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Assignment : 1

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1. The resistance of a certain length of wire is 4.6 Ω at 20°C and 5.68 Ω at 80°C. Determine, a) temperature co-efficient of resistance of the material of wire at 0°C, b) resistance of wire at 60°C.

→ Here, Let $R_1 = 4.6\ \Omega$, $t_1 = 20^\circ C$

$R_2 = 5.68\ \Omega$, $t_2 = 80^\circ C$.

$\alpha_0 = ?$

$R_{60} = ?$

From, $R = R_0 (1 + \alpha_0 t)$ ——— (1)

$$\therefore \frac{R_1}{R_2} = \frac{R_0 (1 + \alpha_0 t_1)}{R_0 (1 + \alpha_0 t_2)}$$

$$\therefore \frac{R_1}{R_2} = \frac{1 + \alpha_0 t_1}{1 + \alpha_0 t_2}$$

$$\therefore R_1 + R_1 \alpha_0 t_2 = R_2 + R_2 \alpha_0 t_1$$

$$\therefore \alpha_0 (R_1 t_2 - R_2 t_1) = R_2 - R_1$$

$$\therefore \alpha_0 = \frac{R_2 - R_1}{R_1 t_2 - R_2 t_1}$$

$$= \frac{5.68 - 4.6}{(4.6)(80) - (5.68)(20)}$$

$$= \frac{1.08}{368 - 113.6}$$

$$\therefore \boxed{\alpha_0 = 0.00425\ ^\circ C^{-1}}$$

→ To find resistance of wire at 60°C,

$$\frac{R_{60}}{R_{20}} = \frac{1 + \alpha_0 (60)}{1 + \alpha_0 (20)} \quad (\because \text{eq. } (1))$$

$$\therefore R_{60} = (4.6) \frac{[1 + (0.00425)(60)]}{[1 + (0.00425)(20)]}$$

$$\therefore R_{60} = \frac{5.773}{1.085} \qquad \therefore \boxed{R_{60} = 5.32\ \Omega}$$

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2. The resistance of a wire of 3 mm^2 cross sectional area and 6 m length is 0.15 Ω at 0°c. When the temperature of the wire is raised to 65°c the resistance is found to be 0.2 Ω. Calculate the temperature co-efficient of resistance of the wire and its resistivity at 0°c

→ Here, let. $R_1 = 0.15\ \Omega$, $t_1 = 0°C$, $l = 6$ m

$R_2 = 0.2\ \Omega$, $t_2 = 65°C$, $\alpha_0 = ?$

$A = 3\ mm^2 = 3 \times 10^{-6}\ m^2$, $\rho_0 = ?$

→ From, $R = R_0(1 + \alpha_0 t)$

$$\frac{R_1}{R_2} = \frac{R_0[1 + \alpha_0 t_1]}{R_0[1 + \alpha_0 t_2]}$$

$$\therefore \frac{R_1}{R_2} = \frac{1 + \alpha_0 t_1}{1 + \alpha_0 t_2}$$

$$\therefore R_1 + R_1 \alpha_0 t_2 = R_2 + R_2 \alpha_0 t_1$$

$$\therefore \alpha_0 (R_1 t_2 - R_2 t_1) = R_2 - R_1$$

$$\therefore \alpha_0 = \frac{R_2 - R_1}{R_1 t_2 - R_2 t_1}$$

$$= \frac{0.2 - 0.15}{(0.15)(65) - (0.2)(0)}$$

$$= \frac{0.05}{9.75}$$

$$\therefore \boxed{\alpha_0 = 0.0051\ °c^{-1}}$$

→ From, $R = \frac{\rho l}{A}$ $\quad \rho_0 = \frac{R_0 A}{l}$

Where, $R_0 = 0.15$, $A = 3 \times 10^{-6}\ m^2$, $l = 6$ m

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$\therefore \rho_0 = \frac{R_0 A}{l} = \frac{0.15 \times 3 \times 10^{-6}}{6} = 0.075 \times 10^{-6}\ \Omega \cdot m$

$\boxed{\therefore \rho_0 = 7.5 \times 10^{-8}\ \Omega \cdot m}$

3. A copper wire has a resistivity of $1.6 \times 10^{-6}\ \Omega$-cm at $0^\circ C$ and at $20^\circ C$, the temperature co-efficient of resistance is $\frac{1}{254.5^\circ}\ {}^\circ C^{-1}$. Find the restivity and temperature co-efficient of resistance at $60^\circ C$.

→ Here, $\rho_0 = 1.6 \times 10^{-6}\ \Omega \cdot cm$, $\alpha_{20} = \frac{1}{254.5}\ {}^\circ C^{-1}$

$= 1.6 \times 10^{-8}\ \Omega \cdot m.$

$\rho_{60} = ?$, $\alpha_{60} = ?$

→ First we need to find α_0,

from $\alpha_t = \frac{\alpha_0}{1 + \alpha_0 t}$ —— (1)

$\alpha_{20} = \frac{\alpha_0}{1 + \alpha_0 \cdot 20}$

$\therefore \alpha_{20} + \alpha_{20} \cdot \alpha_0 \cdot 20 = \alpha_0$

$\therefore \alpha_0 (\alpha_{20} 20 - 1) = -\alpha_{20}$

$\therefore \alpha_0 = \frac{-\alpha_{20}}{\alpha_{20} \cdot 20 - 1} = \frac{\alpha_{20}}{1 - 20 \cdot \alpha_{20}}$

$\therefore \alpha_0 = \frac{1/254.5}{1 - 20 \cdot (1/254.5)}$

$\boxed{\therefore \alpha_0 = 4.27 \times 10^{-3}\ {}^\circ C^{-1}}$

→ Now, $\rho_{60} = \rho_0 (1 + \alpha_0 \cdot 60)$

$= 1.6 \times 10^{-8} (1 + (4.27 \times 10^{-3})(60))$

$\boxed{\therefore \rho_{60} = 2.01 \times 10^{-8}\ \Omega \cdot m}$

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→ And now again from eq. ①,

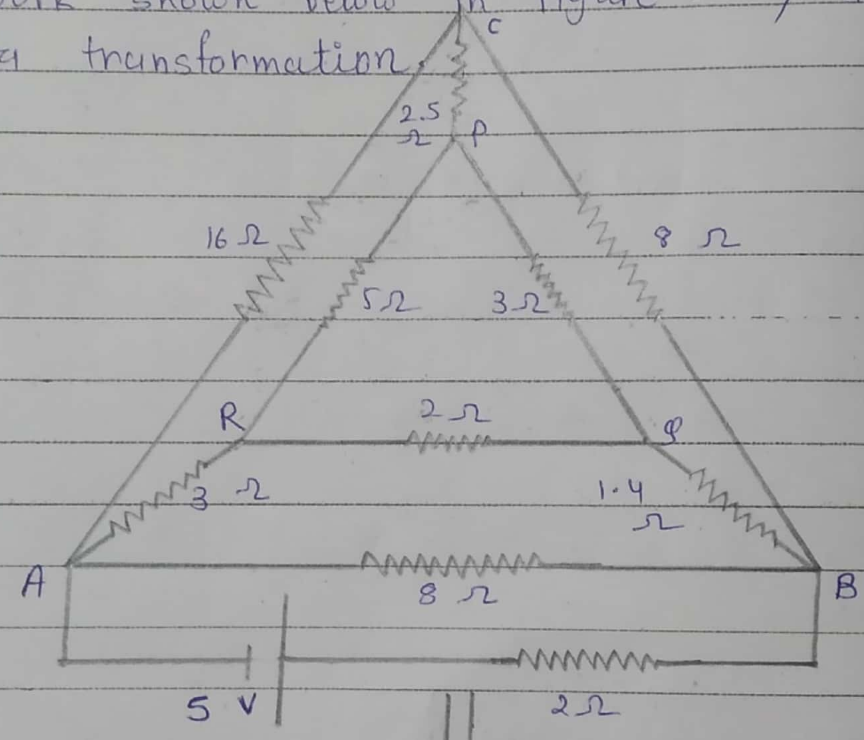
$$\alpha_{60} = \frac{\alpha_0}{1+\alpha_0 t} = \frac{4.27\times10^{-3}}{1+(4.27\times10^{-3})(60)}$$

