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Optimizing blood assignment in a donation–transfusion system

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Abstract

A multi-product, multi-period, multi-objective linear programming model has been built as a contribution to good management of a blood donation–transfusion system in order to determine the best assignment of blood resources to demand, which minimizes the quantity of blood imported from outside the system and stabilizes the quantities assigned daily. The model has been applied to the Italian Red Cross (CRI) blood donation–transfusion system in Rome and to each hospital belonging to such a system, producing interesting results.

Keywords: donation-transfusion system, multi-objective linear programming

1. Introduction

By blood donation–transfusion system we mean a well defined set of transfusion centres (TCs) and hospitals under the same management. All blood requests made by hospitals must be satisfied ideally by such TCs; importation of blood from outside the system is allowed only due to scarcity and in this case we assume that there is always sufficient supply of the external resource.

The system input, the blood resource, is perishable: a unit of donated blood can only be used within a thirty-day period, unless it is frozen in a very costly process, otherwise it must be incinerated. Most donations are unscheduled.

The system output depends on the demand, and on the deadline after which units of un-utilized blood must be incinerated (see Fig. 1). Most demand arises in an unplanned way, but some planning can take place in the way in which demand is satisfied, if it is not very urgent. Requests have different degrees of urgency; if the request is very urgent, the blood must be assigned immediately; if the request is not very urgent, the blood can be assigned later, within a period of time which varies according to the seriousness of the case.

The units of blood stored in the system must always be classified according to their donation date. The not-very-urgent requests can be satisfied later than their date of arrival in the system. Therefore,

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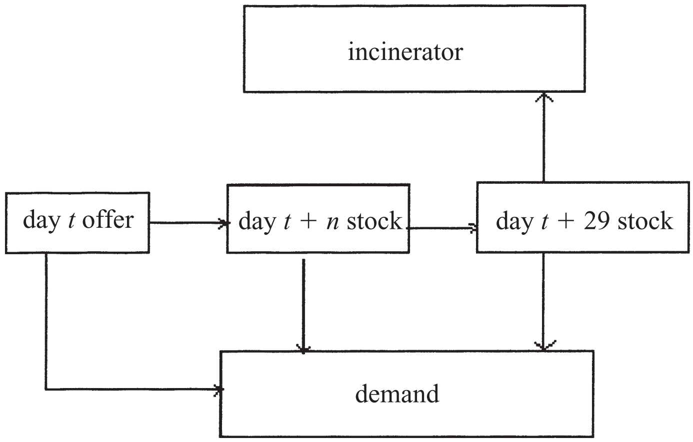

Fig. 1. Life cycle of a donated unit.

they too must be stored and classified according to their arrival date and urgency degree. In this sense we speak of a multi-product and multi-period problem.

Good management of the system means controlling all components as much as possible, in order to make the best use of the blood resource to minimize the quantity of blood imported from outside the system. Increasing blood donations and/or decreasing blood demand through the use of blood saving techniques is certainly important but also good day-to-day management is fundamental, especially when blood resources are scarce. This involves the planning of medical and surgical interventions according to urgency and availability, delaying blood assignment to not-very-urgent requests within tolerance limits, without influencing demand and offer flows, which can only be modified through changes in policies and practices that are beyond our control.

For this purpose we have formulated a multi-objective linear programming problem (Steuer, 1986) that minimizes the quantity of blood imported from outside the system, and the range of the quantities of system blood assigned daily, through a lexicographic minimization process (Steuer, 1986). The solution gives the best assignment flow, conditioned by the donation and request flows.

The model has been applied to the CRI (Italian Red Cross) system in Rome, and to the systems constituted by each single hospital belonging to the CRI in Rome and its transfusion centre, and interesting results have been obtained. Although our application is based on data only known a posteriori, the results suggest rules for day-to-day decisions.

