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SMSA air pollution Data Analysis

Researchers at General Motors collected data on some U.S. Standard Metropolitan Statistical Areas (SMSA's) in a study of whether air pollution contributes to mortality. The dependent variable for analysis is age adjusted mortality ("Mortality"). The data include variables measuring demographic characteristics of the cities, variables measuring climate characteristics, and variables recording the pollution potential of three different air pollutants. As a simple initial test, we will use regression models to determine whether air pollution is significantly related to mortality.

#	Variable	Description
1	city	City ID
2	JanTemp	Mean January temperature (F)
3	JulyTemp	Mean July temperature (F)
4	RelHum	Relative Humidity
5	Rain	Annual rainfall (inches)
6	Education	Median education
7	PopDensity	Population density
8	NW	Percentage of non-whites
9	WC	Percentage of white collar workers
10	pop	Population
11	HHSiz	Average household size
12	income	Median income
13	HCPot	HC pollution potential
14	NOxPot	Nitrous Oxide pollution potential
15	SO2Pot	Sulfur Dioxide pollution potential
16	Mortality	Age adjusted mortality

1.> Started R. Installed some packages and loaded some libraries.

```
install.packages("readxl")
install.packages("graphics")

library(Hmisc) #Contents and Describe
library(leaps) #Variable selection
library(MASS)
```

2.> Set up my working repository where .csv files are saved for both Mortality and Transactions data. Used the functions setwd() and getwd() as specified.

```
> setwd("C:/Users/hitpr/Desktop/1st semester/Business Analytics/Homework")
>
> getwd()
[1] "C:/Users/hitpr/Desktop/1st semester/Business Analytics/Homework"
> |
```

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3.> Loaded data from Mortality.csv.

```
> mortality_data <- read.csv("mortality.csv", header=TRUE)
> head(mortality_data)
```

	City	JanTemp	JulyTemp	RelHum	Rain	Education	PopDensity	Nw	WC	pop	HHSiz	income	HCPot	NOxPot	S02Pot	Mortality
1	1	27	71	59	36	11.4	3243	8.8	43	660328	3.3	29560	21	15	59	922
2	2	23	72	57	35	11.0	4281	3.5	51	835880	3.1	31458	8	10	39	998
3	3	29	74	54	44	9.8	4260	0.8	39	635481	3.2	31856	6	6	33	962
4	4	45	79	56	47	11.1	3125	27.1	50	2138231	3.4	32452	18	8	24	982
5	5	35	77	55	43	9.6	6441	24.4	44	2199531	3.4	32368	43	38	206	1071
6	6	45	80	54	53	10.2	3325	38.5	43	883946	3.4	27835	30	32	72	1030

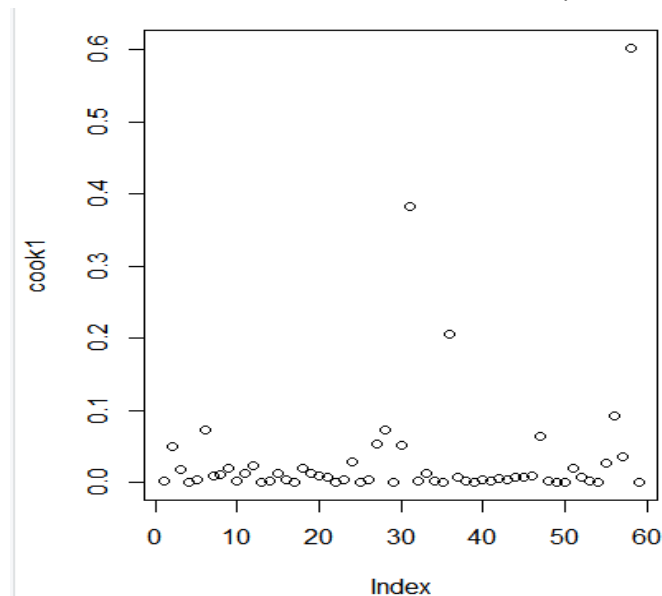
Removed City by assigning null as it is an identifier column.

```
> mortality_data$City <- NULL ##dropping the city variable
> head(mortality_data)
```

