Project: Heat Recovery Device

By

P. Desai

Contact details:

Email: priperfect2207@gmail.com

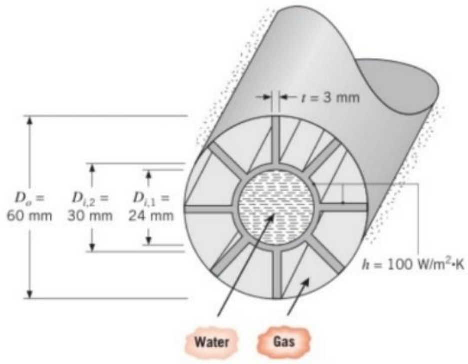
Any suggestions are welcome!!

Problem statement:

The objective of the project is to validate the analytical result of the following heat transfer problem using OpenFOAM. The problem is taken from Problem 11.5, Chapter 11: Heat Exchangers, Heat And Mass Transfer, Incropera and Dewitt:

A heat recovery device involves transferring energy from the hot flue gases passing through an annular region to pressurized water flowing through the inner tube of the annulus. The inner tube has inner and outer diameters of 24 and 30 mm and is connected by eight struts to an insulated outer tube of 60-mm diameter. Each strut is 3 mm thick and is integrally fabricated with the inner tube from carbon steel (k = 50 W/mK).

Consider conditions for which water at 300 K flows through the inner tube at 0.161 kg/s while flue gases at 800 K flow through the annulus, maintaining a convection coefficient of 100 W/m^2 K on both the struts and the outer surface of the inner tube. What is the rate of heat transfer per unit length of tube from gas to the water?



Analytical solution

Assumptions: Steady state condition, constant properties, one-dimensional conduction in strut, adiabatic outer surface conditions, negligible gas-side radiation, fully developed internal flow, negligible fouling.

D_o=60mm

$D_{i,1}$=24mm

$D_{i,2}$=30mm

T=3mm=0.003m

L (length of strut) = (60-30)/2 mm = 0.015m

w (length of tube) =1m

Properties of water is taken from Table A-6: Water (300K): k=0.613 W/m-K , Pr=5.83, η= 855x10^{-6} N-s/m^2

The heat rate is given by

q= (UA) ($T_{m,h}$ - $T_{m,c}$)

$T_{m,h}$: bulk temperature of hot fluid (hot gas) = 800K, $T_{m,c}$: bulk temperature of cold fluid (water) = 300K

R_w: conduction resistance

Where

$$\frac{1}{UA} = \frac{1}{(hA)_c} + R_w + \frac{1}{(\eta hA)_h}$$

$$R_w = \frac{\ln\left(D_{i,2}/D_{i,1}\right)}{2\,\pi\, kL}$$

With

$Re_D = \frac{4\,m}{\pi\, D_{i,1}\mu} = \frac{4*0.161}{3.14*0.024*855*10^{-6}}$=9990

Therefore, internal flow is turbulent

Using Dittus-Boelter Correlation

$$h_c = \left(\frac{k}{D_{i,2}}\right) 0.023\, {Re_D}^{4/5} Pr^{0.4} = \left(\frac{0.163\frac{W}{m.K}}{0.024m}\right) 0.023 *\ 9990^{\frac{4}{5}} * 5.83^{0.4} = 1883\frac{W}{m^2.K}$$

Fin efficiency is determined using:

$$\eta_o = 1 - \left(A_f/A\right)\left(1-\eta_f\right)$$

$$A_f = 8 * 2(L\,.w) = 8 * 2(0.0015 * 1) = 0.24\, m^2$$

$$A = A_f + \left(\pi\, D_{i,2} - 8t\right)w = 0.24 + (\pi * 0.03 - 8 * 0.003) = 0.31 m^2$$

