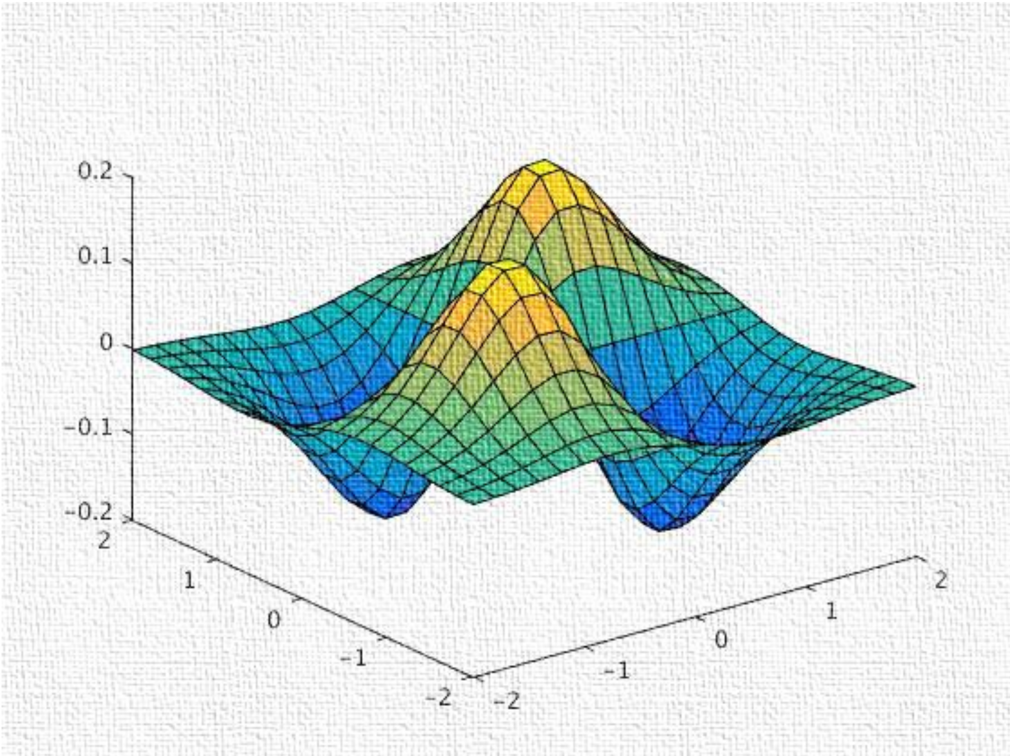
Atma Ram Sanatan Dharma College

University of Delhi



Practical File for

Differential Equations

Paper Code – 32355301

Submitted By -

Anshul Verma
Roll No. – 19/78065
B.Sc. (Hons.)
Computer Science

Submitted To -

Mrs. Shilpi Jain
Mr. Ashutosh Meena

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Practical 1
Solution of first order differential equations.

Written By Anshul Verma (19/78065) for GE-III Practicals
B.Sc.(Hons.) Computer Science

1 Find the solution of first order differential equation dy/dx = 1+4y^2 and also solve the initial value problem dy/dx = 1+4y^2, y(2)=2

```
eq: 'diff(y,x) = 1+4·y^2;
```

(%o1) $\frac{d}{d\,x}\,y=4\,y^2+1$

```
ode2(eq,y,x);
```

(%o2) $\frac{\operatorname{atan}(2\,y)}{2}=x+\%c$

```
sol: ic1(%,x=0,y=0);
```

(%o3) $\frac{\operatorname{atan}(2\,y)}{2}=x$

```
solve(sol, y);
```

(%o4) $[y=\frac{\tan(2\,x)}{2}]$

2 Solve the first order differential equation dy/dx = y*tan(x) and also solve the initial value problem dy/dx = y*tan(x), y(0)=1/(2pi)

```
eq: 'diff(y,x) = y·tan(x)+x^2;
```

(%o5) $\frac{d}{d\,x}\,y = \tan(x)\,y + x^2$

```
ode2(eq,y,x);
```

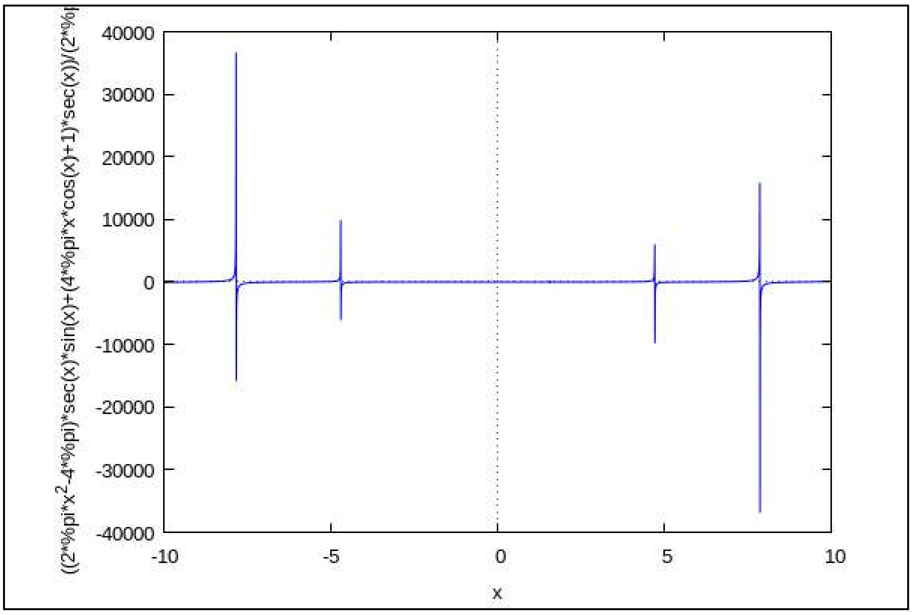
(%o6) $y = \sec(x)\left((x^2-2)\sin(x) + 2\,x\cos(x) + \%c\right)$

```
sol: ic1(%,x=0,y=1/(2·%pi));
```

(%o7) $y = ((2\,\%pi\,x^2 - 4\,\%pi)\sec(x)\sin(x) + (4\,\%pi\,x\cos(x)+1)\sec(x)) / (2\,\%pi)$

```
wxplot2d(rhs(sol),[x,-10,10]);
```

(%t8)



(%o8)

3 Find the solution of first order differential equation dy/dx = y*tanhx and also solve the initial value problem dy/dx = y*tanhx, y(1)=1

```
eq: 'diff(y,x) = y·atan(x);
```

(%o9) $\frac{d}{d\,x}\,y=\operatorname{atan}(x)\,y$

```
ode2(eq,y,x);
```

(%o10) $y=\%c\,\%e^{x\,\operatorname{atan}(x)-\frac{\log\left(x^{2}+1\right)}{2}}$

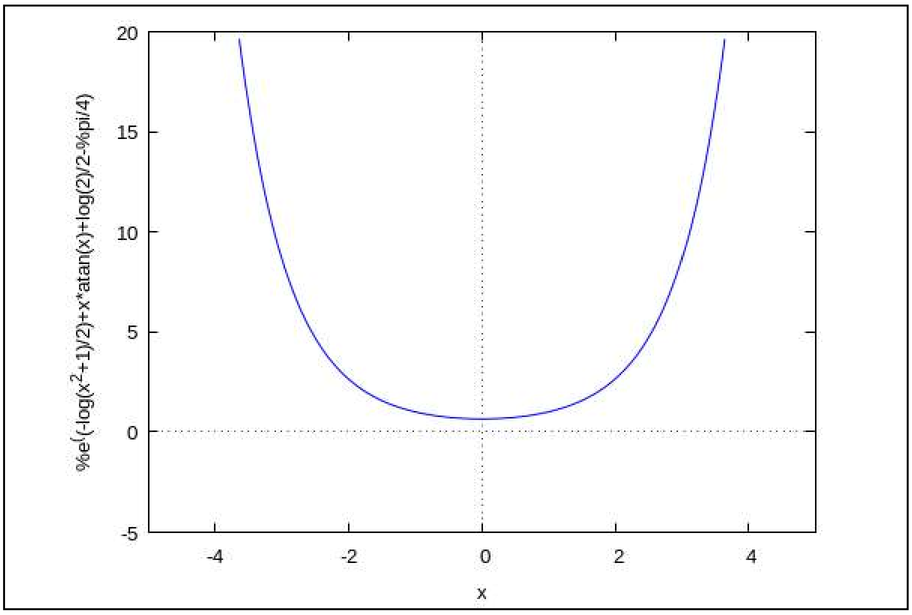
```
sol: ic1(%,x=1,y=1);
```

(%o11) $y=\%e^{-\frac{\log\left(x^{2}+1\right)}{2}+x\,\operatorname{atan}(x)+\frac{\log(2)}{2}-\frac{\%pi}{4}}$

```
wxplot2d(rhs(sol),[x,-5,5],[y,-5,20]);
```

plot2d: some values were clipped.

(%t12)



(%o12)

4 Find the solution of first order differential equation dy/dx = (pi*sin(pi*x)*cosh(3*y))/(3cos(pi*x)*sinh(3*y)

and also solve the initial value problem dy/dx = (pi*sin(pi*x)*cosh(3*y))/(3cos(pi*x)*sinh(3*y) y(1)=1

```
eq: 'diff(y,x) = (%pi·sin(%pi·x)·cosh(3·y))/(3·cos(%pi·x)·sinh(3·y));
```

(%o13) $$\frac{d}{d\,x}\,y=\frac{\%pi\,\sin(\%pi\,x)\,\cosh(3\,y)}{3\,\cos(\%pi\,x)\,\sinh(3\,y)}$$

```
ode2(eq,y,x);
```

(%o14) $$\frac{\log(\cosh(3\,y))}{\%pi}=\%c-\frac{\log(\cos(\%pi\,x))}{\%pi}$$

```
sol: ic1(%,x=1,y=1);
```

(%o15) $$\frac{\log(\cosh(3\,y))}{\%pi}=-\frac{\log(\cos(\%pi\,x))-\log(\cosh(3))-\log(-1)}{\%pi}$$

```
solve(sol,y);
```

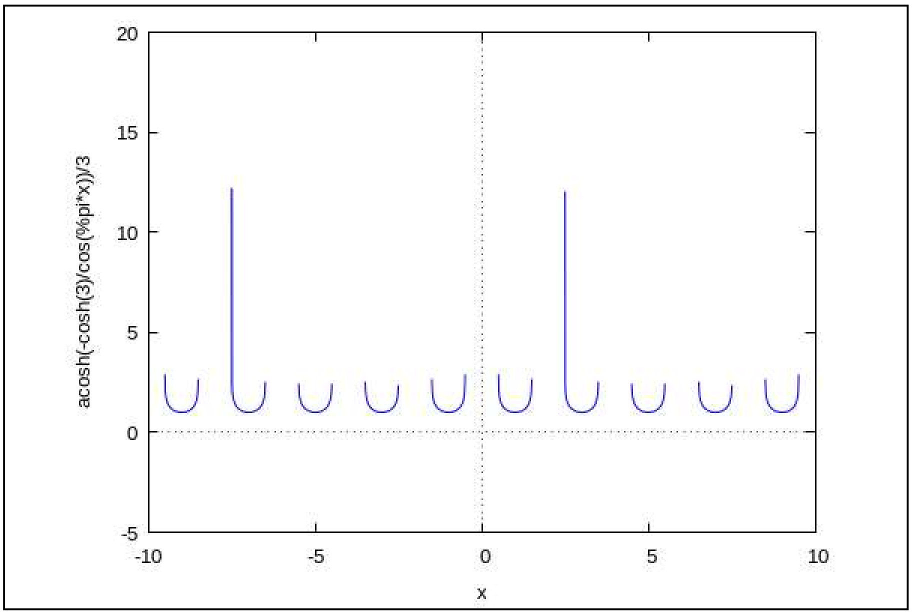
solve: using arc-trig functions to get a solution.
Some solutions will be lost.

(%o16) $$[y=\frac{\operatorname{acosh}\left(-\frac{\cosh(3)}{\cos(\%pi\,x)}\right)}{3}]$$

```
→ wxplot2d(acosh(-cosh(3)/cos(%pi·x))/3,[x,-10,10],[y,-5,20]);
```

plot2d: expression evaluates to non-numeric value somewhere in plottin

(%t17)



(%o17)

Practical 2 Plotting of second order solution family of differential equation.

Written By Anshul Verma (19/78065) for GE-III Practicals
B.Sc.(Hons.) Computer Science

1 *Find the solution of y"-4*y'+40*y=0 by assigning different values to k1 and k2 and plot the solution.*

```
→ kill(all)$depends(y,x);
(%o1) [y(x)]
```

```
eqn:diff(y,x,2)-4·diff(y,x)+40·y=0;
sol:ode2(eqn,y,x);
gr1:ev(sol,%k1=0,%k2=1);
gr2:ev(sol,%k1=1,%k2=0);
gr3:ev(sol,%k1=1,%k2=1);
gr4:ev(sol,%k1=1,%k2=-1);
gr5:ev(sol,%k1=-1,%k2=1);
gr6:ev(sol,%k1=-1,%k2=-1);
wxplot2d(
    [rhs(gr1),rhs(gr2),rhs(gr3),rhs(gr4),rhs(gr5),rhs(gr6)],
    [x,-1,1],[y,-10,10]);
```

(%o2) $\frac{d^2}{d x^2} y - 4 \left(\frac{d}{d x} y\right) + 40 y = 0$

(%o3) $y = \%e^{2x} (\%k1 \sin(6x) + \%k2 \cos(6x))$

(%o4) $y = \%e^{2x} \cos(6x)$

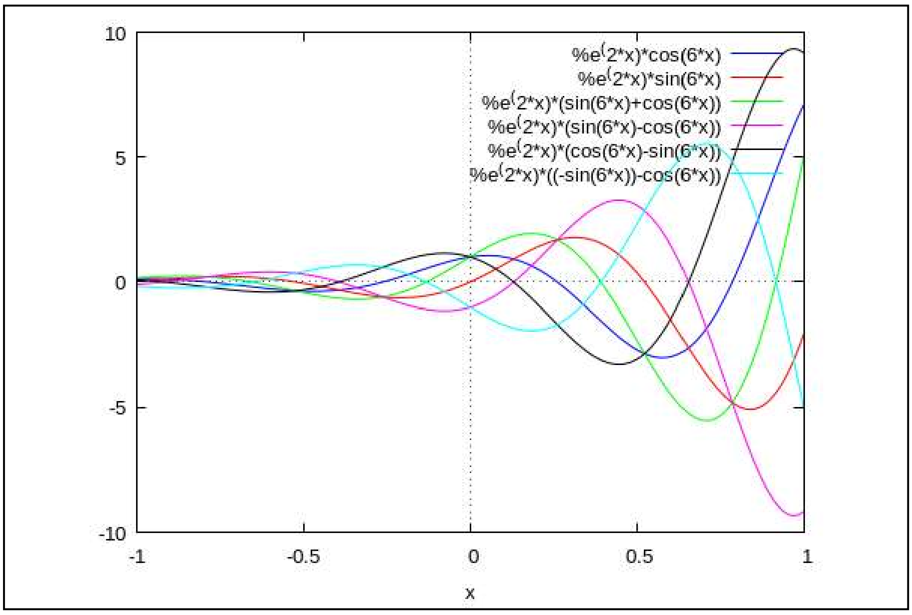
(%o5) $y = \%e^{2x} \sin(6x)$

(%o6) $y = \%e^{2x} (\sin(6x) + \cos(6x))$

(%o7) $y = \%e^{2x} (\sin(6x) - \cos(6x))$

(%o8) $y = \%e^{2x} (\cos(6x) - \sin(6x))$

(%o9) $y = \%e^{2x} (-\sin(6x) - \cos(6x))$

(%t10) 

(%o10)

2 Find the solution of y"-9*pi*y'+6*y=0 with initial conditions y(0)=0,y'(0)=b an plot the solution.

```
kill(all)$depends(y,x);
```

(%o1) $[y(x)]$

```
eqn:diff(y,x,2)-9·%pi·diff(y,x)+6·y=0;
sol:ode2(eqn,y,x);
solx:ic2(sol,x=0,y=0,diff(y,x)=b);
```

(%o2) $$\frac{d^2}{d\,x^2}\,y-9\,\%pi\left(\frac{d}{d\,x}\,y\right)+6\,y=0$$

(%o3) $$y=\%k1\,\%e^{\frac{(\sqrt{81\,\%pi^2-24}+9\,\%pi)\,x}{2}}+\%k2\,\%e^{\frac{(9\,\%pi-\sqrt{81\,\%pi^2-24})\,x}{2}}$$

(%o4) $$y=\frac{b\,\%e^{\frac{(\sqrt{81\,\%pi^2-24}+9\,\%pi)\,x}{2}}}{\sqrt{81\,\%pi^2-24}}-\frac{b\,\%e^{\frac{(9\,\%pi-\sqrt{81\,\%pi^2-24})\,x}{2}}}{\sqrt{81\,\%pi^2-24}}$$

```
gr1:ev(solx,b=-10);
gr2:ev(solx,b=-7);
gr3:ev(solx,b=-3);
gr4:ev(solx,b=0);
gr5:ev(solx,b=3);
gr6:ev(solx,b=7);
gr7:ev(solx,b=10);
```

(%o5) $$y=\frac{10\ \%e^{\frac{(9\ \%pi-\sqrt{81\ \%pi^2-24})\ x}{2}}}{\sqrt{81\ \%pi^2-24}}-\frac{10\ \%e^{\frac{(\sqrt{81\ \%pi^2-24}+9\ \%pi)\ x}{2}}}{\sqrt{81\ \%pi^2-24}}$$

(%o6) $$y=\frac{7\ \%e^{\frac{(9\ \%pi-\sqrt{81\ \%pi^2-24})\ x}{2}}}{\sqrt{81\ \%pi^2-24}}-\frac{7\ \%e^{\frac{(\sqrt{81\ \%pi^2-24}+9\ \%pi)\ x}{2}}}{\sqrt{81\ \%pi^2-24}}$$

(%o7) $$y=\frac{3\ \%e^{\frac{(9\ \%pi-\sqrt{81\ \%pi^2-24})\ x}{2}}}{\sqrt{81\ \%pi^2-24}}-\frac{3\ \%e^{\frac{(\sqrt{81\ \%pi^2-24}+9\ \%pi)\ x}{2}}}{\sqrt{81\ \%pi^2-24}}$$

(%o8) $$y=0$$

(%o9) $$y=\frac{3\ \%e^{\frac{(\sqrt{81\ \%pi^2-24}+9\ \%pi)\ x}{2}}}{\sqrt{81\ \%pi^2-24}}-\frac{3\ \%e^{\frac{(9\ \%pi-\sqrt{81\ \%pi^2-24})\ x}{2}}}{\sqrt{81\ \%pi^2-24}}$$

(%o10) $$y=\frac{7\ \%e^{\frac{(\sqrt{81\ \%pi^2-24}+9\ \%pi)\ x}{2}}}{\sqrt{81\ \%pi^2-24}}-\frac{7\ \%e^{\frac{(9\ \%pi-\sqrt{81\ \%pi^2-24})\ x}{2}}}{\sqrt{81\ \%pi^2-24}}$$

