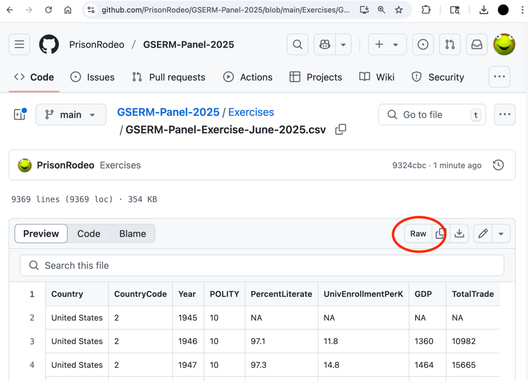
GSERM - St. Gallen 2025

Analyzing Panel Data

June 18, 2025

Data on Github

Download via the "Raw" button...



Can also use (e.g.) `read.csv`:

```
> Data<-read.csv("https://raw.githubusercontent.com/PrisonRodeo/GSERM-Panel-2025/main/
              Exercises/GSERM-Panel-Exercise-June-2025.csv")
```

Generalized Least Squares Models

Start with a focus on residuals...

For:

$$Y_{it} = \mathbf{X}_{it}\beta + u_{it}$$

i.i.d. OLS u_{it}s require:

$$\begin{aligned}
\mathbf{u}\mathbf{u}' \equiv \mathbf{\Omega} &= \sigma^2\mathbf{I} \\
&= \begin{pmatrix} \sigma^2 & 0 & \cdots & 0 \\ 0 & \sigma^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma^2 \end{pmatrix}
\end{aligned}$$

This means that <u>within</u> units:

- $\text{Var}(u_{it}) = \text{Var}(u_{is}) \; \forall \; t \neq s$ (temporal homoscedasticity)
- $\text{Cov}(u_{it}, u_{is}) = 0 \; \forall \; t \neq s$ (no within-unit autocorrelation)

and <u>between</u> units:

- $\text{Var}(u_{it}) = \text{Var}(u_{jt}) \; \forall \; i \neq j$ (cross-unit homoscedasticity)
- $\text{Cov}(u_{it}, u_{jt}) = 0 \; \forall \; i \neq j$ (no between-unit / spatial correlation)

Estimator:

$$\hat{\beta}_{GLS} = (\mathbf{X}'\boldsymbol{\Omega}^{-\mathbf{1}}\mathbf{X})^{-1}\mathbf{X}'\boldsymbol{\Omega}^{-\mathbf{1}}\mathbf{Y}$$

with:

$$\widehat{\mathrm{V}(\beta_{GLS})} = (\mathbf{X}'\boldsymbol{\Omega}^{-\mathbf{1}}\mathbf{X})^{-1}$$

Two approaches:

- Use OLS $\hat{u}_{it}$s to get $\hat{\boldsymbol{\Omega}}$ ("feasible GLS" / " weighted least squares")
- Use substantive knowledge about the data to structure $\boldsymbol{\Omega}$

The variance-covariance matrix is:

$$\begin{aligned}\text{Var}(\hat{\beta}_{WLS}) &= \sigma^2(\mathbf{X}'\mathbf{W}^{-1}\mathbf{X})^{-1} \\ &\equiv (\mathbf{X}'\boldsymbol{\Omega}^{-1}\mathbf{X})^{-1}\end{aligned}$$

A common case is:

$$\text{Var}(u_i) = \sigma^2 \frac{1}{N_i}$$

where N_i is the number of observations upon which (aggregate) observation i is based.

Recall that, if $\sigma_i^2 \neq \sigma_j^2 \; \forall \; i \neq j$,

$$\begin{aligned}
\text{Var}(\beta_{\text{Het.}}) &= (\mathbf{X}'\mathbf{X})^{-1}(\mathbf{X}'\boldsymbol{\Omega}^{-1}\mathbf{X})(\mathbf{X}'\mathbf{X})^{-1} \\
&= (\mathbf{X}'\mathbf{X})^{-1}\,\mathbf{Q}\,(\mathbf{X}'\mathbf{X})^{-1}
\end{aligned}$$

where $\mathbf{Q} = (\mathbf{X}'\boldsymbol{\Omega}^{-1}\mathbf{X})$ and $\boldsymbol{\Omega} = \sigma^2\mathbf{W}$.

We can rewrite $\mathbf{Q}$ as:

$$\begin{aligned}
\mathbf{Q} &= \sigma^2(\mathbf{X}'\mathbf{W}^{-1}\mathbf{X}) \\
&= \sum_{i=1}^{N} \sigma_i^2 \mathbf{X}_i \mathbf{X}_i'
\end{aligned}$$

Estimate $\hat{\mathbf{Q}}$ as:

$$\widehat{\mathbf{Q}} = \sum_{i=1}^{N} \hat{u}_i^2 \mathbf{X}_i \mathbf{X}_i'$$

Yields:

$$\begin{aligned}
\widehat{\text{Var}(\beta)}_{\text{Robust}} &= (\mathbf{X}'\mathbf{X})^{-1}(\mathbf{X}'\widehat{\mathbf{Q}}^{-1}\mathbf{X})(\mathbf{X}'\mathbf{X})^{-1} \\
&= (\mathbf{X}'\mathbf{X})^{-1}\left[\mathbf{X}'\left(\sum_{i=1}^{N} \hat{u}_i^2 \mathbf{X}_i \mathbf{X}_i'\right)^{-1}\mathbf{X}\right](\mathbf{X}'\mathbf{X})^{-1}
\end{aligned}$$

"Robust" VCV estimates:

- are heteroscedasticity-consistent, but
- are biased in small samples, and
- are less efficient than "naive" estimates when $\text{Var}(u) = \sigma^2 \mathbf{I}$.

In practice, they've become the "default;" but it's important to understand their limitations

“Clustering”

Huber / White

?????????