For adiabatic condition on tip of fin, fin efficiency is given by:

$$\eta_f = \frac{\tanh\,(mL)}{mL}$$

Where m= $\frac{2h}{kt}^{1/2}$=[2 x 100 W/m^{2}K /50 W/mK (0.003m)]= 36.5m^{-1}

mL= $\frac{2h}{kt}^{1/2}$ L = 36.5 x 0.015 = 0.55

tanh($\frac{2h}{kt}^{1/2}$ L) = 0.499

Hence $\eta_f = \frac{0.499}{0.55}$=0.911

$\eta_o = 1 - \left(A_f/A\right)\left(1-\eta_f\right)$= 1-(0.24/0.31) (1-0.911) =0.931

$$\frac{1}{(\eta hA)_h} = 0.931 * 100\frac{W}{m^2K} * 0.31 m^2 = 0.0347\, K/W$$

Hence,

$\frac{1}{UA_c} = (7.043 * 10^{-3}$+ 7.1.x 10^{-4} + 0.0347) K/W

UA$_c$ = 23.6 W/K

And q = 23.6 W/K (800 K- 300 K) = 11,800W for 1 m long section

Reference table A-6:

TABLE A.6 Thermophysical Properties of Saturated Water[a]

Tempera-ture, T (K)	Pressure, p (bars)[b]	Specic Volume (m³/kg) $v \cdot 10^3$	v_g	Heat of Vapor-ization, h_{g} (kJ/kg)	Specic Heat (kJ/kg · K) $c_{p,}$	$c_{p,g}$	Viscosity (N · s/m²) $\mu \cdot 10^6$	$\mu_g \cdot 10^6$	Thermal Conductivity (W/m · K) $k \cdot 10^3$	$k_g \cdot 10^3$	Prandtl Number Pr	Pr_g	Surface Tension, $\sigma \cdot 10^3$ (N/m)	Expansion Coef-cient, $\beta \cdot 10^6$ (K^{-1})	Temper-ature, T (K)
273.15	0.00611	1.000	206.3	2502	4.217	1.854	1750	8.02	569	18.2	12.99	0.815	75.5	−68.05	273.15
275	0.00697	1.000	181.7	2497	4.211	1.855	1652	8.09	574	18.3	12.22	0.817	75.3	−32.74	275
280	0.00990	1.000	130.4	2485	4.198	1.858	1422	8.29	582	18.6	10.26	0.825	74.8	46.04	280
285	0.01387	1.000	99.4	2473	4.189	1.861	1225	8.49	590	18.9	8.81	0.833	74.3	114.1	285
290	0.01917	1.001	69.7	2461	4.184	1.864	1080	8.69	598	19.3	7.56	0.841	73.7	174.0	290
295	0.02617	1.002	51.94	2449	4.181	1.868	959	8.89	606	19.5	6.62	0.849	72.7	227.5	295
300	0.03531	1.003	39.13	2438	4.179	1.872	855	9.09	613	19.6	5.83	0.857	71.7	276.1	300
305	0.04712	1.005	29.74	2426	4.178	1.877	769	9.29	620	20.1	5.20	0.865	70.9	320.6	305

Equations

Continuity equation:

For incompressible fluid:

$$\frac{\partial u_i}{\partial x_i} = 0$$

Momentum equation:

$$\frac{\partial(\rho u_i)}{\partial t} + \frac{\partial(\rho u_i u_j)}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \mu\frac{\partial^2 u_i}{\partial x_j \partial x_j} + \rho g_i$$

Energy conservation for water:

$$\frac{D}{Dt}\left(\rho\left(e + \frac{1}{2}u_i^2\right)\right) = -\frac{\partial q_i}{\partial x_i} + \frac{\partial^2 \tau_{ij} u_i}{\partial x_j} + \rho g_i u_i + S$$

Energy conservation for solid:

$$\frac{\partial(\rho e)}{\partial t} = \frac{\partial}{\partial x_j}\left(\alpha\frac{\partial e}{\partial x_j}\right)$$

Coupling equations:

At the interface, temperature of water (fluid) and surface of inner tube in contact with water (solid) is the same.