2. The data

The systems considered here are:

(i) the CRI system in Rome, composed of the Centro Nazionale Trasfusioni Sangue—National Blood Transfusion Centre (NTC), the CRI mobile units and the six TCs of S. Camillo, S. Eugenio, S. Filippo Neri, S. Giacomo, Regina Elena, S. Spirito Hospitals, and the above mentioned hospitals as clients;
(ii) the TC of each of the six hospitals, and the hospital itself as a client.

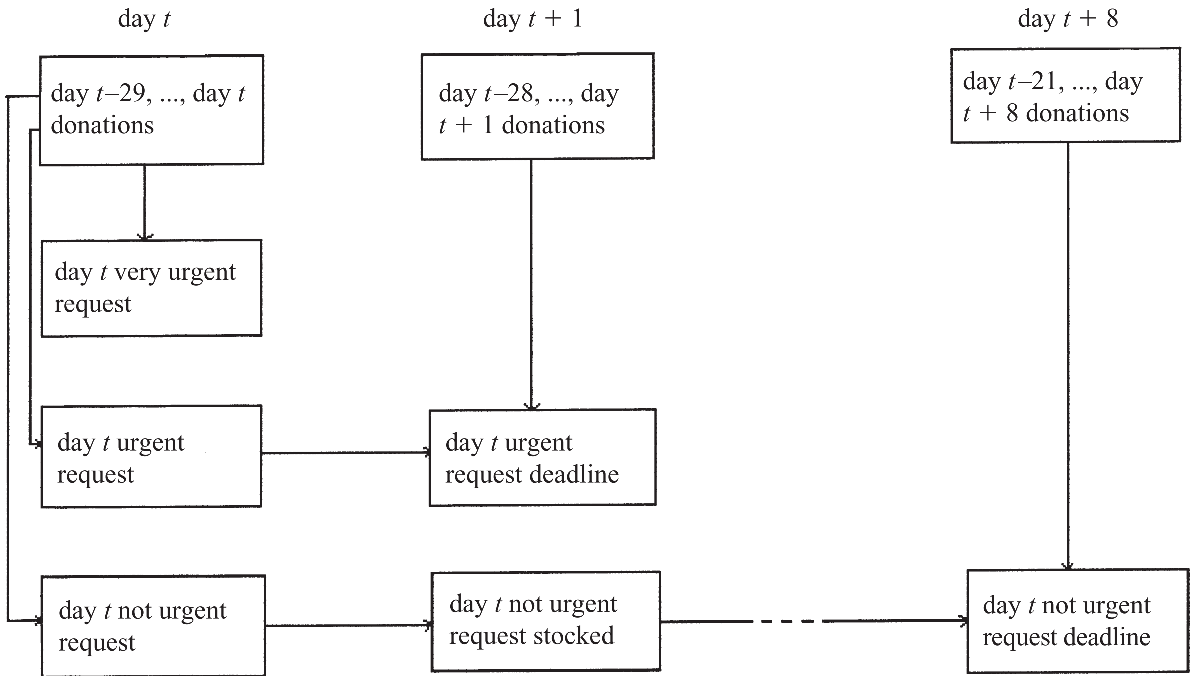

Fig. 2. Life cycle of a request.

Data was available on blood donations, classified by blood group, date, and TC, and on red blood cell requests, classified by blood group, pathology, urgency degree, hospital, hospital ward, quantity requested, date of request, quantity assigned, date of assignment, and quantity used. We introduced three degrees of urgency: very urgent requests, that must be satisfied immediately, urgent requests, that must be satisfied by the day after, and not-urgent requests that must be satisfied within eight days (see Fig. 2). The data refers only to red blood cells, but the validity of our model is general. For clarity, we point out that a red blood cell unit is equivalent to a blood unit.

The data refers to November 1996, which was recent at the time of collection, and not biased by the phenomena of holidays, festivities and so on.

We have also used data from the Italian Health Ministry MSBOS (Maximum Surgical Blood Order Schedule) list, that suggests quantities of blood necessary for each type of surgical intervention. This helped us to control, at least indirectly, the flow of devolved blood that is produced by the blood assigned to the requests but not used, that can be reassigned only if it has not reached its expiry date and has been stored properly the whole time.

