ(dq) >= 2 && !isin(p, dq[SZ(dq) - 2], dq.back()))
            dq.pop_back();
        while (SZ(dq) >= 2 && !isin(p, dq[0], dq[1]))
            dq.pop_front();
        dq.pb(p);
    }
    while (SZ(dq) >= 3 && !isin(dq[0], dq[SZ(dq) - 2], dq.back()))
        dq.pop_back();
    while (SZ(dq) >= 3 && !isin(dq.back(), dq[0], dq[1]))
        dq.pop_front();
    return vector<Line>(iter(dq));
}

```

}

4.6 Min Circle Cover

```

typedef pair<int, int> pii;
#define ld double
#define pdd Pt<ld>

ld len(pdd k) {
    return sqrt(k*k);
}
pdd excenter(pdd p0, pdd p1, pdd p2) {
    p1 = p1 - p0;
    p2 = p2 - p0;
    ld x1 = p1.x, y1 = p1.y, x2 = p2.x, y2 = p2.y;
    ld m = 2.0 * (x1*y2 - y1*x2);
    pdd center;
    center.x = (x1*x1*y2 - x2*x2*y1 + y1*y2*(y1-y2)) / m;
    center.y = (x1*x2*(x2-x1) - y1*y1*x2 + x1*y2*y2) / m;
    return center + p0;
}

pdd Minimum_Enclosing_Circle(vector<pdd> dots, ld &r) {
    mt19937 seed(time(0));
    shuffle(dots.begin(), dots.end(), seed);
    pdd cent;
    cent = dots[0], r = 0;
    for(int i = 1; i < dots.size(); i++) {
        if (len(dots[i] - cent) > r) {
            cent = dots[i], r = 0;
            for(int j = 0; j < i; j++) {
                if (len(dots[j] - cent) > r) {
                    cent = (dots[i] + dots[j]);
                    cent.x /= 2, cent.y /= 2;
                    r = len(dots[i] - cent);
                    for(int k = 0; k < j; k++) {
                        if (len(dots[k] - cent) > r) {
                            cent = excenter(dots[i], dots[j], dots[k]);
                            r = len(dots[k] - cent);
                        }
                    }
                }
            }
        }
    }
    return cent;
}

```

5 String

5.1 Z Algorithm

```

template <typename T>
struct Z_alg {
    void operator()(T a, int n, int *z) {
        z[0] = 0;
        int l = 0;
        for (int i = 1; i <= n; i++) {
            for (z[i] = max(0, min(z[i - l], l + z[l] - i));
                i + z[i] < n && a[i + z[i]] == a[z[i]]; z[i]++);
            if (i + z[i] > l + z[l]) l = i;
        }
    }
};

```

5.2 KMP

```
template <typename T>
struct KMP {
    void operator()(T a, int n, int *pi) {
        pi[0] = -1, pi[1] = 0;
        for (int i = 1; i < n; i++) {
            int j = pi[i];
            while (j >= 0 && a[i] != a[j]) j = pi[j];
            pi[i + 1] = j + 1;
        }
    }
};
```

5.3 Aho Corasick

```
// only construct the automaton
struct AC {
    int nc;
    char c[MXN];
    int pi[MXN], p[MXN], nxt[MXN][MXC];
    void init() {
        nc = 2;
        fill(nxt[0], nxt[0] + MXC, 1);
        fill(nxt[1], nxt[1] + MXC, -1);
    }
    int add_node(int par, char _c) {
        c[nc] = _c;
        p[nc] = par;
        fill(nxt[nc], nxt[nc] + MXC, -1);
        return nc++;
    }
    int push(string &s) {
        int now = 1;
        for (auto &i : s) {
            if (nxt[now][i - 'a'] == -1) nxt[now][i - 'a'] =
add_node(now, i);
            now = nxt[now][i - 'a'];
        }
        return now;
    }
    void build() {
        queue<int> q;
        pi[1] = 0;
        FOR(i, 0, MXC) {
            if (nxt[1][i] == -1) nxt[1][i] = nxt[pi[1]][i];
            else q.push(nxt[1][i]);
        }
        while (q.size()) {
            int id = q.front();
            q.pop();
            pi[id] = nxt[pi[p[id]]][c[id] - 'a'];
            FOR(i, 0, MXC) {
                if (nxt[id][i] == -1) nxt[id][i] =
nxt[pi[id]][i];
                else q.push(nxt[id][i]);
            }
        }
    }
};
```

5.4 Manacher