WLS / GLS

I know very little about my error variances...

I know a great deal about my error variances...

A common case:

$$Y_{ij} = \mathbf{X}_{ij}\boldsymbol{\beta} + u_{ij}$$

with

$$\sigma^2_{ij} = \sigma^2_{ik}.$$

“Robust, clustered” estimator:

$$\widehat{\text{Var}(\boldsymbol{\beta})}_{\text{Clustered}} = (\mathbf{X}'\mathbf{X})^{-1} \left\{ \mathbf{X}' \left[\sum_{i=1}^{N} \left(\sum_{j=1}^{n_j} \hat{u}^2_{ij} \mathbf{X}_{ij} \mathbf{X}'_{ij} \right) \right]^{-1} \mathbf{X} \right\} (\mathbf{X}'\mathbf{X})^{-1}$$

Robust / Clustered SEs: A Simulation

```
> set.seed(3844469)
> X <- rnorm(10)
> Y <- 1 + X + rnorm(10)
> df10 <- data.frame(ID=seq(1:10),X=X,Y=Y)

> fit10 <- feols(Y~X,data=df10)
> summary(fit10)

OLS estimation, Dep. Var.: Y
Observations: 10
Standard-errors: IID
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   0.9536     0.3114   3.062 0.015534 *
X             0.5893     0.2907   2.027 0.077193 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
RMSE: 0.880655   Adj. R2: 0.256783


> fit10robust <- feols(Y~X,data=df10,vcov="hetero")
> summary(fit10robust)

OLS estimation, Dep. Var.: Y
Observations: 10
Standard-errors: Heteroskedasticity-robust
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   0.9536     0.3147   3.030 0.016298 *
X             0.5893     0.2850   2.067 0.072517 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
RMSE: 0.880655   Adj. R2: 0.256783
```

Robust / Clustered SEs: A Simulation (continued)

```
> # "Clone" each observation 100 times:
>
> df1K <- df10[rep(seq_len(nrow(df10)),each=100),]
> df1K <- pdata.frame(df1K, index="ID")

> fit1K <- feols(Y~X,data=df1K)
> summary(fit1K)

OLS estimation, Dep. Var.: Y
Observations: 1,000
Standard-errors: IID
            Estimate Std. Error t value  Pr(>|t|)
(Intercept)   0.9536    0.02788   34.20 < 2.2e-16 ***
X             0.5893    0.02603   22.64 < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
RMSE: 0.880655   Adj. R2: 0.3387

> # Robust, clustered SEs:
>
> fit1Krobust <- feols(Y~X,data=df1K,cluster="ID")
> summary(fit1Krobust)

OLS estimation, Dep. Var.: Y
Observations: 1,000
Standard-errors: Clustered (ID)
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   0.9536     0.2968   3.213 0.010613 *
X             0.5893     0.2689   2.192 0.056090 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
RMSE: 0.880655   Adj. R2: 0.3387
```

Example:

$$\begin{aligned} Y_t &= \beta_0 + \beta_1 X_t + u_t \\ u_t &= \rho u_{t-1} + e_t \end{aligned}$$

with $e_t \sim i.i.d.\ N(0, \sigma_u^2)$ and $\rho \in [-1, 1]$ (typically).

$\rightarrow$ "First-order autoregressive" ["AR(1)"] errors.

Serially Correlated Errors and OLS

Detection

- *Plot* of residuals vs. lagged residuals
- *Runs* test (Geary test)
- Durbin-Watson d
  - Calculated as:

$$d = \frac{\sum_{t=2}^{N} (\hat{u}_t - \hat{u}_{t-1})^2}{\sum_{t=1}^{N} \hat{u}_t^2}$$

  - Non-standard distribution ($d \in [0, 4]$)
  - Null: No autocorrelation
  - Only detects first-order autocorrelation

What to do about it?

- GLS, incorporating ρ / $\hat{\rho}$ into the equation
- *First-difference* models (regressing changes of Y on changes of $\mathbf{X}$)
- Cochrane-Orcutt / Prais-Winsten:
    1. Estimate the basic equation via OLS, and obtain residuals
    2. Use the residuals to consistently estimate $\hat{\rho}$ (i.e. the empirical correlation between u_t and u_{t-1})
    3. Use this estimate of $\hat{\rho}$ to estimate the *difference equation*:

    $$(Y_t - \rho Y_{t-1}) = \beta_0(1-\rho) + \beta_1(X_t - \rho X_{t-1}) + (u_t - \rho u_{t-1})$$

    4. Save the residuals, and use them to estimate $\hat{\rho}$ again
    5. Repeat this process until successive estimates of $\hat{\rho}$ differ by a very small amount

Running Example Redux

The World Development Indicators:

- Cross-national country-level time series data
- $N = 215$ countries, $T = 65$ years (1960-2024) + missingness
- Full descriptions are listed in the Github repo here

Regression model:

$$\begin{aligned} \text{WBLI}_{it} \quad = \quad & \beta_0 + \beta_1 \text{Population Growth}_{it} + \beta_2 \text{Urban Population}_{it} + \beta_3 \text{Fertility Rate}_{it} + \\ & \beta_4 \ln(\text{GDP Per Capita})_{it} + \beta_5 \text{Natural Resource Rents}_{it} + \beta_6 \text{Post-Cold War}_t + u_{it} \end{aligned}$$