	JanTemp	JulyTemp	RelHum	Rain	Education	PopDensity	Nw	WC	pop	HHSiz	income	HCPot	NOxPot	S02Pot	Mortality
1	27	71	59	36	11.4	3243	8.8	43	660328	3.3	29560	21	15	59	922
2	23	72	57	35	11.0	4281	3.5	51	835880	3.1	31458	8	10	39	998
3	29	74	54	44	9.8	4260	0.8	39	635481	3.2	31856	6	6	33	962
4	45	79	56	47	11.1	3125	27.1	50	2138231	3.4	32452	18	8	24	982
5	35	77	55	43	9.6	6441	24.4	44	2199531	3.4	32368	43	38	206	1071
6	45	80	54	53	10.2	3325	38.5	43	883946	3.4	27835	30	32	72	1030

4.> Removing outliers: Used cook's distance to remove the outliers. It is used to find the influence of the data points on the regression model. Data points with large residuals may distort the outcome and accuracy of a regression. Cook's distance depends on the error values y minus \hat{y} . Larger the cook's distance more is the effect of that observation on the regression that leads to skewness.

So, creating a dummy regression model using the variables specified in question to calculate cook's distance. Calculate cook's distance and plot them.

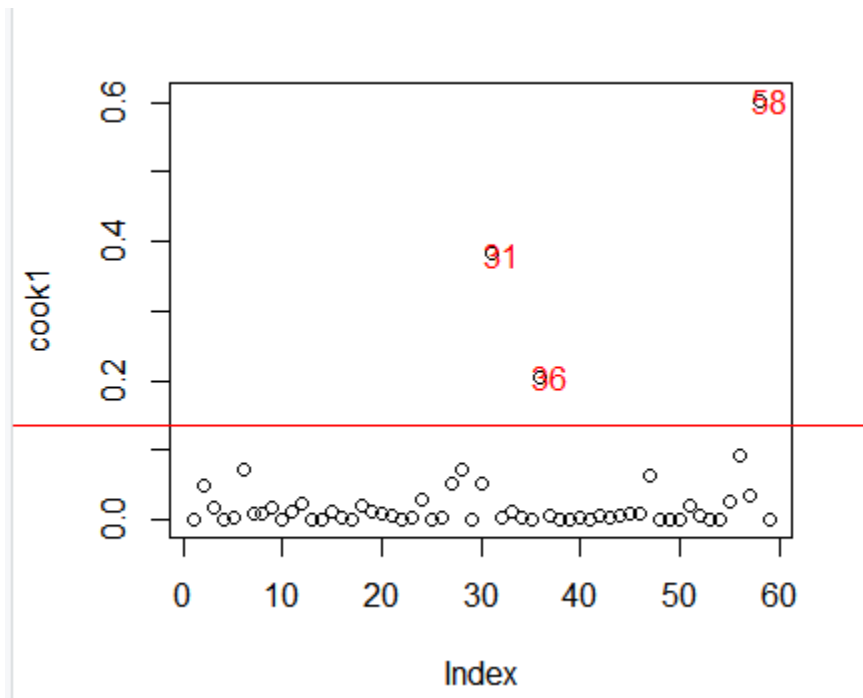


Now for better visualization and understanding drawing a threshold line (using `abline()`) in red in the plot to show what I am considering as outliers. The threshold selected is 4 times the mean of cook's distance. Also labeling the outliers (using `text()`) with the row number or the observation number in red.

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```
plot(cook1)
abline(h = 4*mean(cook1, na.rm=T), col="red")
text(x=1:length(cook1)+1, y=cook1, labels=ifelse(cook1>4*mean(cook1, na.rm=T),names(cook1),""), col="red")
```



Select the outlier row numbers. Check the exact outlier rows and remove the outliers by taking compliment of the rows from the actual data i.e. mortality_data and store it into mortality_data1. Now if we check the data in Mortality_data1, we can see that the rows 31, 36 and 58 are missing. Therefore, the outliers are removed. We now need to use Mortality_data1 for further steps and not Mortality_data.