$$= \frac{4.27\times10^{-3}}{1.2562}$$

$$= 3.399\times10^{-3} \ {}^\circ C^{-1}$$

$$\therefore \boxed{\alpha_{60} = \frac{1}{294.2} \ {}^\circ C^{-1}}$$

4. Find the current supplied by the battery in the network shown below in figure-1 by using star delta transformation.



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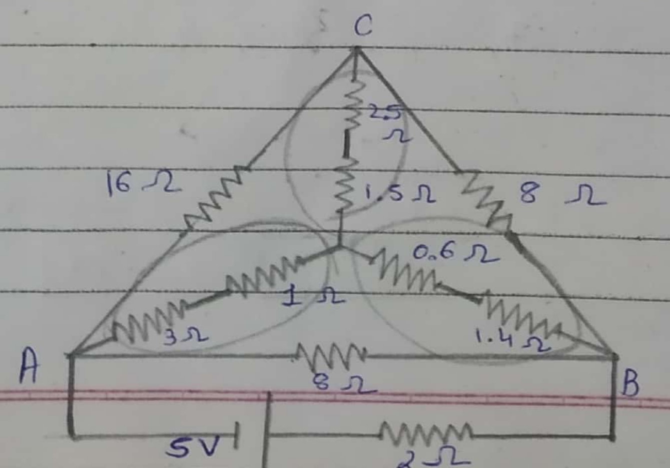

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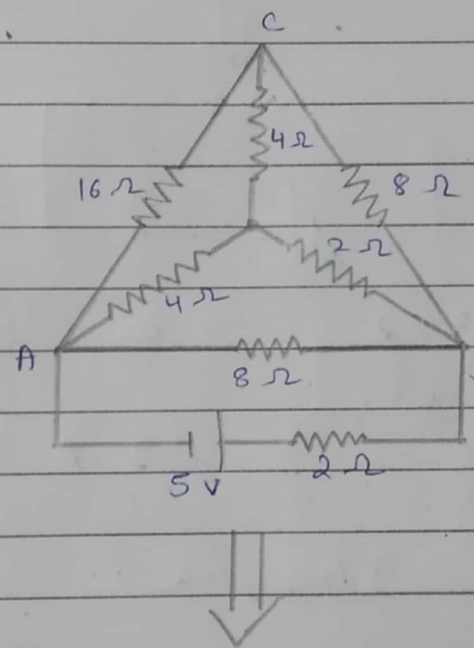
→ Here, let $R_{PQ} = 3\ \Omega$, $R_{QR} = 2\ \Omega$, $R_{PR} = 5\ \Omega$

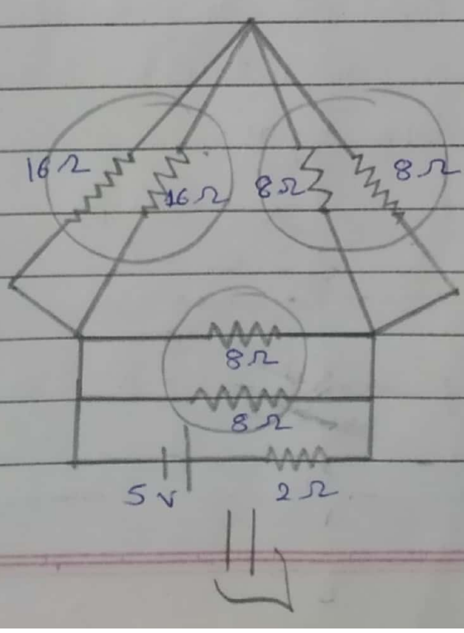
↳ $R_P = \dfrac{R_{PQ}\ R_{PR}}{R_{PQ} + R_{QR} + R_{PR}} = \dfrac{3 \times 5}{3 + 2 + 5} = 1.5\ \Omega$

↳ $R_Q = \dfrac{R_{PQ}\ R_{PQ}}{R_{PQ} + R_{QR} + R_{PR}} = \dfrac{3 \times 2}{3 + 2 + 5} = 0.6\ \Omega$

↳ $R_R = \dfrac{R_{PR} * R_{QR}}{R_{PQ} + R_{QR} + R_{PR}} = \dfrac{5 \times 2}{3 + 2 + 5} = 1\ \Omega$

→ After substituting series connections, we get...





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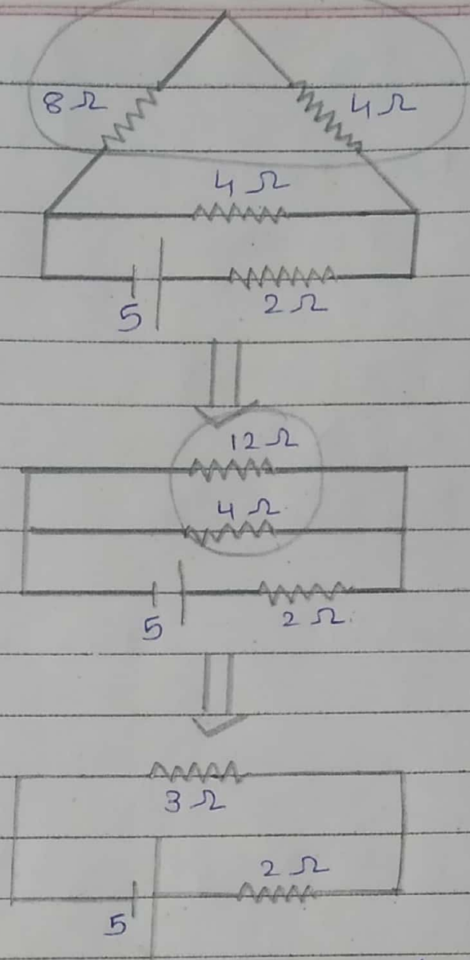


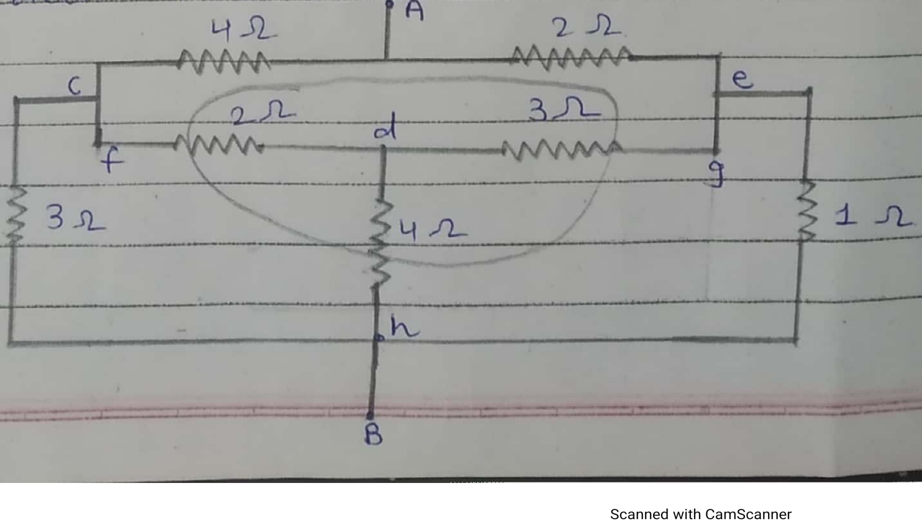
→ Here, $R_{eq} = 3 + 2 = 5\ \Omega$ & $V = 5\ V$.

→ For the current,

$$I = \frac{V}{R} = \frac{5}{5} = 1\ A$$

$$\therefore \boxed{I = 1\ A}$$

5. Determine equivalent resistance between terminal A and B shown in network (figure-2) by using star delta transformation.