(%o11) $$y=\frac{10\ \%e^{\frac{(\sqrt{81\ \%pi^2-24}+9\ \%pi)\ x}{2}}}{\sqrt{81\ \%pi^2-24}}-\frac{10\ \%e^{\frac{(9\ \%pi-\sqrt{81\ \%pi^2-24})\ x}{2}}}{\sqrt{81\ \%pi^2-24}}$$

```
→ wxplot2d(
      [rhs(gr1),rhs(gr2),rhs(gr3),rhs(gr4),rhs(gr5),rhs(gr6),rhs(7)],
      [x,-0.5,0.2],[y,-10,10],
      [legend,"1","2","3","4","5","6","7"]);
```

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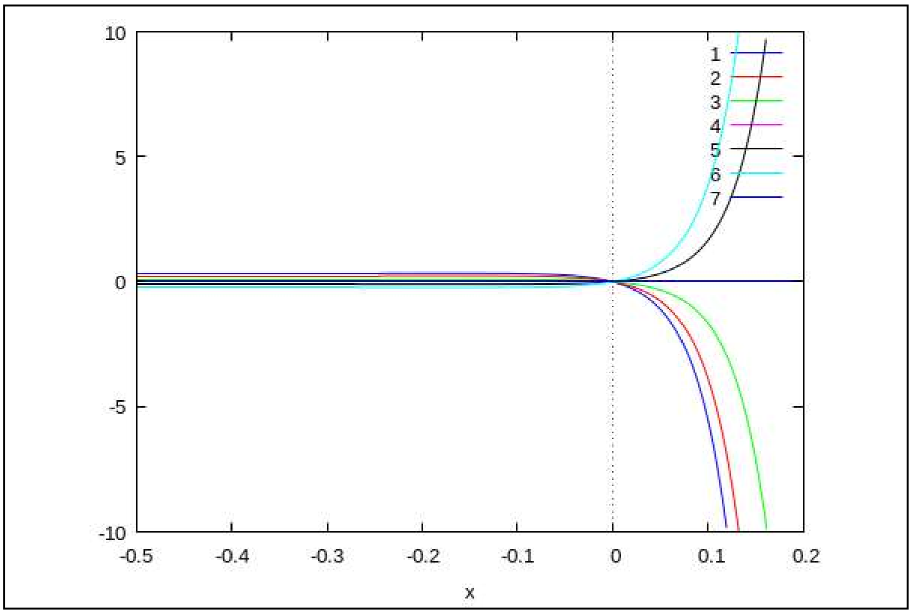
(%t12)



(%o12)

3 Find the solution of 10y"+50*y'-65*y=0 with initial conditions y(0)=b,y'(0)=1.5 and plot the solution.

```
→ kill(all)$depends(y,x);
```

(%o1) [y(x)]

```
eqn:10·diff(y,x,2)+50·diff(y,x)-65·y=0;
sol:ode2(eqn,y,x);
solx:ic2(sol,x=0,y=b,diff(y,x)=1.5);
```

(%o2) $$10\left(\frac{d^2}{d\,x^2}\,y\right)+50\left(\frac{d}{d\,x}\,y\right)-65\,y=0$$

(%o3) $$y=\%k1\,\%e^{\frac{(\sqrt{51}-5)\,x}{2}}+\%k2\,\%e^{\frac{(-\sqrt{51}-5)\,x}{2}}$$

rat: replaced 1.5 by 3/2 = 1.5

(%o4) $$y=\frac{((5\sqrt{51}+51)\,b+3\sqrt{51})\,\%e^{\frac{(\sqrt{51}-5)\,x}{2}}}{102}-\frac{((5\sqrt{51}-51)\,b+3\sqrt{51})\,\%e^{\frac{(-\sqrt{51}-5)\,x}{2}}}{102}$$

```
gr1:ev(solx,b=-3);
gr2:ev(solx,b=-2);
gr3:ev(solx,b=-1);
gr4:ev(solx,b=0);
gr5:ev(solx,b=1);
gr6:ev(solx,b=2);
gr7:ev(solx,b=3);
```

(%o5) $$y=\frac{(3\sqrt{51}-3(5\sqrt{51}+51))\,\%e^{\frac{(\sqrt{51}-5)x}{2}}}{102}-\frac{(3\sqrt{51}-3(5\sqrt{51}-51))\,\%e^{\frac{(-\sqrt{51}-5)x}{2}}}{102}$$

(%o6) $$y=\frac{(3\sqrt{51}-2(5\sqrt{51}+51))\,\%e^{\frac{(\sqrt{51}-5)x}{2}}}{102}-\frac{(3\sqrt{51}-2(5\sqrt{51}-51))\,\%e^{\frac{(-\sqrt{51}-5)x}{2}}}{102}$$

(%o7) $$y=\frac{(-2\sqrt{51}-51)\,\%e^{\frac{(\sqrt{51}-5)x}{2}}}{102}-\frac{(51-2\sqrt{51})\,\%e^{\frac{(-\sqrt{51}-5)x}{2}}}{102}$$

(%o8) $$y=\frac{\sqrt{51}\,\%e^{\frac{(\sqrt{51}-5)x}{2}}}{34}-\frac{\sqrt{51}\,\%e^{\frac{(-\sqrt{51}-5)x}{2}}}{34}$$

(%o9) $$y=\frac{(8\sqrt{51}+51)\,\%e^{\frac{(\sqrt{51}-5)x}{2}}}{102}-\frac{(8\sqrt{51}-51)\,\%e^{\frac{(-\sqrt{51}-5)x}{2}}}{102}$$

(%o10) $$y=\frac{(2(5\sqrt{51}+51)+3\sqrt{51})\,\%e^{\frac{(\sqrt{51}-5)x}{2}}}{102}-\frac{(2(5\sqrt{51}-51)+3\sqrt{51})\,\%e^{\frac{(-\sqrt{51}-5)x}{2}}}{102}$$

(%o11) $$y=\frac{(3(5\sqrt{51}+51)+3\sqrt{51})\,\%e^{\frac{(\sqrt{51}-5)x}{2}}}{102}-\frac{(3(5\sqrt{51}-51)+3\sqrt{51})\,\%e^{\frac{(-\sqrt{51}-5)x}{2}}}{102}$$

```
→ wxplot2d(
      [rhs(gr1),rhs(gr2),rhs(gr3),rhs(gr4),rhs(gr5),rhs(gr6),rhs(7)],
      [x,-2,5],[y,-10,10],
      [legend,"1","2","3","4","5","6","7"]);
```

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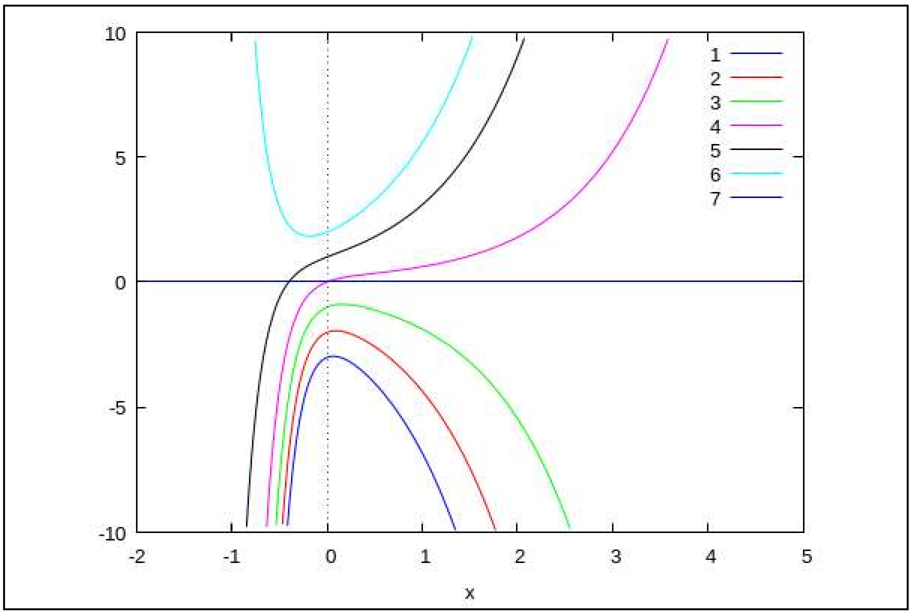
(%t12)



(%o12)

4 Find the solution of x^2y"-x*y'-24*y=0 with initial conditions y(1)=b,y'(1)=-21 and plot the solution.

```
→ kill(all)$depends(y,x);
```

(%o1) [y (x)]

```
eqn:x^2·diff(y,x,2)-x·diff(y,x)-24·y=0;
sol:ode2(eqn,y,x);
solx:ic2(sol,x=1,y=b,diff(y,x)=-21);
gr1:ev(solx,b=-3);
gr2:ev(solx,b=-2);
gr3:ev(solx,b=-1);
gr4:ev(solx,b=0);
gr5:ev(solx,b=1);
gr6:ev(solx,b=2);
gr7:ev(solx,b=3);
wxplot2d(
    [rhs(gr1),rhs(gr2),rhs(gr3),rhs(gr4),rhs(gr5),rhs(gr6),rhs(7)],
    [x,-2,2],[y,-20,40],
    [legend,"1","2","3","4","5","6","7"]);
```

(%o2) $x^2\left(\frac{d^2}{d x^2} y\right)-x\left(\frac{d}{d x} y\right)-24 y=0$

(%o3) $y=\%k1\, x^6+\frac{\%k2}{x^4}$

(%o4) $y=\frac{(4 b-21)\, x^6}{10}+\frac{6 b+21}{10 x^4}$

(%o5) $y=\frac{3}{10 x^4}-\frac{33 x^6}{10}$

(%o6) $y=\frac{9}{10 x^4}-\frac{29 x^6}{10}$

(%o7) $y=\frac{3}{2 x^4}-\frac{5 x^6}{2}$

(%o8) $y=\frac{21}{10 x^4}-\frac{21 x^6}{10}$

(%o9) $y=\frac{27}{10 x^4}-\frac{17 x^6}{10}$

(%o10) $y=\frac{33}{10 x^4}-\frac{13 x^6}{10}$

(%o11) $y=\frac{39}{10 x^4}-\frac{9 x^6}{10}$

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Practical 3
Plot the family of solution of third order differential equation.

Written By Anshul Verma (19/78065) for GE-III Practicals
B.Sc. (Hons) Computer Science

1 Find the solution of the third-order differential equation Y'''-Y''+100*y'-100*Y=0 with initial conditions y(0)=k3,y'(0)=k2,y"(0)=k1. Also, plot the solutions.

```
kill(all)$
eqn:diff(y(x),x,3)-diff(y(x),x,2)+100·diff(y(x),x)-100·y(x)=0;
atvalue(diff(y(x),x,2),x=0,%k1);
atvalue(diff(y(x),x),x=0,%k2);
atvalue(y(x),x=0,%k3);
sol:desolve(eqn,y(x));
```

(%o1) $\frac{d^3}{d x^3} y(x) - \frac{d^2}{d x^2} y(x) + 100\left(\frac{d}{d x} y(x)\right) - 100\, y(x) = 0$

(%o2) %k1

(%o3) %k2

(%o4) %k3

(%o5) $y(x) = -\frac{(100\,\%k3 - 101\,\%k2 + \%k1)\sin(10x)}{1010} + \frac{(\%k3 - \%k1)\cos(10x)}{101} + \frac{(100\,\%k3 + \%k1)\,\%e^{x}}{101}$

```
gr1:ev(sol,%k1=5,%k2=0,%k3=0);
gr2:ev(sol,%k1=0,%k2=5,%k3=0);
gr3:ev(sol,%k1=0,%k2=0,%k3=5);
gr4:ev(sol,%k1=5,%k2=5,%k3=5);
wxplot2d([rhs(gr1),rhs(gr2),rhs(gr3),rhs(gr4)],[x,-5,4],[y,-1,4]);
```

(%o6) $y(x)=-\frac{\sin(10x)}{202}-\frac{5\cos(10x)}{101}+\frac{5\%e^{x}}{101}$

(%o7) $y(x)=\frac{\sin(10x)}{2}$

(%o8) $y(x)=-\frac{50\sin(10x)}{101}+\frac{5\cos(10x)}{101}+\frac{500\%e^{x}}{101}$

(%o9) $y(x)=5\%e^{x}$

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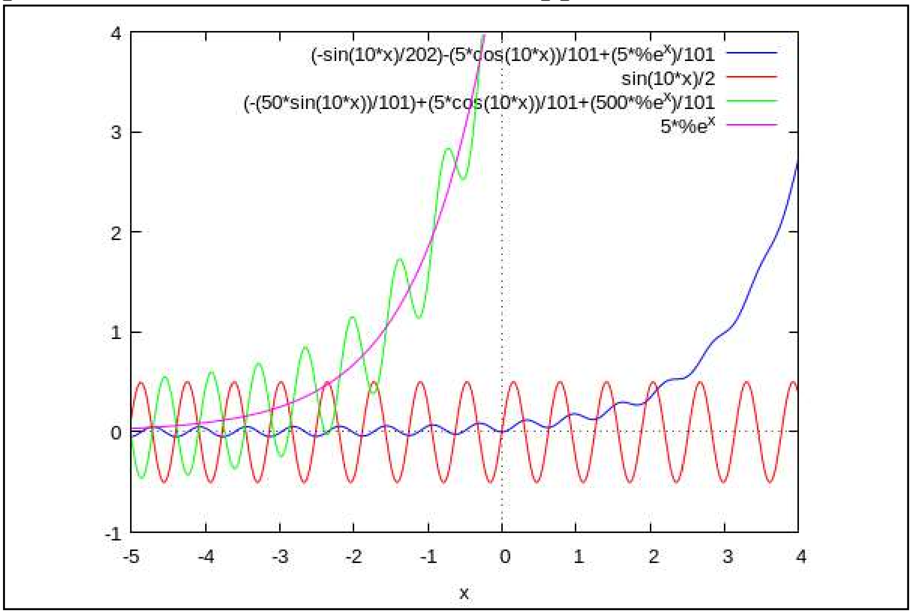
plot2d: some values were clipped.

(%t10) 

(%o10)

2 Find the solution of the third-order differential equation Y'''+3.2*Y''+4.81*y'=0 with initial conditions y(0)=a,y'(0)=-4.6,y"(0)=9.91. Also, plot the solutions.

```
kill(all)$
eq:4·'diff(y(x),x,3)+3.2·'diff(y(x),x,2)+4.81·'diff(y(x),x)=0;
sol:desolve(eq,y(x));
solx:ev(sol,at(diff(y(x),x,2),x=0)=-9.91,at(diff(y(x),x),x=0)=-4.6,y(0
```

(%o1) $$4\left(\frac{d^3}{d\,x^3}\,y(x)\right)+3.2\left(\frac{d^2}{d\,x^2}\,y(x)\right)+4.81\left(\frac{d}{d\,x}\,y(x)\right)=0$$

rat: replaced 4.81 by 481/100 = 4.81
rat: replaced 3.2 by 16/5 = 3.2
rat: replaced 4.81 by 481/100 = 4.81
rat: replaced 3.2 by 16/5 = 3.2
rat: replaced 4.81 by 481/100 = 4.81
rat: replaced 3.2 by 16/5 = 3.2

(%o2) $$y(x)=\frac{\%e^{-\frac{2x}{5}}}{400}+\frac{400\left(\left.\frac{d^2}{d\,x^2}\,y(x)\right|_{x=0}\right)+320\left(\left.\frac{d}{d\,x}\,y(x)\right|_{x=0}\right)+481\,y(0)}{481}$$

(%o3) $$y(x)=\left(\%e^{-\frac{2x}{5}}\left(4520.582120582121\cos\left(\frac{\sqrt{417}\,x}{20}\right)-\frac{635.3430353430333\sin\left(\frac{\sqrt{417}\,x}{20}\right)}{\sqrt{417}}\right)\right)/400+\frac{481\,a-5436.0}{481}$$

```
gr1:ev(solx,a=-3.4)$
gr2:ev(solx,a=-2)$
gr3:ev(solx,a=-1);
gr4:ev(solx,a=0);
gr5:ev(solx,a=2);
gr6:ev(solx,a=3.4);
wxplot2d([rhs(gr1),rhs(gr2),rhs(gr3),rhs(gr4),rhs(gr5),rhs(gr6)],
[x,-2,10],[y,-20,5],[legend,"1","2","3","4","5","6"]);
```

(%o6) $y(x)=\left(\%e^{-\frac{2x}{5}}\left(4520.582120582121\cos\left(\frac{\sqrt{417}\,x}{20}\right)-\frac{635.3430353430333\sin\left(\frac{\sqrt{417}\,x}{20}\right)}{\sqrt{417}}\right)\right)/400-12.3014553014553$

(%o7) $y(x)=\left(\%e^{-\frac{2x}{5}}\left(4520.582120582121\cos\left(\frac{\sqrt{417}\,x}{20}\right)-\frac{635.3430353430333\sin\left(\frac{\sqrt{417}\,x}{20}\right)}{\sqrt{417}}\right)\right)/400-11.3014553014553$

(%o8) $y(x)=\left(\%e^{-\frac{2x}{5}}\left(4520.582120582121\cos\left(\frac{\sqrt{417}\,x}{20}\right)-\frac{635.3430353430333\sin\left(\frac{\sqrt{417}\,x}{20}\right)}{\sqrt{417}}\right)\right)/400-9.301455301455303$

(%o9) $y(x)=\left(\%e^{-\frac{2x}{5}}\left(4520.582120582121\cos\left(\frac{\sqrt{417}\,x}{20}\right)-\frac{635.3430353430333\sin\left(\frac{\sqrt{417}\,x}{20}\right)}{\sqrt{417}}\right)\right)/400-7.901455301455303$

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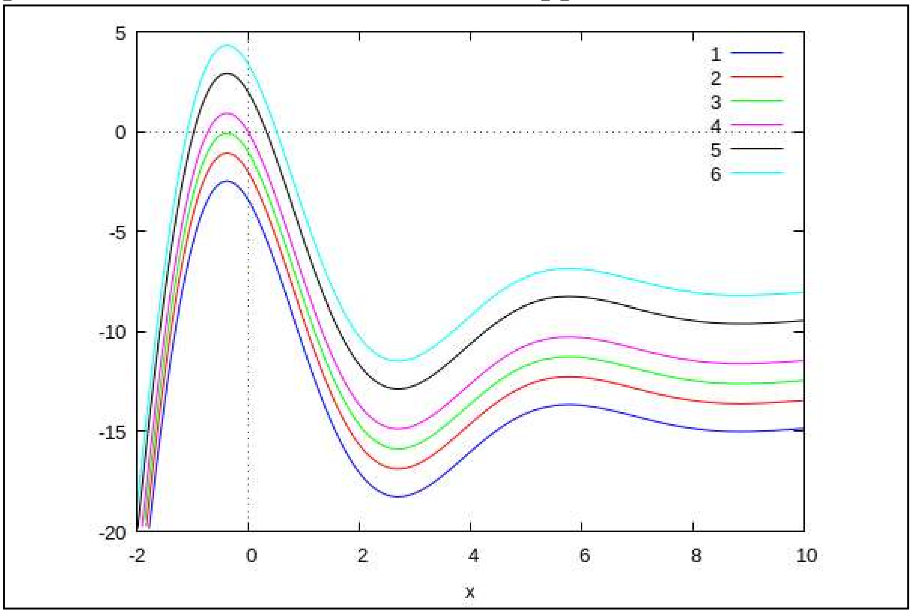
(%t10)



