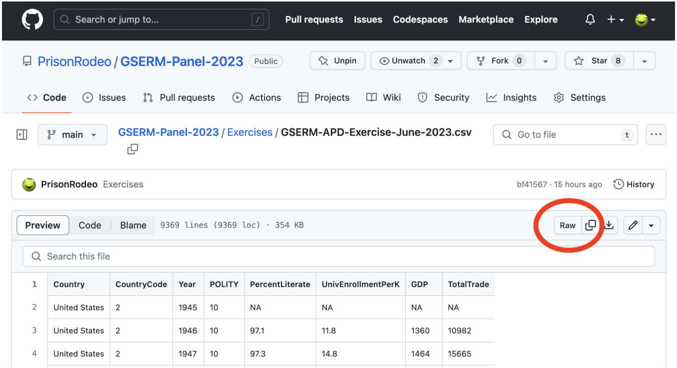
GSERM - St. Gallen 2023
Analyzing Panel Data

June 14, 2023

Data on Github

Download via the “Raw” button...



Can also use (e.g.) `read_csv` (in `readr`):

```
> library(readr)
> Data<-read_csv("https://github.com/PrisonRodeo/GSERM-Panel-2023/raw/main/Exercises/GSERM-APD-Exercise-June-2023.csv")
```

Start with a focus on residuals... For:

$$Y_{it} = \mathbf{X}_{it}\beta + u_{it}$$

i.i.d. OLS u_{it}s require:

$$\begin{aligned}
\mathbf{uu}' \equiv \mathbf{\Omega} &= \sigma^2\mathbf{I} \\
&= \begin{pmatrix} \sigma^2 & 0 & \cdots & 0 \\ 0 & \sigma^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma^2 \end{pmatrix}
\end{aligned}$$

This means that within units:

- $\text{Var}(u_{it}) = \text{Var}(u_{is}) \; \forall \; t \neq s$ (temporal homoscedasticity)
- $\text{Cov}(u_{it}, u_{is}) = 0 \; \forall \; t \neq s$ (no within-unit autocorrelation)

and between units:

- $\text{Var}(u_{it}) = \text{Var}(u_{jt}) \; \forall \; i \neq j$ (cross-unit homoscedasticity)
- $\text{Cov}(u_{it}, u_{jt}) = 0 \; \forall \; i \neq j$ (no between-unit / spatial correlation)

The Key: $\mathbf{\Omega}$

Estimator:

$$\hat{\beta}_{GLS} = (\mathbf{X}'\mathbf{\Omega}^{-\mathbf{1}}\mathbf{X})^{-1}\mathbf{X}'\mathbf{\Omega}^{-\mathbf{1}}\mathbf{Y}$$

with:

$$\widehat{\mathsf{V}(\beta_{GLS})} = (\mathbf{X}'\mathbf{\Omega}^{-\mathbf{1}}\mathbf{X})^{-1}$$

Two approaches:

- Use OLS $\hat{u}_{it}$s to get $\hat{\mathbf{\Omega}}$ ("feasible GLS" / " weighted least squares")
- Use <u>substantive knowledge about the data</u> to structure $\mathbf{\Omega}$

The variance-covariance matrix is:

$$\begin{aligned}\text{Var}(\hat{\boldsymbol{\beta}}_{WLS}) &= \sigma^2(\mathbf{X}'\boldsymbol{\Omega}^{-1}\mathbf{X})^{-1} \\ &\equiv (\mathbf{X}'\mathbf{W}^{-1}\mathbf{X})^{-1}\end{aligned}$$

A common case is:

$$\text{Var}(u_i) = \sigma^2 \frac{1}{N_i}$$

where N_i is the number of observations upon which (aggregate) observation i is based.

Recall that, if $\sigma_i^2 \neq \sigma_j^2 \forall i \neq j$,

$$\begin{aligned} \mathrm{Var}(\beta_{\text{Het.}}) &= (\mathbf{X}'\mathbf{X})^{-1}(\mathbf{X}'\mathbf{W}^{-1}\mathbf{X})(\mathbf{X}'\mathbf{X})^{-1} \\ &= (\mathbf{X}'\mathbf{X})^{-1}\,\mathbf{Q}\,(\mathbf{X}'\mathbf{X})^{-1} \end{aligned}$$

where $\mathbf{Q} = (\mathbf{X}'\mathbf{W}^{-1}\mathbf{X})$ and $\mathbf{W} = \sigma^2\boldsymbol{\Omega}$.

We can rewrite $\mathbf{Q}$ as

$$\begin{aligned} \mathbf{Q} &= \sigma^2(\mathbf{X}'\boldsymbol{\Omega}^{-1}\mathbf{X}) \\ &= \sum_{i=1}^{N} \sigma_i^2 \mathbf{X}_i \mathbf{X}_i' \end{aligned}$$

Estimate $\hat{\mathbf{Q}}$ as:

$$\widehat{\mathbf{Q}} = \sum_{i=1}^{N} \hat{u}_i^2 \mathbf{X}_i \mathbf{X}_i'$$

Yields:

$$\begin{aligned}
\widehat{\text{Var}(\beta)}_{\text{Robust}} &= (\mathbf{X}'\mathbf{X})^{-1}(\mathbf{X}'\widehat{\mathbf{Q}}^{-1}\mathbf{X})(\mathbf{X}'\mathbf{X})^{-1} \\
&= (\mathbf{X}'\mathbf{X})^{-1}\left[\mathbf{X}'\left(\sum_{i=1}^{N} \hat{u}_i^2 \mathbf{X}_i \mathbf{X}_i'\right)^{-1}\mathbf{X}\right](\mathbf{X}'\mathbf{X})^{-1}
\end{aligned}$$

"Robust" VCV estimates:

- are heteroscedasticity-consistent, but
- are biased in small samples, and
- are less efficient than "naive" estimates when $\text{Var}(u) = \sigma^2\mathbf{I}$.