Descriptive Statistics:

	vars	n	mean	sd	median	min	max	range	skew	kurtosis	se
WomenBusLawIndex	1	8318	60.44	19.03	60.62	17.50	100.00	82.50	-0.03	-0.69	0.21
PopGrowth	2	8318	1.67	1.67	1.64	-27.47	21.70	49.17	-0.06	30.06	0.02
UrbanPopulation	3	8318	52.09	24.06	51.93	2.85	100.00	97.16	0.06	-1.04	0.26
FertilityRate	4	8318	3.61	1.90	3.09	0.77	8.61	7.83	0.51	-1.04	0.02
NaturalResourceRents	5	8318	7.19	11.14	2.42	0.00	88.59	88.59	2.53	7.58	0.12
PostColdWar	6	8318	0.69	0.46	1.00	0.00	1.00	1.00	-0.83	-1.31	0.01
lnGDPPerCap	7	8318	8.33	1.46	8.24	4.93	11.68	6.75	0.12	-0.90	0.02

A Digression: Rescaling Covariates

A la Gelman (2008) (and an updated blog post here):

- Continuous = divide by one standard deviation
- Binary = recode to $\{-1, 1\}$

Doing this yields:

	vars	n	mean	sd	median	min	max	range	skew	kurtosis
WomenBusLawIndex	1	8318	60.44	19.03	60.62	17.50	100.00	82.50	-0.03	-0.69
PopGrowth	2	8318	1.00	1.00	0.98	-16.42	12.97	29.39	-0.06	30.06
UrbanPopulation	3	8318	2.17	1.00	2.16	0.12	4.16	4.04	0.06	-1.04
FertilityRate	4	8318	1.90	1.00	1.63	0.41	4.53	4.12	0.51	-1.04
NaturalResourceRents	5	8318	0.65	1.00	0.22	0.00	7.95	7.95	2.53	7.58
PostColdWar	6	8318	0.38	0.92	1.00	-1.00	1.00	2.00	-0.83	-1.31
lnGDPPerCap	7	8318	5.72	1.00	5.65	3.38	8.02	4.63	0.12	-0.90

How Much Autocorrelation in Y?

Note that:

$$d = 2(1 - \rho)$$

which means that we can calculate:

$$\rho = 1 - \frac{d}{2}.$$

So:

```
> WI<-pdwtest(WomenBusLawIndex~1,data=smol)
> WI

 Durbin-Watson test for serial correlation in panel models

data:  WomenBusLawIndex ~ 1
DW = 0.096, p-value <2e-16
alternative hypothesis: serial correlation in idiosyncratic errors

> print(paste("Rho =",round(1 - (WI$statistic/2),3)))
[1] "Rho = 0.952"
```

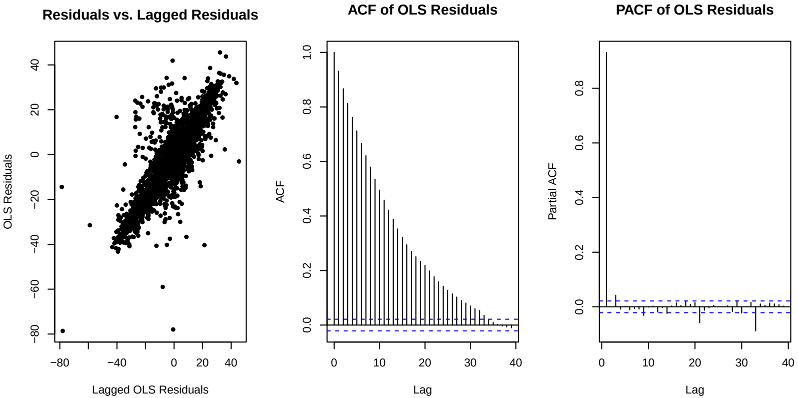
How Much Autocorrelation in $\mathbf{X}$?

Variable	Rho
Population Growth	0.780
Urban Population	0.975
Fertility Rate	0.966
GDP Per Capita	0.978
Natural Resource Rents	0.915
Post Cold War	0.916

Baseline Model: OLS (+ D-W Test)

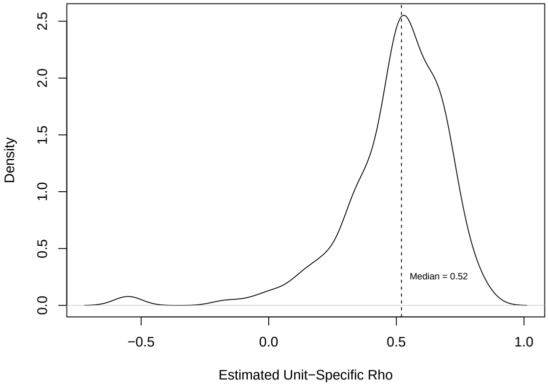
```
> OLS<-plm(WomenBusLawIndex~PopGrowth+UrbanPopulation+FertilityRate+lnGDPPerCap+NaturalResourceRents+
+            PostColdWar,data=smol,model="pooling")

> summary(OLS)
Pooling Model

Unbalanced Panel: n = 187, T = 8-52, N = 8318

Residuals:
   Min. 1st Qu.  Median 3rd Qu.    Max.
 -78.61   -8.48    1.05    9.24   45.58

Coefficients:
                     Estimate Std. Error t-value      Pr(>|t|)
(Intercept)            57.697      1.717   33.60       < 2e-16 ***
PopGrowth              -2.971      0.195  -15.24       < 2e-16 ***
UrbanPopulation        -1.586      0.250   -6.33 0.00000000025 ***
FertilityRate          -5.669      0.293  -19.38       < 2e-16 ***
lnGDPPerCap             3.597      0.278   12.93       < 2e-16 ***
NaturalResourceRents   -4.047      0.167  -24.17       < 2e-16 ***
PostColdWar             5.138      0.183   28.14       < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Total Sum of Squares:    3010000
Residual Sum of Squares: 1500000
R-Squared:      0.5
Adj. R-Squared: 0.5
F-statistic: 1387.87 on 6 and 8311 DF, p-value: <2e-16

> pdwtest(OLS)

 Durbin-Watson test for serial correlation in panel models

data:  WomenBusLawIndex ~ PopGrowth + UrbanPopulation + FertilityRate +  ...
DW = 0.14, p-value <2e-16
alternative hypothesis: serial correlation in idiosyncratic errors
```

Residual Autocorrelation?