```
> outliers0 <- as.numeric(names(cook1)[(cook1 > 4*mean(cook1, na.rm=T))]) # outlier row numbers
> head(mortality_data[outliers0, ]) # outlier observations.
  JanTemp JulyTemp RelHum Rain Education PopDensity NW WC pop HHSiz income HCPot NOxPot SO2Pot Mortality
31      67      82      60      60      11.5      4657 13.5 47 1625781 2.6 32808 3 1 1 861
36      54      81      62      54      9.7      3172 31.4 46 1256256 3.4 32704 20 17 1 1113
58      33      76      54      62      9.0      9699 4.8 62 381255 3.2 28985 8 8 49 912
>
> mortality_data1 <-mortality_data[-c(31,36,58), ]
> mortality_data1
  JanTemp JulyTemp RelHum Rain Education PopDensity NW WC pop HHSiz income HCPot NOxPot SO2Pot Mortality
1      27      71      59      36      11.4      3243 8.8 43 660328 3.3 29560 21 15 59 922
2      23      72      57      35      11.0      4281 3.5 51 835880 3.1 31458 8 10 39 998
3      29      74      54      44      9.8      4260 0.8 39 635481 3.2 31856 6 6 33 962
4      45      79      56      47      11.1      3125 27.1 50 2138231 3.4 32452 18 8 24 982
5      35      77      55      43      9.6      6441 24.4 44 2199531 3.4 32368 43 38 206 1071
6      45      80      54      53      10.2      3325 38.5 43 883946 3.4 27835 30 32 72 1030
7      30      74      56      43      12.1      4679 3.5 49 2805911 3.2 36644 21 32 62 935
8      30      73      56      45      10.6      2140 5.3 40 438557 3.3 47258 6 4 4 900
```

Now we need to run the appropriate regression diagnostics (normality, homoscedasticity) to ensure that the assumptions of OLS are not violated. For that run the regression using the new data frame Mortality_data1 and plot it. Below is the summary of the regression and the plot. As can be seen it is homoskedasticity since the residuals vs fitted is almost linear and horizontal. In

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Normal Q-Q graph it is slightly positively skewed however, since most of the observations are on the line, I have considered it as a linear plot, hence normal.

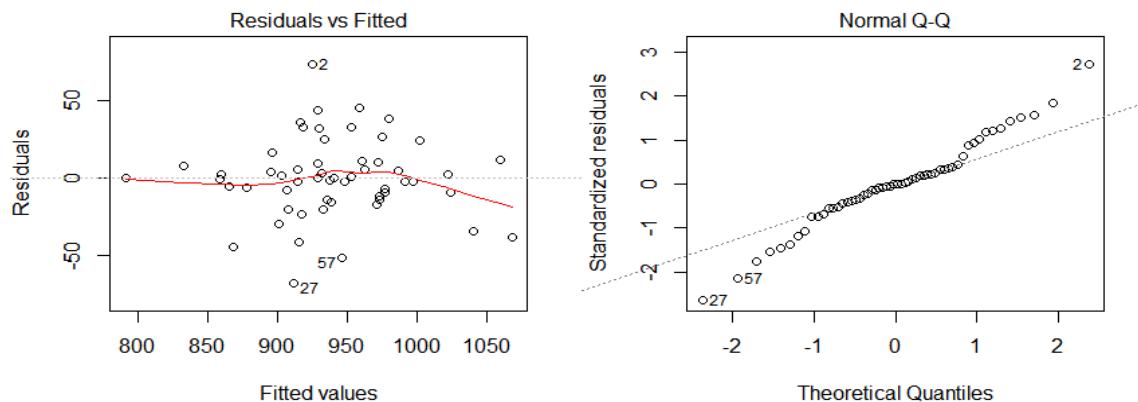
```

Residuals:
    Min       1Q   Median       3Q      Max
-67.44 -12.53   0.01  10.07  72.88

