GSERM - St. Gallen 2024
Analyzing Panel Data

June 12, 2024

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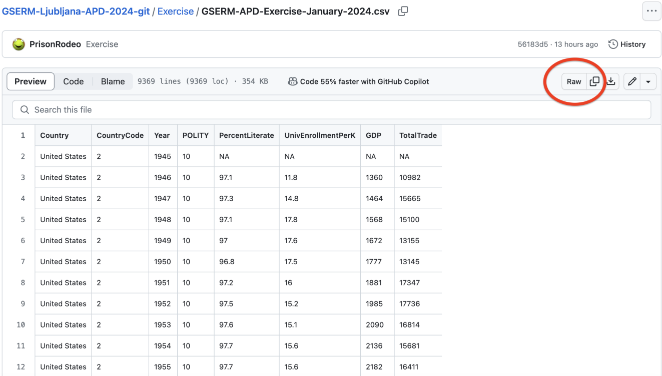


	Country	CountryCode	Year	POLITY	PercentLiterate	UnivEnrollmentPerK	GDP	TotalTrade
2	United States	2	1945	10	NA	NA	NA	NA
3	United States	2	1946	10	97.1	11.8	1360	10982
4	United States	2	1947	10	97.3	14.8	1464	15665
5	United States	2	1948	10	97.1	17.8	1568	15100
6	United States	2	1949	10	97	17.6	1672	13155
7	United States	2	1950	10	96.8	17.5	1777	13145
8	United States	2	1951	10	97.2	16	1881	17347
9	United States	2	1952	10	97.5	15.2	1985	17736
10	United States	2	1953	10	97.6	15.1	2090	16814
11	United States	2	1954	10	97.7	15.6	2136	15681
12	United States	2	1955	10	97.7	15.6	2182	16411

Can also use (e.g.) `read_csv` (in `readr`):

```
> install.packages("readr")
> library(readr)
> Data<-read_csv("https://github.com/PrisonRodeo/GSERM-Ljubljana-APD-2024-git/raw/main/
                Exercises/GSERM-Panel-Exercise-June-2024.csv")
```

Start with a focus on residuals... For:

$$Y_{it} = \mathbf{X}_{it}\beta + u_{it}$$

i.i.d. OLS u_{it}s require:

$$\begin{aligned}
\mathbf{uu}' \equiv \mathbf{\Omega} &= \sigma^2\mathbf{I} \\
&= \begin{pmatrix} \sigma^2 & 0 & \cdots & 0 \\ 0 & \sigma^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma^2 \end{pmatrix}
\end{aligned}$$

This means that within units:

- $\text{Var}(u_{it}) = \text{Var}(u_{is})\ \forall\ t \neq s$ (temporal homoscedasticity)
- $\text{Cov}(u_{it}, u_{is}) = 0\ \forall\ t \neq s$ (no within-unit autocorrelation)

and between units:

- $\text{Var}(u_{it}) = \text{Var}(u_{jt})\ \forall\ i \neq j$ (cross-unit homoscedasticity)
- $\text{Cov}(u_{it}, u_{jt}) = 0\ \forall\ i \neq j$ (no between-unit / spatial correlation)

The Key: $\mathbf{\Omega}$

Estimator:

$$\hat{\beta}_{GLS} = (\mathbf{X}'\mathbf{\Omega}^{-1}\mathbf{X})^{-1}\mathbf{X}'\mathbf{\Omega}^{-1}\mathbf{Y}$$

with:

$$\widehat{V(\beta_{GLS})} = (\mathbf{X}'\mathbf{\Omega}^{-1}\mathbf{X})^{-1}$$

Two approaches:

- Use OLS $\hat{u}_{it}$s to get $\hat{\mathbf{\Omega}}$ ("feasible GLS" / " weighted least squares")
- Use <u>substantive knowledge about the data</u> to structure $\mathbf{\Omega}$

The variance-covariance matrix is:

$$\begin{aligned} \mathrm{Var}(\hat{\beta}_{WLS}) &= \sigma^2(\mathbf{X}'\mathbf{W}^{-1}\mathbf{X})^{-1} \\ &\equiv (\mathbf{X}'\boldsymbol{\Omega}^{-1}\mathbf{X})^{-1} \end{aligned}$$

A common case is:

$$\mathrm{Var}(u_i) = \sigma^2 \frac{1}{N_i}$$

where N_i is the number of observations upon which (aggregate) observation i is based.

Recall that, if $\sigma_i^2 \neq \sigma_j^2 \; \forall \; i \neq j$,

$$\begin{aligned} \text{Var}(\beta_{\text{Het.}}) &= (\mathbf{X}'\mathbf{X})^{-1}(\mathbf{X}'\boldsymbol{\Omega}^{-1}\mathbf{X})(\mathbf{X}'\mathbf{X})^{-1} \\ &= (\mathbf{X}'\mathbf{X})^{-1}\,\mathbf{Q}\,(\mathbf{X}'\mathbf{X})^{-1} \end{aligned}$$

where $\mathbf{Q} = (\mathbf{X}'\boldsymbol{\Omega}^{-1}\mathbf{X})$ and $\boldsymbol{\Omega} = \sigma^2\mathbf{W}$.

We can rewrite $\mathbf{Q}$ as

$$\begin{aligned} \mathbf{Q} &= \sigma^2(\mathbf{X}'\mathbf{W}^{-1}\mathbf{X}) \\ &= \sum_{i=1}^{N} \sigma_i^2 \mathbf{X}_i \mathbf{X}_i' \end{aligned}$$

Estimate $\hat{\mathbf{Q}}$ as:

$$\widehat{\mathbf{Q}} = \sum_{i=1}^{N} \hat{u}_i^2 \mathbf{X}_i \mathbf{X}_i'$$

Yields:

$$\begin{aligned}
\widehat{\text{Var}(\beta)}_{\text{Robust}} &= (\mathbf{X}'\mathbf{X})^{-1}(\mathbf{X}'\widehat{\mathbf{Q}}^{-1}\mathbf{X})(\mathbf{X}'\mathbf{X})^{-1} \\
&= (\mathbf{X}'\mathbf{X})^{-1}\left[\mathbf{X}'\left(\sum_{i=1}^{N} \hat{u}_i^2 \mathbf{X}_i \mathbf{X}_i'\right)^{-1}\mathbf{X}\right](\mathbf{X}'\mathbf{X})^{-1}
\end{aligned}$$

"Robust" VCV estimates:

- are heteroscedasticity-consistent, but
- are biased in small samples, and
- are less efficient than "naive" estimates when $\text{Var}(u) = \sigma^2\mathbf{I}$.

“Clustering”

Huber / White ????????? WLS / GLS

I know very little about my error variances...

I know a great deal about my error variances...

