Optimisation

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Okay. How should I lay this out?

Question 4

Part A

This is a simple knapsack problem. Each variable A, B, C, D, E, F is binary: whether or not the item was taken. This leads to the following function:

$$max(60A + 70B + 40C + 70D + 16E + 100F)$$

in which the constants are the values (£) of each item. The only constraint is equally simple: that the weight of the taken items does not exceed 20kg:

$$6A + 7B + 4C + 9D + 3E + 8F <= 20$$

in which the constants are the weights (kg) of each item. An optimal solution is to take items B, C, and F, resulting in a total weight of $19kg$ and a total value of £210.

Part B

This part adds a new constraint: that taking C only makes sense if D is also taken, but not vice versa. This can be elegantly expressed as:

$$D - C >= 0$$

This condition is only unsatisfied if $D = 0$ and $C = 1$. With this constraint, an optimal solution is to take items D, E, and F, resulting in a total weight of $20kg$ and a total value of £186.

Part C

This part adds a further modification. It is now possible to exceed the $20kg$ limit, but with a penalty of £15 for each kg over. A new variable, w, is necessary. The objective function is modified to:

$$max(60A + 70B + 40C + 70D + 16E + 100F - 15w)$$

to capture the cost of exceeding the weight limit. An additional constraint is also required:

$$w == 6A + 7B + 4C + 9D + 3E + 8F - 20$$

to set w to number of kg over the weight limit the solution is. An optimal solution is to take items A, B, and F, resulting in a total weight of $21kg$ and a total value of £215.