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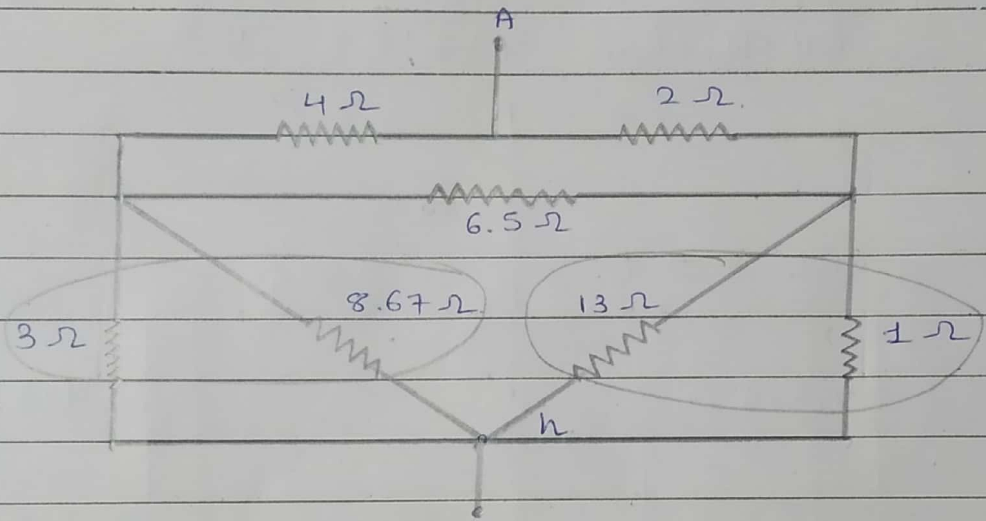
→ Star to delta transformation :-

let, $R_f = 2\,\Omega$, $R_g = 3\,\Omega$, $R_h = 4\,\Omega$.

⇒ $R_{fg} = 2 + 3 + \frac{2 \times 3}{4} = 5 + 1.5 = 6.5\,\Omega$

⇒ $R_{fh} = 2 + 4 + \frac{2 \times 4}{3} = 6 + 8/3 = 8.67\,\Omega$

⇒ $R_{gh} = 3 + 4 + \frac{3 \times 4}{2} = 7 + 6 = 13\,\Omega$



→ Here, L.H.S 3 Ω and 8.67 Ω are in parallel,

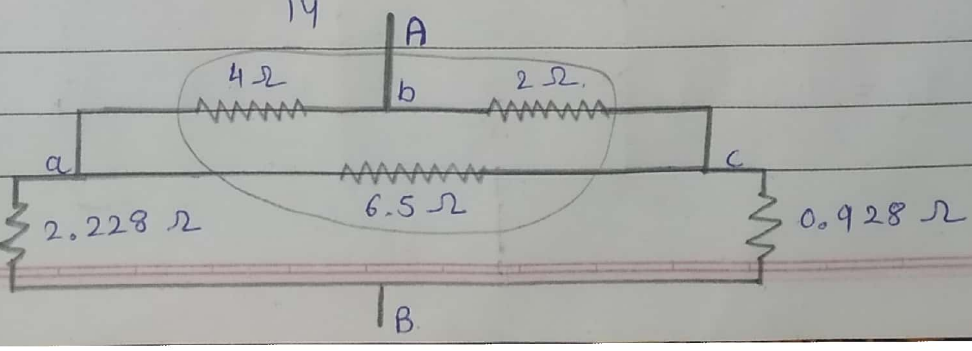
$\therefore \frac{1}{R_x} = \frac{1}{3} + \frac{1}{8.67} = \frac{8.67 + 3}{8.67 \times 3}$

$\therefore R_x = \frac{26.01}{11.67} = 2.228\,\Omega$

& R.H.S 13 Ω and 1 Ω are in parallel,

$\therefore \frac{1}{R_y} = \frac{1}{1} + \frac{1}{13} = \frac{13 + 1}{13}$

$\therefore R_y = \frac{13}{14} = 0.928\,\Omega$



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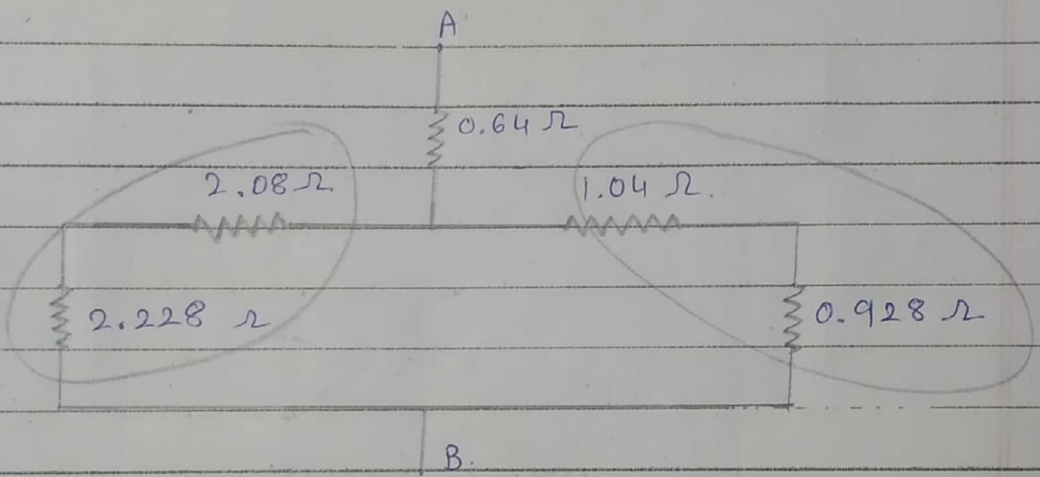
→ Now, Delta to star conversion :-

let $R_{ab} = 4$, $R_{bc} = 2$, $R_{ca} = 6.5\ \Omega$

$$\therefore R_a = \frac{R_{ab} \cdot R_{ac}}{R_{ab} + R_{bc} + R_{ca}} = \frac{4 \times 6.5}{4+2+6.5} = \frac{26}{12.5} = 2.08\ \Omega$$

$$\& \ R_b = \frac{R_{ab} R_{bc}}{R_{ab} + R_{bc} + R_{ca}} = \frac{4 \times 2}{4+2+6.5} = \frac{8}{12.5} = 0.64\ \Omega$$

$$\& \ R_c = \frac{R_{ac} R_{bc}}{R_{ab} + R_{bc} + R_{ca}} = \frac{6.5 \times 2}{4+2+6.5} = \frac{13}{12.5} = 1.04\ \Omega$$

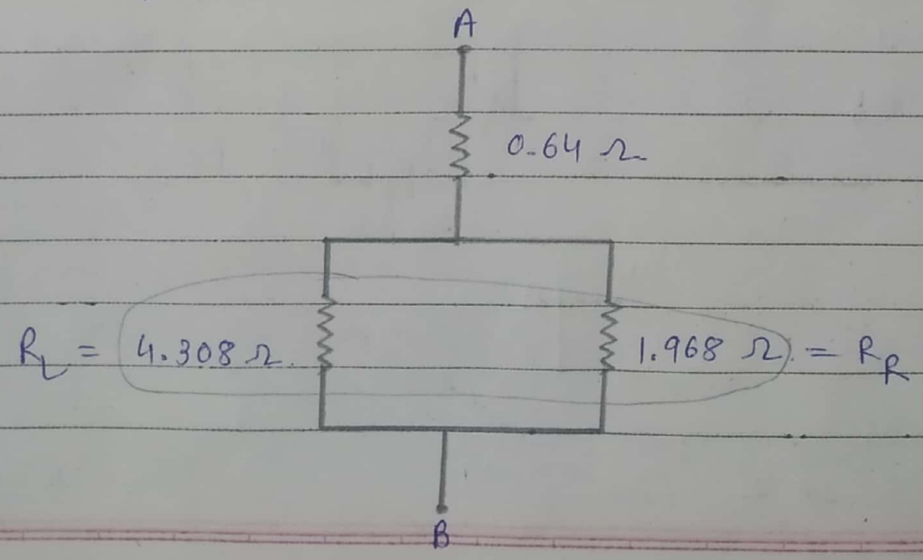


→ Here, 2.08 Ω and 2.228 Ω are in series connection,

$\therefore R_L = 2.08 + 2.228\ \Omega = 4.308\ \Omega$

and 1.04 Ω and 0.928 Ω are in series connection,

$\therefore R_R = 1.04 + 0.928\ \Omega = 1.968\ \Omega$



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→ Here, R_L & R_P is in parallel connection,

$$\therefore R_{RL} = \frac{4.308 \times 1.968}{4.308 + 1.968} = \frac{8.478}{6.276} = 1.35\ \Omega$$