“Clustering”

A common case:

$$Y_{ij} = \mathbf{X}_{ij}\boldsymbol{\beta} + u_{ij}$$

with

$$\sigma_{ij}^2 = \sigma_{ik}^2.$$

“Robust, clustered” estimator:

$$\widehat{\text{Var}(\boldsymbol{\beta})}_{\text{Clustered}} = (\mathbf{X}'\mathbf{X})^{-1}\left\{\mathbf{X}'\left[\sum_{i=1}^{N}\left(\sum_{j=1}^{n_j}\hat{u}_{ij}^2\mathbf{X}_{ij}\mathbf{X}_{ij}'\right)\right]^{-1}\mathbf{X}\right\}(\mathbf{X}'\mathbf{X})^{-1}$$

Robust / Clustered SEs: A Simulation

```
url_robust <- "https://raw.githubusercontent.com/IsidoreBeautrelet/economictheoryblog/master/robust_summary.R"
eval(parse(text = getURL(url_robust, ssl.verifypeer = FALSE)),
     envir=.GlobalEnv)

> set.seed(3844469)
> X <- rnorm(10)
> Y <- 1 + X + rnorm(10)
> df10 <- data.frame(ID=seq(1:10),X=X,Y=Y)
> fit10 <- lm(Y~X,data=df10)
> summary(fit10)

Call:
lm(formula = Y ~ X, data = df10)

Residuals:
   Min     1Q Median     3Q    Max
-1.318 -0.766  0.195  0.378  1.590

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)    0.954      0.311    3.06    0.016 *
X              0.589      0.291    2.03    0.077 .
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.985 on 8 degrees of freedom
Multiple R-squared:  0.339, Adjusted R-squared:  0.257
F-statistic: 4.11 on 1 and 8 DF,  p-value: 0.0772

> rob10 <- vcovHC(fit10,type="HC1")
> sqrt(diag(rob10))
(Intercept)           X
      0.315       0.285
```

Robust / Clustered SEs: A Simulation (continued)

```
> # "Clone" each observation 100 times:
>
> df1K <- df10[rep(seq_len(nrow(df10)),each=100),]
> df1K <- pdata.frame(df1K, index="ID")
> fit1K <- lm(Y~X,data=df1K)

> summary(fit1K)

Call:
lm(formula = Y ~ X, data = df1K)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   0.9536     0.0279    34.2   <2e-16 ***
X             0.5893     0.0260    22.6   <2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.882 on 998 degrees of freedom
Multiple R-squared:  0.339, Adjusted R-squared:  0.339
F-statistic:  513 on 1 and 998 DF,  p-value: <2e-16


> summary(fit1K, cluster="ID")

Call:
lm(formula = Y ~ X, data = df1K)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)    0.954      0.297    3.21   0.0014 **
X              0.589      0.269    2.19   0.0286 *
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.882 on 998 degrees of freedom
Multiple R-squared:  0.339, Adjusted R-squared:  0.339
F-statistic:  4.8 on 1 and 9 DF,  p-value: 0.0561
```

Example:

$$\begin{aligned} Y_t &= \beta_0 + \beta_1 X_t + u_t \\ u_t &= \rho u_{t-1} + e_t \end{aligned}$$

with $e_t \sim i.i.d.\ N(0, \sigma_u^2)$ and $\rho \ \in [-1, 1]$ (typically).

$\rightarrow$ "First-order autoregressive" ("AR(1)") errors.

Detection

- *Plot* of residuals vs. lagged residuals
- *Runs* test (Geary test)
- Durbin-Watson d
  - Calculated as:

$$d = \frac{\sum_{t=2}^{N} (\hat{u}_t - \hat{u}_{t-1})^2}{\sum_{t=1}^{N} \hat{u}_t^2}$$

  - Non-standard distribution ($d \in [0, 4]$)
  - Null: No autocorrelation
  - Only detects first-order autocorrelation

Serially Correlated Errors and OLS

What to do about it?

- GLS, incorporating ρ / $\hat{\rho}$ into the equation
- *First-difference* models (regressing changes of Y on changes of $\mathbf{X}$)
- Cochrane-Orcutt / Prais-Winsten:
  1. Estimate the basic equation via OLS, and obtain residuals
  2. Use the residuals to consistently estimate $\hat{\rho}$ (i.e. the empirical correlation between u_t and u_{t-1})
  3. Use this estimate of $\hat{\rho}$ to estimate the *difference equation*:

$$(Y_t - \rho Y_{t-1}) = \beta_0(1-\rho) + \beta_1(X_t - \rho X_{t-1}) + (u_t - \rho u_{t-1})$$

  4. Save the residuals, and use them to estimate $\hat{\rho}$ again
  5. Repeat this process until successive estimates of $\hat{\rho}$ differ by a very small amount

Running Example Redux

The World Development Indicators:

- Cross-national country-level time series data
- $N = 215$ countries, $T = 64$ years (1960-2023) + missingness
- Full descriptions are listed in the Github repo here

Regression model:

$$\begin{aligned} \text{WBLI}_{it} &= \beta_0 + \beta_1 \text{Population Growth}_{it} + \beta_2 \text{Urban Population}^2_{it} + \beta_3 \text{Fertility Rate}_{it} + \\ &\quad \beta_4 \ln(\text{GDP Per Capita})_{it} + \beta_5 \text{Natural Resource Rents}_{it} + \beta_6 \text{Post-Cold War}_t + u_{it} \end{aligned}$$

Descriptive Statistics:

	vars	n	mean	sd	median	min	max	range	skew
WomenBusLawIndex	1	8127	60.64	18.98	60.62	17.50	100.00	82.50	-0.03
PopGrowth	2	8127	1.65	1.54	1.65	-16.88	19.36	36.24	1.18
UrbanPopulation	3	8127	51.64	23.82	51.50	2.85	100.00	97.16	0.07
FertilityRate	4	8127	3.61	1.90	3.07	0.77	8.61	7.83	0.52
NaturalResourceRents	5	8127	7.03	10.76	2.45	0.00	88.59	88.59	2.52
PostColdWar	6	8127	0.70	0.46	1.00	0.00	1.00	1.00	-0.86
lnGDPPerCap	7	8127	8.30	1.44	8.20	4.92	11.68	6.76	0.14

A Digression: Rescaling Covariates

A la Gelman (2008) (and an updated blog post here):

- Continuous = divide by one standard deviation
- Binary = recode to $\{-1, 1\}$

Doing this yields:

	vars	n	mean	sd	median	min	max	range	skew
WomenBusLawIndex	1	8127	60.64	18.98	60.62	17.50	100.00	82.50	-0.03
PopGrowth	2	8127	1.07	1.00	1.07	-10.94	12.55	23.49	1.18
UrbanPopulation	3	8127	2.17	1.00	2.16	0.12	4.20	4.08	0.07
FertilityRate	4	8127	1.90	1.00	1.62	0.41	4.54	4.13	0.52
NaturalResourceRents	5	8127	0.65	1.00	0.23	0.00	8.23	8.23	2.52
PostColdWar	6	8127	0.40	0.92	1.00	-1.00	1.00	2.00	-0.86
lnGDPPerCap	7	8127	5.78	1.00	5.71	3.43	8.13	4.70	0.14

How Much Autocorrelation in Y?

Note that:

$$d = 2(1 - \rho)$$

which means that we can calculate:

$$\rho = 1 - \frac{d}{2}.$$

So:

```
> WI<-pdwtest(WomenBusLawIndex~1,data=smol)
> WI

 Durbin-Watson test for serial correlation in panel models

data:  WomenBusLawIndex ~ 1
DW = 0.099, p-value <2e-16
alternative hypothesis: serial correlation in idiosyncratic errors

> print(paste("Rho =",round(1 - (WI$statistic/2),3)))
[1] "Rho = 0.951"
```

How Much Autocorrelation in $\mathbf{X}$?