3. The model

The multi-objective, multi-period, multi-product (Williams, 1993) linear programming model built here is based on variables that represent the units of blood coming from inside the system, the units of blood coming from outside the system, and the units of blood assigned daily to the three types of

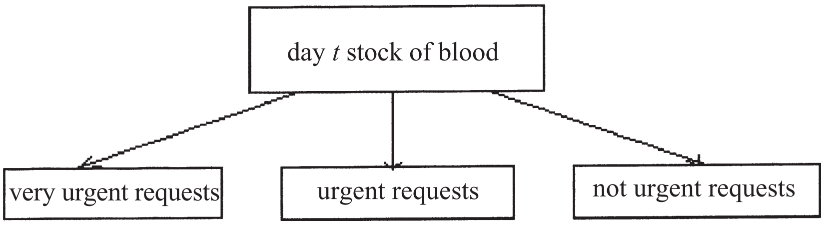

Fig. 3 The three daily assignments.

demand, and on constraints that compel the requests to be satisfied within their urgency limits and the blood to be disposed of when its validity expires. The choice of separating the flows of units assigned into the three types of demand (see Fig. 3) greatly simplifies the model described in De Angelis, Ricciardi and Storchi (1998), Bucciarelli et al. (2000) and De Angelis, Ricciardi and Storchi (1999).

The first goal of our lexicographic optimization is scheduling operations within tolerance limits while producing a schedule of blood orders from outside the system in such a way that their total amount is minimized. The second goal is redistributing the blood more evenly among the days of the month, while keeping the quantity of blood coming from outside the system to its minimum value.

The model deals with one of the blood groups A$\pm$, B$\pm$, AB$\pm$, O$\pm$ at a time.

3.1. The data

The daily demand in November, and the daily red blood cell demand of the last days of October which remained unsatisfied on the first day of November, for the blood group considered, are represented by:

- ${}_t d^0$: very urgent day t request, to be satisfied in the very day, for $t = 1, \ldots, 30$;
- ${}_t d^1$: urgent day t request, to be satisfied by day $t + 1$, for $t = 1, \ldots, 30$; urgent day t request still unsatisfied, for $t = 0$ (i.e., October 31);
- ${}_t d^8$: not urgent day t request, to be satisfied within day $t + 8$, for $t = 1, \ldots, 30$; not urgent day t request still unsatisfied, for $t = -7, \ldots, -1, 0$, where $t = -7$ refers to October 24 and $t = 0$ to October 31.

The daily donations in November and the remainders of the daily October donations (i.e., the initial stock of blood) are represented by:

- ${}_t r$: number of blood units collected in the system on day t, for $t = 1, \ldots, 30$; remainder of blood units collected in the system, on day t, for $t = -28, \ldots, -1, 0$, where $t = -28$ represents October 3.

3.2. The variables

The variables introduced are, for $t = 1, \ldots, 30$:

- ${}_t c$: number of red blood cell system units supplied in day t;
- ${}_t n$: number of red blood cell units supplied in day t coming from outside the system;

- ${}_ty^u$: total number of red blood cell units supplied in day t, from whatever origin, for requests with urgency degree u, for $u = 0, 1, 8$;
- ${}_tm$: number of red blood cell system units, collected in day $t - 29$, that are incinerated in day t because they have not been used by day t;
- ${}_ts$: number of red blood cell system units assigned in day t and taken from day $t - 28, \ldots, t - 1, t$ donations, whose deadline is later than day t;
- l: lower bound for the variables ${}_tc$;
- u: upper bound for the variables ${}_tc$.