3 Find the solution of the third-order differential equation Y'''+7.5*Y''+14.25*y'-9.125*Y=0 with initial conditions y(0)=a,y'(0)=-54.97,y"(0)=257.51. Also, plot the solutions.

```
kill(all)$
eq:'diff(y(x),x,3)+7.5·'diff(y(x),x,2)+14.25·'diff(y(x),x)-9.125·y(x)=
sol:desolve(eq,y(x));
solx:ev(sol,at(diff(y(x),x,2),x=0)=257.51,at(diff(y(x),x),x=0)=-54.97,
```

(%o1) $\frac{d^3}{d x^3} y(x)+7.5\left(\frac{d^2}{d x^2} y(x)\right)+14.25\left(\frac{d}{d x} y(x)\right)-9.125\, y(x)=0$

rat: replaced -9.125 by -73/8 = -9.125
rat: replaced 14.25 by 57/4 = 14.25
rat: replaced 7.5 by 15/2 = 7.5
rat: replaced -9.125 by -73/8 = -9.125
rat: replaced 14.25 by 57/4 = 14.25
rat: replaced 7.5 by 15/2 = 7.5
rat: replaced -9.125 by -73/8 = -9.125
rat: replaced 14.25 by 57/4 = 14.25
rat: replaced 7.5 by 15/2 = 7.5

(%o2) $y(x)=\frac{\%e^{-4 x}}{4}+\frac{\%e^{x/2}\left(4\left(\left.\frac{d^2}{d x^2} y(x)\right|_{x=0}\right)+32\left(\left.\frac{d}{d x} y(x)\right|_{x=0}\right)+73\, y(0)\right)}{90}$

(%o3) $y(x)=\left(\%e^{-4 x}\left(\frac{\left(\frac{32(-34 a-1458.0)}{45}-\frac{8(-199 a-2498.399999999998)}{45}\right)\sin\left(\frac{3 x}{2}\right)}{12}-\frac{(-34 a}{}\right.\right.$

$/4+\frac{(73 a-729.0)\, \%e^{x/2}}{90}$

```
gr1:ev(solx,a=-10.5);
gr2:ev(solx,a=-5.25);
gr3:ev(solx,a=-1.5);
gr4:ev(solx,a=0);
gr5:ev(solx,a=5.25);
gr6:ev(solx,a=10.05);
wxplot2d([rhs(gr1),rhs(gr2),rhs(gr3),rhs(gr4),rhs(gr5),rhs(gr6)],
[x,-0.5,3],[y,-15,15],[legend,"1","2","3","4","5","6"]);
```

(%o4) $y(x)=$
$$\left(\%e^{-4x}\left(24.46666666666667\cos\left(\frac{3x}{2}\right)-59.18666666666667\sin\left(\frac{3x}{2}\right)\right)\right)/4-16.61666666666666\,\%e^{x/2}$$

(%o5) $y(x)=$
$$\left(\%e^{-4x}\left(28.43333333333333\cos\left(\frac{3x}{2}\right)-54.28666666666667\sin\left(\frac{3x}{2}\right)\right)\right)/4-12.35833333333333\,\%e^{x/2}$$

(%o6) $y(x)=$
$$\left(\%e^{-4x}\left(31.26666666666667\cos\left(\frac{3x}{2}\right)-50.78666666666667\sin\left(\frac{3x}{2}\right)\right)\right)/4-9.316666666666667\,\%e^{x/2}$$

(%o7) $y(x)=$
$$\frac{\%e^{-4x}\left(32.4\cos\left(\frac{3x}{2}\right)-49.38666666666669\sin\left(\frac{3x}{2}\right)\right)}{4}-8.1\,\%e^{x/2}$$

(%o8) $y(x)=$
$$\left(\%e^{-4x}\left(36.36666666666667\cos\left(\frac{3x}{2}\right)-44.48666666666669\sin\left(\frac{3x}{2}\right)\right)\right)/4-3.841666666666667\,\%e^{x/2}$$

(%o9) $y(x)=$
$$\left(\%e^{-4x}\left(39.99333333333334\cos\left(\frac{3x}{2}\right)-40.00666666666672\sin\left(\frac{3x}{2}\right)\right)\right)/4+0.05166666666666768\,\%e^{x/2}$$

plot2d: some values were clipped.
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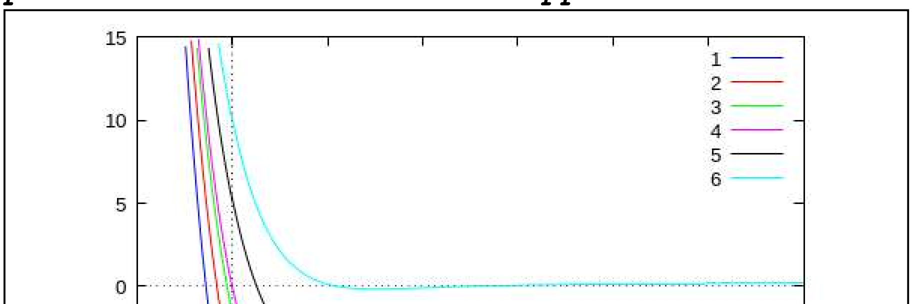
plot2d: some values were clipped.



4 Find the solution of the third-order differential equation 0.45*Y'''-0.165*Y''+0.0045*y'-0.00175*Y=0 with initial conditions y(0)=a,y'(0)=-2.82,y"(0)=2.0485. Also, plot the solutions.

```
kill(all)$
eq:0.45·'diff(y(x),x,3)+0.165·'diff(y(x),x,2)+0.0045·'diff(y(x),x)-0.0
sol:desolve(eq,y(x));
solx:ev(sol,at(diff(y(x),x,2),x=0)=2.0485,at(diff(y(x),x),x=0)=-2.82,y
gr1:ev(solx,a=-17.4);
gr2:ev(solx,a=-10.25);
gr3:ev(solx,a=-6.54);
gr4:ev(solx,a=0);
gr5:ev(solx,a=10.25);
gr6:ev(solx,a=17.4);
wxplot2d([rhs(gr1),rhs(gr2),rhs(gr3),rhs(gr4),rhs(gr5),rhs(gr6)],
[x,-5,20],[y,-20,15],[legend,"1","2","3","4","5","6"]);
```

(%o1) $0.45\left(\frac{d^3}{d\,x^3}\,y(x)\right)+0.165\left(\frac{d^2}{d\,x^2}\,y(x)\right)+0.0045\left(\frac{d}{d\,x}\,y(x)\right)-0.00175\,y(x)=0$

rat: replaced -0.00175 by -7/4000 = -0.00175
rat: replaced 0.0045 by 9/2000 = 0.0045
rat: replaced 0.165 by 33/200 = 0.165
rat: replaced 0.45 by 9/20 = 0.45
rat: replaced -0.00175 by -7/4000 = -0.00175
rat: replaced 0.0045 by 9/2000 = 0.0045
rat: replaced 0.165 by 33/200 = 0.165
rat: replaced 0.45 by 9/20 = 0.45
rat: replaced -0.00175 by -7/4000 = -0.00175
rat: replaced 0.0045 by 9/2000 = 0.0045
rat: replaced 0.165 by 33/200 = 0.165
rat: replaced 0.45 by 9/20 = 0.45

(%o2) $$y(x)=\frac{\%e^{-\frac{x}{10}}}{300}-\frac{\%e^{-\frac{x}{6}}\left(2700\left(\left.\frac{d^2}{d\,x^2}\,y(x)\right|_{x=0}\right)+540\left(\left.\frac{d}{d\,x}\,y(x)\right|_{x=0}\right)-63\,y(0)\right)}{78}$$

(%o3) $$y(x)=(\%e^{-\frac{x}{10}}\,(\left(\left(\frac{600\,(675\,a-4317.749999999998)}{13}-\frac{60\,(750\,a+200407.5)}{13}\right)\sinh\left(\frac{x}{\sqrt{30}}\right)\right)/(20\,\sqrt{30})+\frac{(750\,a+200407.5)\cosh\left(\frac{x}{\sqrt{30}}\right)}{13}))/300-\frac{(4008.15-63\,a)\,\%e^{-\frac{x}{6}}}{78}$$

(%o4) $y(x)=$

$$\Bigg(\%e^{-\frac{x}{10}}\Bigg(\quad\Bigg(\frac{x}{}\Bigg)\quad\frac{80304.23076923075\,\sinh\left(\frac{x}{\sqrt{30}}\right)}{}\Bigg)\Bigg)$$

Practical 4 Solution of differential equation by variation of parameter method.

Written By Anshul Verma (19/78065) for GE-III Practicals
B.Sc. (Hons) Computer Science

1 *Solve the diferential equation y''+4y'+4y=e^(-2x)*sin(x) using variation of parameter.*

```
kill(all)$depends(y,x)$
eq:diff(y,x,2)+4·diff(y,x)+4·y=0;
y:ode2(eq,y,x);
yc:second(y);
a:second(first(second(y)));
b:second(second(second(y)));
m:matrix([a,b],[diff(a,x),diff(b,x)]);
W:determinant(m);
yp:-a·integrate((b·(%e^(-2·x)·sin(x)))/W,x)+b·integrate((a·(%e^(-2·x) ·
soll:yc+yp;
```

(%o2) $\frac{d^2}{d x^2} y+4\left(\frac{d}{d x} y\right)+4 y=0$

(%o3) $y=(\%k2\ x+\%k1)\ \%e^{-2 x}$

(%o4) $(\%k2\ x+\%k1)\ \%e^{-2 x}$

(%o5) $\%k1$

(%o6) $-2 x$

(%o7) $\begin{bmatrix} \%k1 & -2 x \\ 0 & -2 \end{bmatrix}$

(%o8) $-2\ \%k1$

(%o9) $\frac{\%e^{-2 x}\left((10 x+3) \sin(x)+(5 x+4) \cos(x)\right)}{25}+\frac{x\ \%e^{-2 x}\left(-2 \sin(x)-\cos(x)\right)}{5}$

(%o10) $\frac{\%e^{-2 x}\left((10 x+3) \sin(x)+(5 x+4) \cos(x)\right)}{25}+\frac{x\ \%e^{-2 x}\left(-2 \sin(x)-\cos(x)\right)}{5}+(\%k2\ x+\%k1)\ \%e^{-2 x}$

```
trigsimp(yp);
```

(%o11) $\frac{\%e^{-2 x}\left(3 \sin(x)+4 \cos(x)\right)}{25}$

1.1 Verification by Another method

```
kill(all)$depends(y,x)$
eq:diff(y,x,2)+4·diff(y,x)+4·y=%e^(-2·x)·sin(x);
sol:ode2(eq,y,x);
```

(%o2) $\frac{d^2}{d x^2} y+4\left(\frac{d}{d x} y\right)+4 y=\%e^{-2 x} \sin(x)$

(%o3) $y=(\%k2\ x+\%k1)\ \%e^{-2 x}-\%e^{-2 x} \sin(x)$

2 Solve the diferential equation y''-16y=19.2*e^(4x)+60*e^x using variation of parameter.

```
kill(all)$depends(y,x)$
eq:diff(y,x,2)-16·y=0;
y:ode2(eq,y,x);
yc:second(y);
a:second(first(second(y)));
b:second(second(second(y)));
m:matrix([a,b],[diff(a,x),diff(b,x)]);
W:determinant(m);
yp:-a·integrate((b·(19.2·%e^(4·x)+60·%e^x))/W,x)+b·integrate((a·(19.2·
soll:yc+yp;
```

(%o2) $$\frac{d^2}{d\,x^2}\,y-16\,y=0$$

(%o3) $$y=\%k1\,\%e^{4\,x}+\%k2\,\%e^{-4\,x}$$

(%o4) $$\%k1\,\%e^{4\,x}+\%k2\,\%e^{-4\,x}$$

(%o5) $$\%e^{4\,x}$$

(%o6) $$\%e^{-4\,x}$$

(%o7) $$\begin{bmatrix}\%e^{4\,x} & \%e^{-4\,x}\\ 4\,\%e^{4\,x} & -4\,\%e^{-4\,x}\end{bmatrix}$$

(%o8) $$-8$$

(%o9) $$\frac{(19.2\,x-20\,\%e^{-3\,x})\,\%e^{4\,x}}{8}-\frac{\%e^{-4\,x}\,(2.4\,\%e^{8\,x}+12\,\%e^{5\,x})}{8}$$

(%o10) $$-\frac{\%e^{-4\,x}\,(2.4\,\%e^{8\,x}+12\,\%e^{5\,x})}{8}+\frac{(19.2\,x-20\,\%e^{-3\,x})\,\%e^{4\,x}}{8}+\%k1\,\%e^{4\,x}+\%k2\,\%e^{-4\,x}$$

```
trigsimp(yp);
```

rat: replaced 19.2 by 96/5 = 19.2
rat: replaced 2.4 by 12/5 = 2.4

(%o11) $$\frac{(24\,x-3)\,\%e^{4\,x}-40\,\%e^{x}}{10}$$

2.1 Verification by Another method

```
kill(all)$depends(y,x)$
eq:diff(y,x,2)-16·y=19.2·%e^(4·x)+60·%e^x;
sol:ode2(eq,y,x);
```

(%o2) $$\frac{d^2}{d x^2} y - 16 y = 19.2 \%e^{4 x} + 60 \%e^{x}$$

rat: replaced -19.2 by -96/5 = -19.2

(%o3) $$y = \frac{(24 x - 3) \%e^{4 x} - 40 \%e^{x}}{10} + \%k1 \%e^{4 x} + \%k2 \%e^{-4 x}$$

3 Solve the diferential equation y''+9y=cos(x)+1/3*cos(3*x) using variation of parameter.

```
kill(all)$depends(y,x)$
eq:diff(y,x,2)+9·y=0;
y:ode2(eq,y,x);
yc:second(y);
a:second(first(second(y)));
b:second(second(second(y)));
m:matrix([a,b],[diff(a,x),diff(b,x)]);
W:determinant(m);
yp:-a·integrate((b·(cos(x)+1/3·cos(3·x)))/W,x)+b·integrate((a·(cos(x)+
soll:yc+yp;
```

(%o2) $\frac{d^2}{d x^2} y + 9 y = 0$

(%o3) $y = \%k1 \sin(3 x) + \%k2 \cos(3 x)$

(%o4) $\%k1 \sin(3 x) + \%k2 \cos(3 x)$

(%o5) $\sin(3 x)$

(%o6) $\cos(3 x)$

(%o7) $\begin{bmatrix} \sin(3 x) & \cos(3 x) \\ 3 \cos(3 x) & -3 \sin(3 x) \end{bmatrix}$

(%o8) $-3 \sin(3 x)^2 - 3 \cos(3 x)^2$

(%o9) $\cos(3 x) \left(\frac{\cos(4 x) + 2 \cos(2 x)}{24} - \frac{\sin(3 x)^2}{54} \right) - \sin(3 x) \left(\frac{-\frac{\tan(3 x)}{18 \tan(3 x)^2 + 18} - \frac{x}{6}}{3} - \frac{\sin(4 x) + 2 \sin(2 x)}{24} \right)$

(%o10) $-\sin(3 x) \left(\frac{-\frac{\tan(3 x)}{18 \tan(3 x)^2 + 18} - \frac{x}{6}}{3} - \frac{\sin(4 x) + 2 \sin(2 x)}{24} \right) + \cos(3 x) \left(\frac{\cos(4 x) + 2 \cos(2 x)}{24} - \frac{\sin(3 x)^2}{54} \right) + \%k1 \sin(3 x) + \%k2 \cos(3 x)$

```
trigsimp(yp);
```

(%o11)

$(3 \sin(3 x) \sin(4 x) + 3 \cos(3 x) \cos(4 x) + (6 \sin(2 x) + 4 x) \sin(3 x) + 6 \cos$
$/ 72$

3.1 Verification by Another method

```
kill(all)$depends(y,x)$
eq:diff(y,x,2)+9·y=cos(x)+1/3·cos(3·x);
sol:ode2(eq,y,x);
```

(%o2) $$\frac{d^2}{d x^2} y + 9 y = \frac{\cos(3 x)}{3} + \cos(x)$$

(%o3) $$y = \frac{12 x \sin(3 x) + 4 \cos(3 x) + 27 \cos(x)}{216} + \%k1 \sin(3 x) + \%k2 \cos(3 x)$$

4 Solve the diferential equation y''+4y'-6.25y=3.125*(x^2+1) using variation of parameter.

```
kill(all)$depends(y,x)$
eq:diff(y,x,2)+4·diff(y,x)-6.25·y=0;
y:ode2(eq,y,x);
yc:second(y);
a:second(first(second(y)));
b:second(second(second(y)));
m:matrix([a,b],[diff(a,x),diff(b,x)]);
W:determinant(m);
yp:-a·integrate((b·(3.125·(x^2+1)))/W,x)+b·integrate((a·(3.125·(x^2+1)
soll:yc+yp;
```

(%o2) $$\frac{d^2}{d x^2} y+4\left(\frac{d}{d x} y\right)-6.25\, y=0$$

rat: replaced -6.25 by -25/4 = -6.25

(%o3) $$y=\%k1\ \%e^{\frac{(\sqrt{41}-4)\, x}{2}}+\%k2\ \%e^{\frac{(-\sqrt{41}-4)\, x}{2}}$$

(%o4) $$\%k1\ \%e^{\frac{(\sqrt{41}-4)\, x}{2}}+\%k2\ \%e^{\frac{(-\sqrt{41}-4)\, x}{2}}$$

(%o5) $$\%e^{\frac{(\sqrt{41}-4)\, x}{2}}$$

(%o6) $$\%e^{\frac{(-\sqrt{41}-4)\, x}{2}}$$

(%o7) $$\begin{bmatrix} \%e^{\frac{(\sqrt{41}-4)\, x}{2}} & \%e^{\frac{(-\sqrt{41}-4)\, x}{2}} \\ \frac{(\sqrt{41}-4)\ \%e^{\frac{(\sqrt{41}-4)\, x}{2}}}{2} & \frac{(-\sqrt{41}-4)\ \%e^{\frac{(-\sqrt{41}-4)\, x}{2}}}{2} \end{bmatrix}$$

(%o8) $$\frac{(-\sqrt{41}-4)\ \%e^{\frac{(\sqrt{41}-4)\, x}{2}+\frac{(-\sqrt{41}-4)\, x}{2}}}{2}-\frac{(\sqrt{41}-4)\ \%e^{\frac{(\sqrt{41}-4)\, x}{2}+\frac{(-\sqrt{41}-4)\, x}{2}}}{2}$$

(%o9) $$3.125\ \%e^{\frac{(-\sqrt{41}-4)\, x}{2}}\ (-$$

$$\left(\left((197936\sqrt{41}+1267794)\, x^2+(-76168\sqrt{41}-487072)\, x+14592\sqrt{41}+93968\right)\ \%e\right.$$

$$/\,(\sqrt{41}\,(1029769\sqrt{41}+6593276))-\frac{\%e^{\frac{\sqrt{41}\, x}{2}+2\, x}}{\sqrt{41}\left(\frac{\sqrt{41}}{2}+2\right)})-3.125$$

$$\%e^{\frac{(\sqrt{41}-4)\, x}{2}}\ (-$$

```
trigsimp(yp);
```

rat: replaced -3.125 by -25/8 = -3.125
rat: replaced 3.125 by 25/8 = 3.125

(%o11) $-\frac{625\,x^{2}+800\,x+1337}{1250}$

4.1 Verification by Another method

```
kill(all)$depends(y,x)$
eq:diff(y,x,2)+4·diff(y,x)-6.25·y=3.125·(x^2+1);
sol:ode2(eq,y,x);
```