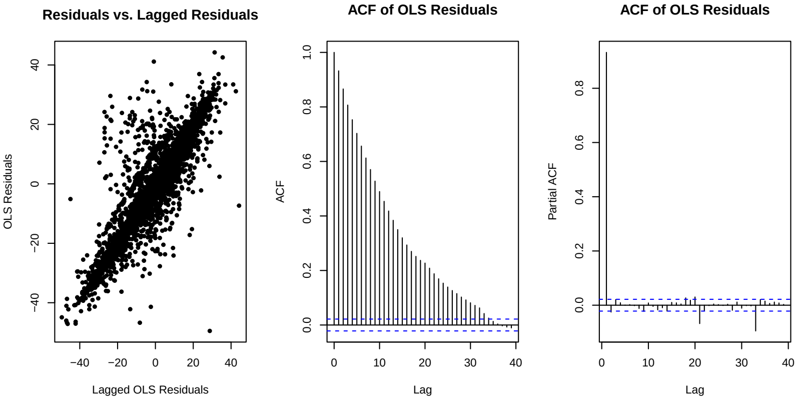
Table: WDI Data - Autocorrelation in the Predictors

Variable	Rho
Population Growth	0.851
Urban Population	0.974
Fertility Rate	0.966
GDP Per Capita	0.977
Natural Resource Rents	0.911
Post Cold War	0.916

Baseline Model: OLS (+ D-W Test)

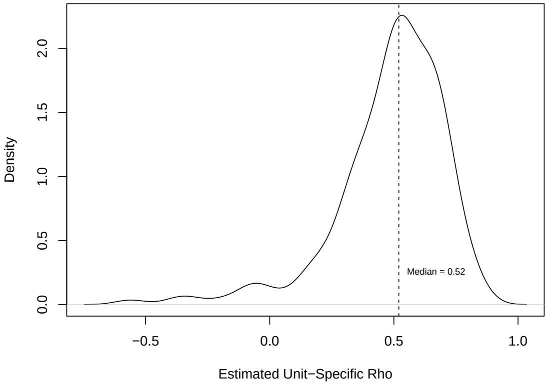
```
> OLS<-plm(WomenBusLawIndex~PopGrowth+UrbanPopulation+FertilityRate+lnGDPPerCap+NaturalResourceRents+
+             PostColdWar,data=smol,model="pooling")

> summary(OLS)
Pooling Model

Call:
plm(formula = WomenBusLawIndex ~ PopGrowth + UrbanPopulation +
    FertilityRate + lnGDPPerCap + NaturalResourceRents + PostColdWar,
    data = smol, model = "pooling")

Unbalanced Panel: n = 187, T = 1-52, N = 8127

Coefficients:
                     Estimate Std. Error t-value    Pr(>|t|)
(Intercept)            55.024      1.755   31.36     < 2e-16 ***
PopGrowth              -3.601      0.202  -17.84     < 2e-16 ***
UrbanPopulation        -1.348      0.251   -5.38 0.000000076 ***
FertilityRate          -4.854      0.301  -16.10     < 2e-16 ***
lnGDPPerCap             3.793      0.279   13.58     < 2e-16 ***
NaturalResourceRents   -3.691      0.167  -22.10     < 2e-16 ***
PostColdWar             5.403      0.185   29.15     < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Total Sum of Squares:    2930000
Residual Sum of Squares: 1470000
R-Squared:      0.499
Adj. R-Squared: 0.499
F-statistic: 1348.85 on 6 and 8120 DF, p-value: <2e-16


> pdwtest(OLS)

 Durbin-Watson test for serial correlation in panel models

data:  WomenBusLawIndex ~ PopGrowth + UrbanPopulation + FertilityRate +  ...
DW = 0.13, p-value <2e-16
alternative hypothesis: serial correlation in idiosyncratic errors
```

Residual Autocorrelation?



Unit-Specific Autocorrelation...

Fit $N = 187$ country-specific regressions, and examine the $\hat{\rho}_i$s...



Another Model: Prais-Winsten

```
> PraisWinsten<-panelAR(WomenBusLawIndex~PopGrowth+UrbanPopulation+FertilityRate+lnGDPPerCap+NaturalResourceRents+
+               PostColdWar, data=smol,panelVar="ISO3",timeVar="Year",autoCorr="ar1",panelCorrMethod="none",
+               rho.na.rm=TRUE)

> summary(PraisWinsten)

Panel Regression with AR(1) Prais-Winsten correction and homoskedastic variance

Unbalanced Panel Design:
 Total obs.:       8127 Avg obs. per panel 43.46
 Number of panels: 187  Max obs. per panel 52
 Number of times:  52   Min obs. per panel 1

Coefficients:
                     Estimate Std. Error t value Pr(>|t|)
(Intercept)           68.3780     3.0007   22.79  < 2e-16 ***
PopGrowth             -0.0640     0.0612   -1.04     0.30
UrbanPopulation       -0.1410     0.5756   -0.24     0.81
FertilityRate         -9.8500     0.4445  -22.16  < 2e-16 ***
lnGDPPerCap            2.0281     0.4997    4.06 0.000050 ***
NaturalResourceRents  -0.0483     0.0856   -0.56     0.57
PostColdWar            0.4674     0.1079    4.33 0.000015 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

R-squared: 0.319
Wald statistic: 1071.481, Pr(>Chisq(6)): 0



> PraisWinsten$panelStructure$rho
[1] 0.9526
```

Better in a Table

WBLI Regressions

	OLS	Prais-Winsten
Intercept	55.02^*	68.38^*
	(1.76)	(3.00)
Population Growth	-3.60^*	-0.06
	(0.20)	(0.06)
Urban Population	-1.35^*	-0.14
	(0.25)	(0.58)
Fertility Rate	-4.85^*	-9.85^*
	(0.30)	(0.44)
ln(GDP Per Capita)	3.79^*	2.03^*
	(0.28)	(0.50)
Natural Resource Rents	-3.69^*	-0.05
	(0.17)	(0.09)
Post-Cold War	5.40^*	0.47^*
	(0.19)	(0.11)
$\hat{\rho}$		0.95
R^2	0.499	0.32
Adj. R^2	0.499	
NT	8100	8100
N panels		187

Variables are standardized a la Gelman (2009). $^*p < 0.05$

Some Panel Data Challenges

Consider the error terms in the model:

$$Y_{it} = \mathbf{X}_{it}\boldsymbol{\beta} + u_{it}$$

Issues:

In Words:	In a Formula:
Variances:	
Unit-Wise Heteroscedasticity	$\text{Var}(u_{it}) \neq \text{Var}(u_{jt})$
Temporal Heteroscedasticity	$\text{Var}(u_{it}) \neq \text{Var}(u_{is})$
Covariances:	
Contemporary Cross-Unit Correlation	$\text{Cov}(u_{it}, u_{jt}) \neq 0$
Within-Unit Serial Correlation	$\text{Cov}(u_{it}, u_{is}) \neq 0$
Non-Contemporaraneous Cross-Unit Correlation	$\text{Cov}(u_{it}, u_{js}) \neq 0$

Parks' (1967) Approach

Assume:

- $\text{Var}(u_{it}, u_{jt}) = \sigma^2$ or σ_i^2 (Common or unit-specific error variances)
- $\text{Var}(u_{it}) = \text{Var}(u_{is}) \;\forall\; t \neq s$ (Temporal homoscedasticity)
- $\text{Cov}(u_{it}, u_{jt}) = \sigma_{ij} \;\forall\; i \neq j$ (Pairwise contemporaneous cross-unit correlation)
- $\text{Cov}(u_{it}, u_{is}) = \rho$ or ρ_i (Common or unit-specific temporal correlation)
- $\text{Cov}(u_{it}, u_{js}) = 0 \;\forall\; i \neq j, t \neq s$ (No non-contemporaneous cross-unit correlation)

(B&K: "panel error assumptions").