$$T_f = T_s$$

Heat flux is same at interface:

$$Q_f = -Q_s$$

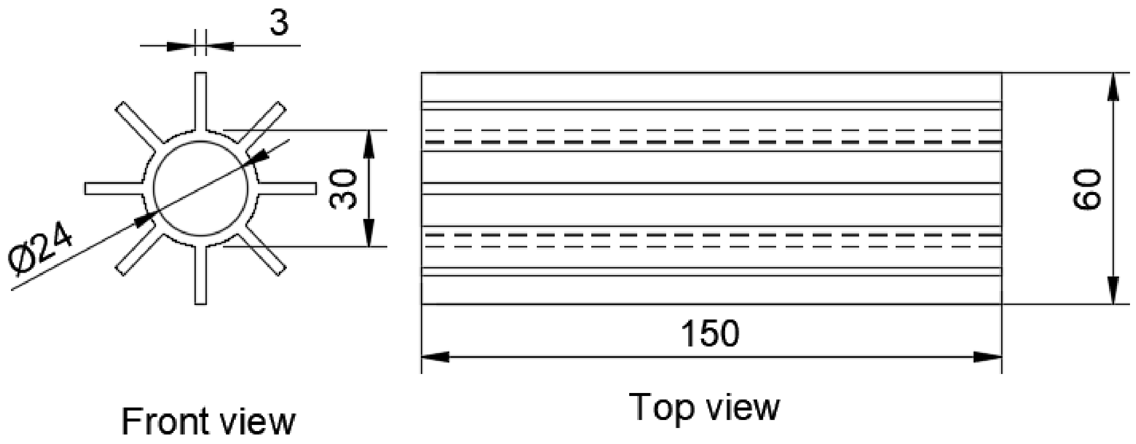
$$k_f\frac{dT_f}{dn} = -k_s\frac{dT_s}{dn}$$

k_f and k_s are thermal conductivities of fluid and solid and n is normal direction to the surface

Geometry and Mesh

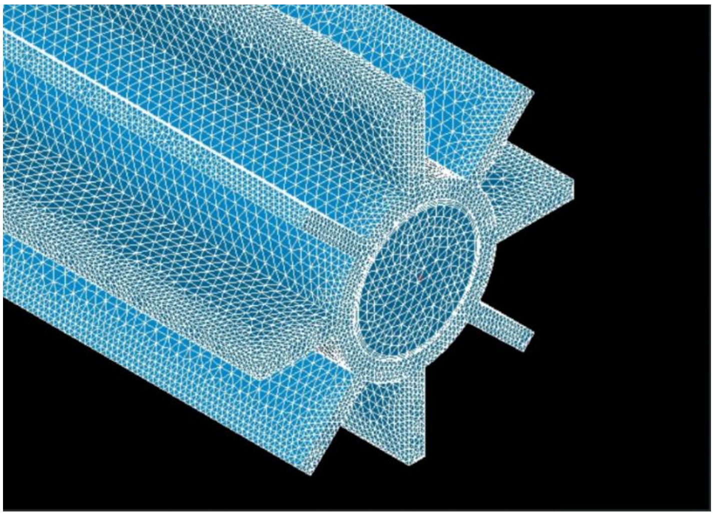
In the given problem, tube of length of 1m is assumed to calculate rate of heat transfer per unit length. For fully developed flow, 15% of length is considered here to compute the result numerically. Hence, 0.15m is taken to be the tube length.

Based on the given problem, an inner tube with struts and water at the center of tube is modelled.



All dimensions are in mm

SALOME software is used for meshing. Tetrahedral mesh is used with inflation layers added. Since the given problem is a multi-region problem, tube and water are meshed independently and combined. This has resulted in non-uniform change in quality of mesh with refinement.



Mesh of tube with struts and water using SALOME

There are two convection and one conduction process. From the given data, the convection coefficients at the outer surface of the inner tube and the struts is given. To model the heat transfer between the gas and the inner tube, boundary condition- externalWallHeatFlux is used. This boundary condition uses the convection coefficient and temperature of hot gas data to determine the temperature of the outer surface of inner tube, i.e, the surface of inner tube in contact with the hot gas.

For convection between inner tube and water, a coupled boundary condition is used to share the temperature data of the cells at the interface between water and inner tube.