Unit-Specific Autocorrelation...

Fit $N = 188$ country-specific regressions, and examine the $\hat{\rho}_i$s...



Another Model: Prais-Winsten

```
> PraisWinsten<-panelAR(WomenBusLawIndex~PopGrowth+UrbanPopulation+FertilityRate+lnGDPPerCap+NaturalResourceRents+
+               PostColdWar, data=smol,panelVar="ISO3",timeVar="Year",autoCorr="ar1",panelCorrMethod="none",rho.na.rm=TRUE)

> summary(PraisWinsten)

Panel Regression with AR(1) Prais-Winsten correction and homoskedastic variance

Unbalanced Panel Design:
 Total obs.:       8318 Avg obs. per panel 44.481
 Number of panels: 187  Max obs. per panel 52
 Number of times:  52   Min obs. per panel 8

Coefficients:
                     Estimate Std. Error t value  Pr(>|t|)
(Intercept)          68.10534    2.88611   23.60   < 2e-16 ***
PopGrowth            -0.00489    0.04634   -0.11      0.92
UrbanPopulation       0.00375    0.58036    0.01      0.99
FertilityRate        -9.70035    0.42217  -22.98   < 2e-16 ***
lnGDPPerCap           1.98821    0.49708    4.00 0.0000640 ***
NaturalResourceRents -0.07378    0.08477   -0.87      0.38
PostColdWar           0.46774    0.10465    4.47 0.0000079 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

R-squared: 0.2949
Wald statistic: 1067.1832, Pr(>Chisq(6)): 0



> PraisWinstenpanelStructurerho
[1] 0.9547
```

Better in a Table

WBLI Regressions

	OLS	Prais-Winsten
Intercept	57.70^*	68.10^*
	(1.72)	(2.89)
Population Growth	-2.97^*	-0.005
	(0.19)	(0.046)
Urban Population	-1.59^*	0.004
	(0.25)	(0.580)
Fertility Rate	-5.67^*	-9.70^*
	(0.29)	(0.422)
ln(GDP Per Capita)	3.60^*	1.99^*
	(0.28)	(0.50)
Natural Resource Rents	-4.05^*	-0.07
	(0.17)	(0.09)
Cold War	5.14^*	0.47^*
	(0.18)	(0.11)
R^2	0.50	0.29
Adj. R^2	0.50	
Num. obs.	8318	8318

Variables are standardized a la Gelman (2009). $^*p < 0.05$

Some Panel Data Challenges

Consider the error terms in the model:

$$Y_{it} = \mathbf{X}_{it}\boldsymbol{\beta} + u_{it}$$

Issues:

In Words:	In a Formula:
Variances:	
Unit-Wise Heteroscedasticity	$\text{Var}(u_{it}) \neq \text{Var}(u_{jt})$
Temporal Heteroscedasticity	$\text{Var}(u_{it}) \neq \text{Var}(u_{is})$
Covariances:	
Contemporary Cross-Unit Correlation	$\text{Cov}(u_{it}, u_{jt}) \neq 0$
Within-Unit Serial Correlation	$\text{Cov}(u_{it}, u_{is}) \neq 0$
Non-Contemporaraneous Cross-Unit Correlation	$\text{Cov}(u_{it}, u_{js}) \neq 0$

Parks' (1967) Approach

Assume:

- $\text{Var}(u_{it}, u_{jt}) = \sigma^2$ or σ_i^2 (Common or unit-specific error variances)
- $\text{Var}(u_{it}) = \text{Var}(u_{is}) \; \forall \; t \neq s$ (Temporal homoscedasticity)
- $\text{Cov}(u_{it}, u_{jt}) = \sigma_{ij} \; \forall \; i \neq j$ (Pairwise contemporaneous cross-unit correlation)
- $\text{Cov}(u_{it}, u_{is}) = \rho$ or ρ_i (Common or unit-specific temporal correlation)
- $\text{Cov}(u_{it}, u_{js}) = 0 \; \forall \; i \neq j, t \neq s$ (No non-contemporaneous cross-unit correlation)

(Beck & Katz (1995): "panel error assumptions").

Then:

1. Use OLS to generate $\hat{u}$s $\rightarrow \hat{\rho}$ ($\rightarrow \hat{\mathbf{\Omega}}$),
2. Use $\hat{\rho}$ for Prais-Winsten.

This method was widely used prior to Beck & Katz (1995)...

The variance-covariance matrix is:

$$\mathbf{\Omega} = \begin{pmatrix} \Sigma & 0 & \cdots & 0 \\ 0 & \Sigma & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Sigma \end{pmatrix} = \Sigma \otimes \mathbf{I}_T$$

where

$$\underset{N \times N}{\Sigma} = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1N} \\ \sigma_{12} & \sigma_2^2 & \cdots & \sigma_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{1N} & \sigma_{2N} & \cdots & \sigma_N^2 \end{pmatrix}$$

Means:

- $\frac{N(N-1)}{2}$ distinct contemporaneous covariances σ_{ij},
- NT observations,
- $\rightarrow 2T/(N+1)$ observations per $\hat{\sigma}$

From PROC PANEL in SAS:

Standard Corrections

For the PARKS option, the first-order autocorrelation coefficient must be estimated for each cross section. Let ρ be the $N \times 1$ vector of true parameters and $R = (r_1, \ldots, r_N)'$ be the corresponding vector of estimates. Then, to ensure that only range-preserving estimates are used in PROC PANEL, the following modification for R is made:

$$r_i = \begin{cases} r_i & \text{if} \quad |r_i| < 1 \\ \max(.95, \text{rmax}) & \text{if} \quad r_i \geq 1 \\ \min(-.95, \text{rmin}) & \text{if} \quad r_i \leq -1 \end{cases}$$

where

$$\text{rmax} = \begin{cases} 0 & \text{if} \quad r_i < 0 \quad \text{or} \quad r_i \geq 1 \quad \forall i \\ \max_j [r_j : 0 \leq r_j < 1] & \text{otherwise} \end{cases}$$

and

$$\text{rmin} = \begin{cases} 0 & \text{if} \quad r_i > 0 \quad \text{or} \quad r_i \leq -1 \quad \forall i \\ \max_j [r_j : -1 < r_j \leq 0] & \text{otherwise} \end{cases}$$

Whenever this correction is made, a warning message is printed.