(%o2) $\frac{d^{2}}{d\,x^{2}}\,y+4\left(\frac{d}{d\,x}\,y\right)-6.25\,y=3.125\,(x^{2}+1)$

rat: replaced -3.125 by -25/8 = -3.125
rat: replaced -6.25 by -25/4 = -6.25

(%o3) $y=\%k1\,\%e^{\frac{(\sqrt{41}-4)\,x}{2}}+\%k2\,\%e^{\frac{(-\sqrt{41}-4)\,x}{2}}-\frac{625\,x^{2}+800\,x+1337}{1250}$

Practical 5
Solution of system of First Order Ordinary Differential Equations

Written By Anshul Verma (19/78065)
B.Sc. (Hons.) Computer Science

1 Case 1: Real and Distinct Roots

1.1 Solve the following system of ordinary differential equations x'=9x + 13.5y y'=1.5x + 9y

```
kill(all)$
```

```
eq1:'diff(x(t),t,1)=9·x(t) + 13.5·y(t);
eq2:'diff(y(t),t,1)=1.5·x(t) + 9·y(t);
```

(%o1) $\frac{d}{d\,t}\,x(t)=13.5\,y(t)+9\,x(t)$

(%o2) $\frac{d}{d\,t}\,y(t)=9\,y(t)+1.5\,x(t)$

1.1.1 Method 1 - Coeficient matrix

```
A:matrix([9,13.5],[1.5,9]);
```

(%o3) $\begin{bmatrix} 9 & 13.5 \\ 1.5 & 9 \end{bmatrix}$

```
/*Eigenvalues and Eigenvectors*/
eigenvalues(A);
eigenvectors(A);
```

rat: replaced -20.25 by -81/4 = -20.25

(%o4) $[[\frac{27}{2},\frac{9}{2}],[1,1]]$

rat: replaced -20.25 by -81/4 = -20.25

(%o5) $[[[\frac{27}{2},\frac{9}{2}],[1,1]],[[[1,\frac{1}{3}]],[[1,-\frac{1}{3}]]]$
$]$

```
/*The general solution for the case of real and distinct eigenvalues i
X = c1*K1*e^(r1*t) + c2*K2*e^(r2*t)
where r1 and r2 are eigenvalues and K1 and K2 are the eigenvectors*/
soln:[x,y]=c1·[1,1/3]·e^(27/2·t) + c2·[1,-1/3]·e^(9/2·t);
```

(%o6) $[x,y]=[c1\,e^{\frac{27\,t}{2}}+c2\,e^{\frac{9\,t}{2}},\frac{c1\,e^{\frac{27\,t}{2}}}{3}-\frac{c2\,e^{\frac{9\,t}{2}}}{3}]$

```
/*First numeric number denotes LHS(1) OR RHS(2) and second entry denot
part(soln,1,1)=part(soln,2,1);
part(soln,1,2)=part(soln,2,2);
```

(%o7) $x=c1\,e^{\frac{27\,t}{2}}+c2\,e^{\frac{9\,t}{2}}$

(%o8) $y=\frac{c1\,e^{\frac{27\,t}{2}}}{3}-\frac{c2\,e^{\frac{9\,t}{2}}}{3}$

1.1.2 Method 2 - Solving directly using desolve

```
desolve([eq1,eq2],[x(t),y(t)]);
```

rat: replaced 13.5 by 27/2 = 13.5
rat: replaced 13.5 by 27/2 = 13.5
rat: replaced 1.5 by 3/2 = 1.5
rat: replaced 1.5 by 3/2 = 1.5
rat: replaced -13.5 by -27/2 = -13.5
rat: replaced -1.5 by -3/2 = -1.5

(%o9) $$\left[x(t)=\frac{(3\,y(0)+x(0))\,\%e^{\frac{27\,t}{2}}}{2}+\frac{(x(0)-3\,y(0))\,\%e^{\frac{9\,t}{2}}}{2},\ y(t)=\frac{(3\,y(0)+x(0))\,\%e^{\frac{27\,t}{2}}}{6}+\frac{(3\,y(0)-x(0))\,\%e^{\frac{9\,t}{2}}}{6}\right]$$

1.2 Solve the following system of ordinary differential equations x'=-x + y + 0.4*z y'=x - 0.1*y +1.4*z z'=0.4*x + 1.4*y + 0.2*z

```
kill(all)$
```

```
eq1:'diff(x(t),t)=-x(t) + y(t) + 0.4·z(t);
eq2:'diff(y(t),t)=x(t) - 0.1·y(t) +1.4·z(t);
eq3:'diff(z(t),t)=0.4·x(t) + 1.4·y(t) + 0.2·z(t);
```

(%o1) $$\frac{d}{d\,t}\,x(t)=0.4\,z(t)+y(t)-x(t)$$

(%o2) $$\frac{d}{d\,t}\,y(t)=1.4\,z(t)-0.1\,y(t)+x(t)$$

(%o3) $$\frac{d}{d\,t}\,z(t)=0.2\,z(t)+1.4\,y(t)+0.4\,x(t)$$

1.2.1 Method 1 - Coeficient matrix

```
A:matrix([-1,1,0.4],[1,-0.1,1.4],[0.4,1.4,0.2]);
```

(%o4) $$\begin{bmatrix}-1 & 1 & 0.4\\ 1 & -0.1 & 1.4\\ 0.4 & 1.4 & 0.2\end{bmatrix}$$

```
→ eigenvalues(A);
rat: replaced 0.3599999999999999 by 9/25 = 0.36
rat: replaced 0.4 by 2/5 = 0.4
rat: replaced 1.4 by 7/5 = 1.4
rat: replaced -0.4 by -2/5 = -0.4
rat: replaced -0.1 by -1/10 = -0.1
rat: replaced -1.959999999999999 by -49/25 = -1.96
rat: replaced -0.1 by -1/10 = -0.1
rat: replaced 0.2 by 1/5 = 0.2
```

(%o5) $[[-\frac{9}{10},\frac{9}{5},-\frac{9}{5}],[1,1,1]]$

```
→ eigenvectors(A);
rat: replaced 0.3599999999999999 by 9/25 = 0.36
rat: replaced 0.4 by 2/5 = 0.4
rat: replaced 1.4 by 7/5 = 1.4
rat: replaced -0.4 by -2/5 = -0.4
rat: replaced -0.1 by -1/10 = -0.1
rat: replaced -1.959999999999999 by -49/25 = -1.96
rat: replaced -0.1 by -1/10 = -0.1
rat: replaced 0.2 by 1/5 = 0.2
```

(%o6) $[[[-\frac{9}{10},\frac{9}{5},-\frac{9}{5}],[1,1,1]],[[[1,\frac{1}{2},-1]],[[1,2,2]],[[1,-1,\frac{1}{2}]]]]$

```
→ soln:[x,y,z]=c1·[1,1/2,-1]·e^(-9/10·t) + c2·[1,2,2]·e^(9/5·t)+ c3·[1,-
```

(%o7) $$[x,y,z]=[c2\,e^{\frac{9t}{5}}+\frac{c1}{e^{\frac{9t}{10}}}+\frac{c3}{e^{\frac{9t}{5}}},2\,c2\,e^{\frac{9t}{5}}+\frac{c1}{2\,e^{\frac{9t}{10}}}-\frac{c3}{e^{\frac{9t}{5}}},2\,c2\,e^{\frac{9t}{5}}-\frac{c1}{e^{\frac{9t}{10}}}+\frac{c3}{2\,e^{\frac{9t}{5}}}]$$

```
part(soln,1,1)=part(soln,2,1);
part(soln,1,2)=part(soln,2,2);
part(soln,1,3)=part(soln,2,3);
```

(%o8) $x=c2\, e^{\frac{9\, t}{5}}+\frac{c1}{e^{\frac{9\, t}{10}}}+\frac{c3}{e^{\frac{9\, t}{5}}}$

(%o9) $y=2\, c2\, e^{\frac{9\, t}{5}}+\frac{c1}{2\, e^{\frac{9\, t}{10}}}-\frac{c3}{e^{\frac{9\, t}{5}}}$

(%o10) $z=2\, c2\, e^{\frac{9\, t}{5}}-\frac{c1}{e^{\frac{9\, t}{10}}}+\frac{c3}{2\, e^{\frac{9\, t}{5}}}$

1.2.2 Desolve Method

```
→ desolve([eq1,eq2,eq3],[x(t),y(t),z(t)]);
```

rat: replaced 0.4 by 2/5 = 0.4
rat: replaced 0.4 by 2/5 = 0.4
rat: replaced -0.1 by -1/10 = -0.1
rat: replaced 1.4 by 7/5 = 1.4
rat: replaced -0.1 by -1/10 = -0.1
rat: replaced 1.4 by 7/5 = 1.4
rat: replaced 0.4 by 2/5 = 0.4
rat: replaced 1.4 by 7/5 = 1.4
rat: replaced 0.2 by 1/5 = 0.2
rat: replaced 0.4 by 2/5 = 0.4
rat: replaced 1.4 by 7/5 = 1.4
rat: replaced 0.2 by 1/5 = 0.2
rat: replaced -0.4 by -2/5 = -0.4
rat: replaced 0.1 by 1/10 = 0.1
rat: replaced -1.4 by -7/5 = -1.4
rat: replaced -0.4 by -2/5 = -0.4
rat: replaced -1.4 by -7/5 = -1.4
rat: replaced -0.2 by -1/5 = -0.2

(%o11)

$$[x(t)=\frac{(10\,z(0)+10\,y(0)+5\,x(0))\,\%e^{\frac{9\,t}{5}}}{45}-\frac{(40\,z(0)-20\,y(0)-40\,x(0))\,\%e^{-\frac{9\,t}{10}}}{90}+\frac{(10\,z(0)-20\,y(0)+20\,x(0))\,\%e^{-\frac{9\,t}{5}}}{45},y(t)=$$

$$\frac{(20\,z(0)+20\,y(0)+10\,x(0))\,\%e^{\frac{9\,t}{5}}}{45}-\frac{(20\,z(0)-10\,y(0)-20\,x(0))\,\%e^{-\frac{9\,t}{10}}}{90}-\frac{(10\,z(0)-20\,y(0)+20\,x(0))\,\%e^{-\frac{9\,t}{5}}}{45},z(t)=$$

$$\frac{(20\,z(0)+20\,y(0)+10\,x(0))\,\%e^{\frac{9\,t}{5}}}{45}+\frac{(40\,z(0)-20\,y(0)-40\,x(0))\,\%e^{-\frac{9\,t}{10}}}{90}+\frac{(5\,z(0)-10\,y(0)+10\,x(0))\,\%e^{-\frac{9\,t}{5}}}{45}]$$

2 Case 2: Real and Equal Eigenvalues

2.1 Solve the following system of ordinary differential equations x' = 8*x - y y' = x + 10*y

```
kill(all)$
```

```
eq1:'diff(x(t),t)=8·x(t) - y(t);
eq2:'diff(y(t),t)= x(t) + 10·y(t);
```

(%o1) $\frac{d}{d\,t}x(t)=8\,x(t)-y(t)$

(%o2) $\frac{d}{d\,t}y(t)=10\,y(t)+x(t)$

```
A:matrix([8,-1],[1,10]);
```

(%o3) $\begin{bmatrix} 8 & -1 \\ 1 & 10 \end{bmatrix}$

```
eigenvalues(A);
```

(%o4) [[9],[2]]

```
/*Repeated eigenvalue = r = 9*/
eigenvectors(A);
```

(%o5) [[[9],[2]],[[[1,-1]]]]

```
/*We get only one eigenvector  [1,-1]*/
/*If we get only a single eigenvector corresponding to the same eigenv
the general solution is :
    c1*k*e^(rt) + c2*(k*t*e^(rt) + n*e^(rt))
where n is obtained as : (A-rI)*n=k*/
r:9;
k1:[1,-1];
```

(%o6) 9

(%o7) [1,-1]

```
desolve([eq1,eq2],[x(t),y(t)]);
```

(%o8) $[x(t)=-y(0)\,t\,\%e^{9\,t}-x(0)\,t\,\%e^{9\,t}+x(0)\,\%e^{9\,t},y(t)=y(0)\,t\,\%e^{9\,t}+x(0)\,t\,\%e^{9\,t}+y(0)\,\%e^{9\,t}]$

2.2 Solve the following system of ordinary differential equations x'=15.5x y'=15.5y

```
kill(all)$
```

```
eq1:'diff(x(t),t)=15.5·x(t);
eq2:'diff(y(t),t)=15.5·y(t);
```

(%o1) $\frac{d}{d\,t}\,x(t)=15.5\,x(t)$

(%o2) $\frac{d}{d\,t}\,y(t)=15.5\,y(t)$

```
A:matrix([15.5,0],[0,15.5]);
```

(%o3) $\begin{bmatrix} 15.5 & 0 \\ 0 & 15.5 \end{bmatrix}$

```
eigenvalues(A);
eigenvectors(A);
```

rat: replaced 15.5 by 31/2 = 15.5

(%o4) $[[\frac{31}{2}],[2]]$

rat: replaced 15.5 by 31/2 = 15.5

(%o5) $[[[\frac{31}{2}],[2]],[[[1,0],[0,1]]]]$

```
/*If we get two linearly independent eigenvectors 'k1', 'k2' correspon
eigenvalue 'r' the general solution is :
    c1*k1*%e^(r*t) + c2*k2*%e^(r*t)*/
r:3;
k1:[1,0];
k2:[0,1];
soln:[x,y]=c1·k1·%e^(r·t) + c2·k2·%e^(r·t);
```

(%o6) 3

(%o7) $[1,0]$

(%o8) $[0,1]$

(%o9) $[x,y]=[c1\,\%e^{3\,t},c2\,\%e^{3\,t}]$

```
part(soln,1,1)=part(soln,2,1);
part(soln,1,2)=part(soln,2,2);
```

(%o10) $x=c1\,\%e^{3\,t}$

(%o11) $y=c2\,\%e^{3\,t}$

```
→ desolve([eq1,eq2],[x(t),y(t)]);
```

rat: replaced 15.5 by 31/2 = 15.5
rat: replaced 15.5 by 31/2 = 15.5
rat: replaced -15.5 by -31/2 = -15.5
rat: replaced -15.5 by -31/2 = -15.5

(%o12) $[x(t)=x(0)\,\%e^{\frac{31\,t}{2}}, y(t)=y(0)\,\%e^{\frac{31\,t}{2}}]$

2.3 y'''' - 5*y'' + 4y = 10 with y(0)=4; y'(0)=3; y''(0)=-9; y'''(0)=-2

```
→ /*using transformation
      x1=y
      x2=x1'=y'
      x3=x2'=y''
      x4=x3'=y'''
      x5=x4'=y''''
      x1(0)=4
      x2(0)=3
      x3(0)=-9
      x4(0)=-2*/
  eq1:'diff(x1(t),t)=x2(t);
  eq2:'diff(x2(t),t)=x3(t);
  eq3:'diff(x3(t),t)=x4(t);
  eq4:'diff(x4(t),t)= 5·x3(t) - 4·x1(t);
```

(%o13) $\frac{d}{d\,t}\,x1(t)=x2(t)$

(%o14) $\frac{d}{d\,t}\,x2(t)=x3(t)$

(%o15) $\frac{d}{d\,t}\,x3(t)=x4(t)$

(%o16) $\frac{d}{d\,t}\,x4(t)=5\,x3(t)-4\,x1(t)$

```
→ A:matrix([0,1,0,0],[0,0,1,0],[0,0,0,1],
  [-4,0,5,0]);
```

(%o17) $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -4 & 0 & 5 & 0 \end{bmatrix}$

```
eigenvalues(A);
eigenvectors(A);
```

(%o18) $[[-2,2,-1,1],[1,1,1,1]]$

(%o19) $[[[-2,2,-1,1],[1,1,1,1]],[[[1,-2,4,-8]],[[1,2,4,8]],[[1,-1,1,-1]],[[1,1,1,1]]]]$

```
/*This is the case of real and distinct eigenvalues*/
r1:-2;r2:2;r3:-1;r4:1;
k1:[1,-2,4,-8];
k2:[1,2,4,8];
k3:[1,-1,1,-1];
k4:[1,1,1,1];
soln:[x1,x2,x3,x4]=c1·k1·e^(r1·t)+
c2·k2·e^(r2·t) + c3·k3·e^(r3·t) +
c4·k4·e^(r4·t);
```

(%o20) -2

(%o21) 2

(%o22) -1

(%o23) 1

(%o24) $[1,-2,4,-8]$

(%o25) $[1,2,4,8]$

(%o26) $[1,-1,1,-1]$

(%o27) $[1,1,1,1]$

(%o28) $[x1,x2,x3,x4]=[c2\,e^{2\,t}+c4\,e^{t}+\frac{c3}{e^{t}}+\frac{c1}{e^{2\,t}},2\,c2\,e^{2\,t}+c4\,e^{t}-\frac{c3}{e^{t}}-\frac{2\,c1}{e^{2\,t}},4\,c2\,e^{2\,t}+c4\,e^{t}+\frac{c3}{e^{t}}+\frac{4\,c1}{e^{2\,t}},8\,c2\,e^{2\,t}+c4\,e^{t}-\frac{c3}{e^{t}}-\frac{8\,c1}{e^{2\,t}}]$

```
e1:part(soln,1,1)=part(soln,2,1);
e2:part(soln,1,2)=part(soln,2,2);
e3:part(soln,1,3)=part(soln,2,3);
e4:part(soln,1,4)=part(soln,2,4);
```

(%o29) $x1=c2\,e^{2\,t}+c4\,e^{t}+\frac{c3}{e^{t}}+\frac{c1}{e^{2\,t}}$

(%o30) $x2=2\,c2\,e^{2\,t}+c4\,e^{t}-\frac{c3}{e^{t}}-\frac{2\,c1}{e^{2\,t}}$

(%o31) $x3=4\,c2\,e^{2\,t}+c4\,e^{t}+\frac{c3}{e^{t}}+\frac{4\,c1}{e^{2\,t}}$

(%o32) $x4=8\,c2\,e^{2\,t}+c4\,e^{t}-\frac{c3}{e^{t}}-\frac{8\,c1}{e^{2\,t}}$

```
t1:subst([x1=4,t=0],e1);
```

(%o33) $4=c4+c3+c2+c1$

```
t2:subst([x2=3,t=0],e2);
```

(%o34) $3=c4-c3+2\,c2-2\,c1$

```
t3:subst([x3=-9,t=0],e3);
```

(%o35) $-9=c4+c3+4\,c2+4\,c1$

```
t4:subst([x4=-2,t=0],e4);
```

(%o36) $-2=c4-c3+8\,c2-8\,c1$

```
solve([t1,t2,t3,t4],[c1,c2,c3,c4]);
```

(%o37) $$[[c1=-\frac{7}{4},c2=-\frac{31}{12},c3=\frac{11}{6},c4=\frac{13}{2}]]$$

```
subst(
[c1=-7/4,c2=-31/12,c3=11/6,c4=13/2],
[e1,e2,e3,e4]);
```

(%o38) $$[x1=-\frac{31\,e^{2\,t}}{12}+\frac{13\,e^{t}}{2}+\frac{11}{6\,e^{t}}-\frac{7}{4\,e^{2\,t}},x2=-\frac{31\,e^{2\,t}}{6}+\frac{13\,e^{t}}{2}-\frac{11}{6\,e^{t}}+\frac{7}{2\,e^{2\,t}},x3=-\frac{31\,e^{2\,t}}{3}+\frac{13\,e^{t}}{2}+\frac{11}{6\,e^{t}}-\frac{7}{e^{2\,t}},x4=-\frac{62\,e^{2\,t}}{3}+\frac{13\,e^{t}}{2}-\frac{11}{6\,e^{t}}+\frac{14}{e^{2\,t}}]$$

```
atvalue(x1(t),t=0,4);
atvalue(x2(t),t=0,3);
atvalue(x3(t),t=0,-9);
atvalue(x4(t),t=0,-2);
desolve([eq1,eq2,eq3,eq4],
[x1(t),x2(t),x3(t),x4(t)]);
```

(%o39) 4

(%o40) 3

(%o41) -9

(%o42) -2

(%o43) $$[x1(t)=-\frac{31\,\%e^{2\,t}}{12}+\frac{13\,\%e^{t}}{2}+\frac{11\,\%e^{-t}}{6}-\frac{7\,\%e^{-2\,t}}{4},x2(t)=-\frac{31\,\%e^{2\,t}}{6}+\frac{13\,\%e^{t}}{2}-\frac{11\,\%e^{-t}}{6}+\frac{7\,\%e^{-2\,t}}{2},x3(t)=-\frac{31\,\%e^{2\,t}}{3}+\frac{13\,\%e^{t}}{2}+\frac{11\,\%e^{-t}}{6}-7\,\%e^{-2\,t},x4(t)=-\frac{62\,\%e^{2\,t}}{3}+\frac{13\,\%e^{t}}{2}-\frac{11\,\%e^{-t}}{6}+14\,\%e^{-2\,t}]$$

Practical 6 Solution of Cauchy Problem of First Order Partial Differential Equations.