→ R_{RL} is in series connection with $0.64\ \Omega$

$\therefore R_{eq} = 0.64 + 1.35$

$\therefore \boxed{R_{eq} = 1.99\ \Omega}$

6. Determine current in XX Ω resistor shown in network by using star-delta transformation.

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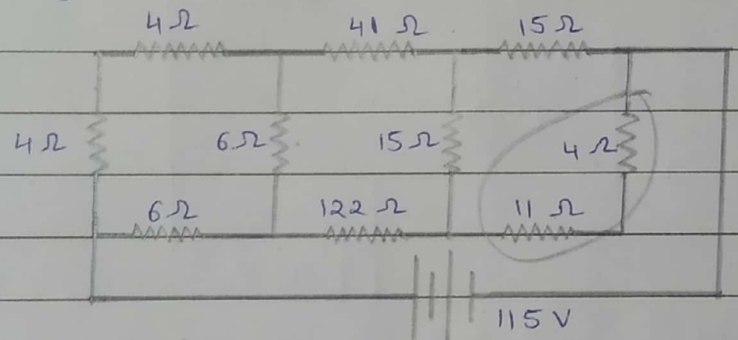
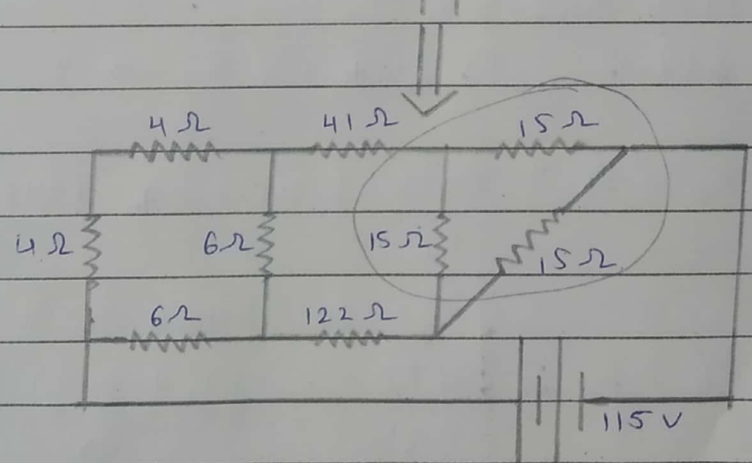
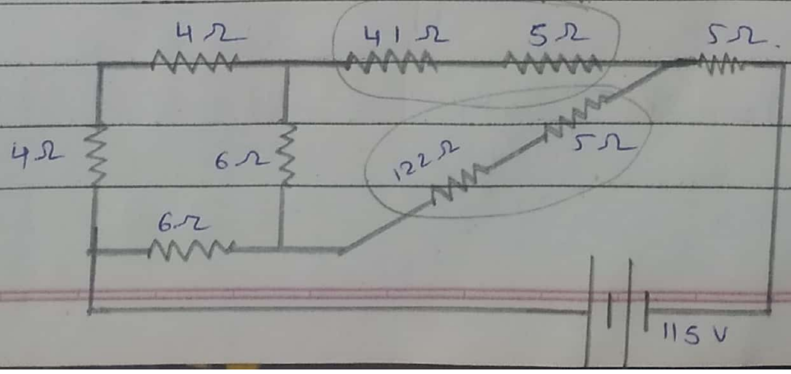
→ Here, XX = 122 Ω







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→ Here, all three restistance are 15 Ω.
Delta to star transformation,

$$R = \frac{15 \times 15}{15+15+15} = \frac{225}{45} = 5\ \Omega$$

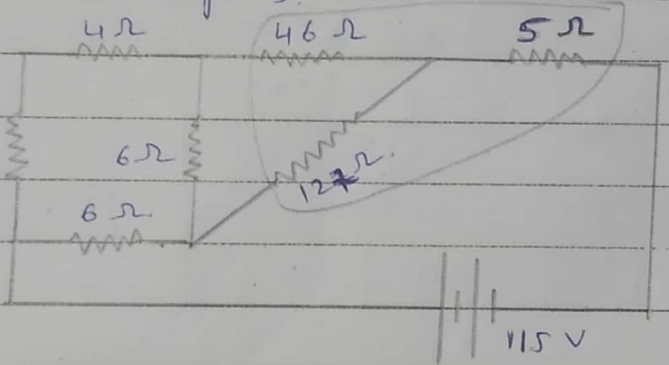
∴ we can put the value 5 Ω to all three star transformed resistance.

→ And 41 Ω & 5 Ω and 122 Ω & 5 Ω are in series connection.

∴ $R_a = 41 + 5 = 46\ \Omega$

& $R_b = 122 + 5 = 127\ \Omega$

Then we can get,



→ star to delta connection...

let $R_a = 46\ \Omega$, $R_b = 127\ \Omega$, $R_c = 5\ \Omega$.

& $R_{ab} = R_a + R_b + \frac{R_a R_b}{R_c} = 46 + 127 + \frac{(46)(127)}{5}$

$= 1341.4\ \Omega$

ııly $R_{bc} = R_b + R_c + \frac{R_b \cdot R_c}{R_a} = 127 + 5 + \frac{(127)(5)}{46}$

$= 145.8\ \Omega$

ııly $R_{ac} = R_a + R_c + \frac{R_a R_c}{R_b} = 46 + 5 + \frac{(46)(5)}{127}$

$= 52.81\ \Omega$

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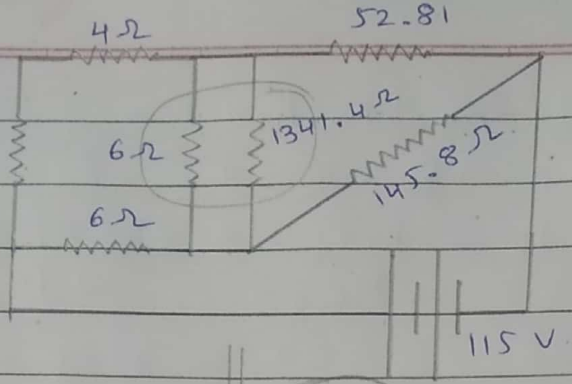
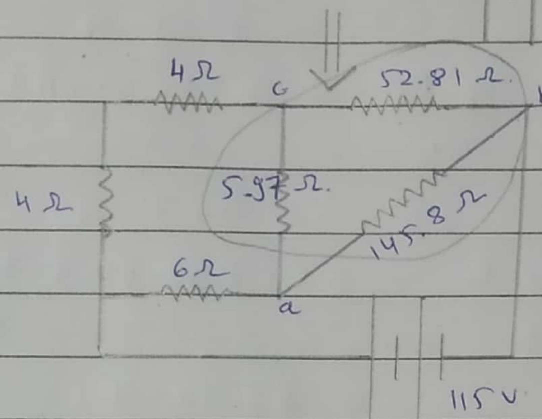




$$\frac{1}{R} = \frac{1}{1341.4} + \frac{1}{6}$$

$$\therefore R = \frac{1341.4 \times 6}{1341.4 + 6}$$

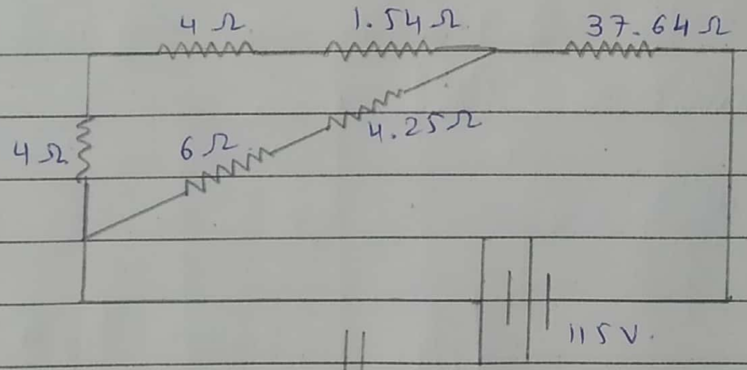
$$\therefore R = 5.97\ \Omega$$

→ Delta to star transformation...

