3/5/2020

Thursday, March 5, 2020 2:29 PM

Example of domain transformation

$$\begin{cases} T(1)=0 \\ T(n)=T\left(\frac{n}{2}\right)+n^2,\ n>1 \end{cases}$$

needs to be $n-1$

will become base of exponent

$n=2^k$

$T(n)=T(2^k)=R(k)$

$T\left(\frac{n}{2}\right)=T\left(\frac{2^k}{2}\right)=T(2^{k-1})=R(k-1)$

$T(1)=T(2^0)=R(0)$

$$\begin{cases} R(0) \\ R(k)=R(k-1)+(2^k)^2,\ n>0 \end{cases}$$

$$R(k)-R(k-1) = 2^{2k}$$
$$R(k-1)-R(k-2) = 2^{2(k-1)}$$
$$\downarrow$$
$$R(1)-R(0) = 2^{2\cdot 1}$$

$$R(k) - R(0) = 2^2 + 2^{2(2)} + \dots 2^{2(k-1)} + 2^{2k}$$

$$R(k)=R(0)+\sum_{i=1}^{k} 2^{2i}$$

$R(0)=0$

$$R(k)=\sum_{i=1}^{k} 2^{2i} = \sum_{i=1}^{k} 4^i$$

$$(a^x)^y = a^{xy} = (a^y)^x$$

$$R(k)=\frac{1-4^{k+1}}{1-4}-1$$
$$=\frac{1-4^{k+1}}{-3}-1$$
$$=-\frac{1}{3}-1+\frac{4}{3}4^k$$
$$=-\frac{4}{3}+\frac{4}{3}4^k$$
$$=\frac{4}{3}(4^k-1)$$

$$R(k)=T(2^k)=\boxed{T(n)=\frac{4}{3}(n^2-1)}$$

prop of exp

$$4^k=2^{2k}=(2^k)^2=n^2$$

$4^k \overset{\text{prop of exp}}{=} 2^{2k} = (2^k)^2 = n^2$

$4^{\log n} = (2^2)^{\log n} = 2^{2\log n} = 2^{\lg n^2} = n^2$

$a^{\lg_a x} = x$

verification:

Basis $n=1$

$T(1) = \frac{4}{3}(1^2-1) = 0$ ✓

induct. hyp: assume that $T(k) = \frac{4}{3}(k^2-1)$

inductive step must show $= T(2k) = \frac{4}{3}((2k)^2-1)$

$T(2k) = T(k) + (2k)^2$

$T(2k) = \frac{4}{3}(k^2-1) + (2k)^2$

$= \frac{4}{3}k^2 - \frac{4}{3} + (2k)^2$

$= \frac{4}{3}k^2 + 4k^2 - 4/3$

$= \frac{4}{3}k^2 + \frac{12k^2}{3} - \frac{4}{3}$

$= \frac{16}{3}k^2 - \frac{4}{3}$

$= \frac{4}{3}4k^2 - \frac{4}{3}$

$= \frac{4}{3}(4k^2-1)$

$= \frac{4}{3}(4k^2-1)$

$= \frac{4}{3}((2k)^2-1)$ ⊠

Range transformation

$\begin{cases} T(1) = 8 \\ T(n) = 3T(n-1) - 15, & n \geq 2 \end{cases}$

constants need to be same

$T(n) = 3T(n-1) - 15 \cdot \frac{1}{3^n}$

$\frac{T(n)}{3^n} = S(n)$

$\frac{3T(n-1)}{3^n} = \frac{T(n-1)}{3^{n-1}} = S(n-1)$

$\frac{T(1)}{3^1} = S(1) = \frac{8}{3}$

transform

$S(1) = \frac{8}{3}$

$S(n) = S(n-1) - \frac{15}{3^n}, \ n>1$

$S(n) - S(n-1) = -\frac{15}{3^n}$

$S(n-1) - S(n-2) = \frac{15}{3^{n-1}}$

$S(2) - S(1) = -\frac{15}{3^2}$

$S(n) - S(1) = -15\left(\frac{1}{3^2} + \frac{1}{3^3} + \cdots + \frac{1}{3^{n-1}} + \frac{1}{3^n}\right)$

