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ASSIGNMENT-2 :- Boolean Algebra

Que: 1: Demonstrate by means of truth table the Validity of both the Distributive laws of Boolean Algebra

The Two Distributive Laws of Boolean Algebra are:

(i) (·) over (+)

(ii) (+) over (·)

If (·) and (+) are two binary operators on a given set, then

(i) (·) is said to be distributive over (+) whenever $A \cdot (B+C) = A \cdot B + A \cdot C$

A	B	C	B+C	A·(B+C)	A·B	A·C	AB + AC
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	1	0	1	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

(ii) (+) is said to be distributive over (·) whenever $A + (B \cdot C) = (A+B) \cdot (A+C)$

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A	B	C	B·C	A+(B·C)	A+B	A+C	(A+B)·(A+C)
0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0
0	1	0	0	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	1	0	0	1	1	1	1
1	1	1	1	1	1	1	1

Que:2 Simplify the following Boolean functions to a minimum number of literals

(a.) $(x+y)(x+y')$

$\Longrightarrow$ $(x+y)\cdot(x+y') = ((x+y)\cdot x) + ((x+y)\cdot y')$

$= (x\cdot x + x\cdot y + x\cdot y + y\cdot y')$

$= x + x\cdot y + x\cdot y' + 0$ $(\because A\cdot A' = 0)$

$= x(1 + y + y')$ $(\because A + A' = 1)(\because A\cdot A = A)$

$= x\cdot(1+1)$ $(\because 1+1 = 1)$

$= x\cdot 1$

$= x$ $(\because A\cdot 1 = A)$

(b) $xyz + x'y + xyz'$

$\Longrightarrow$ $xyz + x'y + xyz' = xy(z+z') + (x'y)$

$= xy\cdot 1 + x'y$ $(\because A + A' = 1)$

$= xy + x'y$ $(\because A\cdot 1 = A)$

$= (x + x')\cdot y$

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$= 1 \cdot y$ ($\because A + A' = 1$)

$= y$ ($\because A \cdot 1 = A$)

(c) $xz + x'yz$

$\Rightarrow x \cdot z + x'yz = z(x + x'y)$

$= z(x + x')(x+y)$

$= z(1)(x+y)$

$= z(x+y)$

(d.) $(a+b)' \cdot (a'+b')'$

$(a+b)' \cdot (a'+b')' = (a'b') \cdot (a \cdot b)$

$= a'a \cdot b'b$

$= 0 \cdot 0$

$= 0$

(e) $y(wz' + wz) + xy$

$\Rightarrow y(wz' + wz) + xy = yw(z' + z) + xy$

$= yw \cdot 1 + xy$

$= yw + xy$

$= y(w + x)$

Que: 3 For the given Boolean function, $F = xy + x'y' + y'z$

1] Implement it with AND(3), OR(2) and NOT(2) gates.