Panel-Corrected Standard Errors

Key to PCSEs:

$$\hat{\sigma}_{ij} = \frac{\sum_{t=1}^{T} \hat{u}_{it}\hat{u}_{jt}}{T}$$

Define:

$$\underset{T\times N}{\mathbf{U}} = \begin{pmatrix} \hat{u}_{11} & \hat{u}_{21} & \cdots & \hat{u}_{N1} \\ \hat{u}_{12} & \hat{u}_{22} & \cdots & \hat{u}_{N2} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{u}_{1T} & \hat{u}_{2T} & \cdots & \hat{u}_{NT} \end{pmatrix}$$

$$\hat{\mathbf{\Sigma}} = \frac{(\mathbf{U}'\mathbf{U})}{T}$$

$$\hat{\mathbf{\Omega}}_{PCSE} = \frac{(\mathbf{U}'\mathbf{U})}{T} \otimes \mathbf{I}_T$$

Correct formula:

$$\mathrm{Cov}(\hat{\beta}_{PCSE}) = (\mathbf{X}'\mathbf{X})^{-1}[\mathbf{X}'\hat{\mathbf{\Omega}}_{PCSE}\mathbf{X}](\mathbf{X}'\mathbf{X})^{-1}$$

General Issues:

- PCSEs do not fix unit-level heterogeneity (a la "fixed" / "random" effects)
- They also do not deal with dynamics
- They depend critically on the "panel data assumptions" of Park / Beck & Katz

Panel Assumptions and Numbers of Parameters

Panel Assumptions	No AR(1)	Common $\hat{\rho}$	Separate $\hat{\rho}_i$s
$\sigma_i^2 = \sigma^2$, $\text{Cov}(\sigma_{it}, \sigma_{jt}) = 0$	$k + 1$	$k + 2$	$k + N + 1$
$\sigma_i^2 \neq \sigma^2$, $\text{Cov}(\sigma_{it}, \sigma_{jt}) = 0$	$k + N$	$k + N + 1$	$k + 2N$
$\sigma_i^2 \neq \sigma^2$, $\text{Cov}(\sigma_{it}, \sigma_{jt}) \neq 0$	$\frac{N(N-1)}{2} + k + N$	$\frac{N(N-1)}{2} + k + N + 1$	$\frac{N(N-1)}{2} + k + 2N$

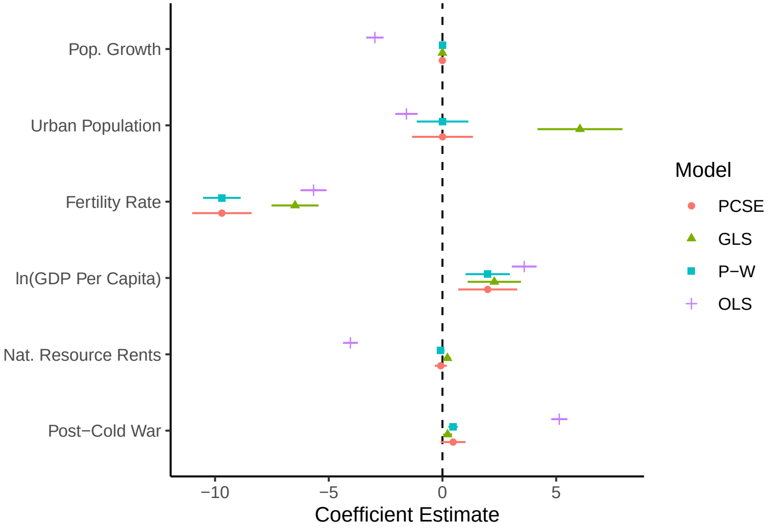
Example: GLS with Homoscedastic AR(1) Errors

```
> GLS<-gls(WomenBusLawIndex~PopGrowth+UrbanPopulation+FertilityRate+lnGDPPerCap+NaturalResourceRents+
+          PostColdWar,data=smol,correlation=corAR1(form=~1|ISO3),na.action="na.omit")

> summary(GLS)
Generalized least squares fit by REML
  Model: WomenBusLawIndex ~ PopGrowth + UrbanPopulation + FertilityRate + lnGDPPerCap + NaturalResourceRents + PostColdWar
  Data: smol
    AIC   BIC logLik
  38995 39058 -19489

Correlation Structure: AR(1)
 Formula: ~1 | ISO3
 Parameter estimate(s):
   Phi
0.9897

Coefficients:
                     Value Std.Error t-value p-value
(Intercept)          47.95     3.867  12.400  0.0000
PopGrowth             0.00     0.043  -0.028  0.9779
UrbanPopulation       6.05     0.954   6.336  0.0000
FertilityRate        -6.48     0.528 -12.285  0.0000
lnGDPPerCap           2.28     0.598   3.813  0.0001
NaturalResourceRents  0.22     0.077   2.836  0.0046
PostColdWar           0.23     0.100   2.277  0.0228

 Correlation:
                     (Intr) PpGrwt UrbnPp FrtltR lGDPPC NtrlRR
PopGrowth             0.037
UrbanPopulation      -0.319 -0.008
FertilityRate        -0.517 -0.055  0.430
lnGDPPerCap          -0.694 -0.032 -0.367  0.028
NaturalResourceRents  0.009 -0.008  0.030  0.036 -0.053
PostColdWar          -0.002  0.029 -0.050  0.057  0.008 -0.019

Residual standard error: 16.87
Degrees of freedom: 8318 total; 8311 residual
```