3.3. The constraints

The following subsets of constraints are introduced in our problem:

1. The total number of red blood cell units assigned on day t is the sum of the number of units that come either from inside or outside the system:

$$\sum_{u=0,1,8} {}_ty^u = {}_tc + {}_tn \quad \text{for } t = 1, \ldots, 30$$

2. *a* The total number of red blood cell units assigned on day t for very urgent requests must satisfy all the day t very urgent requests:

$${}_ty^0 = {}_td^0 \quad \text{for } t = 1, \ldots, 30$$

b. The total number of red blood cell units assigned up to day s for urgent requests must be sufficient to satisfy all the urgent requests whose deadline is not later than day s:

$$\sum_{t \leqslant s} {}_ty^1 \geqslant \sum_{t \leqslant s-1} {}_td^1 \quad \text{for } s = 1, \ldots, 30$$

c. The total number of red blood cell units assigned up to day s for not-urgent requests must be sufficient to satisfy all the not-urgent requests whose deadline is not later than day s:

$$\sum_{t \leqslant s} {}_ty^8 \geqslant \sum_{t \leqslant s-8} {}_td^8 \quad \text{for } s = 1, \ldots, 30$$

3. *a*. The total number of red blood cell units assigned up to day s for urgent requests must not exceed all the urgent units requested up to day s:

$$\sum_{t \leqslant s} {}_ty^1 \leqslant \sum_{t \leqslant s} {}_td^1 \quad \text{for } s = 1, \ldots, 30$$

b. The total number of red blood cell units assigned up to day s for not-urgent requests must not exceed all the not-urgent units requested up to day s:

$$\sum_{t \leqslant s} {}_ty^8 \leqslant \sum_{t \leqslant s} {}_td^8 \quad \text{for } s = 1, \ldots, 30$$

4. The number of red blood cell units assigned on day t from inside the system cannot exceed the number of available units:

$$\begin{cases} {}_1c \leqslant {}_{-28}r + \ldots + {}_0r + {}_1r \\ {}_1c + {}_1m - {}_1s = {}_{-28}r \end{cases}$$

$$\begin{cases} {}_tc \leqslant {}_{t-29}r + \ldots + {}_{t-1}r + {}_tr - {}_{t-1}s \\ {}_tc + {}_tm - {}_ts + {}_{t-1}s = {}_{t-29}r \end{cases} \quad \text{for } t = 2, \ldots, 30$$

5. The unsatisfied demand on the last day of the month must not exceed the unsatisfied demand on the first day of the month:

$$\sum_{t\leqslant 30} \sum_{u=0,1,8} {}_ty^u \geqslant \sum_{t\leqslant 30} \sum_{u=0,1,8} {}_td^u$$

Hence the whole set of constraints is:

$$\sum_{u=0,1,8} {}_ty^u = {}_tc + {}_tn \quad \text{for } t = 1, \ldots, 30$$

$${}_ty^0 = {}_td^0 \quad \text{for } t = 1, \ldots, 30$$

$$\sum_{t\leqslant s} {}_ty^1 \geqslant \sum_{t\leqslant s-1} {}_td^1 \quad \text{for } s = 1, \ldots, 30$$

$$\sum_{t\leqslant s} {}_ty^8 \geqslant \sum_{t\leqslant s-8} {}_td^8 \quad \text{for } s = 1, \ldots, 30$$

$$\sum_{t\leqslant s} {}_ty^1 \leqslant \sum_{t\leqslant s} {}_td^1 \quad \text{for } s = 1, \ldots, 30$$

$$\sum_{t\leqslant s} {}_ty^8 \leqslant \sum_{t\leqslant s} {}_td^8 \quad \text{for } s = 1, \ldots, 30$$

$${}_1c \leqslant {}_{-28}r + \ldots + {}_0r + {}_1r$$

$${}_1c + {}_1m - {}_1s = {}_{-28}r$$

$${}_tc \leqslant {}_{t-29}r + \ldots + {}_{t-1}r + {}_tr - {}_{t-1}s \quad \text{for } t = 2, \ldots, 30$$

$${}_tc + {}_tm - {}_ts + {}_{t-1}s = {}_{t-29}r \quad \text{for } t = 2, \ldots, 30$$

$$\sum_{t\leqslant 30} \sum_{u=0,1,8} {}_ty^u \geqslant \sum_{t\leqslant 30} \sum_{u=0,1,8} {}_td^u$$