Written By Anshul Verma (19/78065) for GE-III Practicals
B.Sc. (Hons.) Computer Science

1 Solve the Cauchy problem 3ux-2uy=1 with u(x,0)=sin(x).

The Characteristic eqn is: dx/3 = dy/(-2) = du/1.
Solving two of these for two constants, we consider:
dy/dx=(-2)/3 and du/dx=1/3

```
eq1:'diff(y,x)=(-2)/3 ;
```

(%o44) $\frac{d}{d\,x}\,y=-\frac{2}{3}$

```
ode2(eq1,y,x);
```

(%o45) $y=\%c-\frac{2\,x}{3}$

```
solve(y=c1-(2·x)/3,c1);
```

(%o46) $[c1=\frac{3\,y+2\,x}{3}]$

```
eq2:'diff(u,x)=1/3;
```

(%o47) $\frac{d}{d\,x}\,u=\frac{1}{3}$

```
ode2(eq2,u,x);
```

(%o48) $u=\frac{x}{3}+\%c$

```
solve(u=x/3+c2,c2);
```

(%o49) $[c2=-\frac{x-3\,u}{3}]$

The general soln of the given PDE is given by c1=f(c2) or c2=f(c1) where f is an arbitrary function.

```
-(x-3·u)/3=f((3·y+2·x)/3);
```

(%o50) $\frac{3\,u-x}{3}=f\left(\frac{3\,y+2\,x}{3}\right)$

```
solve((3·u-x)/3=f((3·y+2·x)/3),u);
```

(%o51) $[u=\frac{3\,f\left(y+\frac{2\,x}{3}\right)+x}{3}]$

```
u(x,y):=(3·f(y+(2·x)/3)+x)/3;
```

(%o52) $u(x,y):=\frac{3\,f\left(y+\frac{2\,x}{3}\right)+x}{3}$

```
u(x,0)=sin(x);
```

(%o53) $\frac{x+3\,f\left(\frac{2\,x}{3}\right)}{3}=\sin(x)$

```
solve(%,f(2·x/3));
```

(%o54) $[f\left(\frac{2\,x}{3}\right)=\frac{3\,\sin(x)-x}{3}]$

```
f(x):=(3·sin(x)-x)/3;
```

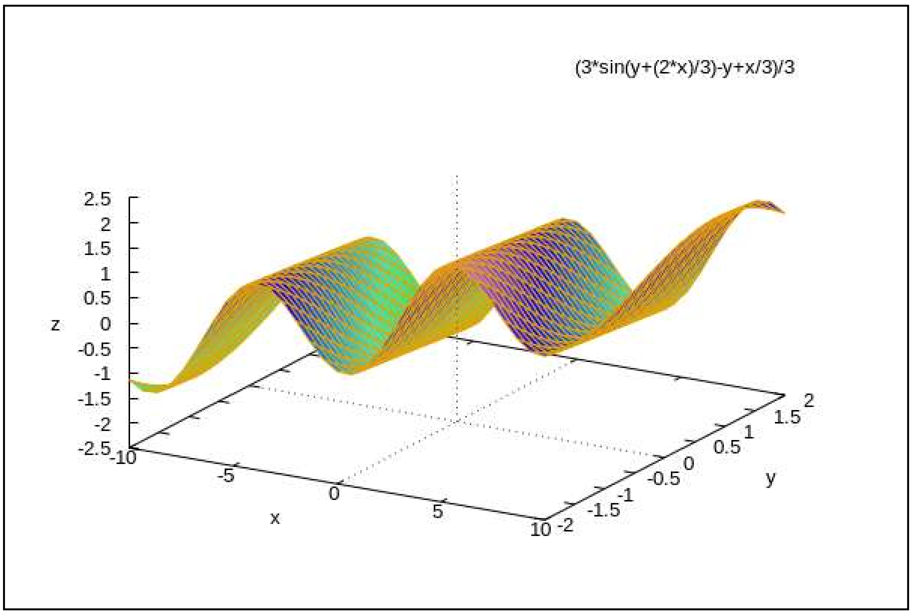
(%o55) $f(x):=\frac{3\,\sin(x)-x}{3}$

```
'u(x,y)=u(x,y);
```

(%o56) $u(x,y)=\frac{3\,\sin\left(y+\frac{2\,x}{3}\right)-y+\frac{x}{3}}{3}$

```
→ wxplot3d(u(x,y),[x,-10,10],[y,-2,2]);
```

(%t57)



(%o57)

2 Solve the Cauchy problem ux+xuy=0 with u(0,y)=sin(y).

The Characteristic eqn is: dx/1 = dy/x = du/0.
Solving two of these for two constants, we consider:
dy/dx=x and du/dx=0

```
→ kill(all)$
```

```
→ eq1:'diff(y,x)=x;
```

(%o1) $\frac{d}{d\,x}\,y=x$

```
→ ode2(eq1,y,x);
```

(%o2) $y=\frac{x^2}{2}+\%c$

```
→ solve(y=x^2/2+c1,c1);
```

(%o3) $[c1=\frac{2\,y-x^2}{2}]$

```
→ eq2:'diff(u,x)=0;
```

(%o4) $\frac{d}{d\,x}\,u=0$

```
→ ode2(eq2,u,x);
```

(%o5) $u=\%c$

```
→ solve(u=c2,c2);
```

(%o6) $[c2=u]$

The general soln of the given PDE is given by c1=f(c2) or c2=f(c1) where f is an arbitrary function.

```
→ u=f((2·y-x^2)/2);
```

(%o7) $u=f\left(\frac{2\,y-x^2}{2}\right)$

```
→ u(x,y):=f((2·y-x^2)/2);
```

(%o8) $u(x,y):=f\left(\frac{2\,y-x^2}{2}\right)$

```
→ u(0,y)=sin(y);
```

(%o9) $f(y)=\sin(y)$

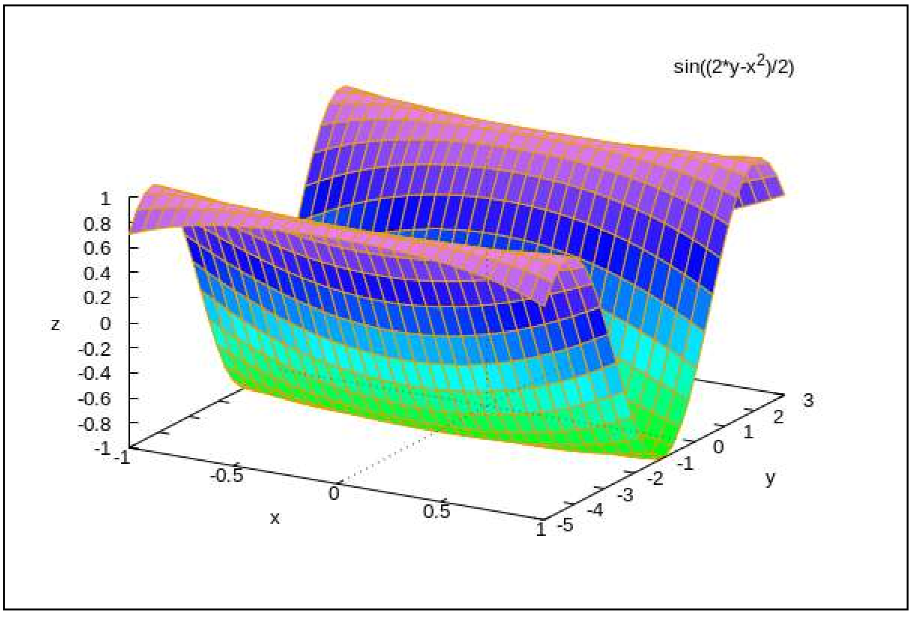
```
→ f(y):=sin(y);
```

(%o10) $f(y):=\sin(y)$

```
→ 'u(x,y)=u(x,y);
```

(%o11) $u(x,y)=\sin\left(\frac{2\,y-x^2}{2}\right)$

```
→ wxplot3d(u(x,y),[x,-1,1],[y,-5,3]);
```

(%t12) 

(%o12)

3 Solve the Cauchy problem x*ux+y*uy=x*exp(-u) ; u(x,x^2)=0.

The Characteristic eqn is: dx/x = dy/(x+y) = du/u+1.
Solving two of these for two constants, we consider:
dy/dx=y/x and du/dx=exp(-u), we get

```
kill(all);
```
(%o0) done

```
eq3:'diff(y,x)=y/x;
```
(%o1) $\frac{d}{d\,x}\,y=\frac{y}{x}$

```
ode2(eq3,y,x);
```
(%o2) $y=\%c\,x$

```
solve(y=c3·x,c3);
```
(%o3) $[c3=\frac{y}{x}]$

```
eq4:'diff(u,x)=exp(-u);
```
(%o4) $\frac{d}{d\,x}\,u=\%e^{-u}$

```
ode2(eq4,u,x);
```
(%o5) $\%e^{u}=x+\%c$

```
solve(exp(u)=x+c4,c4);
```
(%o6) $[c4=\%e^{u}-x]$

The general soln is given by c4=g(c3) where g is an arbitrary function.

```
exp(u)-x=g(y/x);
```
(%o7) $\%e^{u}-x=g\left(\frac{y}{x}\right)$

```
u(x,y):=log(x+g(y/x));
```
(%o8) $u(x,y):=\log\left(x+g\left(\frac{y}{x}\right)\right)$

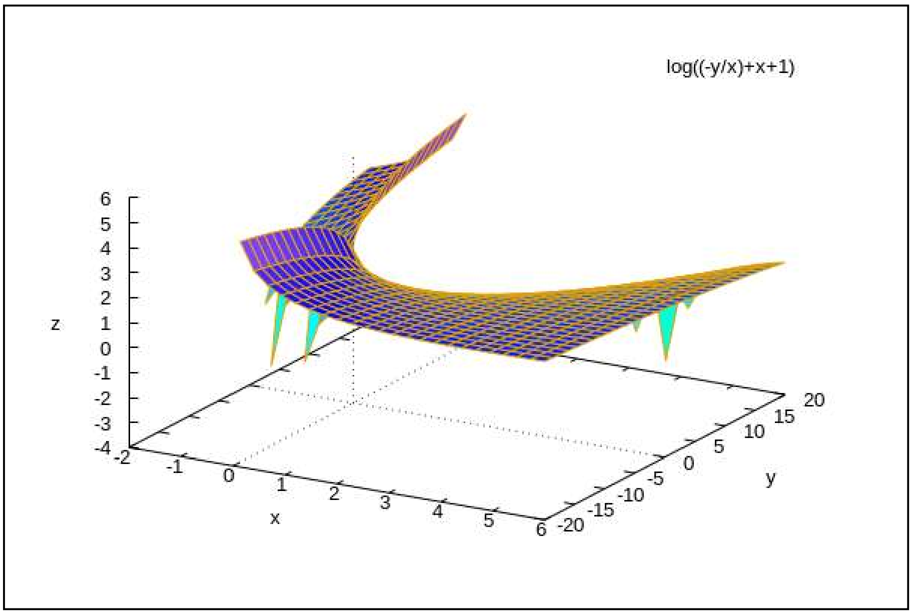
```
→ u(x,x^2)=0;
```
(%o9) $\log(g(x)+x)=0$

```
→ solve(%,g(x));
```
(%o10) $[g(x)=1-x]$

```
→ g(x):=1-x;
```
(%o11) $g(x):=1-x$

```
→ 'u(x,y)=u(x,y);
```
(%o12) $u(x,y)=\log\left(-\frac{y}{x}+x+1\right)$

```
→ wxplot3d(u(x,y),[x,-2,6],[y,-20,20]);
```
(%t13)



(%o13)

4 Solve the Cauchy problem x ux + y uy = 2xy, with u = 2 on y = x^2.

The Characteristic eqn is: dx/x = dy/y = du/x*y.
Solving two of these for two constants, we consider:
dy/dx=y/x and du/dx=y

```
→ kill(all)$
```

```
→ eq1:'diff(y,x)=y/x ;
```
(%o1) $\frac{d}{d\,x}\,y=\frac{y}{x}$

```
→ ode2(eq1,y,x);
```

(%o2) $y = \%c\, x$

```
→ solve(y=c1·x,c1);
```

(%o3) $[c1 = \frac{y}{x}]$

```
→ eq2:'diff(u,x)=y;
```

(%o4) $\frac{d}{d\,x}\, u = y$

```
→ ode2(eq2,u,x);
```

(%o5) $u = x\, y + \%c$

```
→ solve(u=x·y+c2,c2);
```

(%o6) $[c2 = u - x\, y]$

The general soln of the given PDE is given by c1=f(c2) or c2=f(c1) where f is an arbitrary function.

```
→ u-x·y=f(y/x);
```

(%o7) $u - x\, y = f\left(\frac{y}{x}\right)$

```
→ solve(u-x·y=f(y/x),u);
```

(%o8) $[u = f\left(\frac{y}{x}\right) + x\, y]$

```
→ u(x,y):=f(y/x)+x·y;
```

(%o9) $u(x, y) := f\left(\frac{y}{x}\right) + x\, y$

```
→ u(x,x^2)=2;
```

(%o10) $f(x) + x^3 = 2$

```
→ f(x):=2-x^3;
```

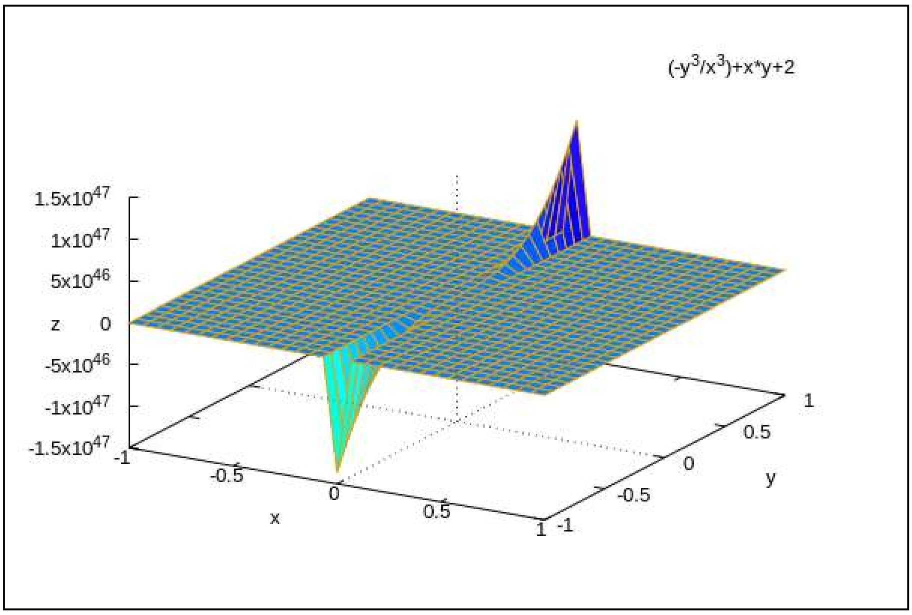
(%o11) $f(x) := 2 - x^3$

```
→ 'u(x,y)=u(x,y);
```

(%o12) $u(x, y) = -\frac{y^3}{x^3} + x\, y + 2$

```
wxplot3d(u(x,y),[x,-1,1],[y,-1,1]);
```

(%t13)



(%o13)

Practical 7 Finding and plotting the Characteristics of a First Order Partial Differential Equations

Written By Anshul Verma (19/78065) for GE-III Practicals
B.Sc. (Hons.) Computer Science

1 (1+x^2)*u_x + u_y = u

Characteristics: dx/(1+x^2) = dy/1 = du/u
The characterstics equations for given PDE will be,
eqn1: dy/dx = 1/(1+x^2)
eqn2: du/dx = u/(1+x^2)

We first consider eqn1: dy/dx = 1/(1+x^2)

```
eq1:'diff(y,x)=1/(1+x^2);
```

(%o14) $\frac{d}{d\,x}\,y=\frac{1}{x^2+1}$

```
ode2(eq1,y,x);
```

(%o15) $y=\mathrm{atan}(x)+\%c$

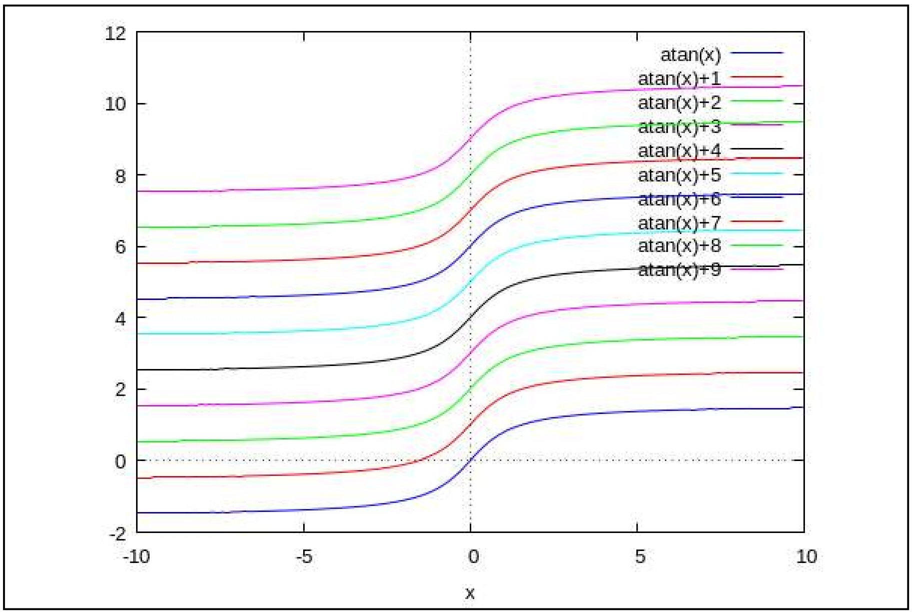
```
solve(y=atan(x)+c1,y);
```

(%o16) $[y=\mathrm{atan}(x)+c1]$

```
psol:makelist(atan(x)+c1,c1,0,9);
```

(%o17) $[\mathrm{atan}(x),\mathrm{atan}(x)+1,\mathrm{atan}(x)+2,\mathrm{atan}(x)+3,\mathrm{atan}(x)+4,\mathrm{atan}(x)+5,\mathrm{atan}(x)+6,\mathrm{atan}(x)+7,\mathrm{atan}(x)+8,\mathrm{atan}(x)+9]$

```
wxplot2d(psol,[x,-10,10]);
```