Example: PCSEs

```
> PCSE<-panelAR(WomenBusLawIndex~PopGrowth+UrbanPopulation+FertilityRate+lnGDPPerCap+NaturalResourceRents+
+             PostColdWar,data=smol,panelVar="ISO3",timeVar="Year",autoCorr="ar1",
+             panelCorrMethod="pcse",rho.na.rm=TRUE)

> summary(PCSE)

Panel Regression with AR(1) Prais-Winsten correction and panel-corrected standard errors

Unbalanced Panel Design:
 Total obs.:       8318 Avg obs. per panel 44.481
 Number of panels: 187  Max obs. per panel 52
 Number of times:  52   Min obs. per panel 8

Coefficients:
                     Estimate Std. Error t value Pr(>|t|)
(Intercept)          68.10534    4.29042   15.87   <2e-16 ***
PopGrowth            -0.00489    0.05089   -0.10   0.9235
UrbanPopulation       0.00375    0.68342    0.01   0.9956
FertilityRate        -9.70035    0.66742  -14.53   <2e-16 ***
lnGDPPerCap           1.98821    0.66281    3.00   0.0027 **
NaturalResourceRents -0.07378    0.13521   -0.55   0.5853
PostColdWar           0.46774    0.27871    1.68   0.0933 .
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

R-squared: 0.2949
Wald statistic: 362.7486, Pr(>Chisq(6)): 0


> PCSE$panelStructure$rho
[1] 0.9547
```

Estimate Comparisons



Dynamics!

Stationarity: A constant d.g.p. over time.[1]

Mean stationarity:

$$E(Y_t) = \mu \; \forall \; t$$

Variance stationarity:

$$Var(Y_t) = E[(Y_t - \mu)^2] \equiv \sigma_Y^2 \; \forall \; t$$

Covariance stationarity:

$$Cov(Y_t, Y_{t-s}) = E[(Y_t - \mu)(Y_{t-s} - \mu)] = \gamma_s \; \forall \; s$$

[1] A stricter form of stationarity requires that the joint probability distribution (in other words, *all* the moments) of series of observations $\{Y_1, Y_2, \dots Y_t\}$ is the same as that for $\{Y_{1+s}, Y_{2+s}, \dots Y_{t+s}\}$ for all t and s.

"ARIMA" $=$ *Autoregressive Integrated Moving Average*...

A (first-order) integrated series ("random walk") is:

$$Y_t = Y_{t-1} + u_t,\ u_t \sim i.i.d.(0, \sigma_u^2)$$

...a/k/a a "random walk":

$$\begin{aligned} Y_t &= Y_{t-2} + u_{t-1} + u_t \\ &= Y_{t-3} + u_{t-2} + u_{t-1} + u_t \\ &= \sum_{t=0}^{T} u_t \end{aligned}$$

I(1) series are not stationary.

Variance:

$$\mathrm{Var}(Y_t) \equiv E(Y_t)^2 \quad = \quad t\sigma^2$$

Autocovariance:

$$\mathrm{Cov}(Y_t, Y_{t-s}) = |t-s|\sigma^2.$$

Both depend on t...

I(1) series (continued)

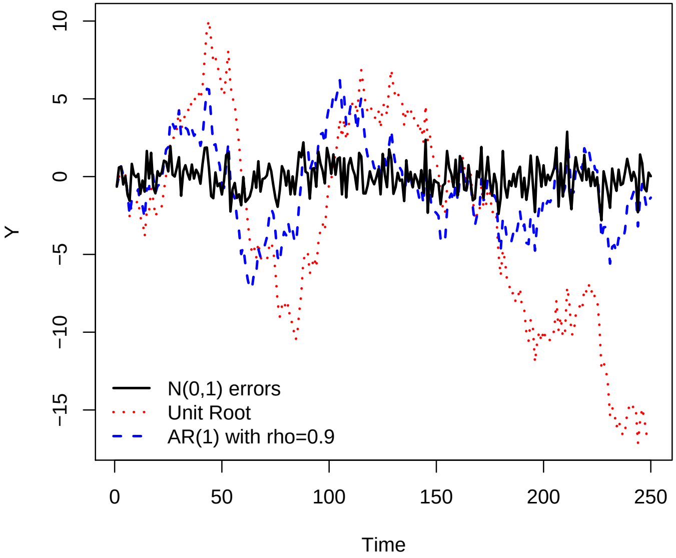
More generally:

- $|\rho| > 1$
  - Series is nonstationary / *explosive*
  - Past shocks have a greater impact than current ones
  - Uncommon

- $|\rho| < 1$
  - *Stationary* series
  - Effects of shocks die out exponentially according to ρ
  - Is mean-reverting

- $|\rho| = 1$
  - Nonstationary series
  - Shocks persist at full force
  - Not mean-reverting; variance increases with t

Time Series Types, Illustrated



I(1) Series: Differencing

For an I(1) series:

$$Y_t - Y_{t-1} = u_t$$

which we often write in terms of the difference operator Δ (or sometimes ∇):

$$\Delta Y_t = u_t$$

The differenced series is just the (stationary, ergodic) white-noise process u_t.

Unit Root Tests Review: Dickey-Fuller

Two steps:

- Estimate $Y_t = \rho Y_{t-1} + u_t$,
- test the hypothesis that $\hat{\rho} = 1$, *but*
- this requires that the *u*s are uncorrelated.