3.4. The first objective function

The main goal of the model is to achieve self-sufficiency for the system through the careful planning of hospital operations according to their urgency degree. So, red blood cell units are supplied in such a way that the deadlines of the various requests are respected, while the quantity of blood to be ordered from outside the system is minimized. This goal is achieved by introducing the following objective function, to be *minimized*:

$$z = \sum_{t=1}^{30} {}_t n$$

3.5. The second objective function

Once the solution that minimizes the total number of red blood cell units coming from outside the system is obtained, the second goal is pursued, aiming to produce a distribution as regular as possible of the number of units assigned and—as a consequence—of the number of hospital operations performed. To this end, the following constraints have been added:

6. The total number of units coming from outside the system must be equal to the optimal value obtained by considering the first objective function:

$$\sum_{t=1}^{30} {}_t n = \textit{optimal value}$$

7. Every variable ${}_t c$ has a lower bound equal to l:

$${}_t c \geqslant l \quad \text{for} \quad t = 1, \ldots, 30$$

8. Every variable ${}_t c$ has an upper bound equal to u:

$${}_t c \leqslant u \quad \text{for} \quad t = 1, \ldots, 30$$

Finally, the following objective function has been introduced, to be *minimized*:

$$z = u - l.$$

4. Applications and results

The available data allowed two applications, using as *quantities to be assigned*:

(*i*) the minimum between the real blood request and the request contained in the MSBOS list suggested by the Health Ministry in the not-very-urgent cases, and the real request in the very-urgent cases;
(*ii*) the quantity of blood used in every case.

The purpose of applying the model to the two different types of data was to measure two different blood shortages: the one due to ‘cautious’ requests, that create a flow of devolved blood that might deteriorate and not be reassigned, and the real one, related to the blood really used.

Our optimal assignment pattern is as follows: the operations are scheduled according to their *expiry date*, i.e., each day, the very-urgent requests of the same day, together with the residual of both the urgent requests of the day before and the not-urgent requests of eight days before, have priority over the urgent-requests of the same day and the residual of the not-urgent requests of seven days before, and so on. Blood is imported from outside the system whenever necessary to satisfy requests whose

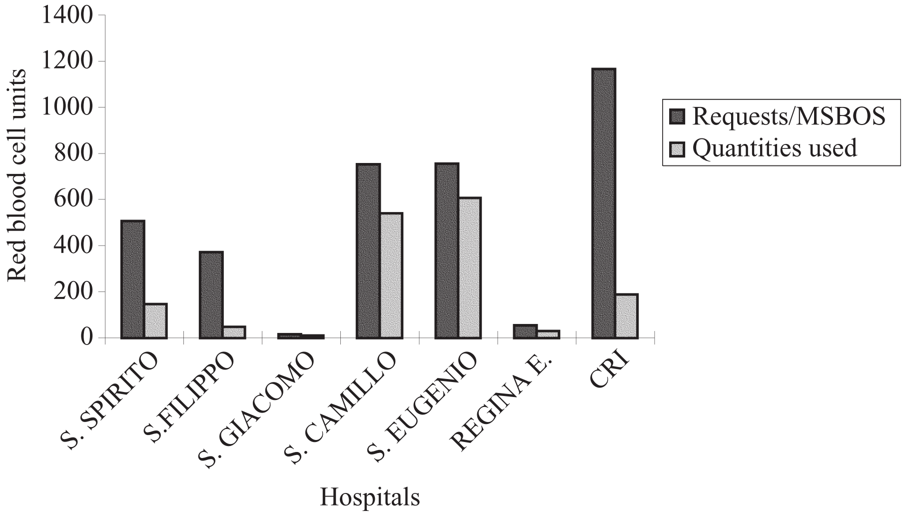

Fig. 4. Shortage of red blood cell units with respect to requests/MSBOS and quantities really used.

urgency expires that day or to stabilize the assignment of blood (and, indirectly, the number of surgical interventions performed).