“Clustering”

Huber / White ????????? WLS / GLS

I know very little about my error variances...

I know a great deal about my error variances...

A common case:

$$Y_{ij} = \mathbf{X}_{ij}\boldsymbol{\beta} + u_{ij}$$

with

$$\sigma_{ij}^2 = \sigma_{ik}^2.$$

"Robust, clustered" estimator:

$$\widehat{\mathrm{Var}(\boldsymbol{\beta})}_{\mathrm{Clustered}} = (\mathbf{X}'\mathbf{X})^{-1} \left\{ \mathbf{X}' \left[\sum_{i=1}^{N} \left(\sum_{j=1}^{n_j} \hat{u}_{ij}^2 \mathbf{X}_{ij} \mathbf{X}_{ij}' \right) \right]^{-1} \mathbf{X} \right\} (\mathbf{X}'\mathbf{X})^{-1}$$

Robust / Clustered SEs: A Simulation

```
url_robust <- "https://raw.githubusercontent.com/IsidoreBeautrelet/economictheoryblog/master/robust_summary.R"
eval(parse(text = getURL(url_robust, ssl.verifypeer = FALSE)),
     envir=.GlobalEnv)

> set.seed(3844469)
> X <- rnorm(10)
> Y <- 1 + X + rnorm(10)
> df10 <- data.frame(ID=seq(1:10),X=X,Y=Y)
> fit10 <- lm(Y~X,data=df10)
> summary(fit10)

Call:
lm(formula = Y ~ X, data = df10)

Residuals:
   Min     1Q Median     3Q    Max
-1.318 -0.766  0.195  0.378  1.590

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)    0.954      0.311    3.06    0.016 *
X              0.589      0.291    2.03    0.077 .
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.985 on 8 degrees of freedom
Multiple R-squared:  0.339,Adjusted R-squared:  0.257
F-statistic: 4.11 on 1 and 8 DF,  p-value: 0.0772

> rob10 <- vcovHC(fit10,type="HC1")
> sqrt(diag(rob10))
(Intercept)           X
      0.315       0.285
```

Robust / Clustered SEs: A Simulation (continued)

```
> # "Clone" each observation 100 times:
>
> df1K <- df10[rep(seq_len(nrow(df10)),each=100),]
> df1K <- pdata.frame(df1K, index="ID")
> fit1K <- lm(Y~X,data=df1K)

> summary(fit1K)

Call:
lm(formula = Y ~ X, data = df1K)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   0.9536     0.0279    34.2   <2e-16 ***
X             0.5893     0.0260    22.6   <2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.882 on 998 degrees of freedom
Multiple R-squared:  0.339,Adjusted R-squared:  0.339
F-statistic:  513 on 1 and 998 DF,  p-value: <2e-16


> summary(fit1K, cluster="ID")

Call:
lm(formula = Y ~ X, data = df1K)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)    0.954      0.297    3.21   0.0014 **
X              0.589      0.269    2.19   0.0286 *
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.882 on 998 degrees of freedom
Multiple R-squared:  0.339,Adjusted R-squared:  0.339
F-statistic:  4.8 on 1 and 9 DF,  p-value: 0.0561
```

Example:

$$
\begin{aligned}
Y_t &= \beta_0 + \beta_1 X_t + u_t \\
u_t &= \rho u_{t-1} + e_t
\end{aligned}
$$

with $e_t \sim i.i.d.\ N(0, \sigma_u^2)$ and $\rho \in [-1, 1]$ (typically).

$\rightarrow$ "First-order autoregressive" ("AR(1)") errors.

Detection

- *Plot* of residuals vs. lagged residuals
- *Runs* test (Geary test)
- Durbin-Watson d
  - Calculated as:

$$d = \frac{\sum_{t=2}^{N}(\hat{u}_t - \hat{u}_{t-1})^2}{\sum_{t=1}^{N}\hat{u}_t^2}$$

  - Non-standard distribution ($d \in [0, 4]$)
  - Null: No autocorrelation
  - Only detects first-order autocorrelation

What to do about it?

- GLS, incorporating ρ / $\hat{\rho}$ into the equation
- *First-difference* models (regressing changes of Y on changes of $\mathbf{X}$)
- Cochrane-Orcutt / Prais-Winsten:
  1. Estimate the basic equation via OLS, and obtain residuals
  2. Use the residuals to consistently estimate $\hat{\rho}$ (i.e. the empirical correlation between u_t and u_{t-1})
  3. Use this estimate of $\hat{\rho}$ to estimate the *difference equation*:

  $$(Y_t - \rho Y_{t-1}) = \beta_0(1-\rho) + \beta_1(X_t - \rho X_{t-1}) + (u_t - \rho u_{t-1})$$

  4. Save the residuals, and use them to estimate $\hat{\rho}$ again
  5. Repeat this process until successive estimates of $\hat{\rho}$ differ by a very small amount

Running Example Redux

The World Development Indicators:

- Cross-national country-level time series data
- $N = 215$ countries, $T = 72$ years (1960-2021) + missingness
- Full descriptions are listed in the Github repo here

Regression model:

$$\begin{aligned} \text{WBLI}_{it} &= \beta_0 + \beta_1 \text{Population Growth}_{it} + \beta_2 \text{Urban Population}^2_{it} + \beta_3 \text{Fertility Rate}_{it} + \\ &\quad \beta_4 \ln(\text{GDP Per Capita})_{it} + \beta_5 \text{Natural Resource Rents}_{it} + \beta_6 \text{Cold War}_t + u_{it} \end{aligned}$$

Descriptive Statistics:

	vars	n	mean	sd	min	max	range	se
WomenBusLawIndex	1	8100	60.69	18.95	17.50	100.00	82.50	0.21
PopGrowth	2	8100	1.65	1.54	-16.88	19.36	36.24	0.02
UrbanPopulation	3	8100	51.56	23.82	2.85	100.00	97.16	0.26
FertilityRate	4	8100	3.61	1.90	0.77	8.61	7.83	0.02
NaturalResourceRents	5	8100	7.04	10.77	0.00	88.59	88.59	0.12
ColdWar	6	8100	0.30	0.46	0.00	1.00	1.00	0.01
lnGDPPerCap	7	8100	8.29	1.44	5.04	11.64	6.60	0.02

How Much Autocorrelation in $\mathbf{X}$?

