

c) We need to invert this transformation. We could take the inverse of the final 3×3 . Let's look at the bits instead.

To return to G from M we have

$$\vec{r}_G = T_{GM} R_{GM} \vec{r}_M$$

where $R_{GM} = R_{MG}^{-1}$ & $T_{GM} = T_{MG}^{-1}$. These matrices are

$$T_{GM} = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_{GM} = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The product becomes

$$T_{GM} R_{GM} = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & -3 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

The point $(3, -2)_M$ becomes

$$\vec{r}_G = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & -3 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{3}{2} + \sqrt{3} & -3 \\ \frac{3\sqrt{3}}{2} & -1 + 1 \\ 1 \end{bmatrix}$$

$$\Rightarrow \vec{r}_G = \begin{bmatrix} \sqrt{3} - \frac{3}{2} \\ \frac{3\sqrt{3}}{2} \\ 1 \end{bmatrix}$$

- This looks about right!