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Linear Algebra Assignment

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Assignment

Q.1 If x and y are vectors in R^n then prove that

$$||x+y|| \leqslant ||x|| + ||y||$$

Do any one out of Q.2 and Q.3

Q.2 Solve the following system by Gauss-Jordan method.

$$\begin{cases} 2x_1 + x_2 + 3x_3 = 16 \\ 3x_1 + 2x_2 + x_4 = 16 \\ 2x_1 + 12x_3 - 5x_4 = 5 \end{cases}$$

or

Q.3 Solve for eigen space E_{-1} corresponding to eigenvalue $\lambda = -1$ for matrix.

$$A = \begin{bmatrix} 0 & -6 & 3 \\ 2 & -13 & 6 \\ 4 & -24 & 11 \end{bmatrix}$$

Q.4 Is $x = [3 \quad 5]$ in row space of $B = \begin{bmatrix} 2 & -4 \\ -1 & 2 \end{bmatrix}$?

Q.5

Do any one out of Q.5 and Q.6

Let x, y be vectors in R^n, Prove that

$$|x \cdot y| \leqslant ||x|| \, ||y||$$

or

Q.6 Solve for eigen space E_6 corresponding to eigen value $\lambda = 6$.

$$A = \begin{bmatrix} 12 & -51 \\ 2 & -11 \end{bmatrix}$$

Sol ① To show: $||x+y||^2 \leq (||x||+||y||)^2$ —①

If we can show Eq. ① to be true, then $||x+y|| \leq ||x||+||y||$ will also be true as we will always get +ve values on both sides of inequality and thus, there will be no change in the inequality after squaring both sides.

Now, Consider $||x+y||^2$

$$||x+y||^2 = (x+y)\cdot(x+y)$$

$$||x+y||^2 = ||x||^2 + x\cdot y + y\cdot x + ||y||^2$$

$$||x+y||^2 = ||x||^2 + 2x\cdot y + ||y||^2 \qquad [\because x\cdot y = y\cdot x]$$

$$||x+y||^2 \leq ||x||^2 + ||y||^2 + 2|x\cdot y|$$

$$\therefore ||x+y||^2 \leq ||x||^2 + ||y||^2 + 2||x||\,||y|| \qquad \left\{ \text{by cauchy-shwartz inequality } |x\cdot y| \leq ||x||\,||y|| \right\}$$

$$\therefore ||x+y||^2 \leq (||x|| + ||y||)^2$$

Therefore, $\boxed{||x+y|| \leq ||x|| + ||y||}$ Hence Proved.

Sol:② Given:

$$\begin{cases} 2x_1 + x_2 + 3x_3 = 16 \\ 3x_1 + 2x_2 + x_4 = 16 \\ 2x_1 + 12x_3 - 5x_4 = 5 \end{cases}$$

Therefore, the corresponding augmented matrix will be,

$$A = \left[\begin{array}{cccc|c} 2 & 1 & 3 & 0 & 16 \\ 3 & 2 & 0 & 1 & 16 \\ 2 & 0 & 12 & -5 & 5 \end{array}\right]$$

Consider B = reduced row echelon form of A,

i.e. row reducing using Gauss-Jordan row reduction method,

$R_1 \longrightarrow R_1 \tfrac{1}{2}$, $\Rightarrow \left[\begin{array}{cccc|c} 1 & 1/2 & 3/2 & 0 & 8 \\ 3 & 2 & 0 & 1 & 16 \\ 2 & 0 & 12 & -5 & 5 \end{array}\right]$

$R_2 \longrightarrow (-3)R_1 + R_2$

$R_3 \longrightarrow (-2)R_1 + R_3$ $\Rightarrow \left[\begin{array}{cccc|c} 1 & 1/2 & 3/2 & 0 & 8 \\ 0 & 1/2 & -9/2 & 1 & -8 \\ 0 & -1 & 9 & -5 & -11 \end{array}\right]$

$R_2 \longrightarrow 2R_2$ $\Rightarrow \left[\begin{array}{cccc|c} 1 & 1/2 & 3/2 & 0 & 8 \\ 0 & 1 & -9 & 2 & -16 \\ 0 & -1 & 9 & -5 & -11 \end{array}\right]$

$R_1 \longrightarrow \frac{1}{2}R_2 + R_1$

$R_3 \longrightarrow R_2 + R_3$ $\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 6 & -1 & 16 \\ 0 & 1 & -9 & 2 & -16 \\ 0 & 0 & 0 & -3 & -27 \end{array}\right]$

$R_3 \longrightarrow \frac{1}{3}R_3$ $\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 6 & -1 & 16 \\ 0 & 1 & -9 & 2 & -16 \\ 0 & 0 & 0 & 1 & 9 \end{array}\right]$

