Optimisation

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Question 1

Question 2

Question 3

For this problem, 9 variables are necessary. Each relates to the amount of paint of one colour (cyan C, magenta M, or yellow Y) used in the creation of paint of another colour (red R, green G, blue B, or black K), e.g. C_K denotes the number of gallons of cyan (C) paint used in the production of black (K) paint. The objective function is therefore:

$$max\left(\frac{10\cdot(Y_R+M_R)}{2} + \frac{15\cdot(Y_G+C_G)}{2} + \frac{25\cdot(M_B+C_B)}{2} + \frac{25\cdot(C_K+M_K+Y_K)}{3}\right)$$

Each term is the amount of paint of a certain colour produced multiplied by the value of a gallon of paint of that colour: 10 for R, 15 for G, 25 for B, and 25 for K. The following constraints capture the limited quantities of paint available: 11 gallons of Y, 10 of C, and 5 of M:

$$Y_R + Y_G + Y_K <= 11$$
$$C_G + C_B + C_K <= 10$$
$$M_B + M_K + M_R <= 5$$

Also necessary are constraints maintaining the correct ratios of paints used:

$$Y_R == M_R$$
$$Y_G == C_G$$
$$M_B == C_B$$
$$C_K == M_K == Y_K$$

These maintain the ratios for R, G, B, and K paint. An optimal solution produces 2 gallons of C paint, 8 gallons of G paint, 2 gallons of B paint, and 3 gallons of K paint, for a total value of £220.

Question 4

Part A

This is a simple knapsack problem. Each variable A, B, C, D, E, F is binary: whether or not the item was taken. This leads to the following function:

$$max(60A + 70B + 40C + 70D + 16E + 100F)$$

in which the constants are the values (£) of each item. The only constraint is equally simple: that the weight of the taken items does not exceed 20kg:

$$6A + 7B + 4C + 9D + 3E + 8F <= 20$$

in which the constants are the weights (kg) of each item. An optimal solution is to take items B, C, and F, resulting in a total weight of $19kg$ and a total value of £210.

Part B

This part adds a new constraint: that taking C only makes sense if D is also taken, but not vice versa. This can be elegantly expressed as:

$$D - C >= 0$$

This condition is only unsatisfied if $D = 0$ and $C = 1$. With this constraint, an optimal solution is to take items D, E, and F, resulting in a total weight of $20kg$ and a total value of £186.

Part C

This part adds a further modification. It is now possible to exceed the $20kg$ limit, but with a penalty of £15 for each kg over. A new variable, w, is necessary. The objective function is modified to:

$$max(60A + 70B + 40C + 70D + 16E + 100F - 15w)$$

to capture the cost of exceeding the weight limit. An additional constraint is also required:

$$w == 6A + 7B + 4C + 9D + 3E + 8F - 20$$

to set w to number of kg over the weight limit the solution is. An optimal solution is to take items A, B, and F, resulting in a total weight of $21kg$ and a total value of £215.