

Deep Learning-III

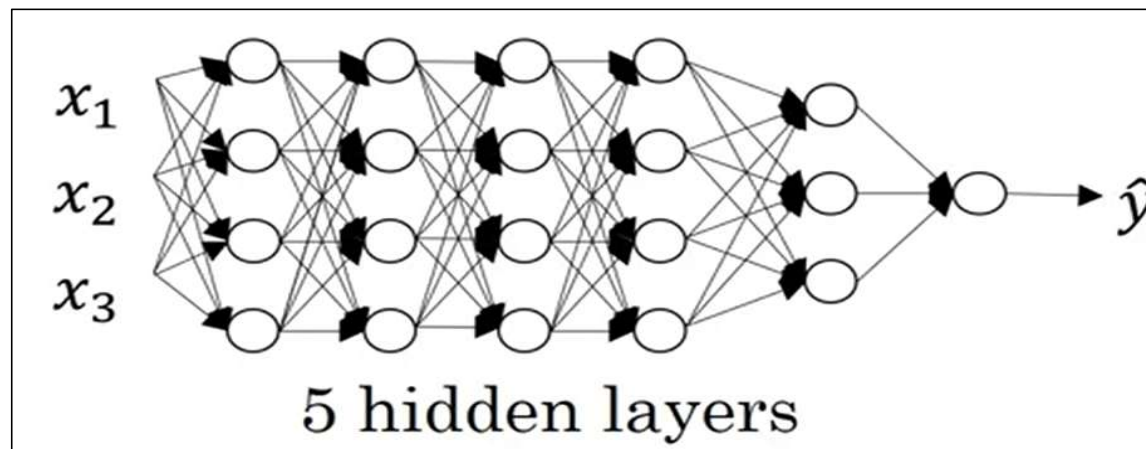
(Deep Neural Networks)

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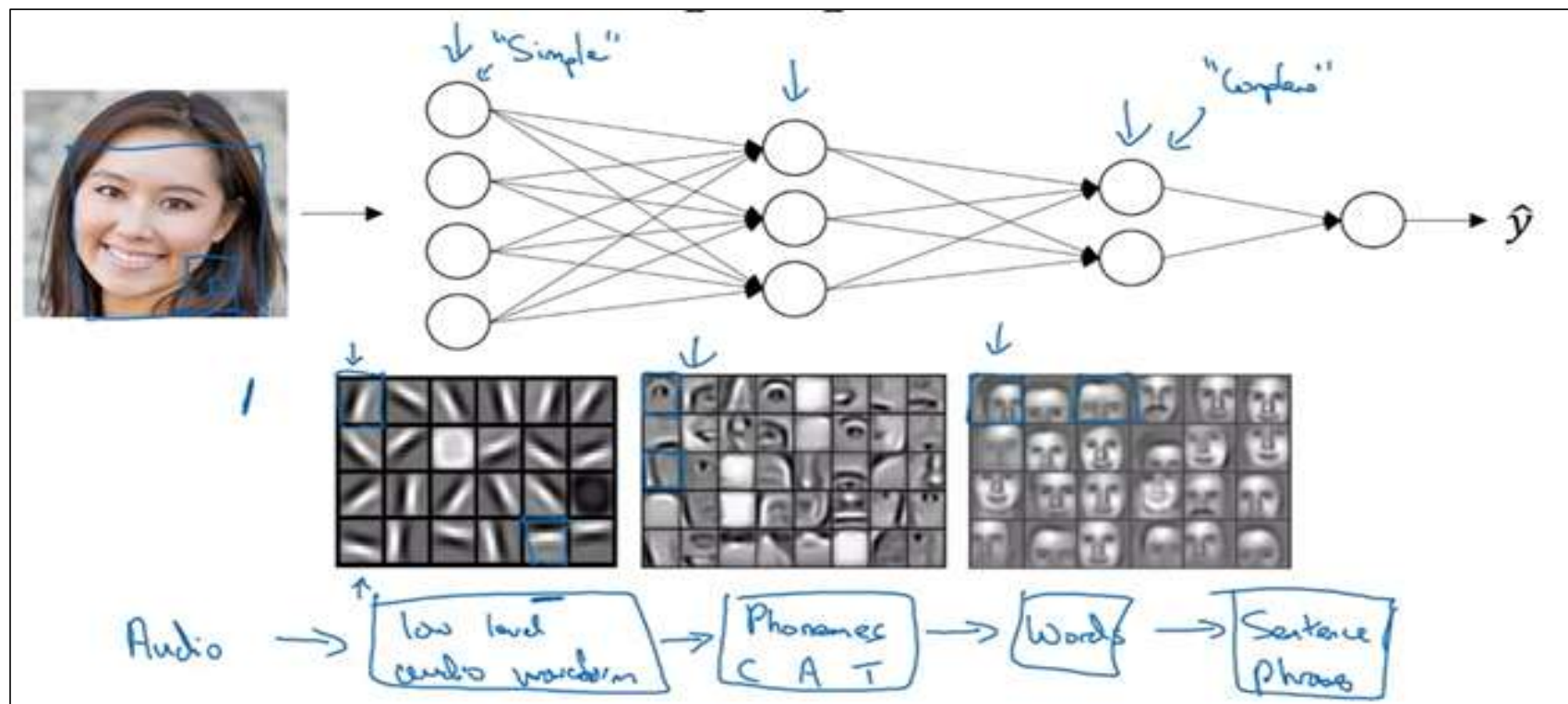