```
template <typename T>
struct MANACHER {
    void operator()(T a, int n, int *mn) {
        int l = 0;
        mn[0] = 0;
        for (int i = 1; i < n; i++) {
```

```
            mn[i] = (l + mn[l] >= i ? min(mn[2 * l - i], l +
mn[l] - i) : 0);
            while (i - mn[i] - 1 >= 0 && i + mn[i] + 1 < n
&& a[i - mn[i] - 1] == a[i + mn[i] + 1]) mn[i]++;
            if (i + mn[i] > l + mn[l]) l = i;
        }
    }
};
```

5.5 Suffix Array

```
int SA[MXN * 2], H[MXN], RA[MXN];
namespace SAIS {
    bool _t[MXN * 2];
    int _s[MXN * 2], _c[MXN * 2], x[MXN], _p[MXN], _q[MXN *
2];
    void pre(int *sa, int *c, int n, int z) {
        fill_n(sa, n, 0);
        copy_n(c, z, x);
    }
    void induce(int *sa, int *c, int *s, bool *t, int n, int
z) {
        copy_n(c, z - 1, x + 1);
        FOR(i, 0, n) {
            if (sa[i] && !t[sa[i] - 1]) {
                sa[x[s[sa[i] - 1]]++] = sa[i] - 1;
            }
        }
        copy_n(c, z, x);
        for (int i = n - 1; i >= 0; i--) {
            if (sa[i] && t[sa[i] - 1]) {
                sa[--x[s[sa[i] - 1]]] = sa[i] - 1;
            }
        }
    }
    void sais(int *s, int *sa, int *p, int *q, bool *t, int
*c, int n, int z) {
        bool uniq = t[n - 1] = true;
        int nn = 0, nmxz = -1, *nsa = sa + n, *ns = s + n,
last = -1;
        fill_n(c, z, 0);
        FOR(i, 0, n) uniq &= ++c[s[i]] < 2;
        partial_sum(c, c + z, c);
        if (uniq) {
            FOR(i, 0, n) sa[--c[s[i]]] = i;
            return;
        }
        for (int i = n - 2; i >= 0; i--) {
            t[i] = (s[i] == s[i + 1] ? t[i + 1] : s[i] < s[i
+ 1]);
        }
        pre(sa, c, n, z);
        FOR(i, 1, n) {
            if (t[i] && !t[i - 1]) {
                sa[--x[s[i]]] = p[q[i] = nn++] = i;
            }
        }
        induce(sa, c, s, t, n, z);
        FOR(i, 0, n) {
            if (sa[i] && t[sa[i]] && !t[sa[i] - 1]) {
                bool neq = last < 0 || !equal(s + sa[i], s +
p[q[sa[i]] + 1], s + last);
                ns[q[last = sa[i]]] = nmxz += neq;
            }
        }
        sais(ns, nsa, p + nn, q + n, t + n, c + z, nn, nmxz
+ 1);
        pre(sa, c, n, z);
        for (int i = nn - 1; i >= 0; i--) {
            sa[--x[s[p[nsa[i]]]]] = p[nsa[i]];
        }
    }
};
```

```

    induce(sa, c, s, t, n, z);
}
void mkhei(int n) {
    for (int i = 0, j = 0; i < n; i++) {
        if (RA[i]) {
            for (; i + j < n && SA[RA[i] - 1] + j < n &&
                _s[i + j] == _s[SA[RA[i] - 1] + j]; ++j);
            H[RA[i]] = j, j = max(0, j - 1);
        }
    }
}
void build(int *s, int n, int mxc) {
    copy_n(s, n, _s), _s[n] = 0;
    sais(_s, SA, _p, _q, _t, _c, n + 1, mxc);
    copy_n(SA + 1, n, SA);
    FOR(i, 0, n) RA[SA[i]] = i;
    mkhei(n);
    copy(H + 1, H + n, H);
}
}

```

6 Math

6.1 Chinese Remainder Theorem