$R_1 \longrightarrow R_3 + R_1$

$R_2 \longrightarrow -2R_3 + R_2$ $\Rightarrow \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \left[\begin{array}{cccc|c} 1 & 0 & 6 & 0 & 25 \\ 0 & 1 & -9 & 0 & -34 \\ 0 & 0 & 0 & 1 & 9 \end{array}\right] \end{array} = B$

$\therefore$ From B, we see that column x_3 is not a pivot column, i.e. x_3 is independent,

$\therefore x_3 = C \in R$

Also from B, corresponding equations will be,

$$\begin{cases} x_1 + 6x_3 = 25 \\ x_2 - 9x_3 = -34 \\ x_4 = 9 \end{cases}$$

Now, as $x_3 = c$, $c \in R$,

$$x_1 = 25 - 6c \quad \text{and} \quad x_2 = -34 + 9c$$

$\therefore$ The complete solution set will be,

$$\{(25-6c, 9c-34, c, 9) \mid c \in R\}$$ Ans.

Sol:4 Given, $B = \begin{bmatrix} 2 & -4 \\ -1 & 2 \end{bmatrix}$, $x = [3, 5]$

To check whether x is in row space of B, we have to row reduce augmented matrix $[B^T \mid x]$ and determine whether its corresponding system has a sol^n.

$$\therefore [B^T \mid x] = \left[\begin{array}{cc|c} 2 & -1 & 3 \\ -4 & 2 & 5 \end{array}\right]$$

row reducing the above matrix,

$$R_1 \longrightarrow \frac{1}{2}R_1 \quad \Rightarrow \left[\begin{array}{cc|c} 1 & -1/2 & 3/2 \\ -4 & 2 & 5 \end{array}\right]$$

$$R_2 \longrightarrow R_1 + \frac{1}{4}R_2 \quad \Rightarrow \left[\begin{array}{cc|c} 1 & -1/2 & 3/2 \\ 0 & 0 & 11/4 \end{array}\right]$$

$$R_2 \longrightarrow \frac{1}{4}R_2 \quad \Rightarrow \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & \end{array} \\ \left[\begin{array}{cc|c} 1 & -1/2 & 3/2 \\ 0 & 0 & 11 \end{array}\right] \end{array}$$

So, the corresponding equations will be,

$$\begin{cases} x_1 - \frac{1}{2}x_2 = \frac{3}{2} \\ 0 = \frac{11}{4} \end{cases} \quad \{\text{not possible}\}$$

Clearly, row 2 have all zeros to left of augmented bar have non-zero entry on right.

Therefore, the corresponding system of equations is inconsistent i.e. <u>have no solutions</u>.

Thus, $X = [3,5]$ is <u>not</u> in row space of $B = \begin{bmatrix} 2 & -4 \\ -1 & 2 \end{bmatrix}$

<u>Sol: 6</u> The charaterstic polynomial of $A = \begin{bmatrix} 12 & -51 \\ 2 & -11 \end{bmatrix}$ is,

Given,

$\cancel{p_A(x) = |x}$ $\quad A = \begin{bmatrix} +12 & -51 \\ 2 & -11 \end{bmatrix}$

for $\lambda_1 = 6$, we need to solve the homogenous system,

$(\lambda I_2 - A)X = 0$ i.e. $(6I_2 - A)X = 0$

Here, $6I_2 - A = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix} - \begin{bmatrix} 12 & -51 \\ 2 & -11 \end{bmatrix}$

$= \begin{bmatrix} -6 & 51 \\ -2 & 17 \end{bmatrix}$

Now, the augmented matrix for the system is,

$[6I_2 - A \mid 0] = \left[\begin{array}{cc|c} -6 & 51 & 0 \\ -2 & 17 & 0 \end{array}\right]$

Row reducing above augmented matrix,

$R_1 \rightarrow -\frac{1}{6}R_1 \Rightarrow \left[\begin{array}{cc|c} 1 & -17/2 & 0 \\ -2 & 17 & 0 \end{array}\right]$

$R_2 \rightarrow R_1 + \frac{1}{2}R_2 \Rightarrow$

$$\begin{array}{cc|c} x_1 & x_2 & \\ \end{array}$$
$$\left[\begin{array}{cc|c} 1 & -17/2 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

we clearly see that x_2 is independent, $\therefore$
let $x_2 = b \in R$

$\therefore$ we get following for $(6I_2 - A)X = 0$,

$$\Rightarrow \begin{bmatrix} 1 & -17/2 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{cases} x_1 - \frac{17}{2}x_2 = 0 \\ 0 = 0 \end{cases}$$

Now, as $x_2 = b$, $b \in R$

$$x_1 = \frac{17}{2}b$$

$\therefore$ Solution set $= \left\{ b\left[\frac{17}{2}, 1\right] \mid b \in R \right\}$

After eliminating fractions,

$$\{ b[17, 2] \mid b \in R \}$$

$\therefore$ $E_6 = \{ b[17,2] \mid b \in R \}$ is eigenspace E_6 for eigenvalue ~~for~~ $\lambda = 6$ for given matrix A.