Note that:

$$d = 2(1 - \rho)$$

which means that we can calculate:

$$\rho = 1 - \frac{d}{2}.$$

Autocorrelation in the Predictors

	Variable	Rho
1	Population Growth	0.852
2	Urban Population	0.974
3	Fertility Rate	0.966
4	GDP Per Capita	0.977
5	Natural Resource Rents	0.911
6	Cold War	0.916

Baseline Model: OLS (+ D-W Test)

```
> OLS<-plm(WomenBusLawIndex~PopGrowth+UrbanPopulation+FertilityRate+log(GDPPerCapita)+NaturalResourceRents+ColdWar,
+         data=WDI,model="pooling")
> summary(OLS)
.
.
.
Coefficients:
                     Estimate Std. Error t-value    Pr(>|t|)
(Intercept)           60.4325     1.6861    35.8     < 2e-16 ***
PopGrowth             -2.3630     0.1306   -18.1     < 2e-16 ***
UrbanPopulation       -0.0587     0.0105    -5.6 0.000000022 ***
FertilityRate         -2.5215     0.1592   -15.8     < 2e-16 ***
log(GDPPerCapita)      2.6533     0.1936    13.7     < 2e-16 ***
NaturalResourceRents  -0.3398     0.0155   -21.9     < 2e-16 ***
ColdWar              -10.9584     0.3715   -29.5     < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Total Sum of Squares:    2910000
Residual Sum of Squares: 1450000
R-Squared:      0.501
Adj. R-Squared: 0.501
F-statistic: 1354.19 on 6 and 8093 DF, p-value: <2e-16
```

```
> pdwtest(OLS)

Durbin-Watson test for serial correlation in panel models

data:  WomenBusLawIndex ~ PopGrowth + UrbanPopulation + FertilityRate +  ...
DW = 0.14, p-value <2e-16
alternative hypothesis: serial correlation in idiosyncratic errors
```

Example: Prais-Winsten

```
> PraisWinsten<-panelAR(WomenBusLawIndex~PopGrowth+UrbanPopulation+
+              FertilityRate+log(GDPPerCapita)+NaturalResourceRents+
+              ColdWar, data=WDI,panelVar="ISO3",timeVar="YearNumeric",
+              autoCorr="ar1",panelCorrMethod="none",
+              rho.na.rm=TRUE)

> summary(PraisWinsten)

Panel Regression with AR(1) Prais-Winsten correction and homoskedastic variance

Unbalanced Panel Design:
 Total obs.:       8100 Avg obs. per panel 43.316
 Number of panels: 187  Max obs. per panel 52
 Number of times:  52   Min obs. per panel 1

Coefficients:
                      Estimate Std. Error t value  Pr(>|t|)
(Intercept)          70.366002   2.977666   23.63   < 2e-16 ***
PopGrowth            -0.038660   0.039708   -0.97   0.33028
UrbanPopulation      -0.000807   0.024026   -0.03   0.97321
FertilityRate        -5.250651   0.233565  -22.48   < 2e-16 ***
log(GDPPerCapita)     1.237712   0.345964    3.58   0.00035 ***
NaturalResourceRents -0.008431   0.007950   -1.06   0.28892
ColdWar              -0.965160   0.216238   -4.46 0.0000082 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

R-squared: 0.3266
Wald statistic: 1094.4684, Pr(>Chisq(6)): 0


> PraisWinsten$panelStructure$rho
[1] 0.9521
```

Better in a Table

	OLS	Prais-Winsten
Intercept	60.43^{*}	70.36^{*}
	(1.69)	(2.98)
Population Growth	-2.36^{*}	-0.04
	(0.13)	(0.04)
Urban Population	-0.06^{*}	-0.0008
	(0.01)	(0.02)
Fertility Rate	-2.52^{*}	-5.25^{*}
	(0.16)	(0.24)
ln(GDP Per Capita)	2.65^{*}	1.24^{*}
	(0.19)	(0.35)
Natural Resource Rents	-0.34^{*}	-0.01
	(0.02)	(0.008)
Cold War	-10.96^{*}	-0.97^{*}
	(0.37)	(0.22)
$\hat{\rho}$		0.95
R^2	0.501	0.33
Adj. R^2	0.501	
NT	8100	8100
N panels		187

$^{*}p < 0.05$

Some Panel Data Challenges

Consider the error terms in the model:

$$Y_{it} = \mathbf{X}_{it}\boldsymbol{\beta} + u_{it}$$

Issues:

In Words:	In a Formula:
Variances:	
Unit-Wise Heteroscedasticity	$\mathrm{Var}(u_{it}) \neq \mathrm{Var}(u_{jt})$
Temporal Heteroscedasticity	$\mathrm{Var}(u_{it}) \neq \mathrm{Var}(u_{is})$
Covariances:	
Contemporary Cross-Unit Correlation	$\mathrm{Cov}(u_{it}, u_{jt}) \neq 0$
Within-Unit Serial Correlation	$\mathrm{Cov}(u_{it}, u_{is}) \neq 0$
Non-Contemporaraneous Cross-Unit Correlation	$\mathrm{Cov}(u_{it}, u_{js}) \neq 0$

Parks' (1967) Approach

Assume:

- $\text{Var}(u_{it}, u_{jt}) = \sigma^2$ or σ_i^2 (Common or unit-specific error variances)
- $\text{Var}(u_{it}) = \text{Var}(u_{is}) \,\forall\, t \neq s$ (Temporal homoscedasticity)
- $\text{Cov}(u_{it}, u_{jt}) = \sigma_{ij} \,\forall\, i \neq j$ (Pairwise contemporaneous cross-unit correlation)
- $\text{Cov}(u_{it}, u_{is}) = \rho$ or ρ_i (Common or unit-specific temporal correlation)
- $\text{Cov}(u_{it}, u_{js}) = 0 \,\forall\, i \neq j, t \neq s$ (No non-contemporaneous cross-unit correlation)

(B&K: "panel error assumptions").