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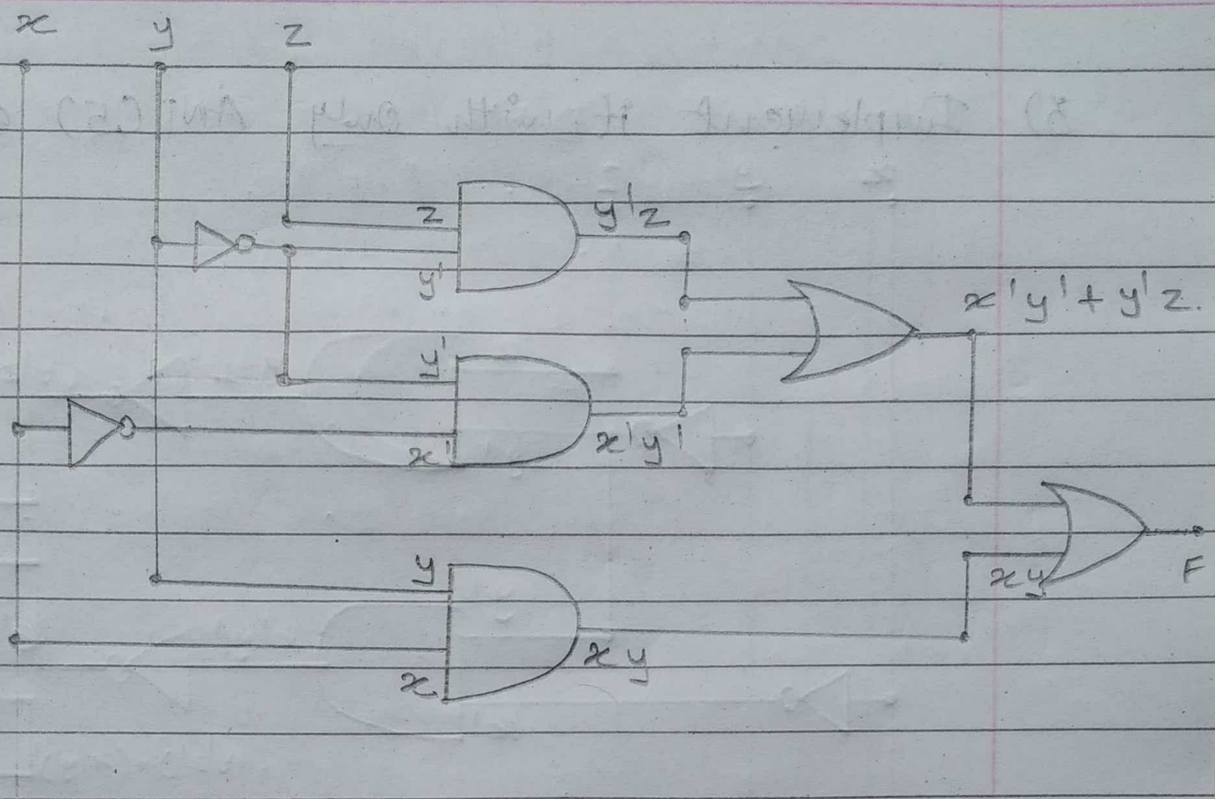
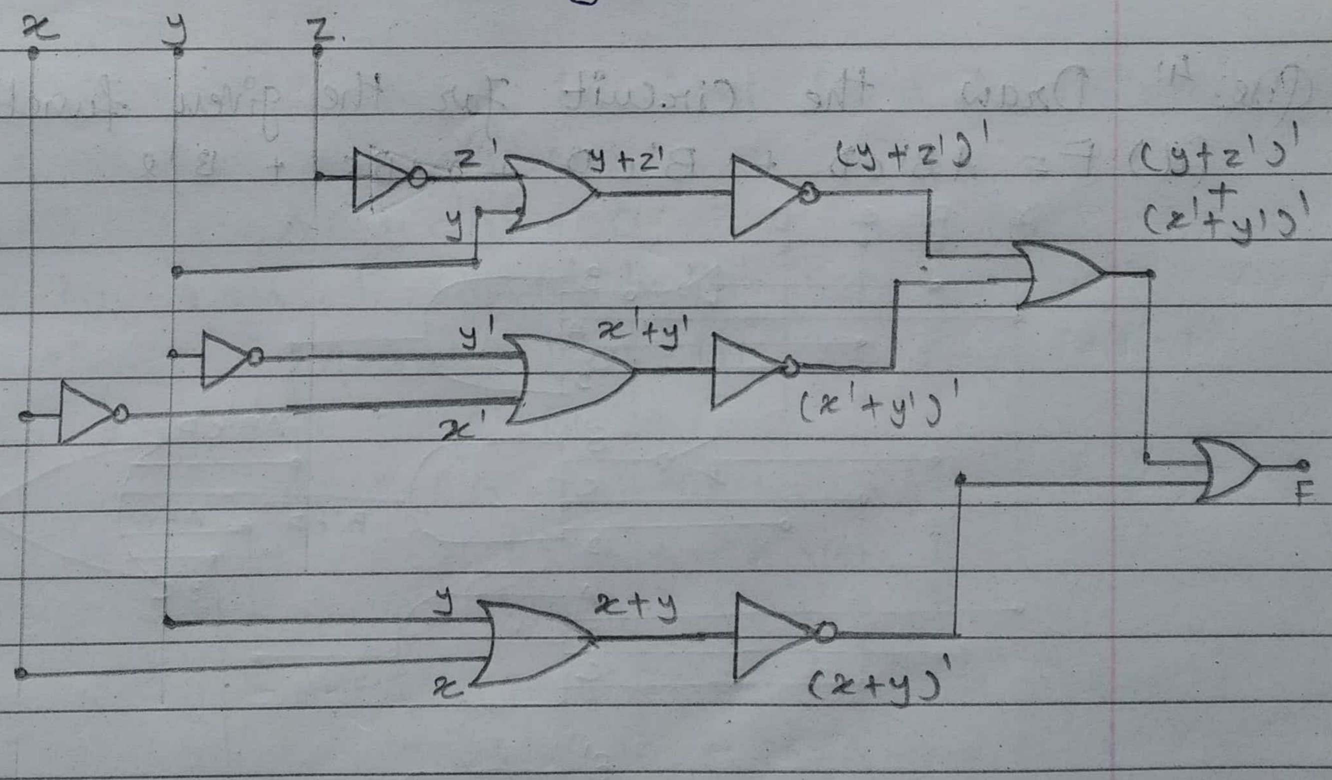