But suppose:

$$\Delta Y_t = \sum_{i=1}^{p} d_i \Delta Y_{t-i} + u_t$$

which yields

$$Y_t = Y_{t-1} + \sum_{i=1}^{p} d_i \Delta Y_{t-i} + u_t.$$

D.F. tests will be incorrect.

Unit Root Alternatives

Augmented Dickey-Fuller Tests:

- Estimate

$$\Delta Y_t = \rho Y_{t-1} + \sum_{i=1}^{p} d_i \Delta Y_{t-i} + u_t$$

- Test $\hat{\rho} = 0$ (vs. alternative that $\hat{\rho} < 0$)

Phillips-Perron Tests:

- Estimate:

$$\Delta Y_t = \alpha + \rho Y_{t-1} + u_t$$

- Calculate modified test statistics (Z_ρ and Z_t)
- Test $\hat{\rho} = 0$

Issues with Unit Roots in Panel Data

- Short series + Asymptotic tests $\rightarrow$ “borrow strength”
- Requires uniform unit roots across cross-sectional units
- Many tests require balanced panels...
- Various alternatives:
    - Maddala and Wu (1999)
    - Hadri (2000)
    - Levin, Lin and Chu (2002)
    - Im, Pesaran, and Shin (2003)
- What to do?
    - Difference the data...
    - Error-correction models

Panel Unit Root Tests: R

```
[data wrangling...]

> purtest(WBLI.W,exo="trend",test="levinlin",pmax=2)

 Levin-Lin-Chu Unit-Root Test (ex. var.: Individual Intercepts and
 Trend)

data:  WBLI.W
z = -2.5, p-value = 0.007
alternative hypothesis: stationarity

> purtest(WBLI.W,exo="trend",test="hadri",pmax=2)

 Hadri Test (ex. var.: Individual Intercepts and Trend) (Heterosked.
 Consistent)

data:  WBLI.W
z = 200, p-value <2e-16
alternative hypothesis: at least one series has a unit root

> purtest(WBLI.W,exo="trend",test="madwu",pmax=2)

 Maddala-Wu Unit-Root Test (ex. var.: Individual Intercepts and Trend)

data:  WBLI.W
chisq = 336, df = 376, p-value = 0.9
alternative hypothesis: stationarity

> purtest(WBLI.W,exo="trend",test="ips",pmax=2)

 Im-Pesaran-Shin Unit-Root Test (ex. var.: Individual Intercepts and
 Trend)

data:  WBLI.W
Wtbar = 2.9, p-value = 1
alternative hypothesis: stationarity
```

Table: Panel Unit Root Tests: WBRI

	Test	Alternative	Statistic	Estimate	P-Value
1	Levin-Lin-Chu	stationarity	z	-2.476	0.0066
2	Hadri	at least one series has a unit root	z	199.634	< 0.0001
3	Maddala-Wu	stationarity	χ^2	335.94	0.9321
4	Im-Pesaran-Shin	stationarity	$\bar{W}_t$	2.851	0.9978

Note: All assume individual intercepts and trends.

Lagged Y?

"Lagged dependent variable":

$$Y_{it} = \phi Y_{it-1} + \mathbf{X}_{it}\boldsymbol{\beta}_{LDV} + \epsilon_{it}$$

If ϵ_{it} is perfect, then:

- $\hat{\beta}_{LDV}$ is biased (but consistent),
- $\text{O(bias)} = \frac{-1+3\boldsymbol{\beta}_{LDV}}{T}$

If ϵ_{it} is autocorrelated...

- $\hat{\beta}_{LDV}$ is biased and <u>inconsistent</u>
- IV is one (bad) option...

Lagged Ys and GLS-ARMA

Can rewrite:

$$
\begin{aligned}
Y_{it} &= \mathbf{X}_{it}\boldsymbol{\beta}_{AR} + u_{it} \\
u_{it} &= \phi u_{it-1} + \eta_{it}
\end{aligned}
$$

as

$$
\begin{aligned}
Y_{it} &= \mathbf{X}_{it}\boldsymbol{\beta}_{AR} + \phi u_{it-1} + \eta_{it} \\
&= \mathbf{X}_{it}\boldsymbol{\beta}_{AR} + \phi(Y_{it-1} - \mathbf{X}_{it-1}\boldsymbol{\beta}_{AR}) + \eta_{it} \\
&= \phi Y_{it-1} + \mathbf{X}_{it}\boldsymbol{\beta}_{AR} + \mathbf{X}_{it-1}\psi + \eta_{it}
\end{aligned}
$$

where $\psi = \phi\beta_{AR}$ and $\psi = 0$ (by assumption).

Lagged Ys and World Domination

In:

$$Y_{it} = \phi Y_{it-1} + \mathbf{X}_{it}\boldsymbol{\beta}_{LDV} + \epsilon_{it}$$

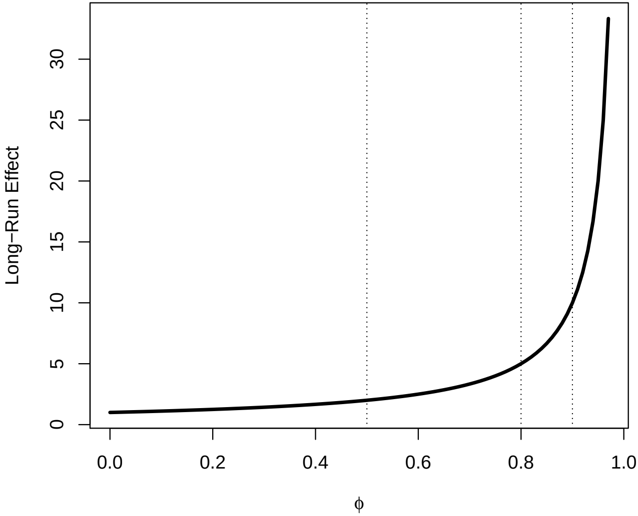
Achen: Bias "deflates" $\hat{\boldsymbol{\beta}}_{LDV}$ relative to $\hat{\phi}$, "suppress" the effects of **X**...