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.34e+03  2.41e+02   5.58  1.7e-06 ***
JanTemp      -7.70e-01  7.11e-01  -1.08  0.2857
JulyTemp     -2.51e+00  1.63e+00  -1.54  0.1323
RelHum       -2.53e-01  9.81e-01  -0.26  0.7980
Rain         1.40e+00  5.05e-01   2.76  0.0085 **
Education    -6.01e+00  9.08e+00  -0.66  0.5117
PopDensity   1.11e-02  4.23e-03   2.62  0.0124 *
NW           4.31e+00  8.04e-01   5.36  3.5e-06 ***
WC          -1.23e+00  1.40e+00  -0.88  0.3852
pop          3.71e-07  3.60e-06   0.10  0.9183
HHSiz       -5.24e+01  3.73e+01  -1.41  0.1674
income      -1.11e-03  1.09e-03  -1.02  0.3147
HCPot       -5.39e-01  3.89e-01  -1.39  0.1729
NOxPot       8.71e-01  7.85e-01   1.11  0.2738
SO2Pot       1.48e-01  1.22e-01   1.22  0.2302
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 29 on 41 degrees of freedom
Multiple R-squared:  0.815,    Adjusted R-squared:  0.751
F-statistic: 12.9 on 14 and 41 DF,  p-value: 7.96e-11

```



5.> a.) R squared value for above model: 0.815

Adjusted R squared for above model: 0.751

R squared means that 81.5 % of the variance is explained by the regression model.

Adjusted R squared is the R squared value that has been adjusted for the number of predictors in the model. Comparison between 2 different models for a dataset is done using the Adjusted R squared value.

b.) There are three significant variables in this model namely Rain (t value=2.76, $\Pr(>|t|)=0.0085$), PopDensity(t value=2.62, $\Pr(>|t|)=0.0124$), NW(t value=5.36, $\Pr(>|t|)=3.5e^{-06}$)

6.> Find the best model using forward and backward methods.

Run another regression model using the final model shown when we do anova.

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Stepwise Model Path
Analysis of Deviance Table

Initial Model:

Mortality ~ JanTemp + JulyTemp + RelHum + Rain + Education +
PopDensity + NW + WC + pop + HHSiz + income + HCPot + NOxPot +
S02Pot

Final Model:

Mortality ~ JanTemp + JulyTemp + Rain + PopDensity + NW + WC +
HHSiz + HCPot + S02Pot

	Step	Df	Deviance	Resid. Df	Resid. Dev	AIC
1				41	35036	391
2	- pop	1	9.1	42	35046	389
3	- RelHum	1	55.4	43	35101	387
4	- Education	1	414.0	44	35515	385
5	- NOxPot	1	956.3	45	36471	385
6	- income	1	1145.1	46	37616	385

> summary(model_mortality3)

Call:

lm(formula = Mortality ~ JanTemp + JulyTemp + Rain + PopDensity +
NW + WC + HHSiz + HCPot + S02Pot, data = mortality_data1)

Residuals:

	Min	1Q	Median	3Q	Max
	-58.13	-17.42	-2.24	13.07	84.89

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)	
(Intercept)	1.22e+03	1.69e+02	7.24	4.0e-09	***
JanTemp	-9.18e-01	6.50e-01	-1.41	0.16470	
JulyTemp	-2.05e+00	1.21e+00	-1.69	0.09761	.
Rain	1.65e+00	4.65e-01	3.55	0.00091	***
PopDensity	1.11e-02	3.79e-03	2.92	0.00534	**
NW	4.37e+00	7.53e-01	5.81	5.6e-07	***
WC	-2.36e+00	9.51e-01	-2.48	0.01674	*
HHSiz	-4.84e+01	3.59e+01	-1.35	0.18377	
HCPot	-1.12e-01	6.83e-02	-1.63	0.10902	
S02Pot	2.55e-01	7.57e-02	3.37	0.00151	**

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 29 on 46 degrees of freedom

Multiple R-squared: 0.801, Adjusted R-squared: 0.762

F-statistic: 20.6 on 9 and 46 DF, p-value: 2.45e-13

- This is an important step as leads to increase in Adjusted R squared with reduced number of variables.
- R squared means that 80.1 % of the variance is explained by the regression model.
Adjusted R squared is the R squared value that has been adjusted for the number of predictors in the model. Comparison between 2 different models for a dataset is done using the Adjusted R squared value.
- There are five significant variables in this model namely Rain (t value=3.55, $\Pr(>|t|)=0.00091$), PopDensity(t value=2.92, $\Pr(>|t|)=0.00534$), NW(t value=5.81, $\Pr(>|t|)=5.6e^{-07}$), WC(t value=-2.48, $\Pr(>|t|)=5.6e^{-07}$), S02Pot(t value=3.37, $\Pr(>|t|)=0.00151$)

7.> Running PCA.

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Store data in another data frame after removing any missing values.

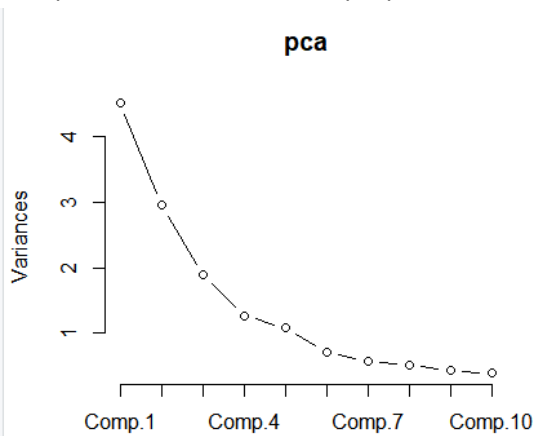
```
mortality_data2<- na.omit(mortality_data1)
pcamortality<-mortality_data2
```

Remove dependent variables.