Then:

1. Use OLS to generate $\hat{u}$s $\rightarrow \hat{\rho}$ ($\rightarrow \hat{\mathbf{\Omega}}$),
2. Use $\hat{\rho}$ for Prais-Winsten.

This method was widely used prior to Beck & Katz (1995)

$$\mathbf{\Omega} = \begin{pmatrix} \Sigma & 0 & \cdots & 0 \\ 0 & \Sigma & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Sigma \end{pmatrix} = \Sigma \otimes \mathbf{I}_T$$

where

$$\underset{N\times N}{\Sigma} = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1N} \\ \sigma_{12} & \sigma_2^2 & \cdots & \sigma_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{1N} & \sigma_{2N} & \cdots & \sigma_N^2 \end{pmatrix}$$

Means:

- $\frac{N(N-1)}{2}$ distinct contemporaneous covariances σ_{ij},
- NT observations,
- $\rightarrow 2T/(N+1)$ observations per $\hat{\sigma}$

More Parks Problems

From PROC PANEL in SAS:

Standard Corrections

For the PARKS option, the first-order autocorrelation coefficient must be estimated for each cross section. Let ρ be the $N \times 1$ vector of true parameters and $R = (r_1, \ldots, r_N)'$ be the corresponding vector of estimates. Then, to ensure that only range-preserving estimates are used in PROC PANEL, the following modification for R is made:

$$r_i = \begin{cases} r_i & \text{if} \quad |r_i| < 1 \\ \max(.95, \text{rmax}) & \text{if} \quad r_i \geq 1 \\ \min(-.95, \text{rmin}) & \text{if} \quad r_i \leq -1 \end{cases}$$

where

$$\text{rmax} = \begin{cases} 0 & \text{if} \quad r_i < 0 \quad \text{or} \quad r_i \geq 1 \quad \forall i \\ \max\limits_j [r_j : 0 \leq r_j < 1] & \text{otherwise} \end{cases}$$

and

$$\text{rmin} = \begin{cases} 0 & \text{if} \quad r_i > 0 \quad \text{or} \quad r_i \leq -1 \quad \forall i \\ \max\limits_j [r_j : -1 < r_j \leq 0] & \text{otherwise} \end{cases}$$

Whenever this correction is made, a warning message is printed.

Panel-Corrected Standard Errors

Key to PCSEs:

$$\hat{\sigma}_{ij} = \frac{\sum_{t=1}^{T} \hat{u}_{it}\hat{u}_{jt}}{T}$$

Define:

$$\underset{T \times N}{\mathbf{U}} = \begin{pmatrix} \hat{u}_{11} & \hat{u}_{21} & \cdots & \hat{u}_{N1} \\ \hat{u}_{12} & \hat{u}_{22} & \cdots & \hat{u}_{N2} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{u}_{1T} & \hat{u}_{2T} & \cdots & \hat{u}_{NT} \end{pmatrix}$$

$$\hat{\mathbf{\Sigma}} = \frac{(\mathbf{U}'\mathbf{U})}{T}$$

$$\hat{\mathbf{\Omega}}_{PCSE} = \frac{(\mathbf{U}'\mathbf{U})}{T} \otimes \mathbf{I}_T$$

Panel-Corrected Standard Errors

Correct formula:

$$\text{Cov}(\hat{\beta}_{PCSE}) = (\mathbf{X}'\mathbf{X})^{-1}[\mathbf{X}'\mathbf{\Omega}\mathbf{X}](\mathbf{X}'\mathbf{X})^{-1}$$

General Issues:

- PCSEs do not fix unit-level heterogeneity (a la “fixed” / “random” effects)
- They also do not deal with dynamics
- They depend critically on the “panel data assumptions” of Park / Beck & Katz

Panel Assumptions and Numbers of Parameters

Panel Assumptions	No AR(1)	Common $\hat{\rho}$	Separate $\hat{\rho}_i$s
$\sigma_i^2 = \sigma^2$, $Cov(\sigma_{it}, \sigma_{jt}) = 0$	$k+1$	$k+2$	$k+N+1$
$\sigma_i^2 \neq \sigma^2$, $Cov(\sigma_{it}, \sigma_{jt}) = 0$	$k+N$	$k+N+1$	$k+2N$
$\sigma_i^2 \neq \sigma^2$, $Cov(\sigma_{it}, \sigma_{jt}) \neq 0$	$\frac{N(N-1)}{2}+k+N$	$\frac{N(N-1)}{2}+k+N+1$	$\frac{N(N-1)}{2}+k+2N$