The results obtained with the two sets of data differ according to the system and blood group considered. Adding up all blood groups, to summarize the results, the shortage is great for each system composed of a single hospital and its TC, while the situation is better for the whole CRI system, as shown in Fig. 4, thanks to the contribution of the NTC and the mobile units. So, talking of the whole CRI system, the total shortage is 32% with the first set of data—which corresponds to an estimated 24% if devolved blood units are considered—and 7.5% with the second one.

Considering each blood group separately, self-sufficiency could have been reached for groups A−, B+, AB+, O+, O− for the whole CRI system in Rome in November 1996 by scheduling operations according to our model, as shown in Fig. 5.

5. Suggestions for day-to-day decisions and conclusions

The results of our applications are very important because they are obtained by applying a tactical rule each day. They show that great attention should be given to the day-to-day management of the system. This is particularly the case, as presented here, where supply is scarce in relation to demand. Managers have been made aware of the possibility of considerably improving results even with the present blood donations level, by good daily management of the system, without waiting for an increase in blood donations.

Apart from showing that, at least theoretically, self-sufficiency might be achieved, the optimal solutions present a pattern that gives useful suggestions for their implementation.

We must consider that, every day, the quantity of blood needed for the next day is partly known: it is

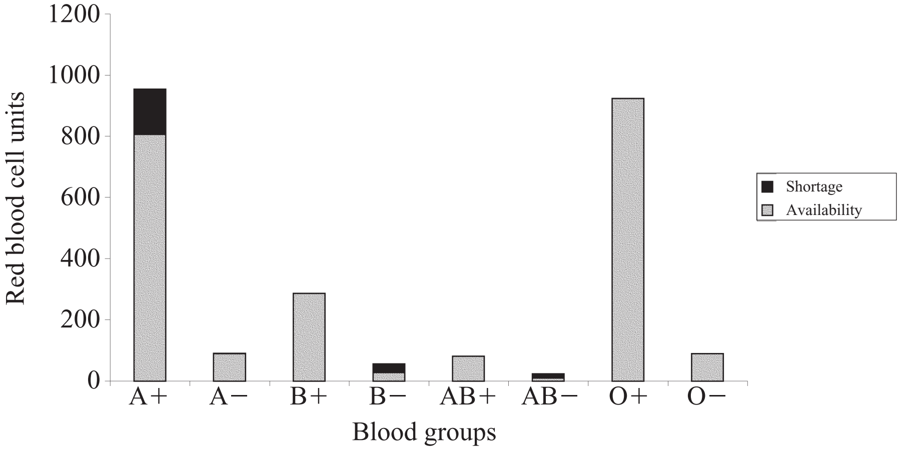

Fig. 5. Shortage of red blood cell units with respect to quantities really used, for the whole CRI system in Rome.

the residual of the not-urgent requests of seven days before plus the residual of the urgent quantity required on the current day (both possibly corrected by means of the MSBOS list), together with the very urgent requests that arise on that day. Knowing the probability distribution of the very urgent requests in a day, a good estimate of the very urgent requests that can arise is given by a suitable quantile of such a distribution. The sum of this quantity with the two residuals previously calculated provides a good estimate of the day-after needs. Then, the residual quantity of blood in stock, calculated after subtracting the blood that deteriorates by the day after, must be compared with such an estimate, and blood must be ordered from outside the system whenever the availability is less than the request.

The scheme suggested is very simple to apply, having all information stored on computers and updated in real time so that the situation can be constantly monitored. It represents a valid alternative to the previous CRI habit of satisfying the not-very-urgent requests on a first-in first-out basis, without taking into account the forecast of requests which may arise in the immediately following days.

Our model solves the problem of the optimal scheduling of medical and surgical interventions according to urgency and availability, without influencing the demand and offer flows. As stated before, policies to increase the number of periodic donors and to make blood donations more regular, or according to request, must be developed as well, together with policies capable of encouraging blood saving techniques and influencing blood requests, but good day-to-day management should never be neglected.

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