2] Implement it with only OR(5) and NOT(6) gates.



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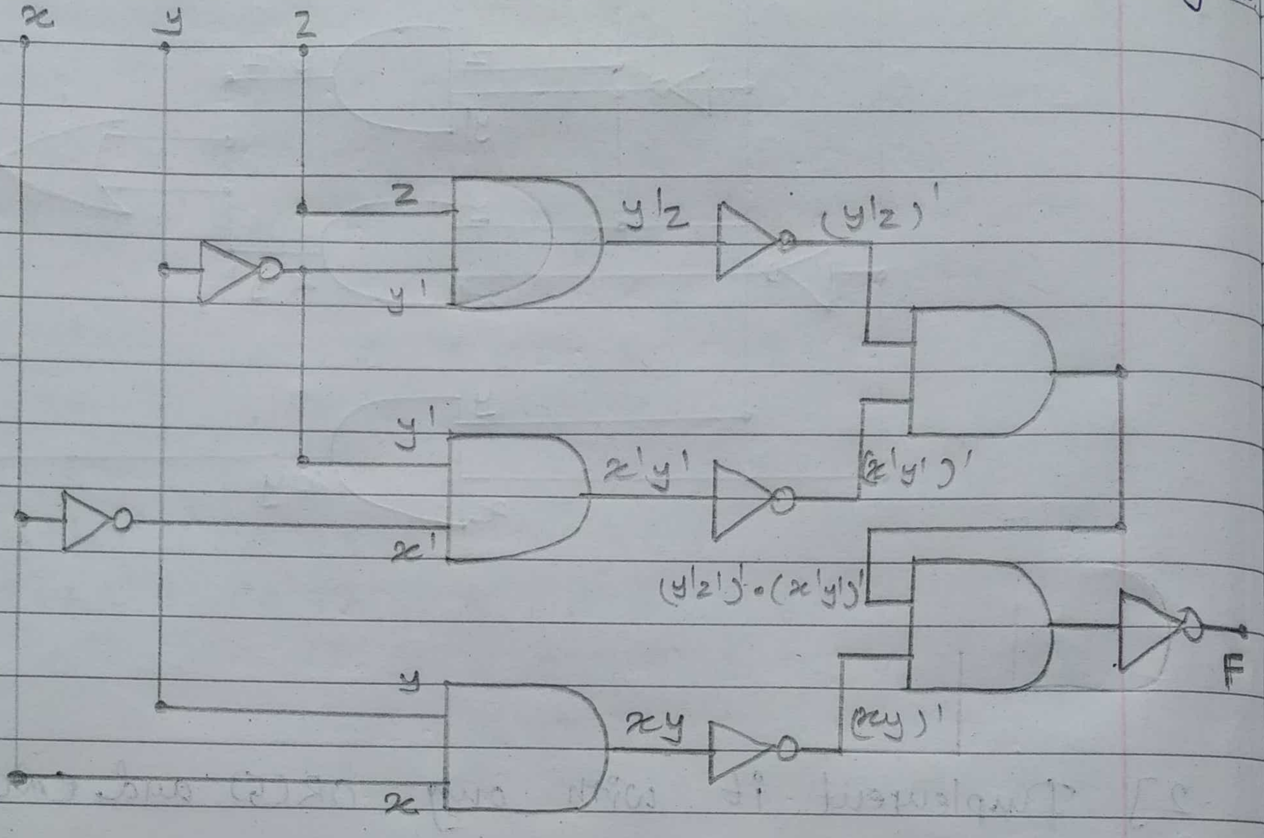
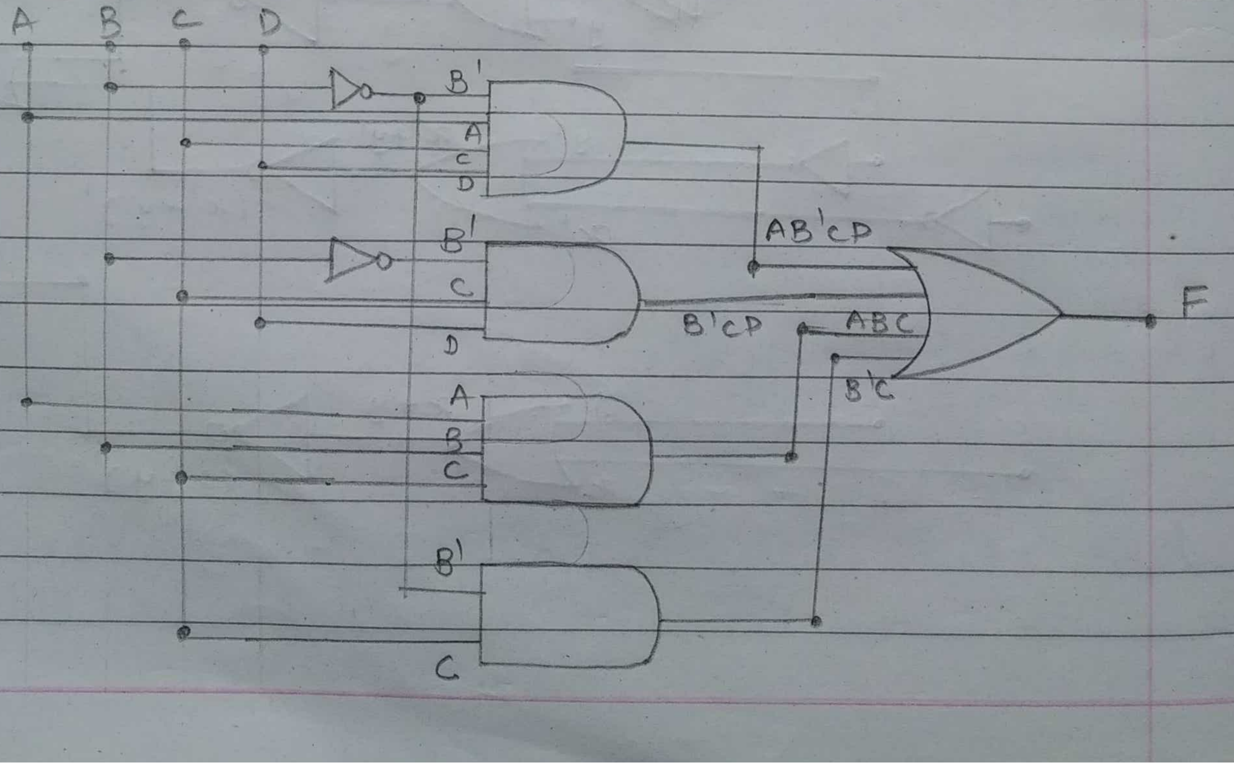
3) Implement it with only AND (5) and NOT (6) gates