(%t18) 

(%o18)

Now, consider eqn2: du/dx = u/(1+x^2)

```
eq2:'diff(u,x)=u/(1+x^2);
```

(%o19) $\frac{d}{d\,x}\,u=\frac{u}{x^2+1}$

```
ode2(eq2,u,x);
```

(%o20) $u=\%c\,\%e^{\mathrm{atan}(x)}$

```
solve(u=c2·%e^atan(x),u);
```

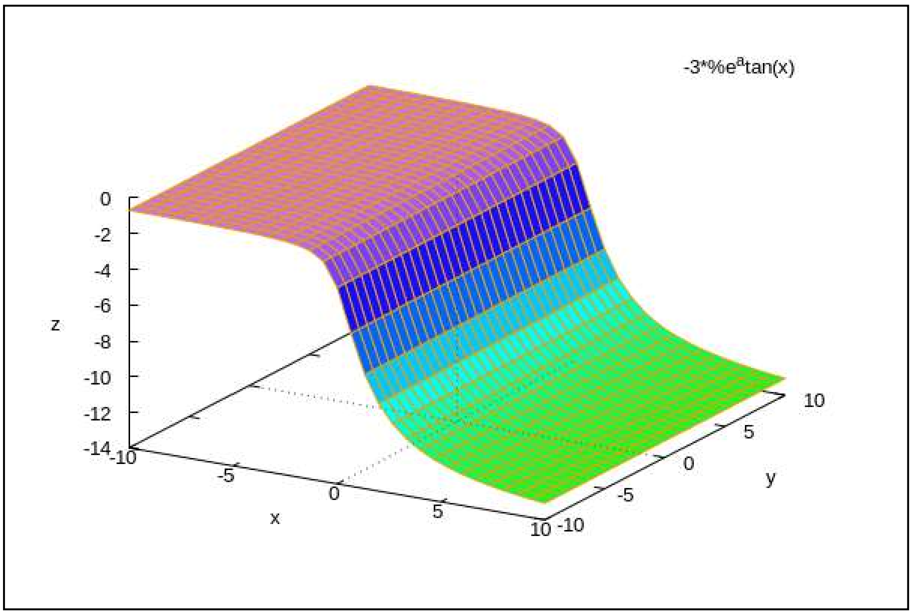
(%o21) $[u=c2\,\%e^{\mathrm{atan}(x)}]$

```
psol2:makelist(c2·%e^atan(x),c2,-3,8);
```

(%o22) $[-3\,\%e^{atan(x)},-2\,\%e^{atan(x)},-\%e^{atan(x)},0,\%e^{atan(x)},2\,\%e^{atan(x)},3\,\%e^{atan(x)},4\,\%e^{atan(x)},5\,\%e^{atan(x)},6\,\%e^{atan(x)},7\,\%e^{atan(x)},8\,\%e^{atan(x)}]$

```
wxplot3d(-3·%e^atan(x),[x,-10,10],[y,-10,10]);
```

(%t23)



(%o23)

2 u_x + 2xy^2u_y = 0

The characterstics equations for given PDE will be,
dx/1 = dy/(2*x*y^2) = du/0
eqn1: dy/dx = 2*x*y^2
eqn2: du/dx = 0

```
kill(all);
```

(%o0) *done*

We first consider eqn1: dy/dx = 2*x*y^

```
eq1:'diff(y,x)=2·x·y^2;
```

(%o1) $\frac{d}{d\,x}\,y=2\,x\,y^{2}$

```
ode2(eq1,y,x);
```

(%o2) $-\frac{1}{2\,y}=\frac{x^{2}}{2}+\%c$

```
→ solve(-1/(2·y)=x^2/2+c1,y);
```

(%o3) $[y=-\frac{1}{x^2+2\,c1}]$

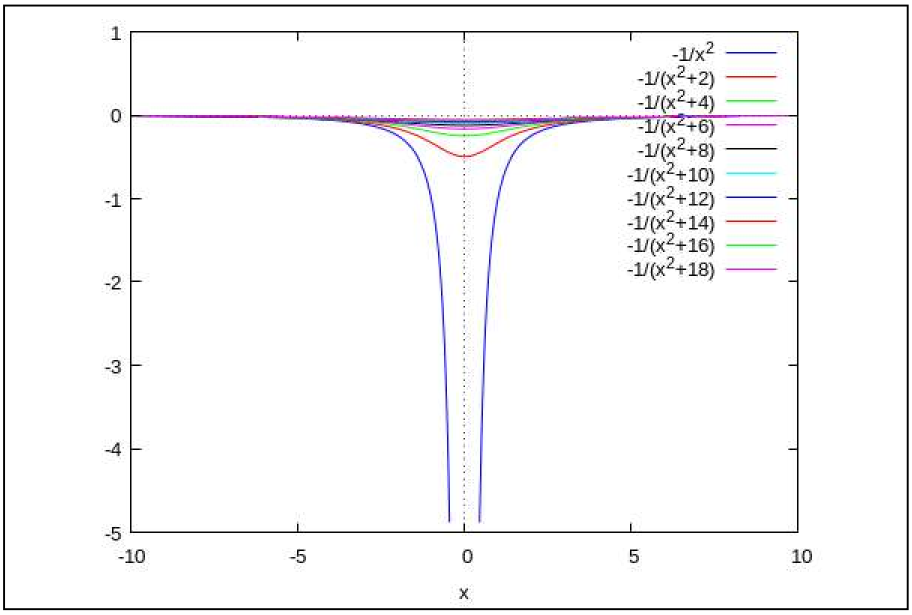
```
→ psol:makelist(-1/(x^2+2·c1),c1,0,9);
```

(%o4) $[-\frac{1}{x^2},-\frac{1}{x^2+2},-\frac{1}{x^2+4},-\frac{1}{x^2+6},-\frac{1}{x^2+8},-\frac{1}{x^2+10},-\frac{1}{x^2+12},-\frac{1}{x^2+14},-\frac{1}{x^2+16},-\frac{1}{x^2+18}]$

```
→ wxplot2d(psol,[x,-10,10],[y,-5,1]);
```

plot2d: expression evaluates to non-numeric value somewhere in plottin
plot2d: some values were clipped.

(%t5)



(%o5)

Now, consider eqn2: du/dx = 0

```
→ eq2:'diff(u,x)=0;
```

(%o6) $\frac{d}{d\,x}\,u=0$

```
→ ode2(eq2,u,x);
```

(%o7) $u=\%c$

```
→ solve(u=c2,u);
```

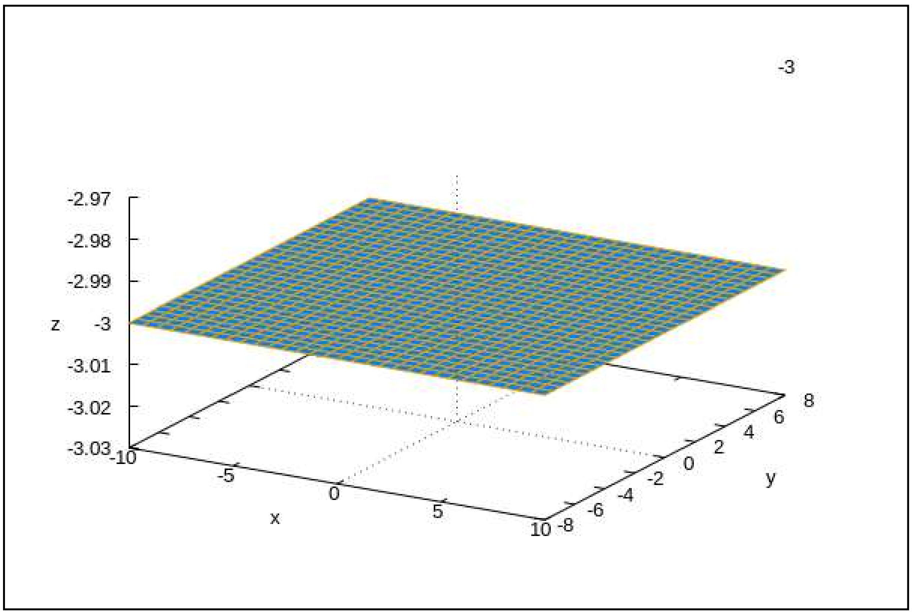
(%o8) $[u=c2]$

```
→ psol2:makelist(c2,c2,-3,6);
```

(%o9) $[-3,-2,-1,0,1,2,3,4,5,6]$

```
wxplot3d(-3,[x,-10,10],[y,-8,8]);
```

(%t10)



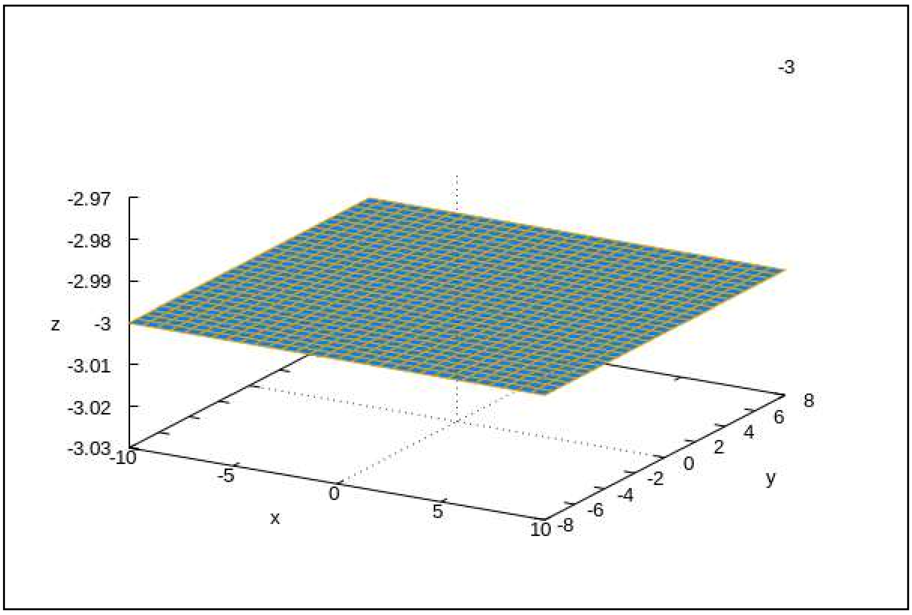
(%o10)

3 x^2*ux+y^2*uy=(x+y)*u

Characteristic eqns are: dx/x^2 = dy/y^2 = du/(x+y)*u
Consider eq1: dy/dx = y^2/x^2

```
eq1: 'diff(y,x)=y^2/x^2;
```

(%t11)



(%o11)

(%o12) $$\frac{d}{d\,x}\,y=\frac{y^2}{x^2}$$

```
ode2(eq1,y,x);
```

```
→ solve(-1/y=c1-1/x,y);
```

(%o13) $\frac{d}{d\,x}\,y=\frac{y^2}{x^2}$

(%o14) $-\frac{1}{y}=\%c-\frac{1}{x}$

```
→ psol:makelist(-x/(c1·x-1),c1,-2,1);
```

```
→ wxplot2d(psol,[x,-8,6],[y,-3,4]);
```

(%o15) $[y=-\frac{x}{c1\,x-1}]$

(%o16) $[-\frac{x}{-2\,x-1},-\frac{x}{-x-1},x,-\frac{x}{x-1}]$

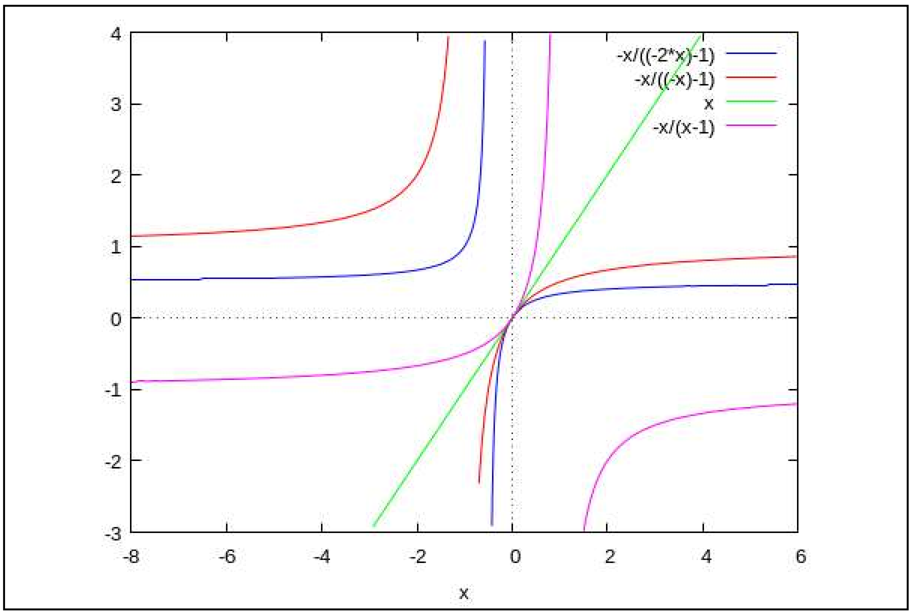
Other Characteristic eqn is eqn2:
dx-dy/(x^2-y^2)=du/(x+y)*u which simplifies
to dx-dy/(x-y)=du/u.
Let t=x-y then dt/t=du/u

```
→ eq2:'diff(u,t)=u/t;
```

```
→ ode2(eq2,u,t);
```

plot2d: some values were clipped.
plot2d: expression evaluates to non-numeric value somewhere in plottin
plot2d: some values were clipped.
plot2d: some values were clipped.
plot2d: some values were clipped.

(%t17)



(%o17)

(%o18) $\frac{d}{d\,t}\,u=\frac{u}{t}$

```
→ subst(x-y,t,%);
```
(%o19) $u=\%c\ t$

```
→ solve(u=c2·(x-y),u);
```

```
→ psol2:makelist(c2·x-c2·y,c2,-3,5);
```
(%o20) $u=\%c\ (x-y)$

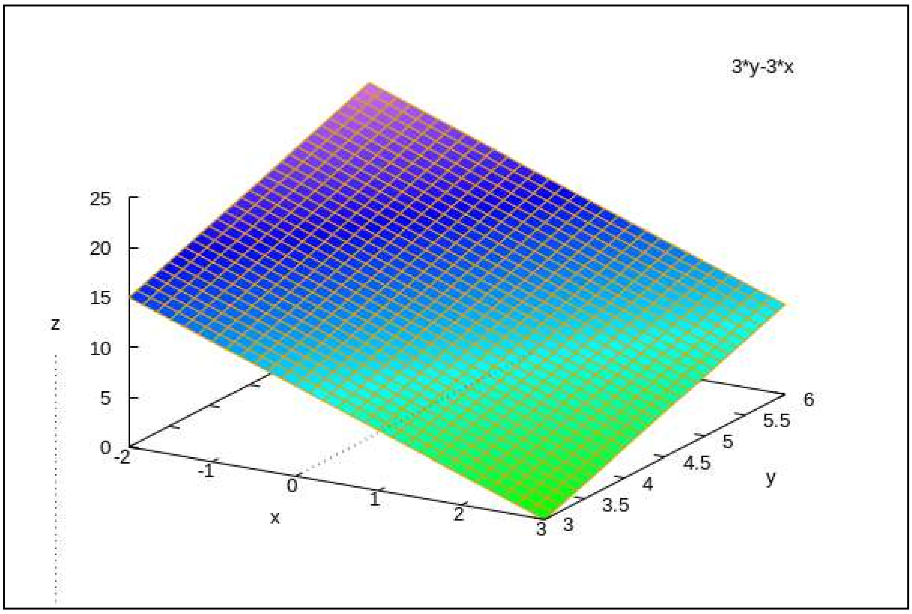
(%o21) $[u=c2\ x-c2\ y]$

```
→ wxplot3d(3·y-3·x,[x,-2,3],[y,3,6]);
```

```
→ kill(all);
```
(%o22) $[3\ y-3\ x, 2\ y-2\ x, y-x, 0, x-y, 2\ x-2\ y, 3\ x-3\ y, 4\ x-4\ y, 5\ x-5\ y]$

4 y*u_x + x*u_y = u

The characterstics equations for given PDE
will be,
dx/y = dy/x = du/u
or
du/u = (dx-dy)/(y-x)
eq1: dy/dx = x/y;
eqn2: du/u = (dx-dy)/(y-x)

```
→ eq1: 'diff(y,x)=x/y;
```
(%t23)


(%o23)

```
→ ode2(eq1,y,x);
```
(%o0) done

(%o1) $\frac{d}{d\,x}\,y=\frac{x}{y}$

```
→ solve(y^2/2=x^2/2+c1,y);
```

```
→ psol1:makelist(-sqrt(x^2+2·c1),c1,-2,1);
```

(%o2) $\frac{y^2}{2}=\frac{x^2}{2}+\%c$

(%o3) $[y=-\sqrt{x^2+2\,c1},y=\sqrt{x^2+2\,c1}]$

```
→ psol2:makelist(sqrt(x^2+2·c1),c1,-2,1);
```

(%o4) $[-\sqrt{x^2-4},-\sqrt{x^2-2},-|x|,-\sqrt{x^2+2}]$

```
→ psol:append(psol1,psol2);
```

```
→ wxplot2d(psol,[x,-6,6],[y,-5,5]);
```

(%o5) $[\sqrt{x^2-4},\sqrt{x^2-2},|x|,\sqrt{x^2+2}]$

(%o6) $[-\sqrt{x^2-4},-\sqrt{x^2-2},-|x|,-\sqrt{x^2+2},\sqrt{x^2-4},\sqrt{x^2-2},|x|,\sqrt{x^2+2}]$

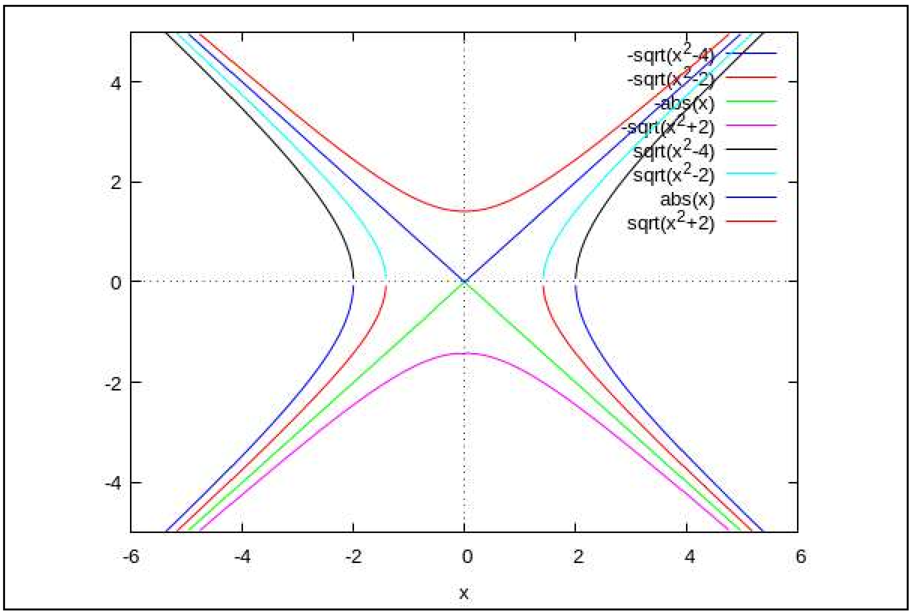
Other Characteristic eqn is eqn2: du/u =
(dx-dy)/(y-x).
Let t=x-y then -dt/t=du/u

```
→ eq2:'diff(u,t)=-u/t;
```

→ `ode2(eq2,u,t);`

plot2d: expression evaluates to non-numeric value somewhere in plottin
plot2d: some values were clipped.
plot2d: expression evaluates to non-numeric value somewhere in plottin
plot2d: some values were clipped.
plot2d: some values were clipped.
plot2d: some values were clipped.
plot2d: expression evaluates to non-numeric value somewhere in plottin
plot2d: some values were clipped.
plot2d: expression evaluates to non-numeric value somewhere in plottin
plot2d: some values were clipped.
plot2d: some values were clipped.
plot2d: some values were clipped.