1. Initial and Final Boundary Conditions

	U velocity	P pressure	p_rgh*	T Temperature	k	omega
Tube : Initial Field	0	1e5		300K	-	-
tube_to_water	0	calculated	-	Coupled	-	-
Gas_to tube	0	calculated	-	externalWallHeatFluxTemperature	-	-
Strut face	0	calculated	-	zeroGradient	-	-
Axial face	0	calculated	-	zeroGradient	-	-
Water : Initial Field	0.3558	1e5	0	300K	4.9e-08	0.2428
water_to _tube	No slip	calculated	Fixed flux pressure	Coupled	kqRWall Function	omegaWallFunction
inlet	0.3558	calculated	Zero gradient	inletOutlet Value:300K	4.9e-08	0.2428
outlet	Zero gradient	calculated	0	inletOutlet Value:300K	zeroGradient	zeroGradient

- inletOutlet: This is a mixed boundary condition that switches between zero gradient when fluid is flowing out of the domain and fixed value (300K) when fluid is flowing into the domain.
- zeroGradient: Adiabatic condition
- Coupled: Implemented using compressible:turbulentTemperatureCoupledBaffleMixed

* p_rgh = p – ρ(g.h)

2. Turbulence modelling

Turbulence parameters for k- ω SST model are calculated using:

Turbulence Intensity (I): $I = 0.16\ Re^{-1/8}$

Turbulence kinetic energy (k): $k = \frac{3}{2}(UI)^2$ Where U is the mean flow velocity

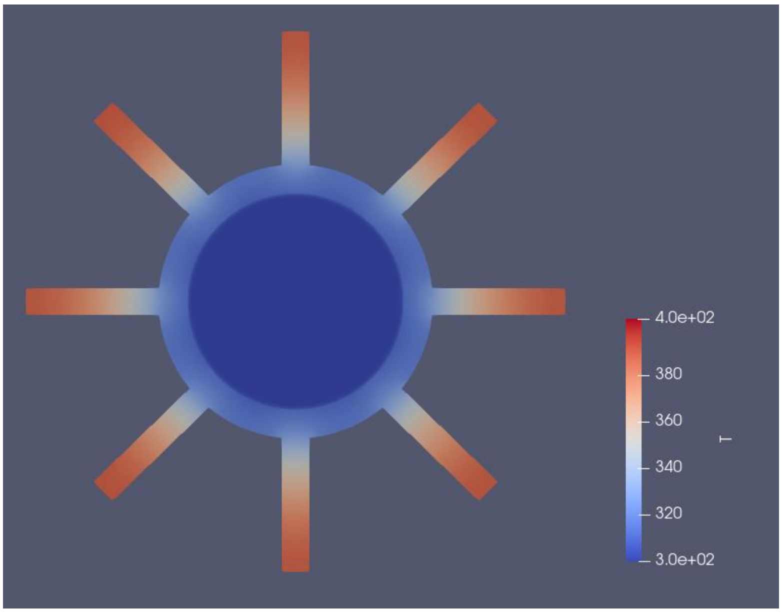
Turbulent dissipation rate (ω): $\omega = C_{\mu}{}^{3/4}\ \frac{k^{1/2}}{l}$ Where C_{μ} =0.09 is turbulence constant, l is turbulence length