Then:

1. Use OLS to generate $\hat{u}$s $\rightarrow \hat{\rho}$ ($\rightarrow \hat{\mathbf{\Omega}}$),
2. Use $\hat{\rho}$ for Prais-Winsten.

This method was widely used prior to Beck & Katz (1995)

$$\boldsymbol{\Omega} = \begin{pmatrix} \Sigma & 0 & \cdots & 0 \\ 0 & \Sigma & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Sigma \end{pmatrix} = \Sigma \otimes \mathbf{I}_T$$

where

$$\underset{N \times N}{\Sigma} = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1N} \\ \sigma_{12} & \sigma_2^2 & \cdots & \sigma_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{1N} & \sigma_{2N} & \cdots & \sigma_N^2 \end{pmatrix}$$

Means:

- $\frac{N(N-1)}{2}$ distinct contemporaneous covariances σ_{ij},
- NT observations,
- $\rightarrow 2T/(N+1)$ observations per $\hat{\sigma}$

More Parks Problems

From PROC PANEL in SAS:

Standard Corrections

For the PARKS option, the first-order autocorrelation coefficient must be estimated for each cross section. Let ρ be the $N \times 1$ vector of true parameters and $R = (r_1, \ldots, r_N)'$ be the corresponding vector of estimates. Then, to ensure that only range-preserving estimates are used in PROC PANEL, the following modification for R is made:

$$r_i = \begin{cases} r_i & \text{if} \quad |r_i| < 1 \\ \max(.95, \text{rmax}) & \text{if} \quad r_i \geq 1 \\ \min(-.95, \text{rmin}) & \text{if} \quad r_i \leq -1 \end{cases}$$

where

$$\text{rmax} = \begin{cases} 0 & \text{if} \quad r_i < 0 \quad \text{or} \quad r_i \geq 1 \quad \forall i \\ \max\limits_j [r_j : 0 \leq r_j < 1] & \text{otherwise} \end{cases}$$

and

$$\text{rmin} = \begin{cases} 0 & \text{if} \quad r_i > 0 \quad \text{or} \quad r_i \leq -1 \quad \forall i \\ \max\limits_j [r_j : -1 < r_j \leq 0] & \text{otherwise} \end{cases}$$

Whenever this correction is made, a warning message is printed.

Panel-Corrected Standard Errors

Key to PCSEs:

$$\hat{\sigma}_{ij} = \frac{\sum_{t=1}^{T} \hat{u}_{it}\hat{u}_{jt}}{T}$$

Define:

$$\underset{T\times N}{\mathbf{U}} = \begin{pmatrix} \hat{u}_{11} & \hat{u}_{21} & \cdots & \hat{u}_{N1} \\ \hat{u}_{12} & \hat{u}_{22} & \cdots & \hat{u}_{N2} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{u}_{1T} & \hat{u}_{2T} & \cdots & \hat{u}_{NT} \end{pmatrix}$$

$$\hat{\mathbf{\Sigma}} = \frac{(\mathbf{U}'\mathbf{U})}{T}$$

$$\hat{\mathbf{\Omega}}_{PCSE} = \frac{(\mathbf{U}'\mathbf{U})}{T} \otimes \mathbf{I}_T$$

Correct formula:

$$\text{Cov}(\hat{\beta}_{PCSE}) = (\mathbf{X}'\mathbf{X})^{-1}[\mathbf{X}'\hat{\mathbf{\Omega}}_{PCSE}\mathbf{X}](\mathbf{X}'\mathbf{X})^{-1}$$

General Issues:

- PCSEs do not fix unit-level heterogeneity (a la "fixed" / "random" effects)
- They also do not deal with dynamics
- They depend critically on the "panel data assumptions" of Park / Beck & Katz

Panel Assumptions and Numbers of Parameters

Panel Assumptions	No AR(1)	Common $\hat{\rho}$	Separate $\hat{\rho}_i$s
$\sigma_i^2 = \sigma^2$, $\text{Cov}(\sigma_{it}, \sigma_{jt}) = 0$	$k+1$	$k+2$	$k+N+1$
$\sigma_i^2 \neq \sigma^2$, $\text{Cov}(\sigma_{it}, \sigma_{jt}) = 0$	$k+N$	$k+N+1$	$k+2N$
$\sigma_i^2 \neq \sigma^2$, $\text{Cov}(\sigma_{it}, \sigma_{jt}) \neq 0$	$\frac{N(N-1)}{2} + k + N$	$\frac{N(N-1)}{2} + k + N + 1$	$\frac{N(N-1)}{2} + k + 2N$

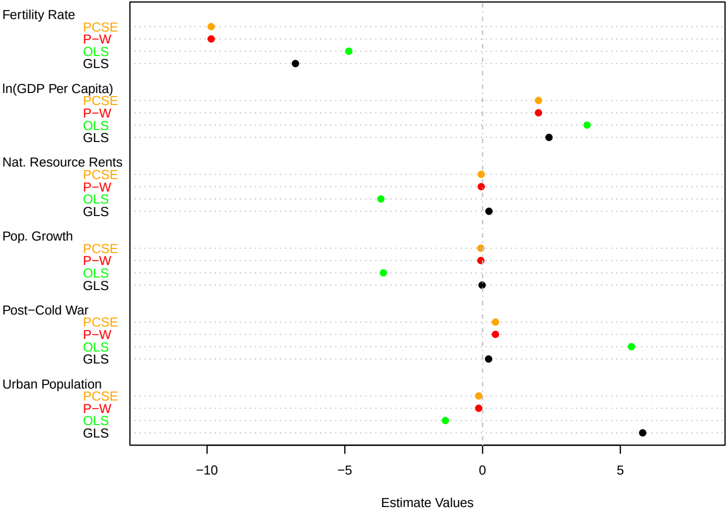
Example: GLS with Homoscedastic AR(1) Errors

```
> GLS<-gls(WomenBusLawIndex~PopGrowth+UrbanPopulation+FertilityRate+lnGDPPerCap+NaturalResourceRents+
+          PostColdWar,data=smol,correlation=corAR1(form=~1|ISO3),na.action="na.omit")

> summary(GLS)
Generalized least squares fit by REML
  Model: WomenBusLawIndex ~ PopGrowth + UrbanPopulation + FertilityRate + lnGDPPerCap + NaturalResourceRents + PostColdWar
  Data: smol
    AIC   BIC logLik
  38224 38287 -19103

Correlation Structure: AR(1)
 Formula: ~1 | ISO3
 Parameter estimate(s):
   Phi
0.9897

Coefficients:
                     Value Std.Error t-value p-value
(Intercept)          48.14     3.997  12.044  0.0000
PopGrowth            -0.02     0.057  -0.320  0.7486
UrbanPopulation       5.81     0.972   5.977  0.0000
FertilityRate        -6.79     0.556 -12.211  0.0000
lnGDPPerCap           2.41     0.604   3.989  0.0001
NaturalResourceRents  0.23     0.077   3.009  0.0026
PostColdWar           0.22     0.102   2.145  0.0320
.
.
.
Residual standard error: 16.94
Degrees of freedom: 8127 total; 8120 residual
```