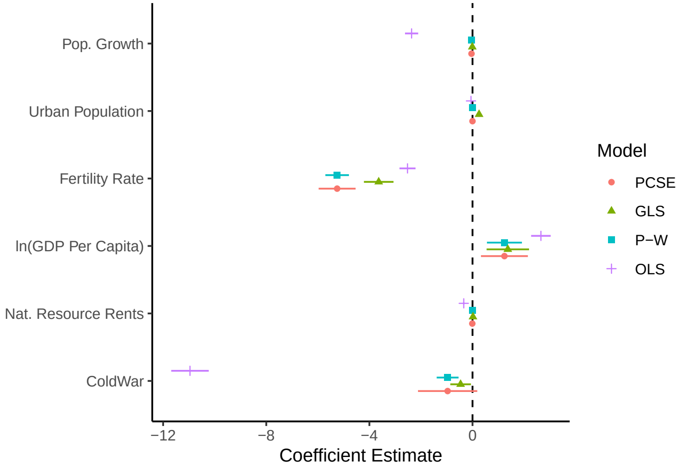
Example: GLS with Homoscedastic AR(1) Errors

```
> GLS<-gls(WomenBusLawIndex~PopGrowth+UrbanPopulation+FertilityRate+log(GDPPerCapita)+NaturalResourceRents+ColdWar,
+          data=WDI,correlation=corAR1(form=~1|ISO3),na.action="na.omit")
>
> summary(GLS)
Generalized least squares fit by REML
  Model: WomenBusLawIndex~PopGrowth+UrbanPopulation+FertilityRate+log(GDPPerCapita)+NaturalResourceRents+ColdWar
  Data: WDI
    AIC   BIC logLik
  38070 38133 -19026

Correlation Structure: AR(1)
 Formula: ~1 | ISO3
 Parameter estimate(s):
   Phi
0.9897

Coefficients:
                     Value Std.Error t-value p-value
(Intercept)          50.72     3.986  12.724  0.0000
PopGrowth            -0.01     0.037  -0.228  0.8194
UrbanPopulation       0.25     0.041   6.245  0.0000
FertilityRate        -3.64     0.293 -12.422  0.0000
log(GDPPerCapita)     1.37     0.420   3.262  0.0011
NaturalResourceRents  0.02     0.007   2.500  0.0124
ColdWar              -0.46     0.204  -2.261  0.0238

.
.
.

Residual standard error: 16.92
Degrees of freedom: 8100 total; 8093 residual
```

Example: PCSEs

```
> PCSE<-panelAR(WomenBusLawIndex~PopGrowth+UrbanPopulation+FertilityRate+log(GDPPerCapita)+NaturalResourceRents+ColdWar,
+             data=WDI,panelVar="ISO3",timeVar="YearNumeric",autoCorr="ar1",panelCorrMethod="pcse",rho.na.rm=TRUE)

> summary(PCSE)

Panel Regression with AR(1) Prais-Winsten correction and panel-corrected standard errors

Unbalanced Panel Design:
 Total obs.:       8100 Avg obs. per panel 43.316
 Number of panels: 187  Max obs. per panel 52
 Number of times:  52   Min obs. per panel 1

Coefficients:
                     Estimate Std. Error t value Pr(>|t|)
(Intercept)         70.366002   4.482539   15.70   <2e-16 ***
PopGrowth           -0.038660   0.044896   -0.86   0.3892
UrbanPopulation     -0.000807   0.025989   -0.03   0.9752
FertilityRate       -5.250651   0.365619  -14.36   <2e-16 ***
log(GDPPerCapita)    1.237712   0.464299    2.67   0.0077 **
NaturalResourceRents -0.008431   0.012149   -0.69   0.4877
ColdWar             -0.965160   0.586562   -1.65   0.0999 .
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

R-squared: 0.3266
Wald statistic: 400.9446, Pr(>Chisq(6)): 0


> PCSE$panelStructure$rho
[1] 0.9521
```

Model Comparisons



Dynamics!

Stationarity: A constant d.g.p. over time.[1]

Mean stationarity:

$$E(Y_t) = \mu \ \forall \ t$$

Variance stationarity:

$$Var(Y_t) = E[(Y_t - \mu)^2] \equiv \sigma_Y^2 \ \forall \ t$$

Covariance stationarity:

$$Cov(Y_t, Y_{t-s}) = E[(Y_t - \mu)(Y_{t-s} - \mu)] = \gamma_s \ \forall \ s$$

[1] A stricter form of stationarity requires that the joint probability distribution (in other words, *all* the moments) of series of observations $\{Y_1, Y_2, \ldots Y_t\}$ is the same as that for $\{Y_{1+s}, Y_{2+s}, \ldots Y_{t+s}\}$ for all t and s.

"ARIMA" $=$ *Autoregressive Integrated Moving Average*...

A (first-order) integrated series ("random walk") is:

$$Y_t = Y_{t-1} + u_t,\ u_t \sim i.i.d.(0, \sigma_u^2)$$

...a/k/a a "random walk":

$$\begin{aligned} Y_t &= Y_{t-2} + u_{t-1} + u_t \\ &= Y_{t-3} + u_{t-2} + u_{t-1} + u_t \\ &= \sum_{t=0}^{T} u_t \end{aligned}$$

I(1) series are not stationary.

Variance:

$$\mathrm{Var}(Y_t) \equiv E(Y_t)^2 \quad = \quad t\sigma^2$$

Autocovariance:

$$\mathrm{Cov}(Y_t, Y_{t-s}) = |t-s|\sigma^2.$$

Both depend on t...

I(1) series (continued)

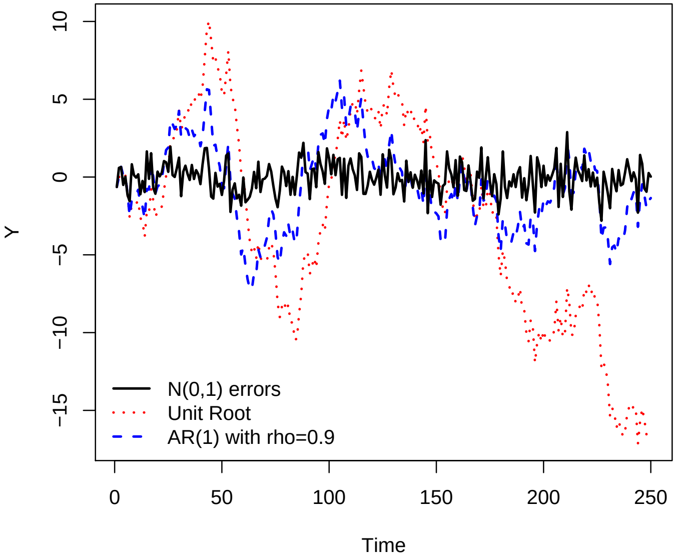
More generally:

- $|\rho| > 1$
  - Series is nonstationary / *explosive*
  - Past shocks have a greater impact than current ones
  - Uncommon

- $|\rho| < 1$
  - *Stationary* series
  - Effects of shocks die out exponentially according to ρ
  - Is mean-reverting

- $|\rho| = 1$
  - Nonstationary series
  - Shocks persist at full force
  - Not mean-reverting; variance increases with t

Time Series Types, Illustrated



I(1) Series: Differencing

For an I(1) series:

$$Y_t - Y_{t-1} = u_t$$

which we often write in terms of the difference operator Δ (or sometimes ∇):

$$\Delta Y_t = u_t$$

The differenced series is just the (stationary, ergodic) white-noise process u_t.