Deep Neural Networks

- A deep neural network (DNN) is an **artificial neural network (ANN)** with **multiple hidden layers between the input and output layers**.
- A deep neural network can learn more complex functions, which shallow neural networks fails to learn.



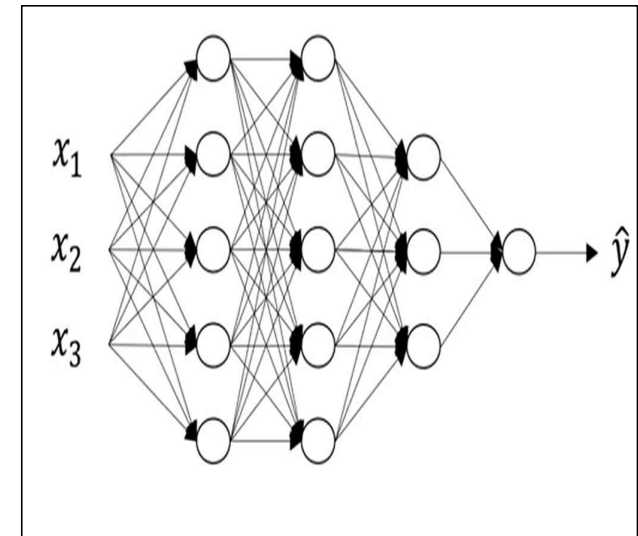
Why Deep Neural Networks?



Notations for Deep Neural Networks

- Consider a deep neural network with four layers and 5, 5, 3, and 1 neurons in each layer. Following notations are used for the network:
- L - layer number ($L=0$ to 4 in this case)
- n_L - number of neurons in each layer (In this case, $n_0=3$, $n_1=n_2=5$, $n_3=3$, $n_4=1$)
- a_L - activations in layer L - produced by applying activation function on the weighted sum of inputs i.e. $a_L=g_L(z_L)$
- W_L - weight matrix at layer L
- B_L - bias matrix at layer L

Weights and bias are used to compute z values of the layer as follows: $z_L=W_L a_{L-1}+b_L$



Matrix Dimensions

For one example,

$$W_L = (n_L, n_{L-1})$$

$$b_L = (n_L, 1)$$

$$z_L = a_L = (n_L, 1)$$

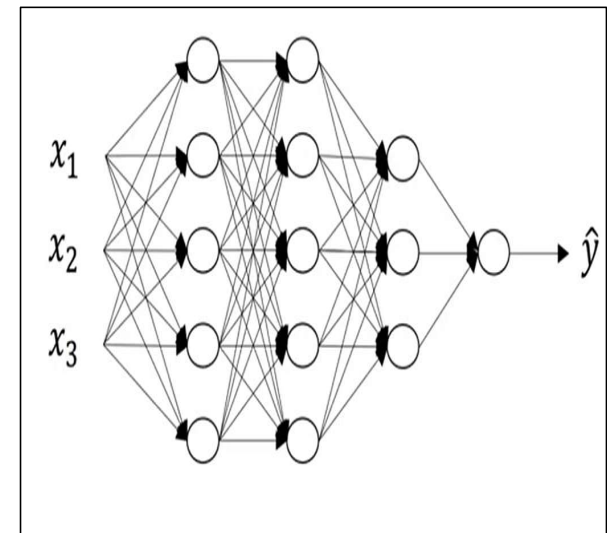
For example, for the neural network shown in fig:

$$W_1 = (5, 3), W_2 = (5, 5), W_3 = (3, 5), W_4 = (1, 3)$$

$$b_1 = (5, 1), b_2 = (5, 1), b_3 = (3, 1), b_4 = (1, 1)$$

$$z_1 = (5, 1), z_2 = (5, 1), z_3 = (3, 1), z_4 = (1, 1)$$

$$a_0 = (3, 1), a_1 = (5, 1), a_2 = (5, 2), a_3 = (3, 1), a_4 = (1, 1)$$



Matrix Dimensions (Contd...)

For n examples/Vectorized implementation,

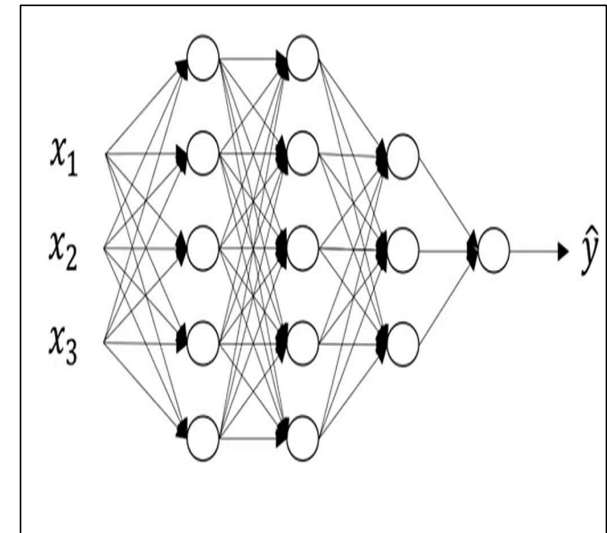
The dimensions of Weights and bias would remain same:

$$W_L = (n_L, n_{L-1})$$

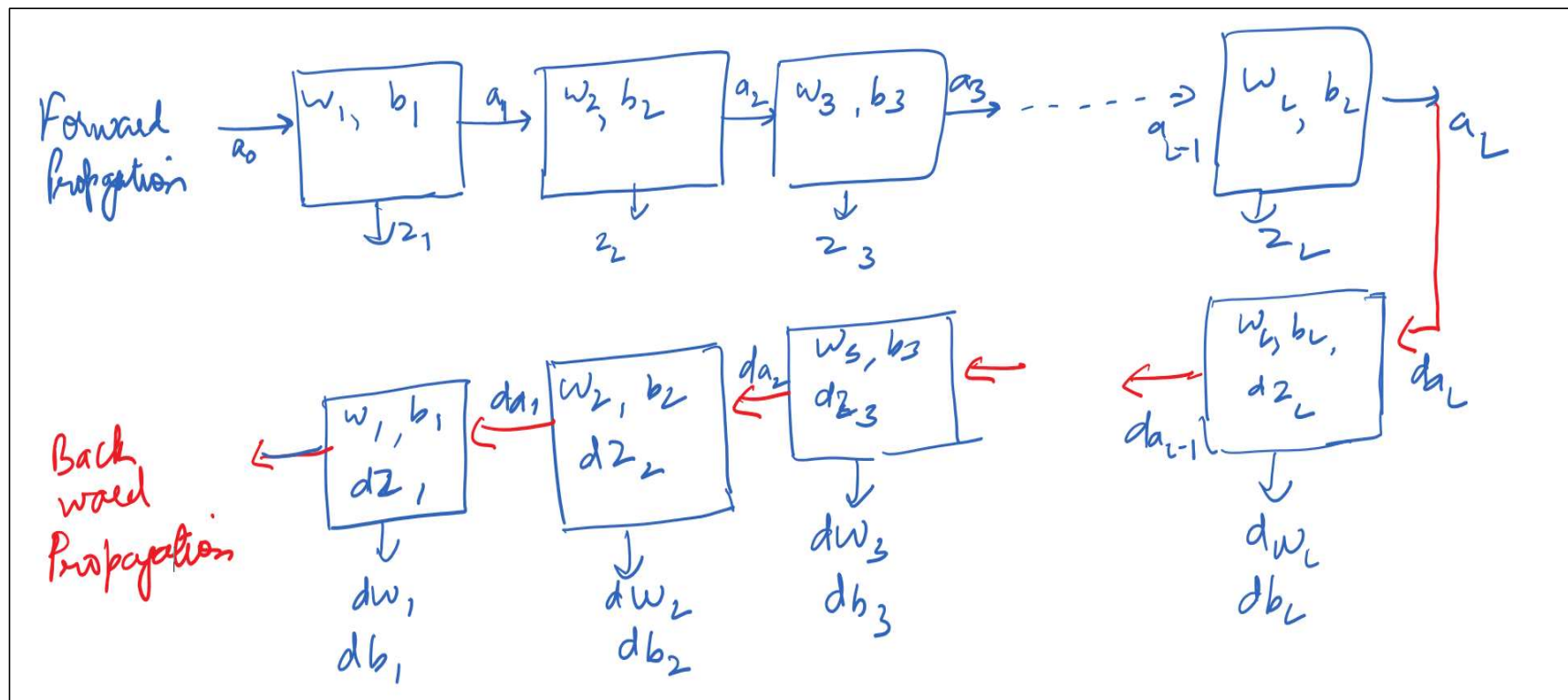
$$b_L = (n_L, 1)$$

The dimensions of Z_L and A_L will be updated according to n examples.

$$Z_L = A_L = (n_L, n)$$



Forward and Backpropagation Functions



Forward Propagation in a deep network

For one example,

$$\text{First Layer: } z_1 = W_1 a_0 + b_1; a_1 = g_1(z_1)$$

$$\text{Second Layer: } z_2 = W_2 a_1 + b_2; a_2 = g_2(z_2)$$

$$\text{Third Layer: } z_3 = W_3 a_2 + b_3; a_3 = g_3(z_3)$$

$$\text{Fourth Layer: } z_4 = W_4 a_3 + b_4; a_4 = g_4(z_4)$$

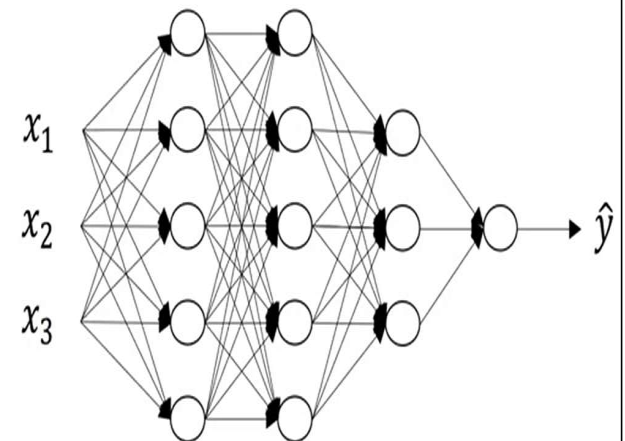
For n training examples,

$$\text{First Layer: } Z_1 = W_1 A_0 + b_1; A_1 = g_1(Z_1)$$

$$\text{Second Layer: } Z_2 = W_2 A_1 + b_2; A_2 = g_2(Z_2)$$

$$\text{Third Layer: } Z_3 = W_3 A_2 + b_3; A_3 = g_3(Z_3)$$

$$\text{Fourth Layer: } \hat{y} = Z_4 = W_4 A_3 + b_4; A_4 = g_4(Z_4)$$



In general forward propagation can be implemented as:

for $l=1$ to L :

$$Z_l = W_l A_{l-1} + b_l$$

$$A_l = g_l(Z_l)$$

Backward Propagation at Layer L

- One training example

$$dz_L = da_L * g'_L(z_L)$$

$$dW_L = dz_L \cdot a_{L-1}^T$$

$$db_L = dz_L$$

$$da_{L-1} = W_L^T \cdot dz_L$$

- For n training examples:

$$dZ_L = dA_L * g'_L(Z_L)$$

$$dW_L = \frac{1}{n} dZ_L \cdot A_{L-1}^T$$

$$db_L = \frac{1}{n} \text{np.sum}(dZ_L, \text{axis} = 1, \text{keepdims} = \text{True})$$

$$dA_{L-1} = W_L^T \cdot dZ_L$$