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Homework 2

Due Date: October 17, 2017

Problem 1: (20 points)

Consider an array x[0..n-1] of ascii characters. The Burrows-Wheeler Transform (BWT) of x works as follows:

1. It forms a matrix A of all the n rotations of x;
2. It sorts the rows of A lexicographically where every row is treated as a word; the outcome is a matrix B;
3. It returns (y,L) where y is the last column of B, and L is the location of the original x in B;

a. Apply BWT on the string *babacaab*, and also on the string ``The hat on that cat", where the blank character ' ' is less than any alphabetic character. Note that you can use Matlab ``sortrows" command to sort the rows of A.
b. Give a general algorithm that constructs B from (y,L).
c. Give an algorithm to reconstruct x from (y,L) using the algorithm in (b).
d. Illustrate how the reconstruction algorithm works on the (y,L) produced in part (a) for *babacaab.*

Problem 2: (20 points)

Let x and y be two vectors of 32 components each where $x_k = \frac{k^2}{3}$, and $y_k = \sin((2k + 1)\frac{\pi}{32})$, for k=0,1,...,31.

a. Compute the Fourier transform X of x and Y of y.
b. The magnitude of a number t is its absolute value, denoted $|t|$. This applies to both real numbers and complex numbers. Recall that the absolute value of a complex number z, where $z=a+ib$, is $\sqrt{a^2 + b^2}$. If V is a vector (row or column), then we denote by $|V|$ the vector of the magnitudes of the components of V. For example, if $V = [z_1, z_2, z_3]$, then $V = [|z_1|, |z_2|, |z_3|]$. Compute $|X|$ and $|Y|$.
c. Identify the 17 smallest-magnitude elements in X and the 17 smallest-magnitude elements in Y.
d. Let $\hat{X}$ be derived from X by replacing each of the 17 smallest-magnitude elements of X by 0, and leaving the other elements intact. Define $\hat{Y}$ similarly. Define by $\hat{x}$ the inverse Fourier transform of $\hat{X}$, and $\hat{y}$ the inverse Fourier transform of $\hat{Y}$. Compute $\hat{x}$ and $\hat{y}$.
e. Plot x and $\hat{x}$ in one figure, and y and $\hat{y}$ in another figure.

Problem 3: (20 points)

Let x and y be as in Problem 2. Let X be the DCT of x, and Y the DCT of y. Let $\hat{X}$ be derived from X by replacing the last 17 elements of X by zeros while keeping the first 15 elements the same, and define $\hat{Y}$ similarly from Y. Finally, let $\hat{x}$ be the inverse DCT of $\hat{X}$, and $\hat{y}$ the inverse DCT of $\hat{Y}$.

a. Compute X, Y, $\hat{x}$, and $\hat{y}$.
b. Plot x and $\hat{x}$ in one figure, and y and $\hat{y}$ in another figure.

Problem 4: (20 points)

a. Same as Problem 2 with 3 exceptions: (1) the transform is Haar, (2) $\hat{X}$ is derived from X by zeroing out the 17 smallest-magnitude elements of X, and (3) $\hat{Y}$ is derived from Y by zeroing out the 17 smallest-magnitude elements of Y.
b. Same problem as (a) except that the Transform is the Walsh-Hadamard transform. Use the Matlab ``hadamard'' command to generate the Hadamard matrix.

Problem 5: (20 points)

a. Put in one figure the plots of x and the four $\hat{x}$'s of the last three problems.
b. For each of the four $\hat{x}$'s, compute the mean square error relative to x.
c. Which of the four $\hat{x}$'s is the best reconstruction of x?
d. Repeat (a), (b) and (c) for y and the four $\hat{y}$'s.