Mzumbe University

Faculty of Science and Technology

MSS 124:Linear Algebra

Tutorial sheet 3.

1. By cramers rule, find the solution for the following system of linear equations, by first obtaining $det(A)$ by (a) cofactor method and (b) elementary row operation.

$$-2x_1 + 3x_2 - x_3 = 1$$
$$x_1 + 2x_2 - x_3 = 4$$
$$-2x_1 - x_2 + x_3 = -3$$

2. By performing elementary row reduction, compute $det(A)$, given that A is invertible.

$$A = \begin{bmatrix} 4 & 3 & 2 \\ 3 & -2 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

3. By computing the determinants of each of the following matrices, study the relationship between the determinants and say what exist in the three matrices.

$$A = \begin{bmatrix} 2 & 2 & 3 \\ 0 & 3 & 4 \\ 0 & 2 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 2 & 3 \\ 0 & 3 & 4 \\ 1 & -2 & -4 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & 2 & 3 \\ 0 & 3 & 4 \\ 1 & 0 & 0 \end{bmatrix}$$

4. Prove that $det(B^{-1}) = \frac{1}{det(B)}$ and then show that $det(AB^{-1}) = \frac{det(A)}{det(B)}$

5. Evaluate $|A| = \begin{vmatrix} 1 & 2 & -3 & 4 \\ -4 & 2 & 1 & 3 \\ 3 & 0 & 0 & -3 \\ 2 & 0 & -2 & 3 \end{vmatrix}$

by row reduction-cofactor method.

6. By cofactor method find the inverse of the matrix A below.

$$A = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 5 & 6 \\ 7 & 1 & 2 \end{bmatrix}$$

7. In the process of determining the eigenvalues and eigenvectors of any given matrix, we first obtain the **characteristic polynomial** $p(\lambda)$ of A, $p(\lambda) = det(\lambda I - A)$. Thereafter, we equate the characteristic polynomial $p(\lambda)$ of A to zero. Thus the equation $p(\lambda) = det(\lambda I - A) = 0$ is called the **characteristic equation**.

Question: Let $A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{bmatrix}$. Find (a) the characteristic polynomial of A (b) the eigenvalues and the associated eigenvectors of the matrix A

8. Compute the eigenvalues and the corresponding eigenspaces of the matrix B.

$B = \begin{bmatrix} 0 & 0 & 3 \\ 1 & 0 & -1 \\ 0 & 1 & 3 \end{bmatrix}$. Hence obtain the basis of the eigenspaces of the matrix B associated with each of the eigenvalues

9. Find the eigenvalues and the associated eigenvectors of $C = \begin{bmatrix} -1 & 0 & 0 \\ 1 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

10. Find all the eigenvalues and the associated eigenvectors of each of the following matrices.

$$A = \begin{bmatrix} 4 & 2 & -4 \\ 1 & 5 & -4 \\ 0 & 0 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Assignment 2

The assignment should be handed in Friday 30^{th} **May 2014**

Remember that, excuses shall not be entertained

11. Compute the eigenvalues and the associated eigenspaces of the matrix P.

$$P = \begin{bmatrix} 6 & 0 & 0 & 0 & 0 \\ 9 & -4 & 0 & 0 & 0 \\ -2 & 0 & 11 & 0 & 0 \\ 1 & -1 & 3 & 0 & 0 \\ 0 & 1 & -7 & 4 & 8 \end{bmatrix}$$

Hence obtain the basis of the eigenspaces of the matrix P associated with each of the eigenvalues.