Unit Root Tests Review: Dickey-Fuller

Two steps:

- Estimate $Y_t = \rho Y_{t-1} + u_t$,
- test the hypothesis that $\hat{\rho} = 1$, *but*
- this requires that the *u*s are uncorrelated.

But suppose:

$$\Delta Y_t = \sum_{i=1}^{p} d_i \Delta Y_{t-i} + u_t$$

which yields

$$Y_t = Y_{t-1} + \sum_{i=1}^{p} d_i \Delta Y_{t-i} + u_t.$$

D.F. tests will be incorrect.

Unit Root Alternatives

Augmented Dickey-Fuller Tests:

- Estimate

$$\Delta Y_t = \rho Y_{t-1} + \sum_{i=1}^{p} d_i \Delta Y_{t-i} + u_t$$

- Test $\hat{\rho} = 1$

Phillips-Perron Tests:

- Estimate:

$$\Delta Y_t = \alpha + \rho Y_{t-1} + u_t$$

- Calculate modified test statistics (Z_ρ and Z_t)
- Test $\hat{\rho} = 0$

Issues with Unit Roots in Panel Data

- Short series + Asymptotic tests $\rightarrow$ "borrow strength"
- Requires uniform unit roots across cross-sectional units
- Many tests require balanced panels...
- Various alternatives:
    - Maddala and Wu (1999)
    - Hadri (2000)
    - Levin, Lin and Chu (2002)
    - Im, Pesaran, and Shin (2003)
- What to do?
    - Difference the data...
    - Error-correction models

Panel Unit Root Tests: R

```
[data wrangling...]

> purtest(WBLI.W,exo="trend",test="levinlin",pmax=2)

Levin-Lin-Chu Unit-Root Test (ex. var.: Individual Intercepts and Trend)

data:  WBLI.W
z = -2.7, p-value = 0.004
alternative hypothesis: stationarity

> purtest(WBLI.W,exo="trend",test="hadri",pmax=2)

Hadri Test (ex. var.: Individual Intercepts and Trend) (Heterosked. Consistent)

data:  WBLI.W
z = 197, p-value <2e-16
alternative hypothesis: at least one series has a unit root

> purtest(WBLI.W,exo="trend",test="madwu",pmax=2)

Maddala-Wu Unit-Root Test (ex. var.: Individual Intercepts and Trend)

data:  WBLI.W
chisq = 364, df = 376, p-value = 0.7
alternative hypothesis: stationarity

> purtest(WBLI.W,exo="trend",test="ips",pmax=2)

Im-Pesaran-Shin Unit-Root Test (ex. var.: Individual Intercepts and Trend)

data:  WBLI.W
Wtbar = 2.8, p-value = 1
alternative hypothesis: stationarity
```

A Better Table

	Test	Alternative	Statistic	Estimate	P-Value
1	Levin-Lin-Chu Unit-Root Test	stationarity	z	-2.672	0.0038
2	Hadri Test	at least one series has a unit root	z	196.75	$<$0.0001
3	Maddala-Wu Unit-Root Test	stationarity	χ^2	363.89	0.6635
4	Im-Pesaran-Shin Unit-Root Test	stationarity	$\bar{W}_t$	2.77	0.9972

Note: All assume individual intercepts and trends.

Lagged Y?

"Lagged dependent variable":

$$Y_{it} = \phi Y_{it-1} + \mathbf{X}_{it}\boldsymbol{\beta}_{LDV} + \epsilon_{it}$$

If ϵ_{it} is perfect, then:

- $\hat{\beta}_{LDV}$ is biased (but consistent),
- O(bias) $= \frac{-1+3\boldsymbol{\beta}_{LDV}}{T}$

If ϵ_{it} is autocorrelated...

- $\hat{\beta}_{LDV}$ is biased and <u>inconsistent</u>
- IV is one (bad) option...

Lagged Ys and GLS-ARMA

Can rewrite:

$$\begin{aligned} Y_{it} &= \mathbf{X}_{it}\boldsymbol{\beta}_{AR} + u_{it} \\ u_{it} &= \phi u_{it-1} + \eta_{it} \end{aligned}$$

as

$$\begin{aligned} Y_{it} &= \mathbf{X}_{it}\boldsymbol{\beta}_{AR} + \phi u_{it-1} + \eta_{it} \\ &= \mathbf{X}_{it}\boldsymbol{\beta}_{AR} + \phi(Y_{it-1} - \mathbf{X}_{it-1}\boldsymbol{\beta}_{AR}) + \eta_{it} \\ &= \phi Y_{it-1} + \mathbf{X}_{it}\boldsymbol{\beta}_{AR} + \mathbf{X}_{it-1}\psi + \eta_{it} \end{aligned}$$

where $\psi = \phi\beta_{AR}$ and $\psi = 0$ (by assumption).