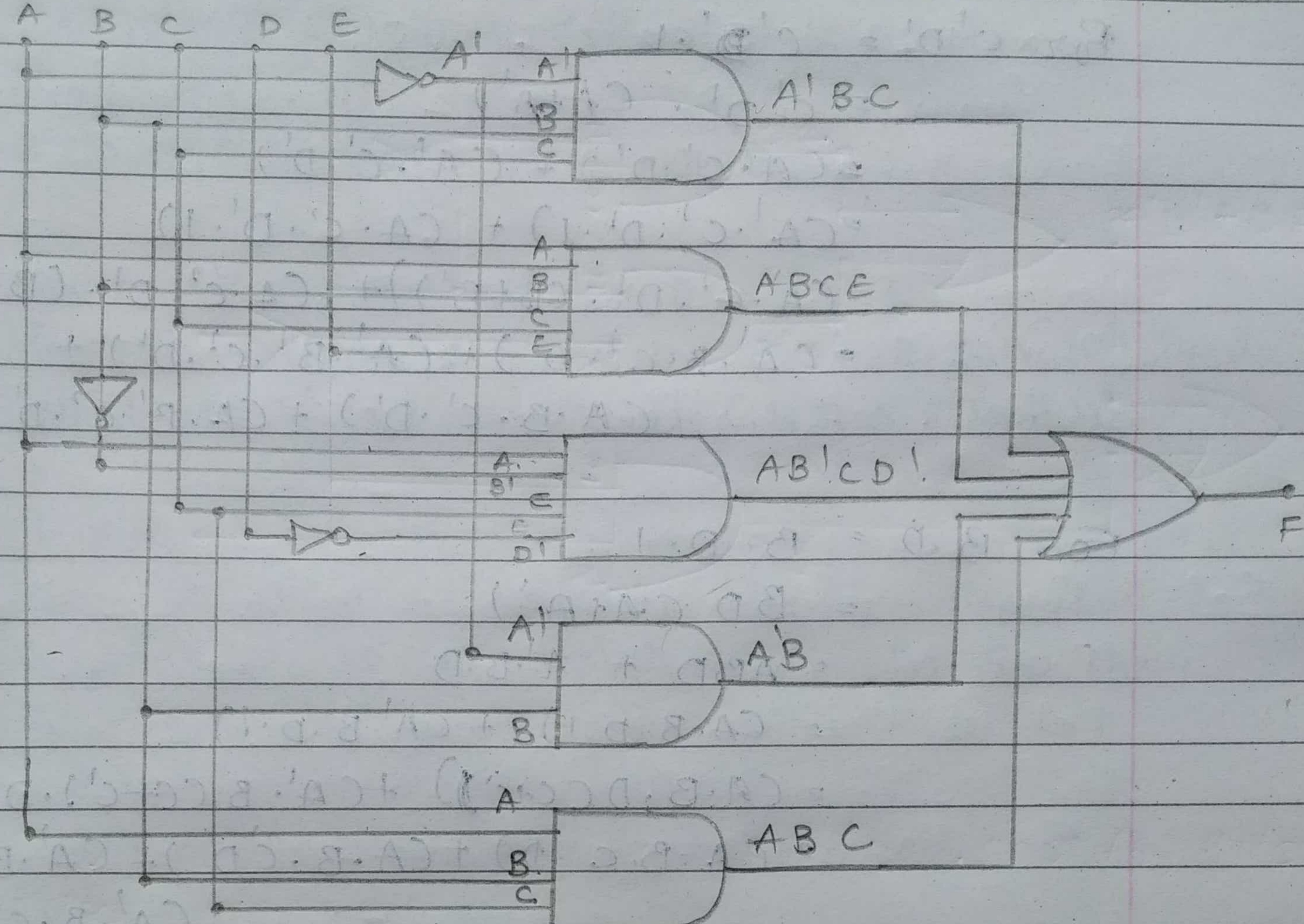


Que: 4: Draw the circuit for the given function

(a) $F = AB'CD + B'CD + ABC + B'C$



(b) $F = A'BC + ABCE + AB'CD' + A'B + ABC$



Que 5: Express the following functions in a sum of minterms and product of maxterms.

(a) $F(A,B,C,D) = A'B + C'D' + BD$

Sol: For Min-terms, F should be in SOP Form

The given $F(A,B,C,D)$ is in SOP Form

Thus, for $A'B = A' \cdot B \cdot 1$

$= A'B \cdot (C + C')$

$= (A' \cdot B \cdot C) + (A' \cdot B \cdot C')$

$= (A' \cdot B \cdot C \cdot 1) + (A' \cdot B \cdot C' \cdot 1)$

$= (A' \cdot B \cdot C (D + D')) + (A' \cdot B\, C' (D + D'))$

$= (A' \cdot B \cdot C \cdot D) + (A' \cdot B \cdot C \cdot D') +$

$(A' \cdot B \cdot C' D) + (A' \cdot B \cdot C' D')$

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For $C'D' = C'D' \cdot 1$

$= C'D' \cdot (A+A')$

$= (A \cdot C' \cdot D') + (A' \cdot C' \cdot D')$

$= (A' \cdot C' \cdot D' \cdot 1) + (A \cdot C' \cdot D' \cdot 1)$

$= (A' \cdot C' \cdot D' \cdot (B+B')) + (A \cdot C' \cdot D' \cdot (B+B'))$

$= (A' \cdot B \cdot C' \cdot D') + (A' \cdot B' \cdot C' \cdot D') +$
$(A \cdot B \cdot C' \cdot D') + (A \cdot B' C' \cdot D')$