Example: PCSEs

```
> PCSE<-panelAR(WomenBusLawIndex~PopGrowth+UrbanPopulation+FertilityRate+lnGDPPerCap+NaturalResourceRents+
+             PostColdWar,data=smol,panelVar="ISO3",timeVar="Year",autoCorr="ar1",
+             panelCorrMethod="pcse",rho.na.rm=TRUE)

> summary(PCSE)

Panel Regression with AR(1) Prais-Winsten correction and panel-corrected standard errors

Unbalanced Panel Design:
 Total obs.:       8127 Avg obs. per panel 43.46
 Number of panels: 187  Max obs. per panel 52
 Number of times:  52   Min obs. per panel 1

Coefficients:
                     Estimate Std. Error t value Pr(>|t|)
(Intercept)           68.3780     4.5026   15.19   <2e-16 ***
PopGrowth             -0.0640     0.0692   -0.92   0.3551
UrbanPopulation       -0.1410     0.6265   -0.22   0.8220
FertilityRate         -9.8500     0.6900  -14.28   <2e-16 ***
lnGDPPerCap            2.0281     0.6714    3.02   0.0025 **
NaturalResourceRents  -0.0483     0.1296   -0.37   0.7095
PostColdWar            0.4674     0.2932    1.59   0.1109
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

R-squared: 0.319
Wald statistic: 407.5469, Pr(>Chisq(6)): 0


> PCSE$panelStructure$rho
[1] 0.9526
```

Estimate Comparisons



Dynamics!

Stationarity: A constant d.g.p. over time.[1]

Mean stationarity:

$$E(Y_t) = \mu \ \forall \ t$$

Variance stationarity:

$$Var(Y_t) = E[(Y_t - \mu)^2] \equiv \sigma_Y^2 \ \forall \ t$$

Covariance stationarity:

$$Cov(Y_t, Y_{t-s}) = E[(Y_t - \mu)(Y_{t-s} - \mu)] = \gamma_s \ \forall \ s$$

[1] A stricter form of stationarity requires that the joint probability distribution (in other words, *all* the moments) of series of observations $\{Y_1, Y_2, \ldots Y_t\}$ is the same as that for $\{Y_{1+s}, Y_{2+s}, \ldots Y_{t+s}\}$ for all t and s.

The "ARIMA" Approach

"ARIMA" $=$ *Autoregressive Integrated Moving Average*...

A (first-order) integrated series ("random walk") is:

$$Y_t = Y_{t-1} + u_t,\ u_t \sim i.i.d.(0, \sigma_u^2)$$

...a/k/a a "random walk":

$$\begin{aligned} Y_t &= Y_{t-2} + u_{t-1} + u_t \\ &= Y_{t-3} + u_{t-2} + u_{t-1} + u_t \\ &= \sum_{t=0}^{T} u_t \end{aligned}$$

I(1) Series Properties

I(1) series are not stationary.

Variance:

$$\text{Var}(Y_t) \equiv E(Y_t)^2 \quad = \quad t\sigma^2$$

Autocovariance:

$$\text{Cov}(Y_t, Y_{t-s}) = |t - s|\sigma^2.$$

Both depend on t...

I(1) series (continued)

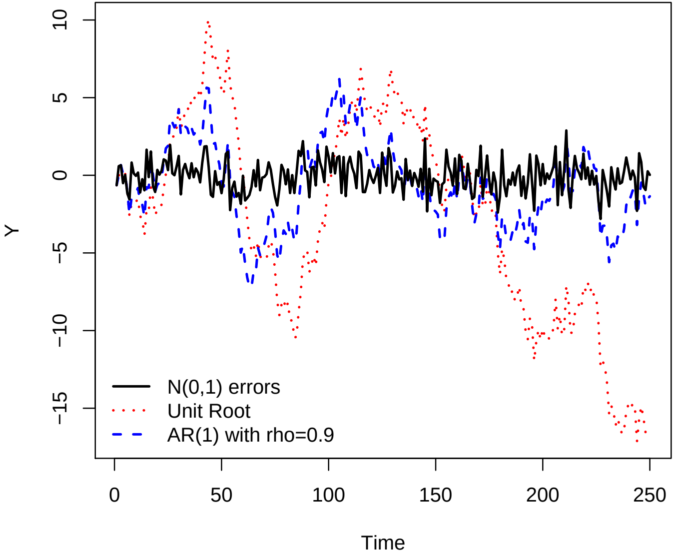
More generally:

- $|\rho| > 1$
  - Series is nonstationary / *explosive*
  - Past shocks have a greater impact than current ones
  - Uncommon

- $|\rho| < 1$
  - *Stationary* series
  - Effects of shocks die out exponentially according to ρ
  - Is mean-reverting

- $|\rho| = 1$
  - Nonstationary series
  - Shocks persist at full force
  - Not mean-reverting; variance increases with t

Time Series Types, Illustrated



I(1) Series: Differencing

For an I(1) series:

$$Y_t - Y_{t-1} = u_t$$

which we often write in terms of the difference operator Δ (or sometimes ∇):

$$\Delta Y_t = u_t$$

The differenced series is just the (stationary, ergodic) white-noise process u_t.

Unit Root Tests Review: Dickey-Fuller

Two steps:

- Estimate $Y_t = \rho Y_{t-1} + u_t$,
- test the hypothesis that $\hat{\rho} = 1$, *but*
- this requires that the us are uncorrelated.

But suppose:

$$\Delta Y_t = \sum_{i=1}^{p} d_i \Delta Y_{t-i} + u_t$$

which yields

$$Y_t = Y_{t-1} + \sum_{i=1}^{p} d_i \Delta Y_{t-i} + u_t.$$

D.F. tests will be incorrect.

Unit Root Alternatives

Augmented Dickey-Fuller Tests:

- Estimate

$$\Delta Y_t = \rho Y_{t-1} + \sum_{i=1}^{p} d_i \Delta Y_{t-i} + u_t$$

- Test $\hat{\rho} = 0$ (vs. alternative that $\hat{\rho} < 0$)

Phillips-Perron Tests:

- Estimate:

$$\Delta Y_t = \alpha + \rho Y_{t-1} + u_t$$

- Calculate modified test statistics (Z_ρ and Z_t)
- Test $\hat{\rho} = 0$

Issues with Unit Roots in Panel Data

- Short series + Asymptotic tests $\rightarrow$ "borrow strength"
- Requires uniform unit roots across cross-sectional units
- Many tests require balanced panels...
- Various alternatives:
  - Maddala and Wu (1999)
  - Hadri (2000)
  - Levin, Lin and Chu (2002)
  - Im, Pesaran, and Shin (2003)
- What to do?
  - Difference the data...
  - Error-correction models

Panel Unit Root Tests: R

```
[data wrangling...]

> purtest(WBLI.W,exo="trend",test="levinlin",pmax=2)

 Levin-Lin-Chu Unit-Root Test (ex. var.: Individual Intercepts and Trend)

data:  WBLI.W
z = -2.5, p-value = 0.007
alternative hypothesis: stationarity

> purtest(WBLI.W,exo="trend",test="hadri",pmax=2)

 Hadri Test (ex. var.: Individual Intercepts and Trend) (Heterosked. Consistent)

data:  WBLI.W
z = 200, p-value <2e-16
alternative hypothesis: at least one series has a unit root

> purtest(WBLI.W,exo="trend",test="madwu",pmax=2)

 Maddala-Wu Unit-Root Test (ex. var.: Individual Intercepts and Trend)

data:  WBLI.W
chisq = 336, df = 376, p-value = 0.9
alternative hypothesis: stationarity

> purtest(WBLI.W,exo="trend",test="ips",pmax=2)

 Im-Pesaran-Shin Unit-Root Test (ex. var.: Individual Intercepts and Trend)

data:  WBLI.W
Wtbar = 2.9, p-value = 1
alternative hypothesis: stationarity
```

A Better Table

Table: Panel Unit Root Tests: WBRI

Test	Alternative	Statistic	Estimate	P-Value
Levin-Lin-Chu	stationarity	z	-2.476	0.0066
Hadri	at least one series has a unit root	z	199.634	< 0.0001
Maddala-Wu	stationarity	χ^2	335.94	0.9321
Im-Pesaran-Shin	stationarity	$\bar{W}_t$	2.851	0.9978

Note: All assume individual intercepts and trends.