$$R_a = \frac{R_{ab} * R_{ac}}{R_{ab} + R_{bc} + R_{ca}} = \frac{145.8 * 52.81}{145.8 + 52.81 + 5.97} = 37.64\ \Omega$$

$$R_b = \frac{R_{ab} * R_{bc}}{R_{ab} + R_{bc} + R_{ca}} = \frac{145.8 * 5.97}{145.8 + 52.81 + 5.97} = 4.25\ \Omega$$

$$R_c = \frac{R_{ac} * R_{bc}}{R_{ab} + R_{bc} + R_{ca}} = \frac{52.81 \cdot 5.97}{145.8 + 52.81 + 5.97} = 1.54\ \Omega$$



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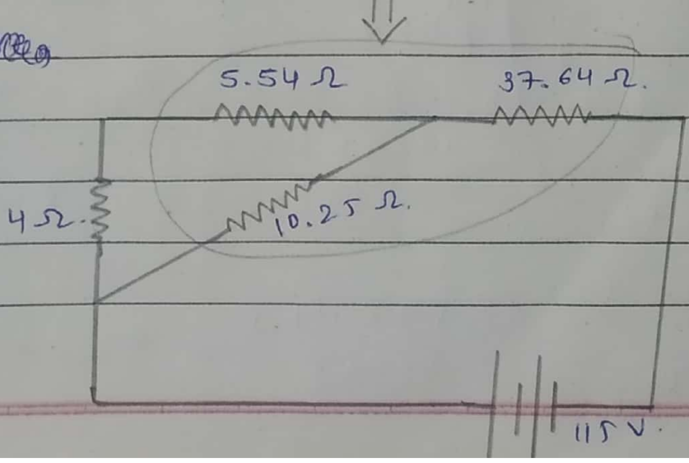

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→ Star to delta transformation :-

let $R_a = 5.54\,\Omega$, $R_b = 10.25\,\Omega$, $R_c = 37.64\,\Omega$

$\therefore R_{ab} = R_a + R_b + \frac{R_a R_b}{R_c} = \frac{5.54 \times 10.25}{37.64} + 5.54 + 10.25$

$= 17.3\,\Omega.$

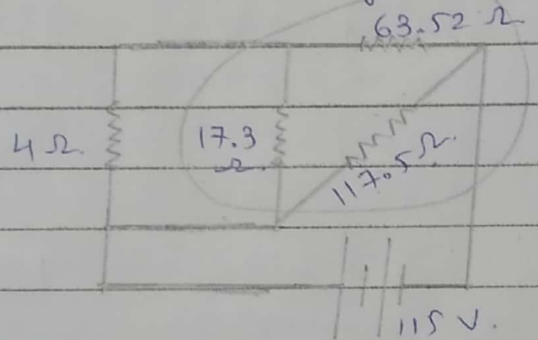
& $R_{bc} = R_b + R_c + \frac{R_b \cdot R_c}{R_a} = \frac{10.25 \times 37.64}{5.54} + 10.25 + 37.64$

$= 117.5\,\Omega$

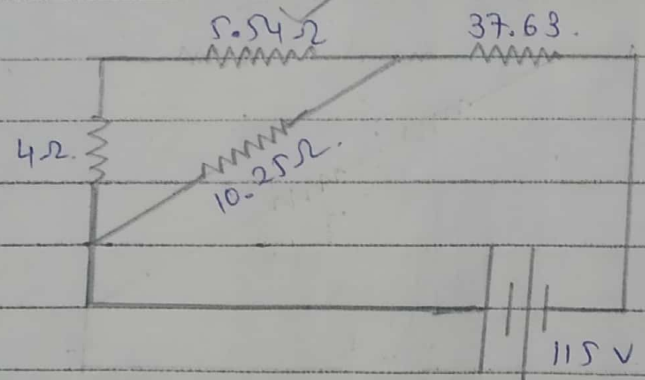
& $R_{ac} = R_a + R_c + \frac{R_a \cdot R_c}{R_b} = 5.54 + 37.64 + \frac{5.54 \times 37.64}{10.25}$

$= 63.52\,\Omega.$

→ Now, we can get...



Delta to star transformation



→ fig. *

let $R_{ab} = 17.3\,\Omega$, $R_{bc} = 63.52\,\Omega$, $R_{ca} = 117.5\,\Omega$.

$R_a = \frac{R_{ab} \cdot R_{ac}}{R_{ab} + R_{bc} + R_c} = \frac{17.3 \times 117.5}{17.3 + 63.52 + 117.5} = \frac{2032.75}{198.32}$

$= 10.25\,\Omega$

$R_c = \frac{R_{bc} \cdot R_{ca}}{R_{ab} + R_{bc} + R_{ca}} = \frac{63.52 \times 117.5}{198.32} = 37.63\,\Omega.$

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$$R_b = \frac{R_{ab} \cdot R_{bc}}{R_{ab}+R_{bc}+R_{ca}} = \frac{17.3 \times 63.52}{198.32} = 5.54\ \Omega.$$

→ From figure (*) $4\ \Omega$ & $5.54\ \Omega$ are in series connection.

∴ R will be $4 + 5.54 = 9.54\ \Omega$.

which is parallel to $10.25\ \Omega$

$$\therefore \frac{1}{R_L} = \frac{1}{10.25} + \frac{1}{9.54}\ \Omega.$$

$$\therefore R_L = \frac{10.25 \times 9.54}{10.25 + 9.54} = \frac{97.785}{19.79} = 4.94\ \Omega$$

→ Then R_L will be in series with ~~4.94 Ω~~. $37.63\ \Omega$.

$$\therefore R_{eq} = 37.63 + 4.94$$

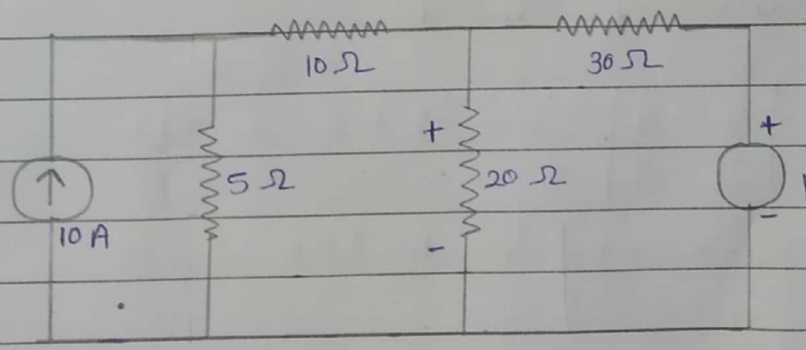
$$\therefore \boxed{R_{eq} = 42.57\ \Omega.}$$

→ To determine the current,

$$I = \frac{V}{R} = \frac{115}{42.57} = 2.70\ A$$

$$\therefore \boxed{I = 2.70\ A.}$$

7. For the circuit, determine the voltage across the $20\ \Omega$ resistor using ~~node analysis~~ mesh analysis.



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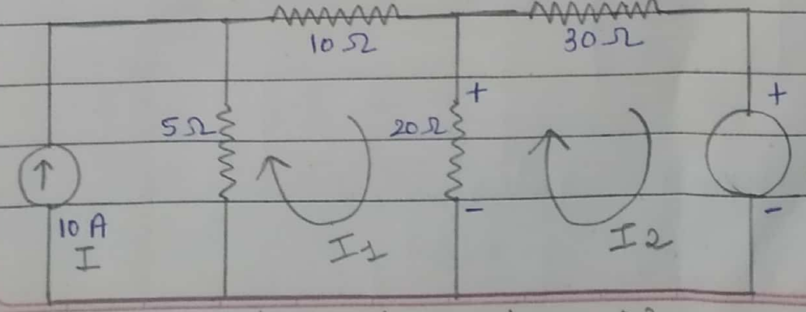

