CS 6351 October 17, 2017

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Homework 3

Due Date: November 7, 2017

Problem 1: (20 points)

Let X=[0, 0.7, 0.7, 0.7, 0.8, 0.8, 0.8, 1, 1, 1, 1, 1.2, 1.2, 1.2, 1.7, 1.74, 1.75, 1.8, 1.83, 1.84, 2.2, 2.2, 2.3, 2.36, 2.37, 2.4, 2.9].

a. Find the decision levels (d's) and reconstruction values (r's) of the 4-level optimal Max-Lloyd quantizer for X. (4-level means the quantizer has four intervals. Assume that d_0=0 and d_4=3.)
b. Quantize X with the optimal quantizer of part (a), then dequantize it. Show the quantized values as an array, and also the dequantized values as an array. Compute the MSE.
c. Do the same as in (b) except this time the quantizer is a 3-level uniform quantizer, where again d_0=0 and d_3=3.
d. An *optimal semi-uniform quantizer* is a quantizer where all the intervals have the same length but the reconstruction values are optimal. It can be shown that the reconstruction value in each interval is the (weighted) average of the data points in that interval. Find the reconstruction values of the 4-level optimal semi-uniform quantizer for X, where d_0=0 and d_4=3. Quantize X and then dequantize it using this quantizer, show the quantized and dequantized values in two arrays, and finally compute the MSE.
e. Compare the three MSEs you obtained in (b-d).

Problem 2: (25 points)

This problem will require you to download the river image from the course Website (river).

Remarks: Most images, which are in gif or other standard formats, are represented as an index matrix "*I*" coupled with a colormap (call it "map"). The color map is a 3-column table: every row consists of three components, representing one color. Thus, if $I(i,j)=k$, then the actual pixel value at (i,j) is the color represented in row k of map. This is true even if the image is a grayscale image. The arrangement of the rows (i.e., colors) in the map is random, that is, two very similar colors are not necessarily near each other in the map; in fact, they could be far apart. The implication of this image representation scheme is that none of the image compression processes (e.g., transforms, quantization, etc.) would give the expected results.

Remedy: To resolve this problem, do the following:

- [I,map]=imread('river.gif'); (or imread('river', 'format');, where format can be gif, tiff, etc.)
- G=ind2gray(I,map);

Now the image in G is in the "right" representation. That is, you can run on G all the compression-related algorithms you have studied, and you will get the expected outcome.

a. Compute the entropy of the river image.
b. Quantize G into G' using a 6-level optimal uniform quantizer, compute the entropy of G', dequantize G' into $\hat{G}_u$, display $\hat{G}_u$, and compute SNR$(G, \hat{G}_u)$. Note that you can take d_0=the min of G, and d_6=(the max of G + 0.001).
c. Repeat (b) except this time you should use a 6-level optimal semi-uniform quantizer. Call the dequantized image $\hat{G}_{su}$.
d. Repeat (b) except this time you should use a 6-level optimal Max-Lloyd quantizer. Call the dequantized image $\hat{G}_{op}$.
e. Compare the entropies of the three schemes (in b, c, and d); also compare the SNRs of the three schemes; finally, compare the subjective (visual) quality of the three reconstructed images $\hat{G}_u$, $\hat{G}_{su}$, and $\hat{G}_{op}$.

Problem 3: (25 points)

a. Using DPCM with parameters (a,b,c), compute the residual image R of river G for each of the following cases of (a,b,c): (1,0,0) , (0,0,1), (.5,0,.5) , (1,-1,1), (.75,-.5,.75). Note: $R(i,j) = floor(G(i,j) - (a * G(i,j-1) + bG(i-1,j-1) + c * G(i-1,j)))$. However, for elements of the first row and first column the following rules apply: $R(1,1)=G(1,1)$, $R(1,j)=G(1,j) - G(1,j-1)$, $R(i,1)=G(i,1)-G(i-1,1)$. You do not to report anything for this part (a) of the question.
b. Compute the entropy of each of the residual images obtained in part (a). Call E the residual image of minimum entropy; display the image E.
c. Quantize E into E' with a 6-level optimal Max-Lloyd quantizer, then apply RLE on the row-wise flattened E' yielding a sequence e, and finally compute the entropy of e. You need to report the entropy only (i.e., do not report E' or e). Note that you can take d_0=the min of E, and d_6=(the max of E + 0.001).
d. Dequantize E' into $\hat{E}$ and reconstruct from $\hat{E}$ an approximation $\hat{G}$ for G. Display $\hat{G}$, and compute the signal-to-noise ratio (SNR) of (G, $\hat{G}$). Note that $\hat{G}(1,1) = \hat{E}(1,1)$, $\hat{G}(1,j) = \hat{G}(1,j-1) + \hat{E}(1,j), \hat{G}(i,1) = \hat{G}(i-1,1) + \hat{E}(i,1),$ and for all other cases $\hat{G}(i,j) = a * \hat{G}(i,j-1) + b * \hat{G}(i-1,j-1) + c * \hat{G}(i-1,j) + \hat{E}(i,j)$.
e. What is the visual quality of $\hat{G}$ (i.e., good or bad)? Why do you think you got that (good or bad) quality?

Problem 4: (30 points)

In this problem you will compress the river image G using 8×8 block-oriented DCT. First apply dct2 on each of the contiguous 8×8 blocks of G. The top leftmost term of each transformed block is called the DC term. Quantize all the DC terms of all the blocks as one data set, using a uniform 16-level quantizer. Afterwards, quantize the 9 elements in the 2nd, 3rd, and 4th counterdiagonals of each block with a separate 8-level uniform quantizer, while the remaining 54 elements of each block are zeroed out.

a. Compute the resulting bitrate and the compression ratio. (Note: you need not carry out any entropy coding beyond the quantization step.)
b. Reconstruct the image (into $\hat{G}_{10}$) by dequantizing and applying the inverse DCT on each separate block. Display the reconstructed image and compute the SNR.
c. Repeat (a) and (b) except that this time you also drop (zero out) the 4th counterdiagonal of each block. Call the reconstructed image $\hat{G}_6$.
d. Repeat (a) and (b) except that this time you also drop (zero out) the 3rd and 4th counterdiagonals of each block. Call the reconstructed image $\hat{G}_3$.
e. Repeat (a) and (b) except that this time only the DC terms are quantized with a 16-level uniform quantizer, while the remaining 63 terms of each block are zeroed out. Call the reconstructed image $\hat{G}_1$.
f. Compare the bitrates of the four schemes; also compare the SNRs of the four schemes; finally, compare the subjective (visual) quality of the four reconstructed images $\hat{G}_{10}$, $\hat{G}_6$, $\hat{G}_3$, and $\hat{G}_1$.