Lagged Ys and World Domination

In:

$$Y_{it} = \phi Y_{it-1} + \mathbf{X}_{it}\boldsymbol{\beta}_{LDV} + \epsilon_{it}$$

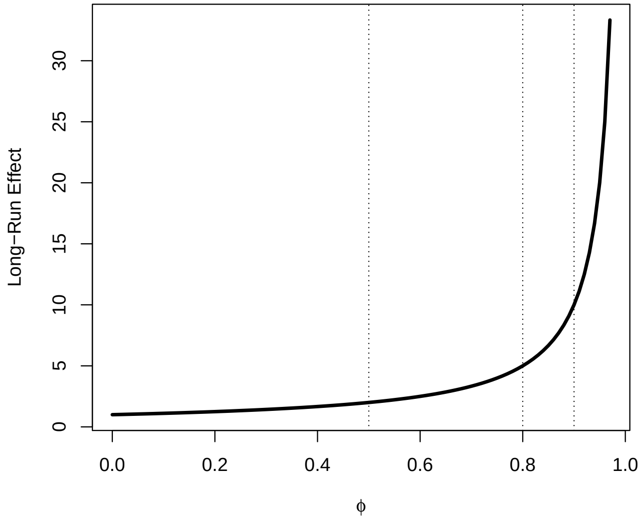
Achen: Bias "deflates" $\hat{\boldsymbol{\beta}}_{LDV}$ relative to $\hat{\phi}$, "suppress" the effects of **X**...

Keele & Kelly (2006):

- Contingent on ϵs having autocorrelation
- Key: In LDV, *long-run impact of a unit change in X is*:

$$\hat{\beta}_{LR} = \frac{\hat{\beta}_{LDV}}{1 - \hat{\phi}}$$

Long-Run Impact for $\hat{\beta} = 1$



Lagged Ys and Unit Effects

Consider:

$$Y_{it} = \phi Y_{it-1} + \mathbf{X}_{it}\boldsymbol{\beta} + \alpha_i + u_{it}.$$

If we omit the unit effects, we have:

$$Y_{it} = \phi Y_{it-1} + \mathbf{X}_{it}\boldsymbol{\beta} + u^*_{it}$$

with

$$u^*_{it} = \alpha_i + u_{it}$$

Lagging yields:

$$Y_{it-1} = \phi Y_{it-2} + \mathbf{X}_{it-1}\boldsymbol{\beta} + \alpha_i + u_{it-1}$$

which means

$$\text{Cov}(Y_{it-1}, u^*_{it}) \neq 0. \rightarrow \text{ bias in } \hat{\phi}, \hat{\boldsymbol{\beta}}$$

“Nickell” Bias

Bias in $\hat{\phi}$ is

- toward zero when $\phi > 0$,
- increasing in ϕ.

Including unit effects still yields bias in $\hat{\phi}$ of $O(\frac{1}{T})$, and bias in $\hat{\boldsymbol{\beta}}$.

Solutions:

- Difference/GMM estimation
- Bias correction approaches

$$
\begin{aligned}
Y_{it} - Y_{it-1} &= \phi(Y_{it-1} - Y_{it-2}) + (\mathbf{X}_{it} - \mathbf{X}_{it-1})\boldsymbol{\beta} + (\alpha_i - \alpha_i) + (u_{it} - u_{it-1}) \\
\Delta Y_{it} &= \phi \Delta Y_{it-1} + \Delta \mathbf{X}_{it}\boldsymbol{\beta} + \Delta u_{it}
\end{aligned}
$$

Anderson/Hsiao: If $\nexists$ autocorrelation, then use ΔY_{it-2} or Y_{it-2} as instruments for ΔY_{it-1}...

- Consistent in theory,
- in practice, the former is preferred, and both have issues if ϕ is high;
- both are inefficient.

Arellano & Bond (also Wawro): Use *all* lags of Y_{it} and $\mathbf{X}_{it}$ from $t-2$ and before.

- "Good" estimates, better as $T \rightarrow \infty$,
- Easy to handle higher-order lags of Y,
- Easy software (`plm` in R , `xtabond` in Stata).
- Model *is* fixed effects...
- $\mathbf{Z}_i$ has $T-p-1$ rows, $\sum_{i=p}^{T-2} i$ columns $\rightarrow$ difficulty of estimation declines in p, grows in T.

Kiviet (1995, 1999; Bun and Kiviet 2003; Bruno 2005a,b): Derive the bias in $\hat{\phi}$ and $\hat{\beta}$, then correct it...

- More accurate than the instrumental-variables/GMM estimators of A&H/A&B...
- ...especially when T is small; but not as T gets reasonably large ($T \approx 20$)

	Lagged Y	First Difference	FE	Lagged Y + FE
Intercept	2.139*	0.629*		
	(0.319)	(0.039)		
Lagged WBLI	0.986*			0.953*
	(0.002)			(0.003)
Population Growth	−0.061*	0.003	−0.089	−0.064*
	(0.023)	(0.036)	(0.095)	(0.028)
Urban Population	0.003	−0.019	0.306*	0.013*
	(0.002)	(0.064)	(0.020)	(0.006)
Fertility Rate	−0.073*	−0.833*	−2.033*	−0.244*
	(0.028)	(0.335)	(0.162)	(0.049)
ln(GDP Per Capita)	−0.034	0.502	8.723*	0.148
	(0.035)	(0.445)	(0.300)	(0.095)
Natural Resource Rents	−0.009*	0.023*	0.065*	−0.003
	(0.003)	(0.007)	(0.017)	(0.005)
Cold War	−0.295*	−0.062	−6.869*	−0.408*
	(0.069)	(0.201)	(0.296)	(0.090)
R^2	0.985	0.002	0.532	0.958
Adj. R^2	0.985	0.002	0.521	0.957
Num. obs.	7996	7913	8100	7996

$^*p < 0.05$

Anderson-Hsiao, Arellano-Bond, etc.

In R:

- Anderson-Hsiao can be fit using lm or (more easily) `plm` in the `plm` package
- Arellano-Bond is most easily fit using `pgmm` ("panel gmm") in the `plm` package
- See Criossant and Millo (2018, Chapter 7) for statistics + code details
- This post is also useful...

Stata:

- `xtabond` / `xtdpdsys` / `xtdpd` fit both A-H and A-B / Blundell-Bond models (among others)
- This is also a good (slightly dated) reference

Trends!