```

using lll = __int128_t;

struct ICRT {
    lll p1, p2, p3;
    lll c1, c2, c3;
    ICRT() {}
    ICRT(lll _p1, lll _p2, lll _p3) : p1(_p1), p2(_p2),
    p3(_p3) {
        auto POW = [&](lll a, lll b, lll mod) -> lll {
            lll ans = 1;
            while (b) {
                if (b & 1) ans = ans * a % mod;
                b >>= 1;
                a = a * a % mod;
            }
            return ans;
        };
        c1 = POW(p2 * p3 % p1, p1 - 2, p1) * p2 * p3;
        c2 = POW(p3 * p1 % p2, p2 - 2, p2) * p3 * p1;
        c3 = POW(p1 * p2 % p3, p3 - 2, p3) * p1 * p2;
    };
    lll operator()(int r1, int r2, int r3) {
        return (c1 * r1 + c2 * r2 + c3 * r3) % (p1 * p2 *
    p3);
    };
};

ICRT icrt(998244353, 104857601, 167772161);

```

6.2 Euclid

```

struct euclid{
    ll x, y, g;
    void ec(ll a, ll b){
        // minimum integer solution of "ax+by=g, x>0"
        if(!b) return void((x=1, y=0, g=a));
        ec(b, a%b);
        swap(x, y);
        y -= a/b*x+a/g;
        x += b/g;
    }
    inline euclid(ll a, ll b){
        ec(abs(a), abs(b));
    }
};

```

```

    if(b<0) y = -y;
    if(a<0) x = -x;
}
};

```

6.3 FFT

```

using cd = complex<double>;
struct PolyF : public vector<cd> {
    static constexpr double PI = 3.14159265358979323;
    PolyF() : vector<cd>() {}
    PolyF(size_t sz) : vector<cd>(sz) {}
    void conv(size_t N, bool inv = 0) {
        assert(size() && N >= size());
        int LG = __lg(N);
        assert(N == (1 << LG));
        resize(N);
        vector<int> r(N);
        FOR(i, 1, N) {
            int i_ = i ^ (1 << __lg(i));
            r[i] = r[i_] << (__lg(i) - __lg(i_)) | 1;
            int j = r[i] << (LG - 1 - __lg(i));
            if (i < j) {
                std::swap(at(i), at(j));
            }
        }
        for (int w = 1; w < N; w <= 1) {
            FOR(ok, 0, w) {
                double th = PI * ok / w * (inv ? -1 : 1);
                cd o(cos(th), sin(th));
                for (int s = 0; s < N; s += (w <= 1)) {
                    cd &L = at(s + ok), &R = at(s + ok + w);
                    cd l = L, r = o * R;
                    L = l + r;
                    R = l - r;
                }
            }
        }
        if (inv) {
            FOR(i, 0, N) {
                at(i) /= N;
            }
        }
    };
};

```

6.4 FWT

```

//      AND      OR      XOR
// | 1 1 | | 1 0 | | 1 1 |
// | 0 1 | | 1 1 | | 1 -1 |

struct FWT {
    // mod operations ADD, SUB, MUL, POW (if needed)
    void btf(int &L, int &R, bool inv) { // sample: XOR
        int l = L, r = R;
        L = ADD(l, r);
        R = SUB(l, r);
    }
    void operator()(int *a, int n, bool inv) {
        // sample: XOR
        for (int w = 1; w < n; w <= 1) {
            FOR(i, 0, n) if (i & w) {
                btf(a[i - w], a[i], inv);
            }
        }
        if (inv) {
            int x = POW(n, mod - 2);
            FOR(i, 0, n) a[i] = MUL(a[i], x);
        }
    }
};

```

```

    }
};

```

6.5 NTT

```

#define FOR(i, j, k) for (int i = j, Z = k; i < Z; i++)