$S(n) - S(1) = -15\sum^{n} \frac{1}{3^i}$

$$\frac{T(1)}{3^1} = S(1) = \frac{8}{3}$$

Verification $T(n) = \frac{1}{2}(3^{n-1} + 15)$

Basis $n=1$

$$T(1) = \frac{1}{2}(3^{1-1} + 15)$$
$$= \frac{1}{2}(1+15)$$
$$= \frac{16}{2}$$
$$= 8$$

Ind hyp: assume that $T(k) = \frac{1}{2}(3^{k-1} + 15)$

Ind step must show: $T(k+1) = \frac{1}{2}(3^k + 15)$

$$T(k+1) = 3T(k) - 15$$
$$= 3\left(\frac{1}{2}(3^{k-1} + 15)\right) - 15$$
$$= \frac{1}{2} \cdot 3 \cdot 3^{k-1} + \frac{3}{2}15 - 15$$
$$= \frac{1}{2}3^k + \frac{1}{2}15$$
$$= \frac{1}{2}(3^k + 15) \boxtimes$$

$$S(n) = S(1) - 15\sum_{i=2}^{n} \frac{1}{3^i}$$

$$S(n) = \frac{8}{3} - 15\sum_{i=2}^{n} \frac{1}{3^i}$$

$$\frac{1}{3^i} = \left(\frac{1}{3}\right)^i = q^i$$

$q \rightarrow 1/3$

two missing terms

$$= \frac{8}{3} - 15\sum_{i=0}^{n} \frac{1}{3^i} + 15(1 + 1/3)$$
$$= \frac{8}{3} + \frac{60}{3} - 15\sum_{i=0}^{n} \frac{1}{3^i}$$
$$= \frac{68}{3} - 15\sum_{i=0}^{n} \frac{1}{3^i}$$

$$\sum_{i=0}^{n} \frac{1}{3^i} = \frac{1 - \frac{1}{3^{n+1}}}{1 - 1/3}$$
$$= \frac{3^{n+1} - 1}{3^{n+1} \cdot \frac{2}{3}}$$
$$= \frac{3^{n+1} - 1}{2 \cdot 3^n}$$

$$S(n) = \frac{68}{3} - 15\left(\frac{3^{n+1} - 1}{2 \cdot 3^n}\right)$$

$$T(n) = 3^n(S(n))$$
$$T(n) = 3^n\left(\frac{68}{3} - 15\left(\frac{3^{n+1} - 1}{2 \cdot 3^n}\right)\right)$$
$$= 68 \cdot 3^{n-1} - \frac{15}{2} \cdot 3^{n+1} + \frac{15}{2}$$

$$-\frac{15}{2} \cdot 3^{n+1} = -\frac{15}{2} 3^{(n-1)+2}$$
$$= -\frac{15}{2} 3^2 \cdot 3^{n-1}$$

$$= 68 \cdot 3^{n-1} - \frac{135}{2} 3^{n-1} + \frac{15}{2}$$
$$= \frac{136}{2} 3^{n-1} - \frac{135}{2} 3^{n-1} + \frac{15}{2}$$
$$= \frac{1}{2} 3^{n-1} + \frac{15}{2}$$
$$= \frac{1}{2}(3^{n-1} + 15)$$

26th Quiz 2 possibly ★

DUE 3/12

HW6

(1) $\begin{cases} T(1) = 3 \\ T(n) = T(n/3) + 3, \; n > 1 \end{cases}$

HW6

1. $\begin{cases} T(1) = 3 \\ T(n) = T(n/3) + 3, \ n > 1 \end{cases}$

2. $\begin{cases} T(1) = 1 \\ T(n) = T(n/2) + bn\lg n, \ n > 1 \end{cases}$

3. $\begin{cases} T(1) = 3 \\ T(n) = T(n-1) + 2(n-3), \ n > 1 \end{cases}$

4. $\begin{cases} T(1) = 5 \\ T(n) = 2T(n-1) + 3n + 1 \quad n > 1 \end{cases}$