(%t7)



(%o7)

(%o8) $\frac{d}{d\,t}\,u=-\frac{u}{t}$

→ `subst(x-y,t,%);`

→ `solve(u=c2/(x-y),u);`

(%o9) $u=\frac{\%c}{t}$

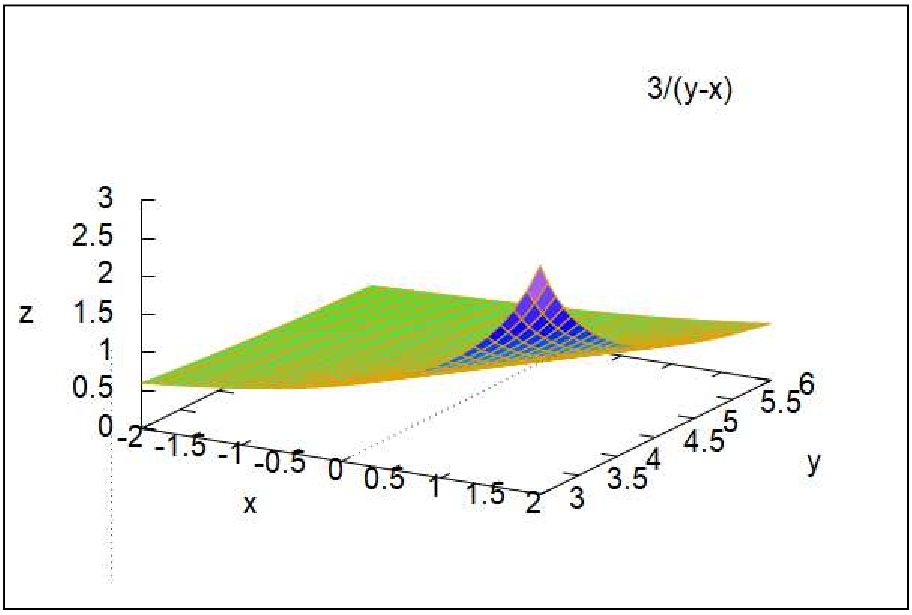
(%o10) $u=\frac{\%c}{x-y}$

→ `psol2:makelist(-c2/(y-x),c2,-3,5);`

(%o1) $[\frac{3}{y-x},\frac{2}{y-x},\frac{1}{y-x},0,-\frac{1}{y-x},-\frac{2}{y-x},-\frac{3}{y-x},-\frac{4}{y-x},-\frac{5}{y-x}]$

```
wxplot3d(3/(y-x),[x,-2,2],[y,3,6]);
```

(%t2)



(%o2)

Practical 8
Plot the integral surfaces of first order partial differential equations with initial data.

Written By Anshul Verma (19/78065)
B.Sc. (Hons.) Computer Science

1 Find the Integral Surface of the eqn 3*u_x-4*u_y=1, so that the surface passes through an initial curve represented parametrically by x(s,0)=2*s^2,y(s,0)=2*s & u(s,0)=0 where is s>0 is a parameter

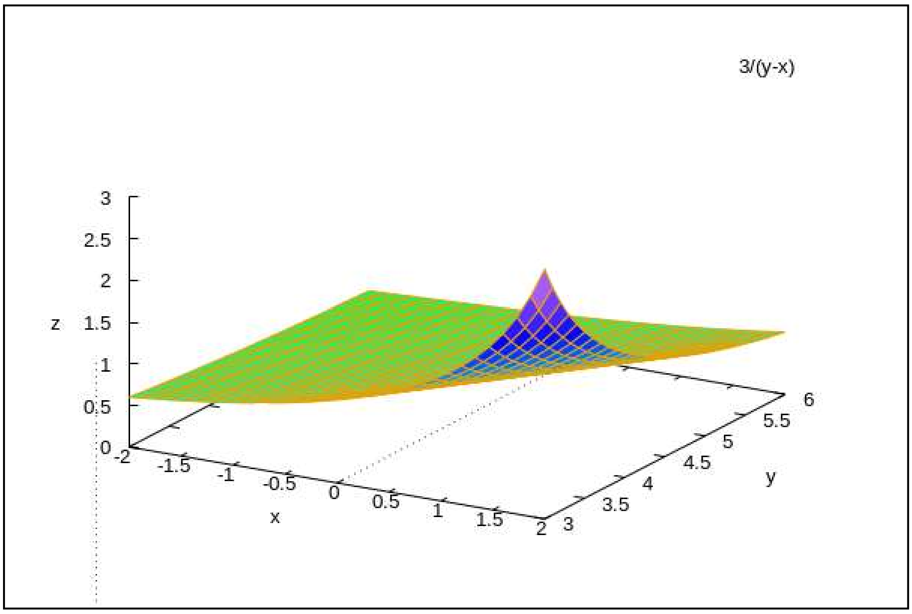
Sol:
The characterstic eqns are: dx/3 =dy/(-4) = du/1 =dt;
where t is another parameter.
So, they reduce parametrically to:
dx/dt=3 ; x(0)= 2*s^2
dy/dt=(-4) ; y(0)=2*s
du/dt=1 ; u(0)=0.

```
kill(all);
```

For dx/dt=3,

```
a:'diff(x,t)=3;
```

(%t13)



(%o13)
(%o0) done

```
→ ode2(a,x,t);
```

(%o1) $\frac{d}{d\,t}\,x=3$

```
→ a1:solve(x=3·t+c1,x);
```

For dy/dt=(-4),

```
→ b:'diff(y,t)=-4;
```

(%o2) $x=3\,t+\%c$

(%o3) $[x=3\,t+c1]$

```
→ ode2(b,y,t);
```

(%o4) $\frac{d}{d\,t}\,y=-4$

```
→ b1:solve(y=c2-4·t,y);
```

For du/dt=1,

```
→ d:'diff(u,t)=1;
```

(%o5) $y=\%c-4\,t$

(%o6) $[y=c2-4\,t]$

```
→ ode2(d,u,t);
```

```
→ d1:solve(u=t+c3,u);
```

(%o7) $\frac{d}{d\,t}\,u=1$

(%o8) $u=t+\%c$

Since, x(0)= 2*s^2; y(0)=2*s; u(0)=0;

```
→ subst([t=0,x=2·s^2],a1);
```

```
→ subst([t=0,y=2·s],b1);
```

(%o9) $[u=t+c3]$

(%o10) $[2\,s^2=c1]$

```
→ subst([t=0,u=0],d1);
```

```
→ 'x = 3·t+2·s^2;
```

(%o11) $[2\,s=c2]$

(%o12) $[0=c3]$

```
→ 'y=2·s-4·t ;
```

```
→ 'u = t;
```

(%o13) $x=3\,t+2\,s^2$

(%o14) $y=2\,s-4\,t$

```
→ load(draw)$
```

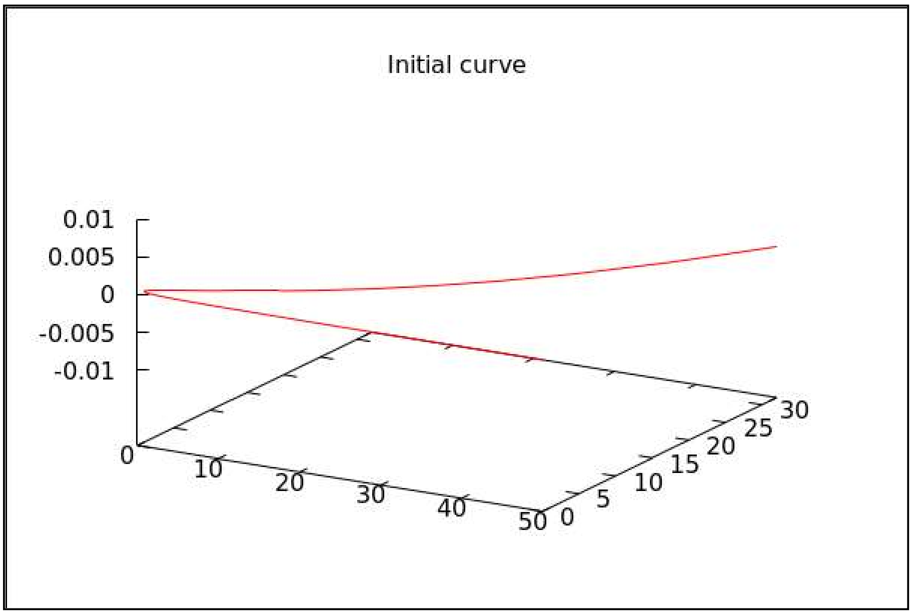
```
→ wxdraw3d(color=red,
      parametric(2·s^2,2^s,0,s,-5,5),title="Initial curve");
```

(%o15) $u=t$

```
→ wxdraw3d(color=blue,parametric_surface(3·t+2·s^2,2·s-4·t,t,
          s,-50,50,
          t,-100,100),title="Integral Surface");
```

```
→ wxdraw3d(
      [parametric_surface(
          3·t+2·s^2,2·s-4·t,t,
          s,-50,50,t,-100,100),
          color=red,parametric(2·s^2,2^s,0,s,-50,50)],
      title="Integral surface with Initial curve");
```

(%t17)



(%o17)

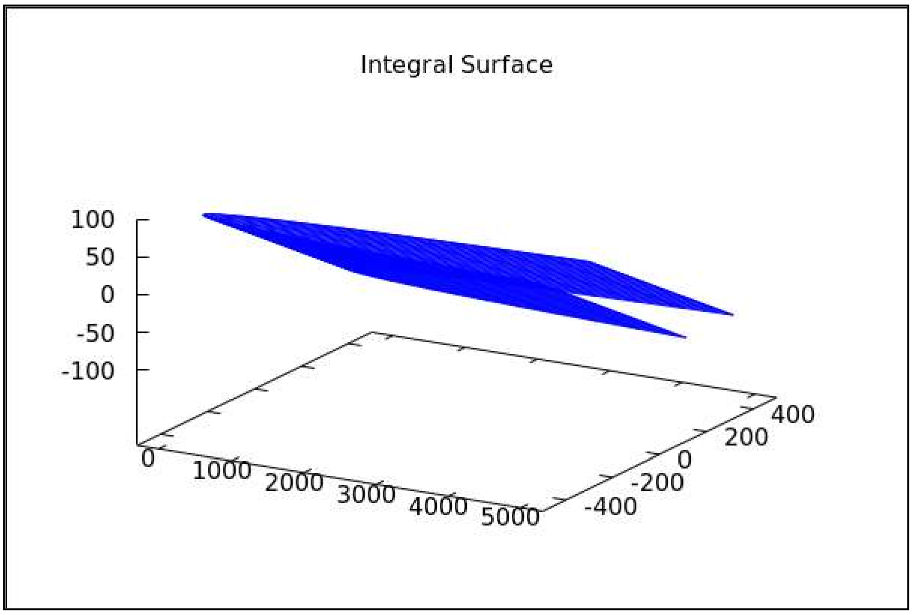
2 Find the Integral Surface of the eqn, (1/2y)u_x + u_y = 2u^2, so that the surface passes through an initial curve represented parametrically by

x(s,0)=4*s, y(s,0)=s & u(s,0)=s^2, where s>0 is a parameter.

Sol: Characterstic equations are: dx/(1/2y) = dy/1 = du/2u^2 = dt, where t is another parameter.
So they reduce parametrically to:
a: dx/dt = 1/2y; x(0)=4*s
b: dy/dt = 1; y(0)=s
d: du/dt = 2u^2; u(0)=s^2

```
kill(all);
```

(%t18)

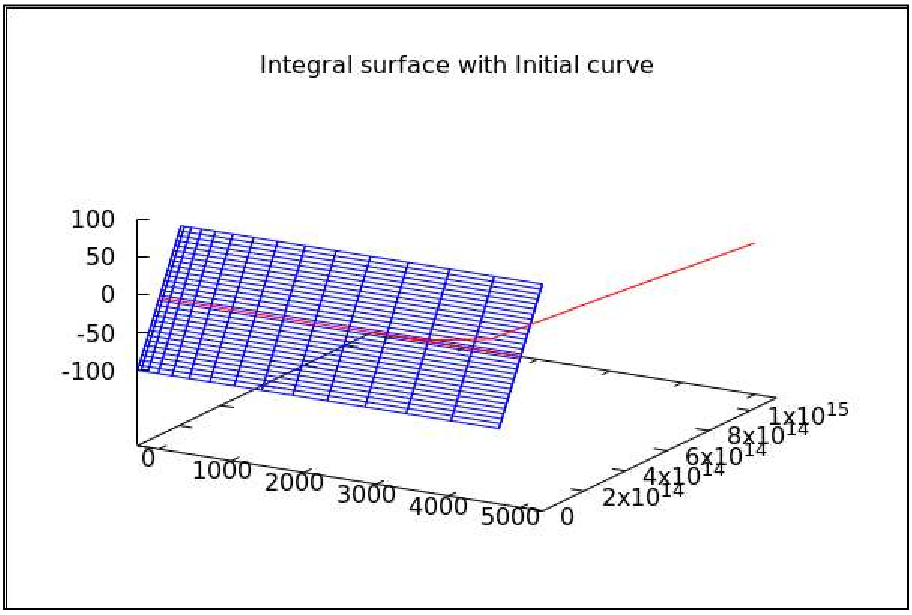


(%o18)

For b: dy/dt = 1; y(0)=s

```
→ b:'diff(y,t)= 1;
```

(%t19)



(%o19)
(%o0) done

```
→ ode2(b,y,t);
```

(%o1) $\frac{d}{d\,t}\,y=1$

```
→ b1:solve(y=t+c2,y);
```

```
→ subst([t=0,y=s],b1);
```

(%o2) $y=t+\%c$

```
→ 'y = t+s;
```

(%o3) $[y=t+c2]$
(%o4) $[s=c2]$

For a: dx/dt = 1/2y; x(0)=4*s

```
→ a:'diff(x,t)=1/(2·(t+c2));
```

(%o5) $y=t+s$

```
→ ode2(a,x,t);
```

(%o6) $\frac{d}{d\,t}\,x=\frac{1}{2\,(t+c2)}$

```
→ a1:solve(x=log(2·t+2·c2)/2+c1,c1);
```

```
subst([t=0,x=4·s,c2=s],a1);
```

(%o7) $x=\frac{\log(2\,t+2\,c2)}{2}+\%c$

(%o8) $[c1=\frac{2\,x-\log(2\,t+2\,c2)}{2}]$

```
'x=log(2·t+2·s)/2 + (8·s-log(2·s))/2;
```

For d: du/dt = 2u^2; u(0)=s^2

```
d:'diff(u,t)=2·u^2;
```

(%o9) $[c1=\frac{8\,s-\log(2\,s)}{2}]$

(%o10) $x=\frac{\log(2\,t+2\,s)}{2}+\frac{8\,s-\log(2\,s)}{2}$

```
ode2(d,u,t);
```

```
d1: solve(-(1/(2·u))=t+c3, c3);
```

(%o11) $\frac{d}{d\,t}\,u=2\,u^2$

(%o12) $-\frac{1}{2\,u}=t+\%c$

```
subst([t=0,u=s^2],d1);
```

```
solve(-(1/(2·u))=t-1/(2·s^2),u);
```

(%o13) $[c3=-\frac{2\,t\,u+1}{2\,u}]$

(%o14) $[c3=-\frac{1}{2\,s^2}]$

```
'u = -(s^2)/(2·s^2·t-1);
```

Therefore,
x=log(2*t+2*s)/2+(8*s-log(2*s))/2
y=t+s
u=-s^2/(2*s^2*t-1)

```
wxdraw3d(color=magenta,parametric(4·s,s,s^2,s,2,5),title="Initial curv
```

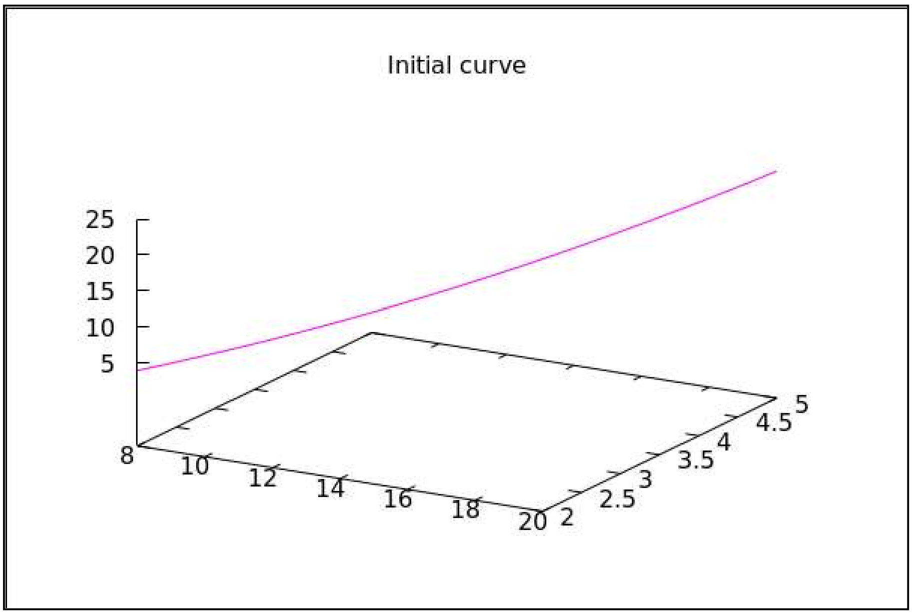
(%o15) $[u=-\frac{s^2}{2\,s^2\,t-1}]$

(%o16) $u=-\frac{s^2}{2\,s^2\,t-1}$

```
→   wxdraw3d(
        color=blue,
        parametric_surface(log(2·t+2·s)/2+(8·s−log(2·s))/2,
            t+s,
            −s^2/(2·s^2·t−1),
            s,−50,50,
            t,−100,100),title="Integral Surface");

→   wxdraw3d(
        [parametric_surface(log(2·t+2·s)/2+(8·s−log(2·s))/2,
                t+s, −s^2/(2·s^2·t−1),s,−50,50,t,−100,100),
            color=magenta,parametric(4·s,s,s^2,s,−50,50)],
        title="Integral surface with Initial curve");
```

(%t17)



(%o17)

3 Find the integral surface of eqn,

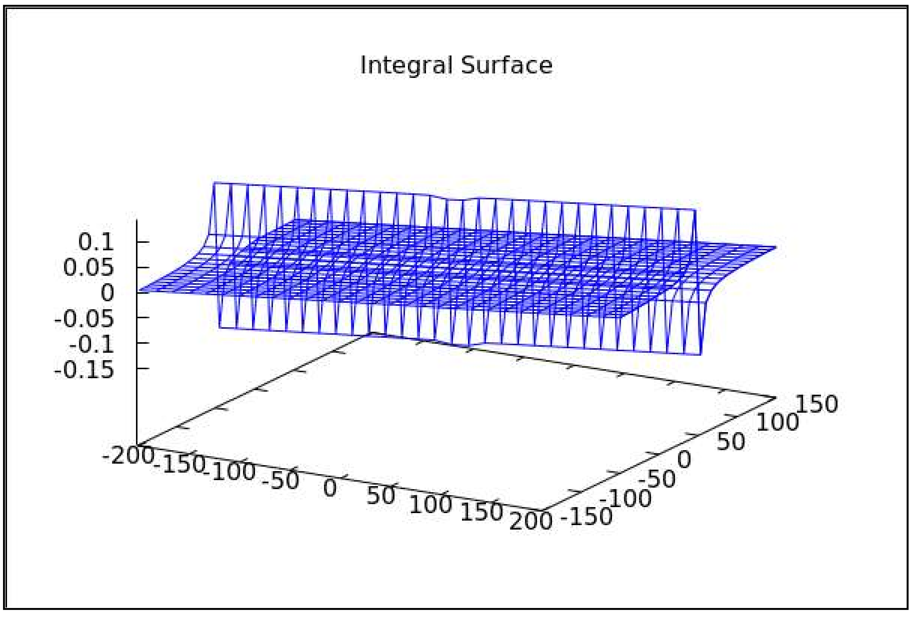
3*U_x − 2*U_y + U = x

so, that the surface passes through an initial curve represented parametrically by x(s,0) = (1−3*s)/2 , y(s,0) = s & u(s,0) = 0, where s>0 is a parameter.