Lagged Y?

"Lagged dependent variable":

$$Y_{it} = \phi Y_{it-1} + \mathbf{X}_{it}\boldsymbol{\beta}_{LDV} + \epsilon_{it}$$

If ϵ_{it} is perfect, then:

- $\hat{\beta}_{LDV}$ is biased (but consistent),
- O(bias) $= \frac{-1+3\boldsymbol{\beta}_{LDV}}{T}$

If ϵ_{it} is autocorrelated...

- $\hat{\beta}_{LDV}$ is biased and inconsistent
- IV is one (bad) option...

Lagged Ys and GLS-ARMA

Can rewrite:

$$\begin{aligned} Y_{it} &= \mathbf{X}_{it}\boldsymbol{\beta}_{AR} + u_{it} \\ u_{it} &= \phi u_{it-1} + \eta_{it} \end{aligned}$$

as

$$\begin{aligned} Y_{it} &= \mathbf{X}_{it}\boldsymbol{\beta}_{AR} + \phi u_{it-1} + \eta_{it} \\ &= \mathbf{X}_{it}\boldsymbol{\beta}_{AR} + \phi(Y_{it-1} - \mathbf{X}_{it-1}\boldsymbol{\beta}_{AR}) + \eta_{it} \\ &= \phi Y_{it-1} + \mathbf{X}_{it}\boldsymbol{\beta}_{AR} + \mathbf{X}_{it-1}\psi + \eta_{it} \end{aligned}$$

where $\psi = \phi\beta_{AR}$ and $\psi = 0$ (by assumption).

Lagged Ys and World Domination

In:

$$Y_{it} = \phi Y_{it-1} + \mathbf{X}_{it}\boldsymbol{\beta}_{LDV} + \epsilon_{it}$$

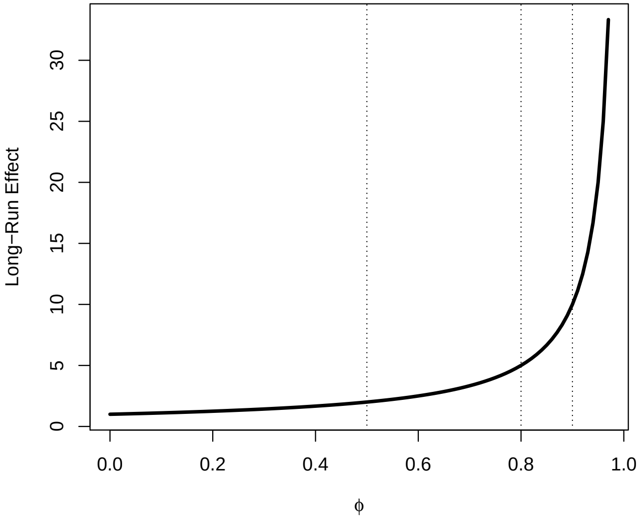
Achen: Bias "deflates" $\hat{\boldsymbol{\beta}}_{LDV}$ relative to $\hat{\phi}$, "suppress" the effects of $\mathbf{X}$...

Keele & Kelly (2006):

- Contingent on ϵs having autocorrelation
- Key: In LDV, *long-run impact of a unit change in X is*:

$$\hat{\beta}_{LR} = \frac{\hat{\beta}_{LDV}}{1 - \hat{\phi}}$$

Long-Run Impact for $\hat{\beta} = 1$



Lagged Ys and Unit Effects

Consider:

$$Y_{it} = \phi Y_{it-1} + \mathbf{X}_{it}\boldsymbol{\beta} + \alpha_i + u_{it}.$$

If we omit the unit effects, we have:

$$Y_{it} = \phi Y_{it-1} + \mathbf{X}_{it}\boldsymbol{\beta} + u_{it}^*$$

with

$$u_{it}^* = \alpha_i + u_{it}$$

Lagging yields:

$$Y_{it-1} = \phi Y_{it-2} + \mathbf{X}_{it-1}\boldsymbol{\beta} + \alpha_i + u_{it-1}$$

which means

$$\text{Cov}(Y_{it-1}, u_{it}^*) \neq 0. \;\rightarrow\; \text{bias in } \hat{\phi}, \hat{\boldsymbol{\beta}}$$

"Nickell" Bias

Bias in $\hat{\phi}$ is

- toward zero when $\phi > 0$,
- increasing in ϕ.

Including unit effects still yields bias in $\hat{\phi}$ of $O(\frac{1}{T})$, and bias in $\hat{\boldsymbol{\beta}}$.

Solutions:

- Difference/GMM estimation
- Bias correction approaches

First Difference Estimation

$$\begin{aligned} Y_{it} - Y_{it-1} &= \phi(Y_{it-1} - Y_{it-2}) + (\mathbf{X}_{it} - \mathbf{X}_{it-1})\boldsymbol{\beta} + (\alpha_i - \alpha_i) + (u_{it} - u_{it-1}) \\ \Delta Y_{it} &= \phi \Delta Y_{it-1} + \Delta \mathbf{X}_{it}\boldsymbol{\beta} + \Delta u_{it} \end{aligned}$$

Anderson/Hsiao: If $\nexists$ autocorrelation, then use ΔY_{it-2} or Y_{it-2} as instruments for ΔY_{it-1}...

- Consistent in theory,
- in practice, the former is preferred, and both have issues if ϕ is high;
- both are inefficient.

Arellano & Bond (also Wawro): Use *all* lags of Y_{it} and $\mathbf{X}_{it}$ from $t-2$ and before.

- "Good" estimates, better as $T \rightarrow \infty$,
- Easy to handle higher-order lags of Y,
- Easy software (`plm` in R , `xtabond` in Stata).
- Model *is* fixed effects...
- $\mathbf{Z}_i$ has $T-p-1$ rows, $\sum_{i=p}^{T-2} i$ columns $\rightarrow$ difficulty of estimation declines in p, <u>grows in T</u>.

Bias-Correction Models

Kiviet (1995, 1999; Bun and Kiviet 2003; Bruno 2005a,b): Derive the bias in $\hat{\phi}$ and $\hat{\beta}$, then correct it...