struct NTT {
    const static int LG = 20;
    int mod;
    int o[(1 << LG) + 1];
    int ADD(int a, int b) {
        // help yourself
    }
    int SUB(int a, int b) {
        // help yourself
    }
    int MUL(int a, int b) {
        // help yourself
    }
    int POW(int a, int b) {
        // help yourself
    }
    NTT(int g, int gap, int _mod) {
        mod = _mod;
        o[0] = 1;
        int pp = POW(g, gap);
        FOR(i, 1, (1 << LG) + 1) o[i] = MUL(o[i - 1], pp);
    }
    void operator()(int *a, int n, bool inv) {
        auto REV = [&](int x) -> int {
            int ans = 0;
            for (int w = 1; w < n; w <= 1) {
                ans = (ans << 1) | (x & 1);
                x >>= 1;
            }
            return ans;
        };
        FOR(i, 0, n) {
            int j = REV(i);
            if (i < j) swap(a[i], a[j]);
        }
        for (int w = 1; w < n; w <= 1) {
            int owo = 1 << (LG - __lg(w) - 1), oid = 0;
            FOR(i, 0, w) {
                int omega = o[inv ? (1 << LG) - oid : oid];
                for (int s = 0; s < n; s += (w << 1)) {
                    int &L = a[s + i], &R = a[s + w + i];
                    int l = L, r = MUL(omega, R);
                    L = ADD(l, r);
                    R = SUB(l, r);
                }
                oid += owo;
            }
        }
        if (inv) {
            int x = POW(n, mod - 2);
            FOR(i, 0, n) a[i] = MUL(a[i], x);
        }
    }
};

NTT ntt1(3, 952, 998244353);
NTT ntt2(3, 100, 104857601);
NTT ntt3(3, 160, 167772161);

namespace POLY {
    const int MXM = 4 * MXN;
    int a[MXN], b[MXN];
    vector<int> VMUL(vector<int> v, vector<int> w, int m) {

```

```

        int N = 4 << __lg(m);
        fill(a, a + N, 0);
        fill(b, b + N, 0);
        int na = min((int) v.size(), m), nb = min((int)
w.size(), m);
        FOR(i, 0, na) a[i] = v[i];
        FOR(i, 0, nb) b[i] = w[i];
        ntt(a, N, false);
        ntt(b, N, false);
        FOR(i, 0, N) a[i] = MUL(a[i], b[i]);
        ntt(a, N, true);
        vector<int> ans;
        FOR(i, 0, m) ans.push_back(a[i]);
        return ans;
    }
}

```

6.6 Pollard Rho

```

// needs mad,mub,mul,pw with changable mod
//!!! use int128 for pw and mul

bool isprime(ll x) {
    if (x <= 2 || ~x & 1) return x == 2;
    auto witn = [&](ll a, int t) {
        for (ll a2; t-- && (a2 = mul(a, a, x)); a = a2)
            if (a2 == 1 && a != 1 && a != x - 1) return true;
        return a > 1;
    };
    int t = __builtin_ctzll(x-1); ll odd = (x-1) >> t;
    for (ll m:
        {2, 325, 9375, 28178, 450775, 9780504, 1795265022})
        if (witn(pw(m % x, odd, x), t)) return false;
    return true;
}

ll pollard_rho(ll n) {
    static mt19937_64 rnd(120821011);
    if (!(n & 1)) return 2;
    ll y = 2, z = y, c = rnd() % n, p = 1, i = 0, t;
    auto f = [&](ll x) {
        return mad(mul(x, x, n), c, n);
    };
    do {
        p = mul(mub(z = f(f(z)), y = f(y), n), p, n);
        if (++i &= 63) if (i == (i & -i)) t = gcd(p, n);
    } while (t == 1);
    return t == n ? pollard_rho(n) : t;
}

vector<ll> factorize(ll k){
    if(k == 1)return {};
    else if(isprime(k))return {k};
    else{
        vector<ll> re;
        function<void(ll)> dc = [&](ll k){
            if(isprime(k)){
                re.push_back(k);
                return;
            }
            ll x = pollard_rho(k);
            dc(x);dc(k/x);
        };
        dc(k);
        sort(re.begin(),re.end());
        return re;
    }
}

```

6.7 Floor Sum


```
ll floor_sum(ll a, ll b, ll c, ll n) {  
    // floor((a * x + b) / c) for x in [0, n]  
    if (n < 0) return 0;  
    if (a == 0) return b / c * (n + 1);  
    if (a >= c || b >= c) return (n * (n + 1) / 2 * (a / c)  
+ (b / c) * (n + 1)) + floor_sum(a % c, b % c, c, n);  
    int m = (a * n + b) / c;  
    return m * n - floor_sum(c, c - b - 1, a, m - 1);  
}
```

7 Notes

- NO PATH COMPRESSION on rollback dsu please