```
> #remove dependent variable
> pcamortality$Mortality <- NULL
> #check for non numeric variables
> str(pcamortality)
'data.frame': 56 obs. of 14 variables:
 $ JanTemp : int 27 23 29 45 35 45 30 30 24 27 ...
 $ JulyTemp : int 71 72 74 79 77 80 74 73 70 72 ...
 $ RelHum : int 59 57 54 56 55 54 56 56 61 59 ...
 $ Rain : int 36 35 44 47 43 53 43 45 36 36 ...
 $ Education : num 11.4 11 9.8 11.1 9.6 10.2 12.1 10.6 10.5 10.7 ...
 $ PopDensity: int 3243 4281 4260 3125 6441 3325 4679 2140 6582 4213 ...
 $ NW : num 8.8 3.5 0.8 27.1 24.4 38.5 3.5 5.3 8.1 6.7 ...
 $ WC : num 42.6 50.7 39.4 50.2 43.7 43.1 49.2 40.4 42.5 41 ...
 $ pop : int 660328 835880 635481 2138231 2199531 883946 2805911 438557 1015472 404421 ...
 $ HHSiz : num 3.34 3.14 3.21 3.41 3.44 3.45 3.23 3.29 3.31 3.36 ...
 $ income : int 29560 31458 31856 32452 32368 27835 36644 47258 31248 29089 ...
 $ HCPot : int 21 8 6 18 43 30 21 6 18 12 ...
 $ NOxPot : int 15 10 6 8 38 32 32 4 12 7 ...
 $ SO2Pot : int 59 39 33 24 206 72 62 4 37 20 ...
```

Running PCA and checking for loadings and scores.

- 8.> Draw the scree plot for PCA. I have selected 8 components as the variance almost stabilizes after Comp8. Also the Cumulative proportion of Variance reaches ~ 0.9 till Comp8.



```
head(pcamortality)
pca <- princomp(pcamortality, cor = TRUE)
summary(pca) # print variance accounted for
pca$loadings
plot(pca,type="lines")
pca$scores
pca$scores[, 1]
```

- 9.> Manipulate data to run regression on above PCA.

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```
pca_mortality_data<-mortality_data2

pca_mortality_data$pc1<-pca$scores[, 1]
pca_mortality_data$pc2<-pca$scores[, 2]
pca_mortality_data$pc3<-pca$scores[, 3]
pca_mortality_data$pc4<-pca$scores[, 4]
pca_mortality_data$pc5<-pca$scores[, 5]
pca_mortality_data$pc6<-pca$scores[, 6]
pca_mortality_data$pc7<-pca$scores[, 7]
pca_mortality_data$pc8<-pca$scores[, 8]

head(pca_mortality_data)
```