Loop : 01

Loop : 02

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→ Here, $I = 10\ A$

Now, applying KVL to loop : 01,

$$-10I_1 - 20(I_1 - I_2) - 5(I_1 - I) = 0$$

$$\therefore -10I_1 - 20I_1 + 20I_2 - 5I_1 + 5I = 0$$

$$\therefore -35I_1 + 20I_2 + 5(10) = 0$$

$$\therefore 35I_1 - 20I_2 = 50$$

$$\therefore 7I_1 - 4I_2 = 10 \quad \text{— (1)}$$

Now, applying KVL to loop : 02,

$$-100 - 20(I_2 - I_1) - 30I_2 = 0$$

$$\therefore -20I_2 + 20I_1 - 30I_2 = 100$$

$$\therefore 20I_1 - 50I_2 = 100$$

$$\therefore 2I_1 - 5I_2 = 10 \quad \text{— (2)}$$

→ From eq. (1) and (2),

$$\begin{bmatrix} 7 & -4 \\ 2 & -5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \end{bmatrix}$$

Now, $\Delta R = \begin{vmatrix} 7 & -4 \\ 2 & -5 \end{vmatrix} = -35 + 8 = -27$

$\Delta R_1 = \begin{vmatrix} 10 & -4 \\ 10 & -5 \end{vmatrix} = -50 + 40 = -10.$

$\Delta R_2 = \begin{vmatrix} 7 & 10 \\ 2 & 10 \end{vmatrix} = 70 - 20 = 50.$

→ Now, using cramer's rule,

$$I_1 = \frac{\Delta R_1}{\Delta R} = \frac{-10}{-27} = 0.37$$

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$I_2 = \frac{\Delta R_2}{\Delta R} = \frac{50}{-27} = -1.85$

→ Current through $20\ \Omega$ is $I_1 - I_2$

$= 0.37 - (-1.85)$

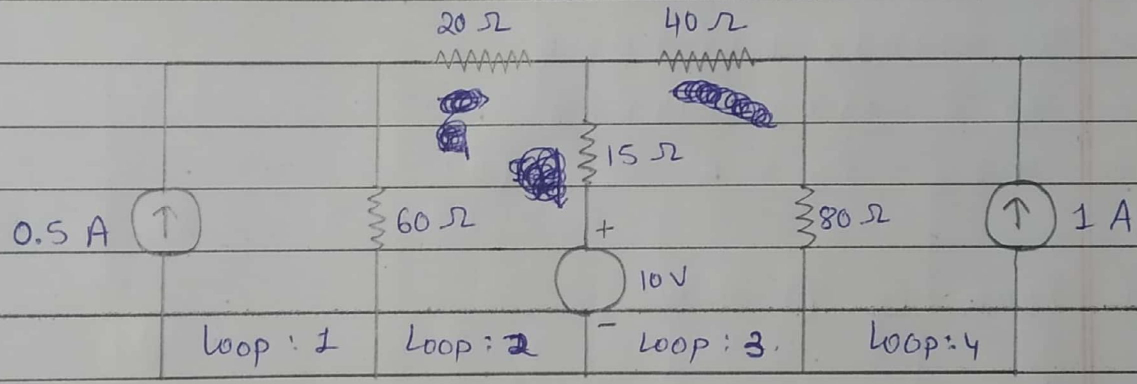
$\boxed{I_x = 2.22\ A}$

& voltage across $20\ \Omega$ is $V = I_x R$

$= 2.22 \times 20$

$\therefore \boxed{V = 44.4\ V}$

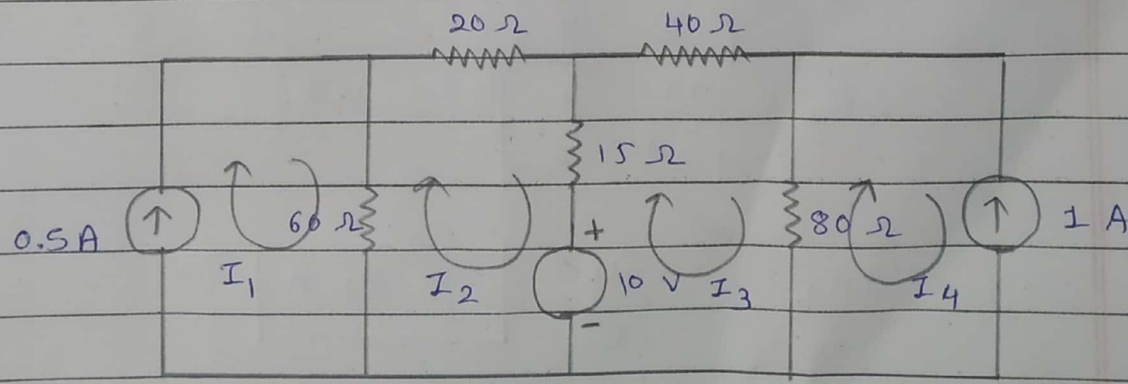
8. Using mesh analysis find currents I_1, I_2 and I_3 for given circuit.



→ ~~Here, $I_L = 0.5\ A$, $I_R = 1\ A$~~

~~Now, Applying KVL to loop : 1~~,

→



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→ Now, applying KVL to loop : 2 ,

$$-20 I_2 - 15(I_2 - I_3) - 10 - 60(I_2 - 0.5) = 0$$

$$\therefore -20 I_2 - 15 I_2 + 15 I_3 - 10 - 60 I_2 + 30 = 0$$

$$\therefore -95 I_2 + 15 I_3 = -20$$

$$\therefore 20 = 95 I_2 - 15 I_3$$

$$\therefore 4 = 19 I_2 - 3 I_3 \quad \text{——} \; (1).$$

& applying KVL to loop : 3 ,

$$-40 I_3 - 80(I_3 - 1) + 10 - 15(I_3 - I_2) = 0$$

$$\therefore -40 I_3 - 80 I_3 + 80 + 10 - 15 I_3 + 15 I_2 = 0.$$

$$\therefore 90 - 135 I_3 + 15 I_2 = 0$$

$$\therefore 90 = -15 I_2 + 135 I_3$$

$$\therefore 9 I_3 - I_2 = 6 \quad \text{——} \; (2).$$

→ From eq. (1) & (2) ,

$$\begin{bmatrix} 19 & -3 \\ -1 & 9 \end{bmatrix} \begin{bmatrix} I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$\therefore$ Now, $\Delta R = \begin{vmatrix} 19 & -3 \\ -1 & 9 \end{vmatrix} = 168$

$$\Delta R_2 = \begin{vmatrix} 4 & -3 \\ 6 & 9 \end{vmatrix} = 54$$

$$\Delta R_3 = \begin{vmatrix} 19 & 4 \\ -1 & 6 \end{vmatrix} = 118.$$

→ $I_2 = \dfrac{\Delta R_2}{\Delta R} = \dfrac{54}{168} \boxed{= 0.32 \text{ A}}$