For $B \cdot D = B \cdot D \cdot 1$

$= BD(A+A')$

$= ABD + A' \cdot B \cdot D$

$= (A \cdot B \cdot D \cdot 1) + (A' \cdot B \cdot D \cdot 1)$

$= (A \cdot B \cdot D(C+C')) + (A' \cdot B(C+C') \cdot D)$

$= (A \cdot B \cdot C \cdot D) + (A \cdot B \cdot C'D) + (A' \cdot B \cdot C \cdot D) +$
$(A' \cdot B \cdot C' \cdot D)$

Now thus,

$F = A'B + C'D' + BD$

$= ABCD + A'BCD + A'BC'D + A'BC'D' + A'BC'D' +$
$A'B'C'D' + ABC'D + AB'CD + ABCD + ABC'D +$
$A'BCD + A'BC'D$

$= A'B'C'D' + ABC'D + ABC'D + AB'C'D + ABCD$
$+ A'BCD + A'BC'D + A'BC'D + ABCD$

$= A'B'C'D' + A'BC'D' + A'BC'D + A'BCD' + A'BCD$
$+ AB'C'D' + ABCD' + ABC'D + ABCD$

$= m_0 + m_4 + m_5 + m_6 + m_7 + m_8 + m_{12}$
$+ m_{13} + m_{15}$

$F(ABCD) = \Sigma(0, 4, 5, 6, 7, 8, 12, 13, 15)$

From $m_j = M_j$

$F(A, B, C, D) = \Pi(1, 2, 3, 9, 10, 11, 14)$

Hence: $F = A'B + C'D' + BD$

Sum of minterms : $\Sigma(0, 4, 5, 6, 7, 8, 12, 13, 15)$ and product of maxterms : $\Pi(1, 2, 3, 9, 10, 11, 14)$

(b) $F(x, y, z) = y'z + xy' + z'$.

Sol: For Minterms, F should be in SOP form. And the $F(x, y, z)$ is also in SOP form.

So for $y'z = y'z \cdot 1$

$= y'z \cdot (x + x')$

$= xy'z + x'y'z$

for $xy' = xy' \cdot 1$

$= xy'(z + z)$

$= xy'z + xy'z'$

for $z' = z' \cdot 1$

$= z'(\cancel{x} + \cancel{x'})$

$= xz' + x'z'$

$= (xz'(y + y')) + (x'z'(y + y'))$

$= xyz' + xy'z' + x'yz' + x'y'z'$

Thus $F = y'z + xy' + z'$

$= xy'z + x'y'z + xy'z + xy'z' + xyz' + xy'z' + x'yz' + x'y'z'$

$= xy'z + x'y'z + xy'z' + xyz' + x'yz' + x'y'z'$

$= m_5 + m_1 + m_4 + m_6 + m_2 + m_0$

$= m_0 + m_1 + m_2 + m_4 + m_5 + m_6$

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$F(x,y,z) = \Sigma(0,1,2,4,5,6)$

From $m_j' = m_j$

$F(x,y,z) = \Pi(3,7)$

Hence, $F = y'z + xy' + z'$

Sum of Minterms are $\Sigma(0,1,2,4,5,6)$ and product of Maxterms are $\Pi(3,7)$

(c) $F(A.B.C.D) = (A'+B)(B'+C)$

Sol: For minterms, F should be in SOP Form. But the given F(A.B.C.D) is in POS Form. Let us consider Maxterms, since it should be in POS form

For $A'+B$, $A'+B = A'+B+0$

$= A'+B+C\cdot C'$

$= (A'+B+C)\cdot(A'+B+C')$

$= (A'+B+C+D)\cdot(A'+B+C'+0)$

$= (A'+B+C+D\cdot D')\cdot(A'+B+C'+D\cdot D')$

$= (A'+B+C+D)\cdot(A'+B+C+D)\cdot$
$(A'+B+C'+D\ \ (A'+B+C'+D')$

For $B'+C$, $B'+C = B'+C+0$

$= B'+C+A\cdot A'$

$= (A+B'+C)\cdot(A'+B'+C)$

$= (A+B'+C+0)\cdot(A'+B'+C+0)$

$= (A+B'+D\cdot D')\cdot(A'+B'+C+D\cdot D')$

$= (A+B'+C+D)\ (A\cdot B'+C+D')$
$(A'+B'+C+D)\ (A'+B'+C+D$

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Now $F = (A'+B)(B'+C)$ $(A'+B+C'+D')$