What if Y is *trending* over time?

- First Question: Why?
  - Organic growth (e.g., populations)
  - Temporary / short-term factors
  - Covariates...
- Second question: Should we care?
  (A: Yes, usually... $\rightarrow$ "spurious regressions")
- Third question: What to do?
  - Ignore it...
  - Include a counter / trend term...

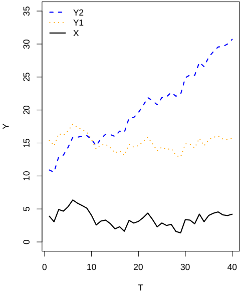
In general, adding a trend term will *decrease* the magnitudes of $\hat{\beta}$...

Trends Matter, Illustrated

Data generating processes:

$$Y_{1t} = 10 + (1 \times X_t) + u_t$$

$$Y_{2t} = 5 + (1 \times X_t) + (0.5 \times T) + u_t$$



	Y_1	Y_2	
		No Trend	Trend
X	0.921***	−0.382	0.874***
	(0.245)	(0.786)	(0.255)
T			0.482***
			(0.026)
Constant	10.300***	20.200***	5.860***
	(0.917)	(2.950)	(1.200)
Observations	40	40	40
R^2	0.272	0.006	0.905
Adjusted R^2	0.253	−0.020	0.900
Residual Std. Error	1.800 (df = 38)	5.790 (df = 38)	1.810 (df = 37)
Note:			*p<0.1; **p<0.05; ***p<0.01

Trends Matter, Part II

Table: FE Models of WBLI

	FE	FE.trend	FE.intx
Population Growth	-0.089	-0.266^{***}	-0.227^{***}
	(0.095)	(0.080)	(0.079)
Urban Population	0.306^{***}	0.019	0.037^{**}
	(0.020)	(0.017)	(0.017)
Fertility Rate	-2.033^{***}	1.276^{***}	1.206^{***}
	(0.162)	(0.146)	(0.146)
ln(GDP Per Capita)	8.723^{***}	2.069^{***}	1.855^{***}
	(0.300)	(0.274)	(0.274)
Natural Resource Rents	0.065^{***}	0.034^{**}	0.037^{***}
	(0.017)	(0.014)	(0.014)
Cold War	-6.869^{***}	1.760^{***}	8.688^{***}
	(0.296)	(0.286)	(0.927)
Trend (1950=0)		0.745^{***}	0.770^{***}
		(0.013)	(0.013)
Cold War x Trend			-0.202^{***}
			(0.026)
Observations	8,100	8,100	8,100
R^2	0.532	0.676	0.678
Adjusted R^2	0.521	0.668	0.670
F Statistic	$1,501.000^{***}$ (df = 6; 7907)	$2,354.000^{***}$ (df = 7; 7906)	$2,083.000^{***}$ (df = 8; 7905)

$^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Another Approach: Orthogonalization

Note: We're rarely substantively interested in the fixed effects $\hat{\alpha}$...

- $\rightarrow$ reparameterize the αs so that they are *information-orthogonal* to the other parameters in the model (including the βs and ϕ)
- Key idea: Transform the αs so that (for example):

$$\mathrm{E}\left(\frac{\partial^2 L_i}{\partial\alpha\partial\beta}\right) = 0$$

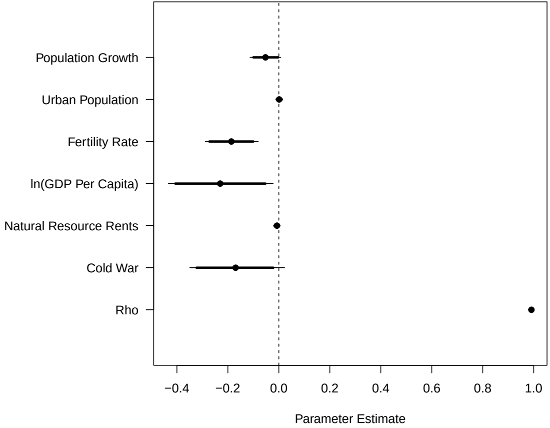
- Can do this via imposition of priors, in a Bayesian framework...
- **In general**, this approach is less assumption-laden and more efficient than the IV/GMM-based approaches discussed above.
- Provides consistent-in-N estimates for T as low as 2...

References:

- Lancaster, T. 2002. "Orthogonal Parameters and Panel Data." *Review of Economic Studies* 69:647-666.
- Pickup et al. (2017) [the "orthogonalized panel model" ("OPM")]

FE + Dynamics Using Orthogonalization

```
> library(OrthoPanels)
> set.seed(7222009)
> OPM.fit <- opm(WomenBusLawIndex~PopGrowth+UrbanPopulation+FertilityRate+
              lnGDPPerCap+NaturalResourceRents+ColdWar,data=WDI,
              index=c("ISO3","Year"),n.samp=1000)
```



OPM Results: Short- and Long-Run Effects

For $\hat{\phi} \approx 0.98$:

Parameter	Short-Run	Long-Run
Population Growth	-0.0526	-5.5623
Urban Population	0.0012	0.1468
Fertility Rate	-0.1864	-20.1788
ln(GDP Per Capita)	-0.2303	-25.5141
Natural Resource Rents	-0.0075	-0.8048
Cold War	-0.1697	-18.3407

R :

- the `plm` package (`purtest` for unit roots; `plm` for first-difference models; `pgmm` for Arellano-Bond)
- the `panelAR` package (GLS-ARMA models)
- the `gls` package (GLS)
- the `dynpanel` package (A&H, A&B; minimal...)

Stata :

- `xtgls` (GLS)
- `xtpcse` (PCSEs)
- `xtabond` / `xtdpd` (A&H A&B dynamic models)

Things to consider:

- N vs. T...
- Are dynamics nuisance or substance?
- What problem(s) do you *really* care about?