→ $I_3 = \dfrac{\Delta R_3}{\Delta R} = \dfrac{118}{168} \boxed{= 0.7}$

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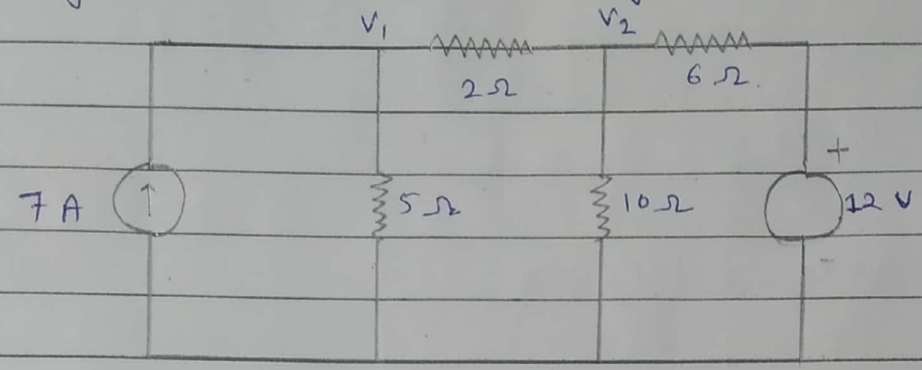
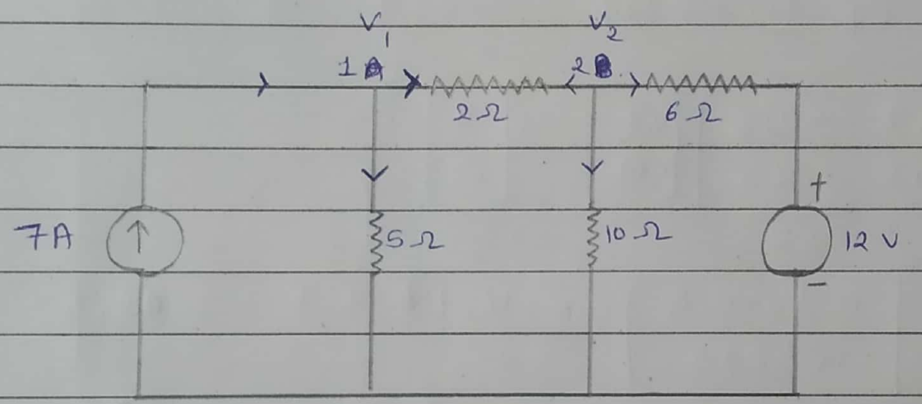
9. For the circuit of figure-6 determine the nodal voltages and current through 2 Ω resistor.



→ Now, identifying the number of nodes in the given circuit.



→ Now, applying KCL at node : 1,

$$\frac{V_1}{5} + \frac{V_1 - V_2}{2} = 7$$

$$\therefore 0.2V_1 + 0.5V_1 - 0.5V_2 = 7$$

$$\therefore 0.7V_1 - 0.5V_2 = 7 \quad \text{——} \quad (1).$$

→ Now, applying KCL at node : 2,

$$\frac{V_2 - 12}{6} + \frac{V_2}{10} + \frac{V_2 - V_1}{2} = 0$$

$$\therefore \frac{V_2}{6} - 2 + \frac{V_2}{10} + \frac{V_2}{2} - \frac{V_1}{2} = 0.$$

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$$\therefore \frac{V_2}{6} + \frac{V_2}{10} + \frac{5V_2}{10} - \frac{V_1}{2} = 2$$

$$\therefore \frac{10V_2 + 6V_2 + 30V_2}{60} - \frac{V_1}{2} = 2$$

$$\therefore \frac{46V_2}{60} - \frac{V_1}{2} = 2$$

$$\therefore \frac{23V_2}{30} - \frac{15V_1}{30} = 2$$

$$\therefore 23V_2 - 15V_1 = 60 \quad \text{— (2)}$$

→ From eq (1) & (2) ...

$$\begin{bmatrix} 0.7 & -0.5 \\ -15 & 23 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 7 \\ 60 \end{bmatrix}$$

$$\Delta R = \begin{vmatrix} 0.7 & -0.5 \\ -15 & 23 \end{vmatrix} = 16.1 - 7.5 = 8.6$$

$$\Delta R_1 = \begin{vmatrix} 7 & -0.5 \\ 60 & 23 \end{vmatrix} = 161 + 30 = 191$$

$$\Delta R_2 = \begin{vmatrix} 0.7 & 7 \\ -15 & 60 \end{vmatrix} = 42 + 105 = 147$$

→ Now, $V_1 = \frac{\Delta R_1}{\Delta R}$ ~~$= \frac{131}{23.6} = 5.55$~~

$$\therefore V_1 = \frac{191}{8.6} \boxed{= 22.2 \text{ V.}}$$

$$\& \ V_2 = \frac{\Delta R_2}{\Delta R} = \frac{147}{8.6} \boxed{= 17.1 \text{ V.}}$$

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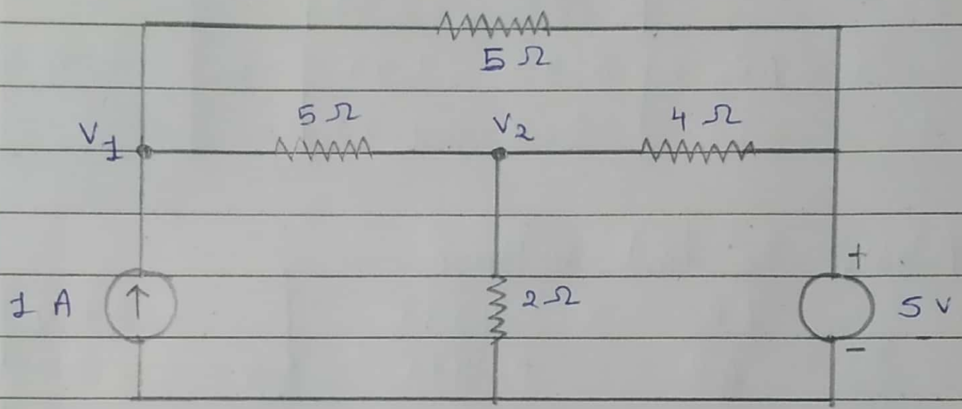
→ Current through $2\,\Omega$ is $\frac{V_1 - V_2}{2} = \frac{22.2 - 17.1}{2}$

$= \frac{5.04}{2}$

$\therefore \boxed{I = 2.55\ A}$

10. For the circuit of figure determine the V_1 and V_2 nodal voltages.



→ Here, applying KCL to node - 1,

$$\therefore 1 = \frac{V_1 - V_2}{5} + \frac{V_1 - 5}{20}$$

$$\therefore 1 = 0.2V_1 - 0.2V_2 + 0.05V_1 - 0.25$$

$$\therefore 1.25 = 0.25V_1 - 0.2V_2 \quad \text{(1)}$$

Now, applying KCL to node - 2,

$$\therefore 0 = \frac{V_2 - V_1}{5} + \frac{V_2 - 5}{4} + \frac{V_2}{2}$$

$$\therefore 0 = 0.2V_2 - 0.2V_1 + 0.25V_2 - 1.25 + 0.5V_2$$

$$\therefore 1.25 = 0.95V_2 - 0.2V_1$$

$$\therefore 1.25 = -0.2V_1 + 0.95V_2 \quad \text{(2)}$$

→ From eq. 1 and 2 ...

$$\begin{bmatrix} 0.25 & -0.2 \\ -0.2 & 0.95 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1.25 \\ 1.25 \end{bmatrix}$$

Now, $\Delta R = \begin{vmatrix} 0.25 & -0.2 \\ -0.2 & 0.95 \end{vmatrix} = 0.1975$

& $\Delta R_1 = \begin{vmatrix} 1.25 & -0.2 \\ 1.25 & 0.95 \end{vmatrix} = 1.4375$.

& $\Delta R_2 = \begin{vmatrix} 0.25 & 1.25 \\ -0.2 & 1.25 \end{vmatrix} = 0.5625$

→ Now, To find voltage...

$$V_1 = \frac{\Delta R_1}{\Delta R} = \frac{1.4375}{0.1975} \quad \boxed{= 7.27 \text{ V.}}$$

$$V_2 = \frac{\Delta R_2}{\Delta R} = \frac{0.5625}{0.1975} \quad \boxed{= 2.85 \text{ V.}}$$

→ Current through 5 Ω is $\dfrac{V_1 - V_2}{5}$

$$= \frac{7.27 - 2.85}{5}$$

$$\boxed{I = 0.89 \text{ A}}$$

11. Two metal plates of area 100 cm² are separated by a dielectric of 2 mm having a relative permittivity of 5. When a dc voltage of 500 V is applied across the capacitor plates, find (i) capacitance, (ii) charge on the capacitor, (iii) electric field strength, (iv) electric flux density.