$= (A'+B+C+D)(A'+B+C+D')(A'+B+C'+D)$

$\cdot (A+B'+C+D)(A+B'+C+D')\cdot(A'+B'+C+D)$
$(A'+B'+C+D')$

$= (A'+B+C+D)(A'+B+C+D')(A'+B+C'+D)$
$(A'+B+C'+D')$ $(A+B'+C+D)$ $(A+B'+C+D')$
$(A'+B'+C+D')$ $(A'+B'+C+D)$

$= M_8\ M_9\ M_{10}\ M_{11}\ M_4\ M_5\ M_{13}\ M_{12}$

$F(A,B,C,D) = \Pi(4,5,8,9,10,11,12,13)$

From $m_j' = m_j$

$F(A,B,C,D) = \Sigma(0,1,2,3,6,7,14,15)$

Hence $F(A,B,C,D) = (A'+B)(B'+C)$
Sum of minterms are $= \Sigma(0,1,2,3,6,7,14,15)$
Product of maxterms are $\Pi(4,5,8,9,10,11,12,13)$

(d) $F(w,x,y,z) = (y+z)(x'+y')(x'+z)$

Sol: For minterms, F should be in SOP form but the given $F(w,x,y,z)$ is in POS form

Let us first convert it to SOP form

$F = (y+z)(x'+y')(x'+z)$

$= [y\cdot(x'\cdot y') + z\cdot(x'+y')](x'+z)$

$= [x'y + yy' + x'z + y'z](x'+z)$

$= (x'y + 0 + x'z + y'z)(x'+z)$

$= x'(x'y + x'z + y'z) + (z(x'y + x'z + y'z)$

$= x'y + x'z + x'y'z + x'yz + x'z + y'z$

$= x'y + x'z + x'y'z + x'yz + y'z$

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$= x'y + x'z + x'y'z + x'yz + y'z$ (SOP Form)

For $x'y = x'y(z+z')(w+w')$

$= wx'yz + w'x'yz + wx'yz' + w'x'yz'$

For $x'z = x'z(y+y')(w+w')$

$= wx'yz + w'x'yz + wx'y'z + w'x'y'z$

For $x'y'z = x'y'z(w+w')$

$= wx'y'z + w'x'y'z$

For $x'yz = x'yz(w+w')$

$= wx'yz + w'x'yz$

For $y'z = y'z(x+x')(w+w')$

$= wxy'z + wx'y'z + w'xy'z + w'x'y'z$

Thus $F = x'y + x'z + x'y'z + x'yz + y'z$

$= wx'yz + w'x'yz + wx'yz' + w'x'yz'$
$+ wx'y'z + w'x'y'z + wxy'z = wxy'z$

$F = m_{11} + m_3 + m_{10} + m_2 + m_9 + m_1 + m_{13} + m_5$

$= m_1 + m_2 + m_3 + m_5 + m_9 + m_{10} + m_{11} + m_{13}$

$F(w,x,y,z) = \Sigma(1,2,3,5,9,10,11,13)$

From $m_j = M_j$

$F(w,x,y,z) = \Pi(0,4,6,7,8,12,14,15)$

Sum of Minterms are $= \Sigma(1,2,3,5,9,10,11,13)$

Product of Maxterms are $: \Pi(0,4,6,7,8,12,14,15)$

Que: 6 Define :-

(i) Fan-out :-

The fanout is the number of gate input driven by the output of another single logic gate Fan-out $= I_{OH} / I_{IH}$ or I_{OL} / I_{IL}

(ii) Power Dissipation:

The power dissipation of logic gates in characterised under two modes. These are static and dynamic.
Under static conditions, the input is held at either logic '1' or '0'

$$\therefore P_{static} = V_{dd} \times I_{supply}$$

Under dynamic conditions, the inputs are changing state and hence the transistors between the supplies will either be both on or require energy to charge and discharge output capacitance.
Hence, the dynamic power dissipation will depend upon the number of times the transistors switch per second.

$$\therefore P_{dynamic} = C_L + V_{dd}^2 \times f$$

C_L = Total capacitance
f = frequency

(iii) Propagation Delay:

Propogation Delay is the length of time which starts when the input to a logic gate becomes stable and valid to change, to the time that the output of that logic gate is stable and valid to change.

(iv) Noise Margin:-

Noise Margin is the amount by which the Signal Exceeds the threshold for a proper '0' or '1'

- The maximum Voltage amplitude of Exteraneous signal that can be algebraically added to the noise-free worst-case input level without Causing the output Voltage to device from the allowable logic voltage level.

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