10.> Running regression.

```
> model_mortality4 <- lm(Mortality ~ pc1 + pc2 + pc3 + pc4 + pc5 + pc6 + pc7 + pc8 ,data=pca_mortality_data)
> summary(model_mortality4)

Call:
lm(formula = Mortality ~ pc1 + pc2 + pc3 + pc4 + pc5 + pc6 +
    pc7 + pc8, data = pca_mortality_data)

Residuals:
    Min       1Q   Median       3Q      Max
-95.2   -15.6    -0.8    18.2    74.4

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   940.05      4.31   218.12 < 2e-16 ***
pc1           -10.11      2.07    -4.89 1.2e-05 ***
pc2            -23.47      2.73   -8.61 3.2e-11 ***
pc3              7.97      3.16    2.52 0.015 *
pc4             -4.50      3.84   -1.17 0.247
pc5            -19.87      4.38   -4.54 3.9e-05 ***
pc6             -9.54      5.17   -1.85 0.071 .
pc7              4.08      5.74    0.71 0.481
pc8             13.26      6.19    2.14 0.037 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 32 on 47 degrees of freedom
Multiple R-squared:  0.742,    Adjusted R-squared:  0.698
F-statistic: 16.9 on 8 and 47 DF,  p-value: 1.83e-11
```

- a.) R squared means that 74.2% of the variance is explained by the regression model. Adjusted R squared is the R squared value that has been adjusted for the number of predictors in the model. Comparison between 2 different models for a dataset is done using the Adjusted R squared value.
- b.) There are five significant variables in this model namely pc1(t value=-4.89, $\Pr(>|t|) = <1.2e-05$), pc2(t value=-8.61, $\Pr(>|t|) = 3.2e-11$), pc3(t value=2.52, $\Pr(>|t|) = 0.015$), pc5(t value=-4.54, $\Pr(>|t|) = 3.9e-05$), pc8(t value=2.14, $\Pr(>|t|) = 0.037$)

11.> Found best model using forward and backward methods and use the variables to run another regression.

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```
> model_mortality6<- lm(Mortality ~pc1 + pc2 + pc3 + pc5 + pc6 + pc8 ,data=pca_mortality_data)
> summary(model_mortality6)
```

Call:

```
lm(formula = Mortality ~ pc1 + pc2 + pc3 + pc5 + pc6 + pc8, data = pca_mortality_data)
```

Residuals:

Min	1Q	Median	3Q	Max
-100.46	-15.34	1.51	16.79	81.50

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)	
(Intercept)	940.05	4.30	218.40	< 2e-16	***
pc1	-10.11	2.07	-4.89	1.1e-05	***
pc2	-23.47	2.72	-8.62	2.2e-11	***
pc3	7.97	3.15	2.53	0.015	*
pc5	-19.87	4.37	-4.54	3.6e-05	***
pc6	-9.54	5.16	-1.85	0.070	.
pc8	13.26	6.18	2.14	0.037	*

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 32 on 49 degrees of freedom
Multiple R-squared: 0.731, Adjusted R-squared: 0.698
F-statistic: 22.2 on 6 and 49 DF, p-value: 1.95e-12

- a.) R squared means that 73.1% of the variance is explained by the regression model.
Adjusted R squared is the R squared value that has been adjusted for the number of predictors in the model. Comparison between 2 different models for a dataset is done using the Adjusted R squared value.
- b.) There are five significant variables in this model namely pc1(t value=-4.89, Pr(>|t|)= <1.1e-05), pc2(t value=-8.62, Pr(>|t|)= 2.2e-11), pc3(t value=2.53, Pr(>|t|)= 0.015), pc5(t value=-4.54, Pr(>|t|)= 3.6e-05), pc8(t value=2.14, Pr(>|t|)= 0.037)

12.> In the above regression models the model in step 6 has the maximum Adjusted R squared value and the model in step 8 has the minimum number of variables. I will personally select model in step 6 as it has decrease number of variables (although not minimum) and has the max adjusted R squared value.