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→ Here, $A = 100\ cm^2 = 100 \times 10^{-4}\ m^2 = 10^{-2}\ m^2.$

$d = 2\ mm = 2 \times 10^{-3}\ m$

$\epsilon_r = 5$

$V = 500\ V$

(i) The capacitance, $C = \frac{\epsilon_0 A}{d}$

$= \epsilon_0 \epsilon_r A/d$

$\therefore C = \frac{8.85 \times 10^{-12} \times 10^{-2} \times 5}{2 \times 10^{-3}}$

$\therefore C = 22.125 \times 10^{-11}$

$\therefore \boxed{C = 221.25\ pF}$

(ii) Charge on the capacitor, $q = CV$

$\therefore q = 22.125 \times 10^{-11} \times 500$

$= 11062.5 \times 10^{-11}$

$\therefore \boxed{q = 0.1107 \times 10^{-6}\ C}$

(iii) Electric field strength, $\mathcal{E} = \frac{V}{d}$

$\therefore \mathcal{E} = \frac{500}{2 \times 10^{-3}}$

$= 250 \times 10^{+3}\ V/m$

$\therefore \boxed{\mathcal{E} = 250\ kV/m}$

(iv) Electric flux density, $D = \frac{q}{A}$

$\therefore D = \frac{11062.5 \times 10^{-11}}{10^{-2}}$

$\therefore D = 0.1107 \times 10^{-4}$ ~~11.07~~

$\therefore \boxed{D = 11.07 \times 10^{-6}\ C/m^2}$

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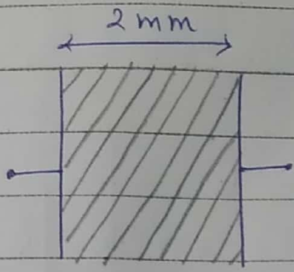
12. A capacitor is made up of two plates with an area of 11 cm² which are separated by a mica sheet 2 mm thick. If the relative permittivity of mica is 6, find its capacitance. Now, if one plate is moved further to give an air gap 0.5 mm wide between the plate and mica, find the new capacitance.



→ $A = 11\ cm^2 = 11 \times 10^{-4}\ m^2$

$\epsilon_r = 6$

$d = 2\,mm = 2 \times 10^{-3}\ m$

$$C_1 = \frac{\epsilon A}{d} = \frac{\epsilon_0 \epsilon_r A}{d}$$

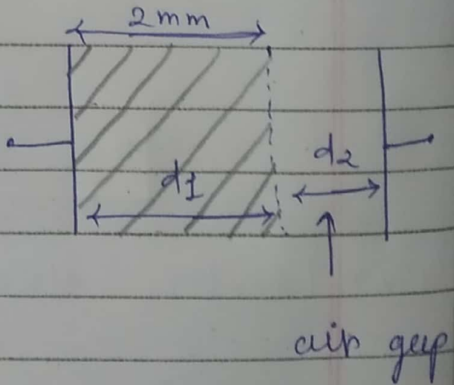
$$= \frac{8.85 \times 10^{-12} \times 6 \times 11 \times 10^{-4}}{2 \times 10^{-3}}$$

$$= 292.05 \times 10^{-13}$$

$$= 29.2 \times 10^{-12}$$

$$\therefore \boxed{C_1 = 29.2\ pF}$$

→ Now, one plate is moved farther...



Here, $d_1 = 2 \times 10^{-3}\ m$

$d_2 = 0.5 \times 10^{-3}\ m$

$$C_2 = \frac{\epsilon_0 A}{\frac{d_1}{\epsilon_{r_1}} + \frac{d_2}{\epsilon_{r_2}}}$$

$$= \frac{8.85 \times 10^{-12} \times 11 \times 10^{-4}}{\frac{2 \times 10^{-3}}{6} + \frac{0.5 \times 10^{-3}}{1}}$$

$$\therefore C_2 = \frac{97.35 \times 10^{-16} \times 6}{2 \times 10^{-3} + 3 \times 10^{-3}}$$

$$= \frac{97.35 \times 6 \times 10^{-16}}{5 \times 10^{-3}}$$

$$= 116.82 \times 10^{-13}$$

$$= 11.68 \times 10^{-12}$$

$$\therefore \boxed{C_2 = 11.68 \text{ pF}}$$

13. Two plates are kept 1.5 cm apart in air and 1 kV supply is connected across them. Calculate the electric field strength in air when a glass sheet 0.5 cm thick with relative permittivity 3 is introduced between the plates without changing the previous distance between the plates.

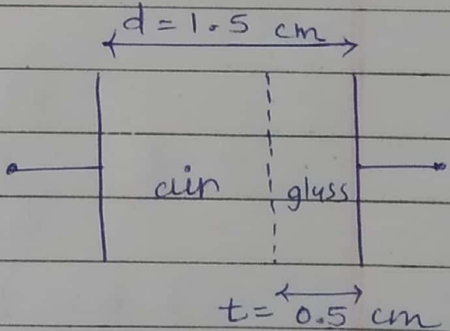
→ Here, $d = 1.5 \text{ cm} = 1.5 \times 10^{-2} \text{ m}$

$V = 10^3 \text{ V}$.

$(d-t) = d_1 = 1 \text{ cm} = 10^{-2} \text{ m}$ (air)

$d_2 = t = 0.5 \text{ cm} = 0.5 \times 10^{-2} \text{ m}$

$\epsilon_r = 3$ (glass)



→ $C = \frac{C_1 C_2}{C_1 + C_2}$

$$\therefore C = \frac{\frac{\epsilon_0 A}{d_1} \times \frac{\epsilon_0 \epsilon_r A}{d_2}}{\frac{\epsilon_0 A}{d_1} + \frac{\epsilon_0 \epsilon_r A}{d_2}}$$

$$\therefore C = \frac{\frac{\cancel{\epsilon_0 A}}{d_1} \cdot \frac{\epsilon_0 \epsilon_r A}{d_2}}{\cancel{\epsilon_0 A}\left(\frac{1}{d_1} + \frac{\epsilon_r}{d_2}\right)}$$

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$$\therefore \frac{q}{V} = \frac{\frac{\epsilon_0 \epsilon_r A}{d_1 d_2}}{\frac{1}{d_1} + \frac{\epsilon_r}{d_2}}$$

$$\therefore \frac{q}{V} = \frac{\frac{\epsilon_0 \epsilon_r A}{d_1 d_2}}{\frac{d_2 + d_1 \epsilon_r}{d_1 d_2}}$$

$$\therefore \frac{q}{V} = \frac{\epsilon_0 \epsilon_r A}{d_2 + d_1 \epsilon_r}$$

$$\therefore q = \frac{V \epsilon_0 \epsilon_r A}{d_2 + d_1 \epsilon_r}$$

→ Now, $E = \frac{Kq}{d^2}$

$$= \frac{1}{4\pi \epsilon_0 \epsilon_r d^2} q$$

$$= \frac{1}{4\pi \epsilon_0 \epsilon_r} \frac{q\, 4\pi}{A}$$

| $A = 4\pi d^2$ |
| $\therefore d^2 = A/4\pi$ |

$$\therefore E = \frac{q}{\epsilon_0 A \epsilon_r}$$

$$\therefore E\, \epsilon_0 \epsilon_r A = q$$

$$\therefore E\, \epsilon_0 \epsilon_r A = \frac{V \epsilon_0 \epsilon_r A}{d_2 + d_1 \epsilon_r}$$

$$\therefore E = \frac{V}{d_2 + d_1 \epsilon_r}$$

$$\therefore E = \frac{1000}{0.5 \times 10^{-2} + 10^{-2} \times 3}$$

$$\therefore E = \frac{1000}{3.5 \times 10^{-2}}$$

$$\boxed{\therefore E = 285.7 \text{ V/m}}$$