- More accurate than the instrumental-variables/GMM estimators of A&H/A&B...
- ...especially when T is small; but not as T gets reasonably large ($T \approx 20$)

Some Dynamic Models

	OLS	Lagged Y	First Difference	Fixed Effects	FE + Lagged Y	Anderson-Hsaio
(Intercept)	55.024	2.041	0.636			−0.156
	(1.755)	(0.330)	(0.039)			(0.279)
Population Growth	−3.601	−0.073	−0.001	−0.163	−0.077	0.038
	(0.202)	(0.037)	(0.056)	(0.147)	(0.045)	(0.328)
Urban Population	−1.348	0.075	−0.791	7.049	0.308	2.268
	(0.251)	(0.045)	(1.515)	(0.472)	(0.147)	(23.435)
Fertility Rate	−4.854	−0.159	−1.427	−3.802	−0.499	−0.969
	(0.301)	(0.055)	(0.637)	(0.307)	(0.094)	(9.045)
ln(GDP Per Capita)	3.793	−0.055	1.225	12.691	0.234	−2.451
	(0.279)	(0.050)	(0.639)	(0.431)	(0.139)	(10.302)
Natural Resource Rents	−3.691	−0.098	0.290	0.715	−0.033	−0.111
	(0.167)	(0.031)	(0.076)	(0.185)	(0.057)	(0.792)
Post-Cold War	5.403	0.138	0.014	3.515	0.197	0.124
	(0.185)	(0.035)	(0.101)	(0.147)	(0.046)	(1.383)
Lagged WBLI		0.986			0.952	1.182
		(0.002)			(0.003)	(0.265)
Num.Obs.	8127	7924	7940	8127	7924	7731
R2	0.499	0.985	0.003	0.533	0.958	0.003
R2 Adj.	0.499	0.985	0.002	0.521	0.957	0.003
Log.Lik.		−18 033.445				
RMSE	13.43	2.36	2.39	7.80	2.31	3.57

Anderson-Hsiao, Arellano-Bond, etc.

In R:

- Anderson-Hsiao can be fit using lm or (more easily) `plm` in the `plm` package
- Arellano-Bond is most easily fit using `pgmm` ("panel gmm") in the `plm` package
- See Criossant and Millo (2018, Chapter 7) for statistics + code details
- This post is also useful...

Stata:

- `xtabond` / `xtdpdsys` / `xtdpd` fit both A-H and A-B / Blundell-Bond models (among others)
- This is also a good (slightly dated) reference

Another Approach: Orthogonalization

Note: We're rarely substantively interested in the fixed effects $\hat{\alpha}$...

- $\rightarrow$ reparameterize the αs so that they are *information-orthogonal* to the other parameters in the model (including the βs and ϕ)
- Key idea: Transform the αs so that (for example):

$$\mathrm{E}\left(\frac{\partial^2 L_i}{\partial\alpha\partial\beta}\right) = 0$$

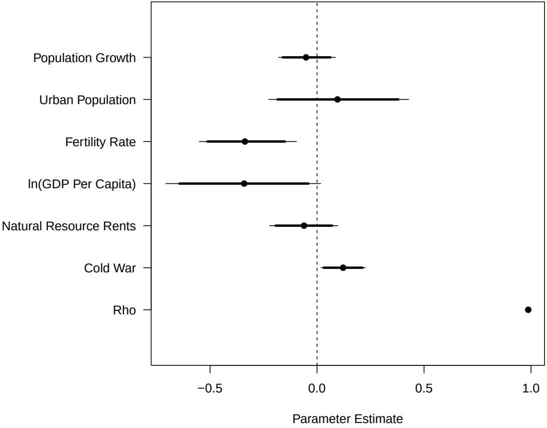
- Can do this via imposition of priors, in a Bayesian framework...
- **In general**, this approach is less assumption-laden and more efficient than the IV/GMM-based approaches discussed above.
- Provides consistent-in-N estimates for T as low as 2...

References:

- Lancaster, T. 2002. "Orthogonal Parameters and Panel Data." *Review of Economic Studies* 69:647-666.
- Pickup et al. (2017) [the "orthogonalized panel model" ("OPM")]

FE + Dynamics Using Orthogonalization

```
> library(OrthoPanels)
> set.seed(7222009)
> OPM.fit <- opm(WomenBusLawIndex~PopGrowth+UrbanPopulation+FertilityRate+
               lnGDPPerCap+NaturalResourceRents+PostColdWar,data=smol,
               index=c("ISO3","Year"),n.samp=1000)
```



OPM Results: Short- and Long-Run Effects

For $\hat{\phi} \approx 0.98$:

Parameter	Short-Run	Long-Run
Population Growth	-0.05	-3.93
Urban Population	0.10	6.87
Fertility Rate	-0.34	-24.90
ln(GDP Per Capita)	-0.34	-25.27
Natural Resource Rents	-0.06	-4.55
Post-Cold War	0.12	9.22

Trends!

What if Y is *trending* over time?

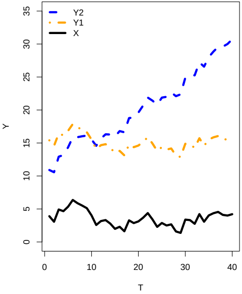
- First Question: Why?
  - Organic growth (e.g., populations)
  - Temporary / short-term factors
  - Covariates...
- Second question: Should we care?
  (A: Yes, usually... $\rightarrow$ "spurious regressions")
- Third question: What to do?
  - Ignore it...
  - Include a counter / trend term...

In general, adding a trend term will *decrease* the magnitudes of $\hat{\beta}$...

Data generating processes:

$$Y_{1t} = 10 + (1 \times X_t) + u_t$$

$$Y_{2t} = 5 + (1 \times X_t) + (0.5 \times T) + u_t$$



	Y_1	Y_2	
		No Trend	Trend
X	0.921***	−0.382	0.874***
	(0.245)	(0.786)	(0.255)
T			0.482***
			(0.026)
Constant	10.300***	20.200***	5.860***
	(0.917)	(2.950)	(1.200)
Observations	40	40	40
R^2	0.272	0.006	0.905
Adjusted R^2	0.253	−0.020	0.900
Residual Std. Error	1.800 (df = 38)	5.790 (df = 38)	1.810 (df = 37)
Note:			*p<0.1; **p<0.05; ***p<0.01

Trends Matter, Part II

Table: FE Models of WBLI

	FE	FE.trend	FE.intx
Population Growth	-0.163	-0.425^{***}	-0.369^{***}
	(0.147)	(0.123)	(0.122)
Urban Population	7.049^{***}	0.198	0.580
	(0.472)	(0.409)	(0.411)
Fertility Rate	-3.802^{***}	2.424^{***}	2.303^{***}
	(0.307)	(0.276)	(0.275)
ln(GDP Per Capita)	12.690^{***}	3.114^{***}	2.831^{***}
	(0.431)	(0.393)	(0.393)
Natural Resource Rents	0.715^{***}	0.393^{**}	0.421^{***}
	(0.185)	(0.154)	(0.153)
Post-Cold War	3.515^{***}	-0.845^{***}	-4.076^{***}
	(0.147)	(0.143)	(0.461)
Trend (1950=0)		0.746^{***}	0.675^{***}
		(0.013)	(0.016)
Post-Cold War x Trend			0.094^{***}
			(0.013)
Observations	8,127	8,127	8,127
R^2	0.533	0.677	0.679
Adjusted R^2	0.521	0.669	0.671

$^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Another Approach: FEIS

"Fixed Effects Individual Slope" models

- Cite: Bruederl, Josef, and Volker Ludwig. 2015. "Fixed-Effects Panel Regression." In *The Sage Handbook of Regression Analysis and Causal Inference*, Eds. Henning Best and Christof Wolf. Los Angeles: Sage, pp. 327-357.
- FE + unit-level slopes for (some / all) predictor variables
- Equivalent to including $N - 1$ interactions between a predictor $\mathbf{X}$ and each of the α_is
- Also can test for homogeneity of estimated slopes (Hausman-like test)
- See the `feisr` R package, and its accompanying vignette, or `xtfeis` in Stata