3. Solver

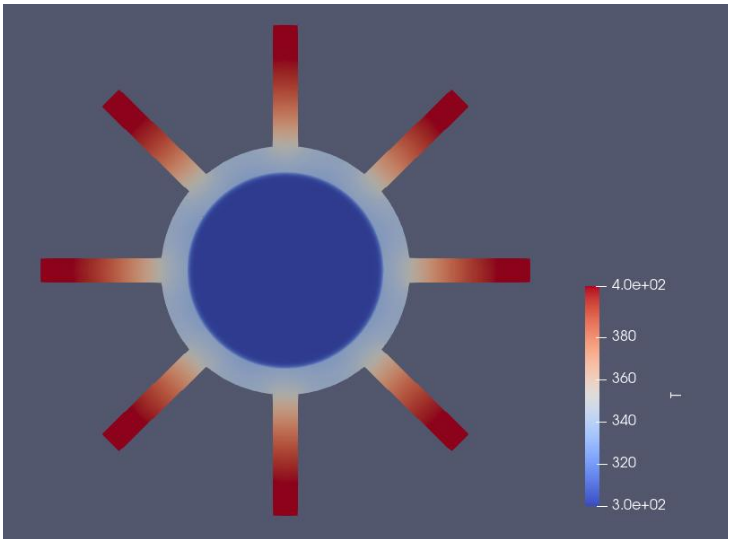
OpenFOAM version 2312 has been used. For flow simulations, chtMultiRegionFoam solver was used with $k-\omega$ turbulent model. chtMultiRegionFoam is a transient solver which solves for flow simulation in multi-region problems. multiRegionHeater tutorial was referred for solver settings. **k-omega SST** model is used as the problem has internal, turbulent flow close to walls.

4. Results

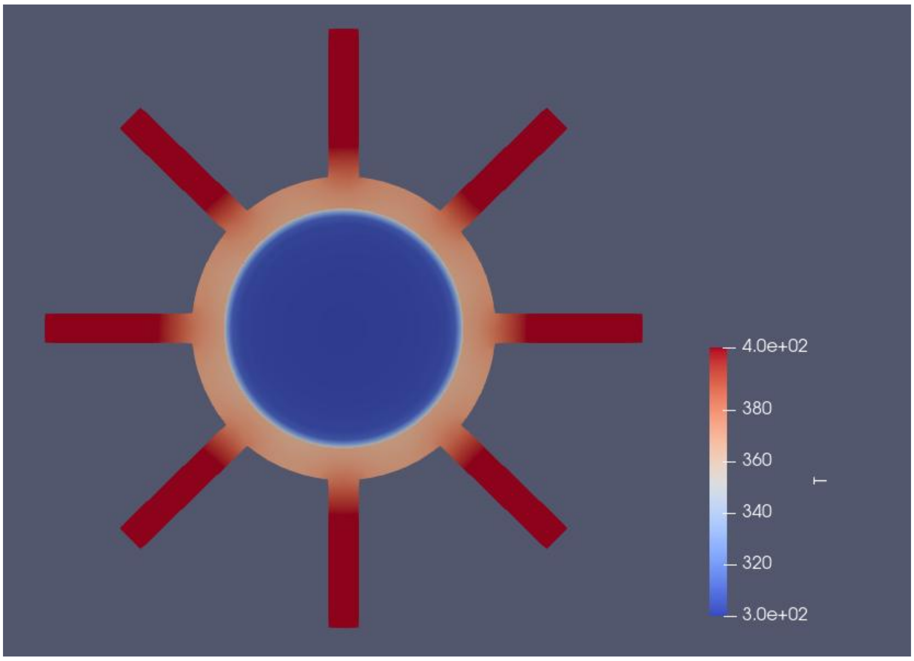
For mesh with 224,349 cells, solution converges at 1.613s



At 0.03s



At 0.125s



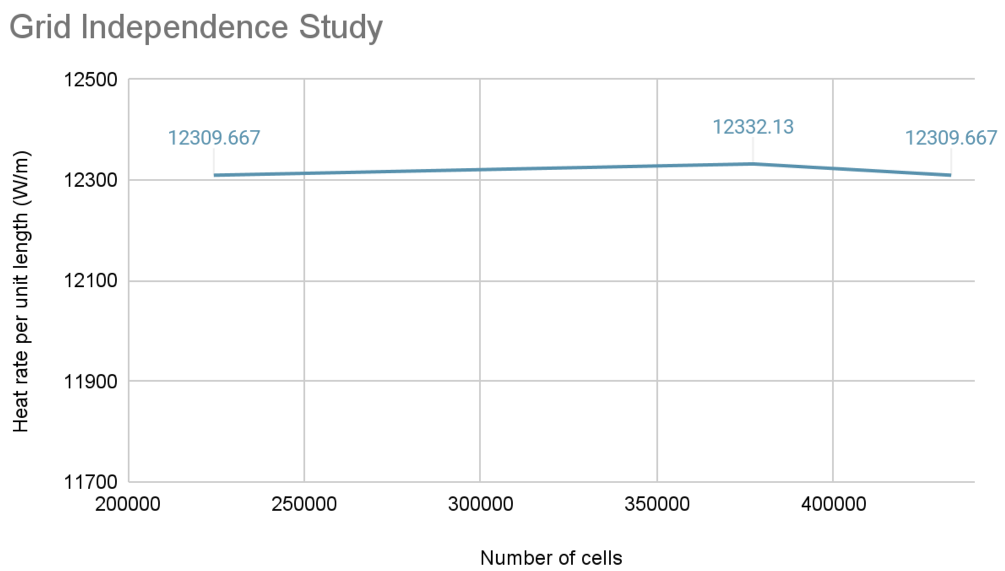
1.645 s (after convergence)