Solution :
Char Equations are,
dx/3 = dy/-2 = du/1 = dt , where we are
using t as a dummy variable.
Char equations affter reducing
parametrically to :
dx/dt = 3 , x(s,0) = (1-3*s)/2
dy/dt = -2 , y(s,0) = s
du/dt = 1 , u(s,0) = 0

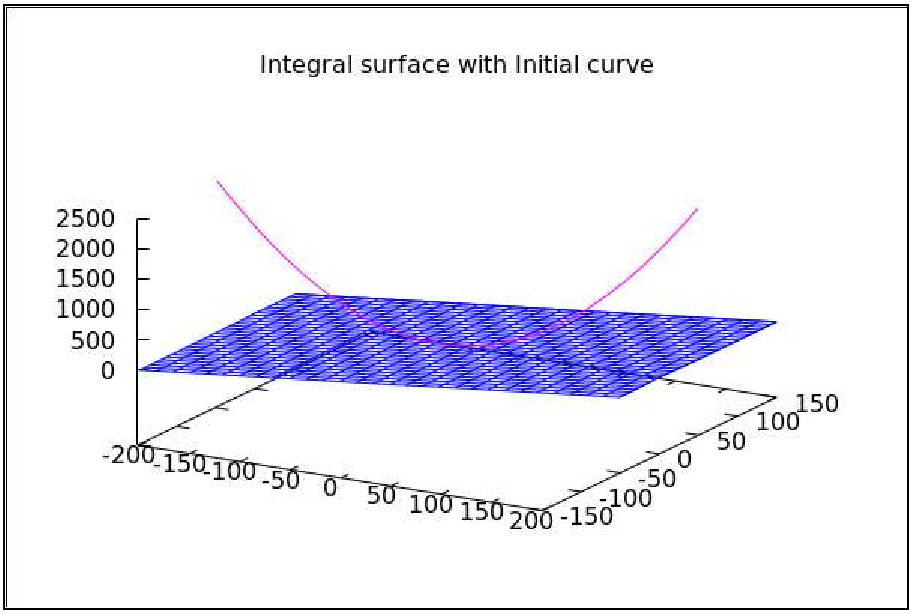
```
kill(all)$
```

(%t18)



(%o18)

```
a:'diff(x,t)=3;
ode2(a,x,t);
a1:solve(x=3·t+c1,x);
subst([t=0,x=(1-3·s)/2],a1);
```

(%t19) 

(%o19)

(%o1) $\frac{d}{d\,t}\,x=3$

(%o2) $x=3\,t+\%c$

(%o3) $[x=3\,t+c1]$

```
b:'diff(y,t)=-2;
ode2(b,y,t);
b1:solve(y=-2·t+c2,y);
subst([t=0,y=s],b1);
```

(%o4) $[\frac{1-3\,s}{2}=c1]$

(%o5) $\frac{d}{d\,t}\,y=-2$

(%o6) $y=\%c-2\,t$

```
c:'diff(u,t)=1;
ode2(c,u,t);
d1:solve(u=t+c3,u);
subst([t=0,u=0],d1);
```

(%o7) $[y=c2-2\,t]$

(%o8) $[s=c2]$

(%o9) $\frac{d}{d\,t}\,u=1$

(%o10) $u=t+\%c$

```
→ 'x =  3·t + (1-3·s)/2 ;
  'y = -2·t + s ;
  'u =  t;
```

(%o11) $[u=t+c3]$

(%o12) $[0=c3]$

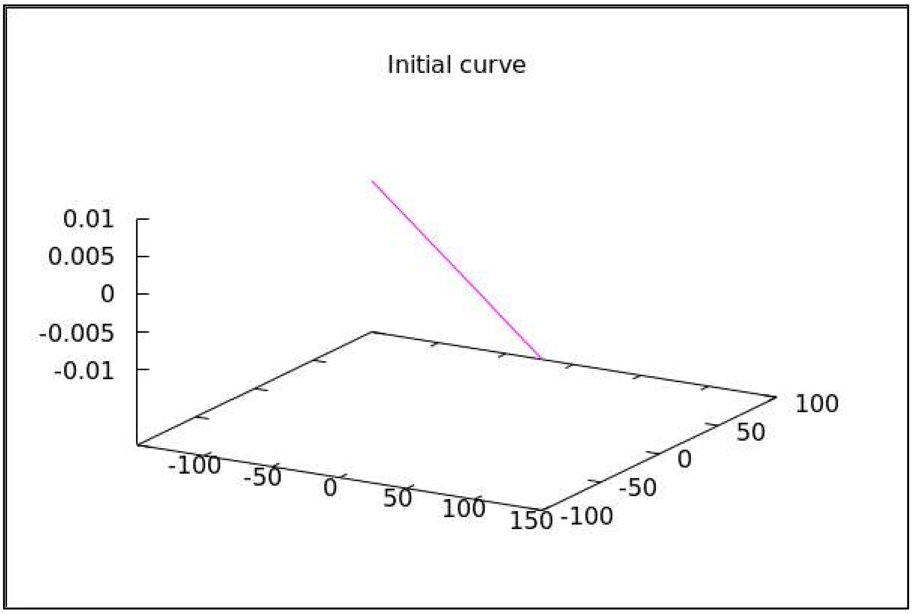
(%o13) $x=3\,t+\frac{1-3\,s}{2}$

(%o14) $y=s-2\,t$

```
→ load(draw)$
```

```
→ wxdraw3d(color=magenta,
           parametric((1-3·s)/2,s,0,s,-100,100),
           title="Initial curve");
  wxdraw3d(color=cyan,
           parametric_surface(3·t+(1-3·s)/2,-2·t+s,t,
              s,-100,100,t,-200,200),
           title="Integral surface");
  wxdraw3d([color=cyan,
            parametric_surface(3·t+(1-3·s)/2,-2·t+s,t,
              s,-100,100,t,-200,200),
            color=magenta,
            parametric((1-3·s)/2,s,0,s,-100,100)],
            title="Integral surface with Initial curve");
```

(%o15) $u=t$

(%t17) 

(%o17)

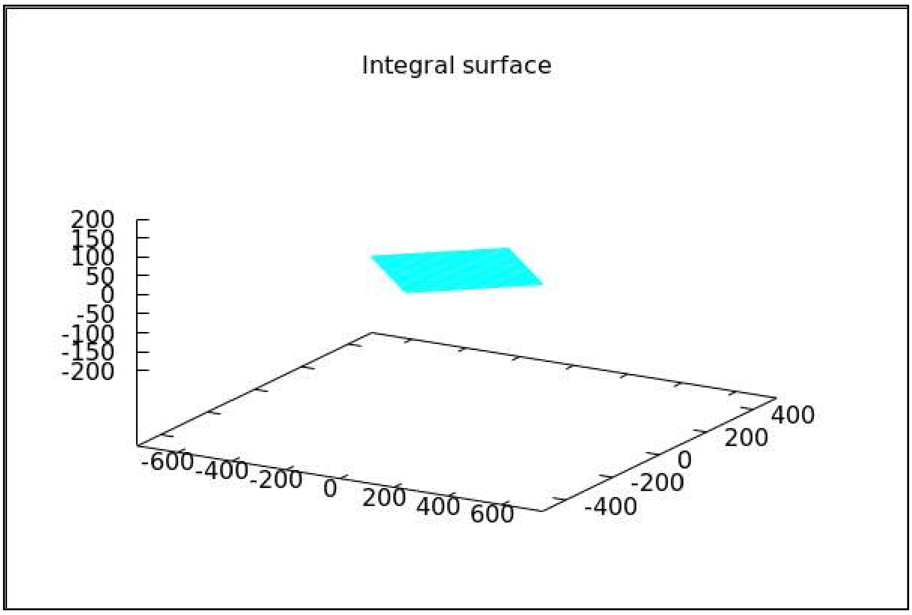
4 Solve the Cauchy problem, x^2u_x + uu_y = 2u with Cauchy data: x(s,0)= x(0)=2*sin(s)/3,y(s,0)=s^2,u(s,0)=3s.

Also plot the integral surface passing through initial curve.

Sol: Characterstic equations are: dx/y^2 =
dy/u = du/2u = dt, where t
is another parameter.
    So they reduce parametrically to:
    a: dx/dt = x^2; x(0)=2*sin(s)/3
    b: dy/dt = u; y(0)=s^2
    d: du/dt = 2u; u(0)=3*s

```
kill(all);
```

(%t18)

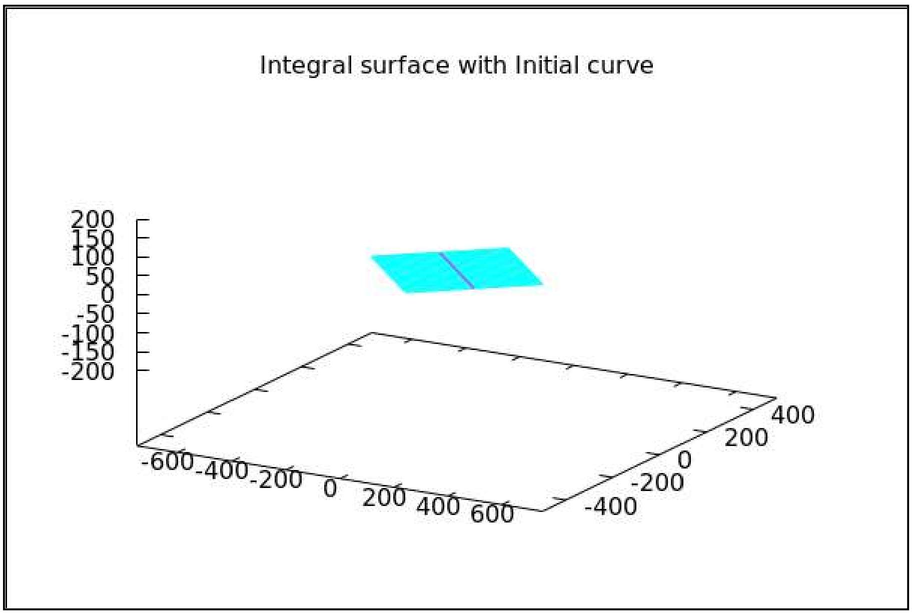


(%o18)

For a: dx/dt = x^2; x(0)=2*sin(s)/3,

```
a:'diff(x,t)=x^2;
```

(%t19)



(%o19)

```
ode2(a,x,t);
```

(%o0) done

(%o1) $\frac{d}{d\,t}\,x=x^{2}$

```
a1: solve(-1/x=t+c1, c1);
```

```
subst([t=0,x=2·sin(s)/3],a1);
```

(%o2) $-\frac{1}{x}=t+\%c$

(%o3) $[c1=-\frac{t\,x+1}{x}]$

```
solve(-1/x=t+-3/(2·sin(s)),x);
```

(%o4) $[c1=-\frac{3}{2\,\sin(s)}]$

```
'x=-(2·sin(s))/(2·sin(s)·t-3);
```

For d: du/dt = 2u; u(0)=3*s,

```
d:'diff(u,t)=2·u;
```

(%o5) $[x=-\frac{2\,\sin(s)}{2\,\sin(s)\,t-3}]$

(%o6) $x=-\frac{2\,\sin(s)}{2\,\sin(s)\,t-3}$

```
ode2(d,u,t);
```

```
→ d1: solve(u=c3·(e^(2·t)), c3);
```

(%o7) $\frac{d}{d\,t}\,u=2\,u$

(%o8) $u=\%c\,\%e^{2\,t}$

```
→ subst([t=0,u=3·s],d1);
```

```
→ solve(u=(3·s)·(e^(2·t)),u);
```

(%o9) $[c3=\frac{u}{e^{2\,t}}]$

(%o10) $[c3=3\,s]$

```
→ 'u=3·e^(2·t)·s;
```

For b: dy/dt = u; y(0)=s^2,

```
→ b:'diff(y,t)= c3·(e^(2·t));
```

(%o11) $[u=3\,e^{2\,t}\,s]$

(%o12) $u=3\,e^{2\,t}\,s$

```
→ ode2(b,y,t);
```

```
→ b1: solve(y=(c3·e^(2·t))/(2·log(e))+c2, c2);
```

(%o13) $\frac{d}{d\,t}\,y=c3\,e^{2\,t}$

(%o14) $y=\frac{c3\,e^{2\,t}}{2\,\log(e)}+\%c$

```
→ subst([t=0,y=s^2,c3=3·s],b1);
```

```
→ solve(y=(3·s·e^(2·t))/(2·log(e))+(2·log(e)·s^2-3·s)/(2·log(e)),y);
```

(%o15) $[c2=\frac{2\,\log(e)\,y-c3\,e^{2\,t}}{2\,\log(e)}]$

(%o16) $[c2=\frac{2\,\log(e)\,s^2-3\,s}{2\,\log(e)}]$

```
→ 'y=(2·log(e)·s^2+(3·e^(2·t)-3)·s)/(2·log(e));
```

Therefore,
x=-(2*sin(s))/(2*sin(s)*t-3),

y=(2*log(e)*s^2+(3*e^(2*t)-3)*s)/(2*log(e)),
u=3*e^(2*t)*s

```
load(draw)$
```

(%o17) $[y=\frac{2\log(e)\,s^2+(3\,e^{2t}-3)\,s}{2\log(e)}]$

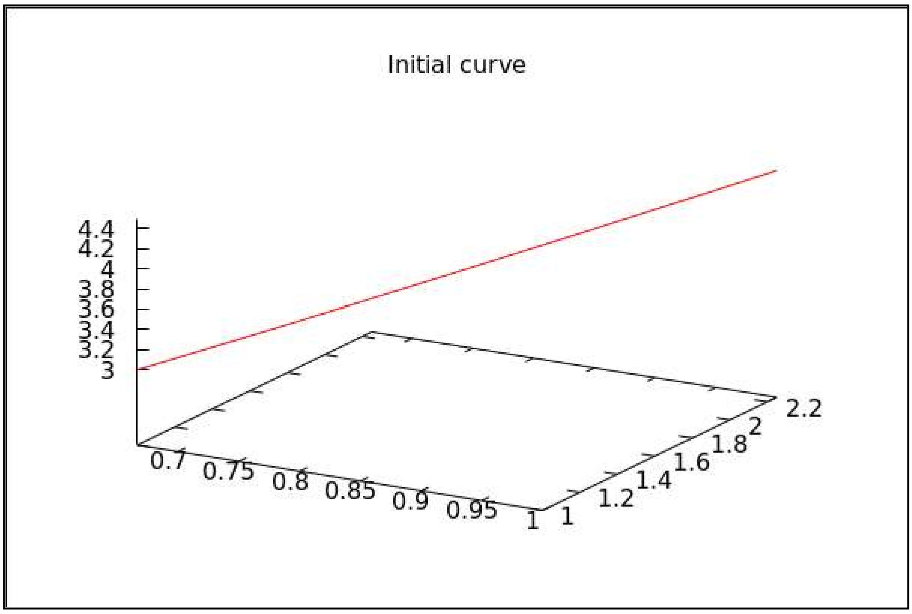
(%o18) $y=\frac{2\log(e)\,s^2+(3\,e^{2t}-3)\,s}{2\log(e)}$

```
wxdraw3d(
    color=red,
    parametric((2·s)/3,s^2,3·s,s,1,1.5),
    title="Initial curve");
```

```
wxdraw3d(
    parametric_surface(
        -(2·sin(s))/(2·sin(s)·t-3),
            (2·log(%e)·s^2+(3·%e^(2·t)-3)·s)/(2·log(%e)),
            3·%e^(2·t)·s,
            s,1,2,t,-1,0),
    title="Integral surface");
```

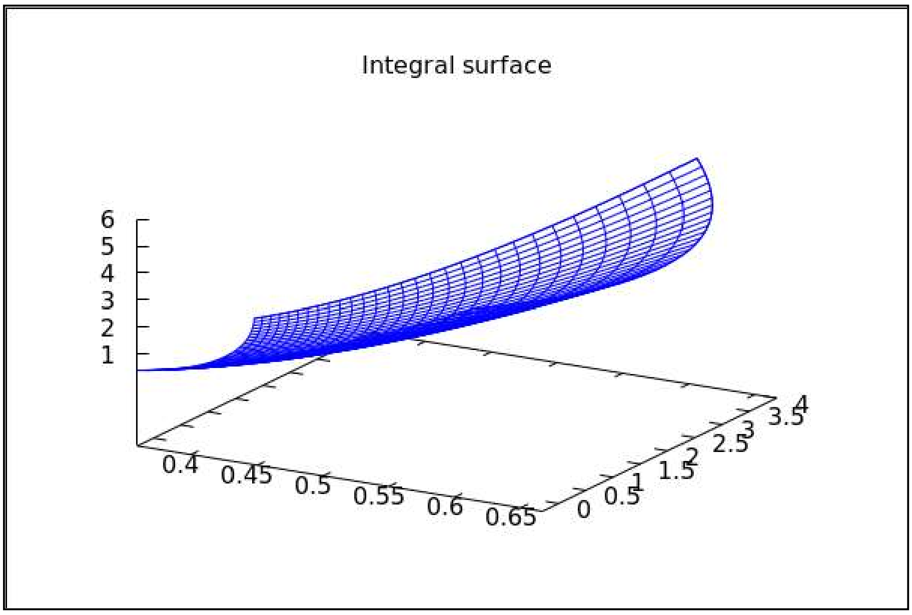
```
→ wxdraw3d(
     [parametric_surface(
           -(2·sin(s))/(2·sin(s)·t-3),
           (2·log(%e)·s^2+(3·%e^(2·t)-3)·s)/(2·log(%e)),
           3·%e^(2·t)·s,
           s,1,2,t,-1,0),
       color=red,
       parametric((2·s)/3,s^2,3·s,s,0,1)],
     title="Integral surface with Initial curve");
```

(%t20)



(%o20)

(%t21)



(%o21)

(%t22)