FEIS Example: Post-Cold War

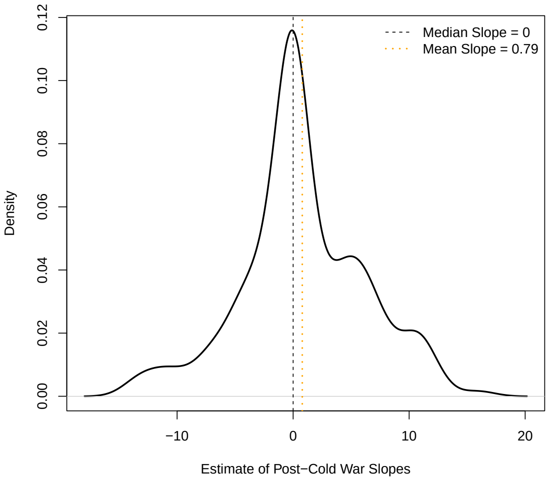
```
> FEIS<-feis(WomenBusLawIndex~PopGrowth+UrbanPopulation+FertilityRate+lnGDPPerCap+NaturalResourceRents | PostColdWar,
+            data=(smol),id="ID",robust=FALSE)

Warning in feis(WomenBusLawIndex ~ PopGrowth + UrbanPopulation + FertilityRate +  :
  FEIS needs at least n(slopes)+1 observations per group.
 You specified 1 slope parameter(s) plus intercept, all groups with t <= 2 dropped

> summary(FEIS)

Call:
feis(formula = WomenBusLawIndex ~ PopGrowth + UrbanPopulation +
    FertilityRate + lnGDPPerCap + NaturalResourceRents | PostColdWar,
    data = (smol), id = "ID", robust = FALSE)

Residuals:
    Min.  1st Qu.   Median  3rd Qu.     Max.
-26.3890  -3.3153   0.0469   3.2897  49.3952

Coefficients:
                     Estimate Std. Error t-value Pr(>|t|)
PopGrowth             -0.4100     0.1204   -3.41  0.00066 ***
UrbanPopulation        8.8151     0.5559   15.86  < 2e-16 ***
FertilityRate         -7.5893     0.3363  -22.57  < 2e-16 ***
lnGDPPerCap           16.0464     0.4823   33.27  < 2e-16 ***
NaturalResourceRents   0.0894     0.1620    0.55  0.58130
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Normal standard errors
Slope parameters:  PostColdWar
Total Sum of Squares:    509000
Residual Sum of Squares: 294000
R-Squared:      0.423
Adj. R-Squared: 0.423
```

FEIS: Testing

```
> FEIS.test<-feistest(FEIS)
> summary(FEIS.test)

Call:
feis(formula = WomenBusLawIndex ~ PopGrowth + UrbanPopulation +
    FertilityRate + lnGDPPerCap + NaturalResourceRents | PostColdWar,
    data = (smol), id = "ID", robust = FALSE)

Artificial Regression Test

FEIS vs. FE:
------------
H0: FEIS and FE estimates consistent
Alternative H1: FE inconsistent
Model constraints: PopGrowth_hat, UrbanPopulation_hat, FertilityRate_hat, lnGDPPerCap_hat,
NaturalResourceRents_hat = 0

Chi-squared test:
Chisq = 1386.6, df = 5, P(> X2) = 0.0


FE vs. RE:
------------
H0: FE and RE estimates consistent
Alternative H1: RE inconsistent
Model constraints: PopGrowth_mean, UrbanPopulation_mean, FertilityRate_mean, lnGDPPerCap_mean,
NaturalResourceRents_mean, PostColdWar_mean = 0

Chi-squared test:
Chisq = 422.2, df = 6, P(> X2) = 0.0


FEIS vs. RE:
------------
H0: FEIS and RE estimates consistent
Alternative H1: RE inconsistent
Model constraints: PopGrowth_hat, UrbanPopulation_hat, FertilityRate_hat, lnGDPPerCap_hat,
NaturalResourceRents_hat = 0

Chi-squared test:
Chisq = 1568.6, df = 5, P(> X2) = 0.0
```

FEIS: Unit-Specific Slopes

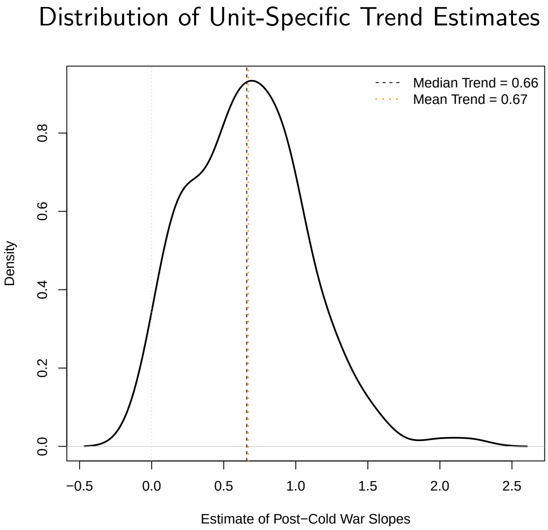
Distribution of Unit-Specific Slopes for *Post-Cold War*



FEIS: Unit-Specific Trends

```
> FEIS2<-feis(WomenBusLawIndex~PopGrowth+UrbanPopulation+FertilityRate+lnGDPPerCap+NaturalResourceRents+PostColdWar | Year,
+            data=(smol),id="ID",robust=FALSE)

Warning in feis(WomenBusLawIndex ~ PopGrowth + UrbanPopulation + FertilityRate +  :
  FEIS needs at least n(slopes)+1 observations per group.
 You specified 1 slope parameter(s) plus intercept, all groups with t <= 2 dropped

> summary(FEIS2)

Call:
feis(formula = WomenBusLawIndex ~ PopGrowth + UrbanPopulation +
    FertilityRate + lnGDPPerCap + NaturalResourceRents + PostColdWar |
    Year, data = (smol), id = "ID", robust = FALSE)

Residuals:
    Min.  1st Qu.   Median  3rd Qu.     Max.
-18.2791  -2.4844  -0.0043   2.3942  41.1899

Coefficients:
                     Estimate Std. Error t-value      Pr(>|t|)
PopGrowth             -0.3784     0.0917   -4.13 0.00003722458 ***
UrbanPopulation       -0.1996     0.7490   -0.27          0.79
FertilityRate         -0.2623     0.3527   -0.74          0.46
lnGDPPerCap            5.1171     0.5587    9.16       < 2e-16 ***
NaturalResourceRents  -0.1468     0.1219   -1.20          0.23
PostColdWar           -0.6833     0.1059   -6.45 0.00000000012 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Normal standard errors
Slope parameters:  Year
Total Sum of Squares:    162000
Residual Sum of Squares: 159000
R-Squared:      0.0191
Adj. R-Squared: 0.0184
```

FEIS: Unit-Specific Trends



Dynamic Models: Software

R :

- the `plm` package (`purtest` for unit roots; `plm` for first-difference models; `pgmm` for Arellano-Bond)
- the `panelAR` package (GLS-ARMA models)
- the `gls` package (GLS)
- the `pdynmc` package (GMM models via moment conditions)
- the `dynpanel` package (A&H, A&B; minimal...)

Stata :

- `xtgls` (GLS)
- `xtpcse` (PCSEs)
- `xtabond` / `xtdpd` (A&H A&B dynamic models)
- Others...

Things to consider:

- N vs. T...
- Are dynamics nuisance or substance?
- What problem(s) do you *really* care about?