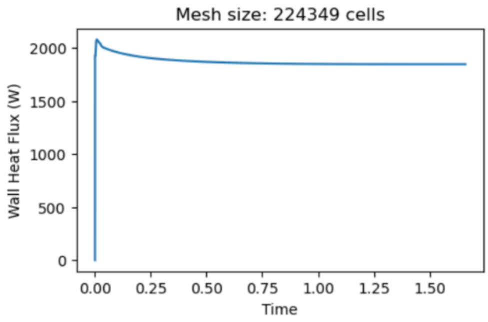
wallHeatFlux utility in OpenFOAM is a function that calculates the wall heat flux at an interface between two regions. It gives three values for each time step: minimum, maximum and integral. The units of minimum and maximum are W/m^2. Integral value is the summation of product of wall heat flux at a cell and area of cell. Hence its unit is W.

In the analytical solution, the result obtained is 11800 W for 1m of the tube length which is 11800 W/m. For the numerical solution, since the length taken is 0.15m, we divide the integral wall heat flux value (in W) with 0.15m.

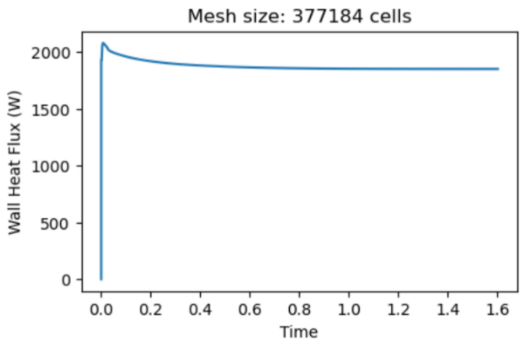
5. Grid Independence Study

Case	Number of cells	Heat rate per meter (W/m)	Error (in %)	Max skewness
A	224349	12309.667	4.31	0.72
B	377184	12332.13	4.5	0.85
C	433358	12309.667	4.31	0.82

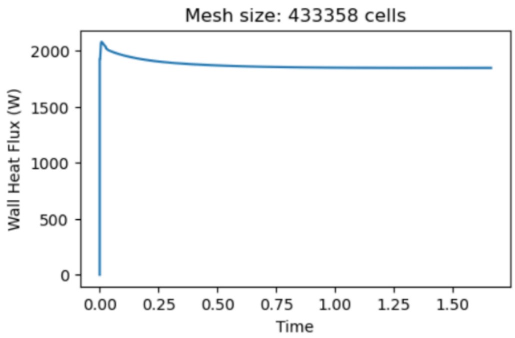




For mesh with 224,349 cells, the value of wall heat flux stabilizes at 1846.45 W. On dividing by the tube length 0.15m we get $= \frac{1846.45}{0.15} = 12309.667\ W/m$



For mesh with 377,184 cells, the value of wall heat flux stabilizes at 1846.45 W. On dividing by the tube length 0.15m we get $= \frac{1849.82}{0.15} = 12332.2\ W/m$



For mesh with 433,358 cells, the value of wall heat flux stabilizes at 1846.45 W. On dividing by the tube length 0.15m we get $= \frac{1846.45}{0.15} = 12309.667\ W/m$