Keele & Kelly (2006):

- Contingent on ϵs having autocorrelation
- Key: In LDV, *long-run impact of a unit change in X is*:

$$\hat{\beta}_{LR} = \frac{\hat{\beta}_{LDV}}{1 - \hat{\phi}}$$

Long-Run Impact for $\hat{\beta} = 1$



Lagged Ys and Unit Effects

Consider:

$$Y_{it} = \phi Y_{it-1} + \mathbf{X}_{it}\boldsymbol{\beta} + \alpha_i + u_{it}.$$

If we omit the unit effects, we have:

$$Y_{it} = \phi Y_{it-1} + \mathbf{X}_{it}\boldsymbol{\beta} + u^*_{it}$$

with

$$u^*_{it} = \alpha_i + u_{it}$$

Lagging yields:

$$Y_{it-1} = \phi Y_{it-2} + \mathbf{X}_{it-1}\boldsymbol{\beta} + \alpha_i + u_{it-1}$$

which means

$$\text{Cov}(Y_{it-1}, u^*_{it}) \neq 0. \rightarrow \text{ bias in } \hat{\phi}, \hat{\boldsymbol{\beta}}$$

Bias in $\hat{\phi}$ is

- toward zero when $\phi > 0$,
- increasing in ϕ.

Including unit effects still yields bias in $\hat{\phi}$ of $O(\frac{1}{T})$, and bias in $\hat{\boldsymbol{\beta}}$.

Solutions:

- Difference/GMM estimation
- Bias correction approaches

$$\begin{aligned} Y_{it} - Y_{it-1} &= \phi(Y_{it-1} - Y_{it-2}) + (\mathbf{X}_{it} - \mathbf{X}_{it-1})\boldsymbol{\beta} + (\alpha_i - \alpha_i) + (u_{it} - u_{it-1}) \\ \Delta Y_{it} &= \phi \Delta Y_{it-1} + \Delta \mathbf{X}_{it}\boldsymbol{\beta} + \Delta u_{it} \end{aligned}$$

Anderson/Hsiao: If $\nexists$ autocorrelation, then use ΔY_{it-2} or Y_{it-2} as instruments for ΔY_{it-1}...

- Consistent in theory,
- in practice, the former is preferred, and both have issues if ϕ is high;
- both are inefficient.

Arellano & Bond (also Wawro): Use *all* lags of Y_{it} and $\mathbf{X}_{it}$ from $t-2$ and before.

- "Good" estimates, better as $T \rightarrow \infty$,
- Easy to handle higher-order lags of Y,
- Easy software (`plm` in R , `xtabond` in Stata).
- Model *is* fixed effects...
- $\mathbf{Z}_i$ has $T-p-1$ rows, $\sum_{i=p}^{T-2} i$ columns $\rightarrow$ difficulty of estimation declines in p, grows in T.

Bias-Correction Models

Kiviet (1995, 1999; Bun and Kiviet 2003; Bruno 2005a,b): Derive the bias in $\hat{\phi}$ and $\hat{\beta}$, then correct it...

- More accurate than the instrumental-variables/GMM estimators of A&H/A&B...
- ...especially when T is small; but not as T gets reasonably large ($T \approx 20$)

Some Dynamic Models

	OLS	Lagged Y	First Difference	Fixed Effects	FE + Lagged Y	Anderson-Hsaio
(Intercept)	57.697	2.121	0.632			−0.162
	(1.717)	(0.322)	(0.037)			(0.271)
Population Growth	−2.971	−0.051	0.005	−0.316	−0.073	0.018
	(0.195)	(0.035)	(0.043)	(0.135)	(0.041)	(0.199)
Urban Population	−1.586	0.066	−0.517	7.459	0.323	3.025
	(0.250)	(0.044)	(1.452)	(0.471)	(0.146)	(22.721)
Fertility Rate	−5.669	−0.183	−1.155	−3.350	−0.476	−0.627
	(0.293)	(0.053)	(0.603)	(0.293)	(0.090)	(9.234)
ln(GDP Per Capita)	3.597	−0.071	1.087	12.789	0.218	−2.406
	(0.278)	(0.050)	(0.632)	(0.428)	(0.137)	(9.027)
Natural Resource Rents	−4.047	−0.104	0.282	0.800	−0.017	−0.085
	(0.167)	(0.031)	(0.076)	(0.185)	(0.057)	(0.652)
Post-Cold War	5.138	0.122	0.009	3.472	0.184	0.078
	(0.183)	(0.034)	(0.098)	(0.144)	(0.044)	(1.229)
Lagged WBLI		0.987			0.953	1.188
		(0.002)			(0.003)	(0.289)
Num.Obs.	8318	8120	8131	8318	8120	7923
R2	0.500	0.985	0.003	0.528	0.958	0.003
R2 Adj.	0.500	0.985	0.002	0.517	0.957	0.003
Log.Lik.		−18 427.767				
RMSE	13.45	2.34	2.37	7.79	2.30	3.56

Anderson-Hsiao, Arellano-Bond, etc.

In R:

- Anderson-Hsiao can be fit using lm or (more easily) `plm` in the `plm` package
- Arellano-Bond is most easily fit using `pgmm` ("panel gmm") in the `plm` package
- See Criossant and Millo (2018, Chapter 7) for statistics + code details
- This post is also useful...

Stata:

- `xtabond` / `xtdpdsys` / `xtdpd` fit both A-H and A-B / Blundell-Bond models (among others)
- This is also a good (slightly dated) reference

Another Approach: Orthogonalization

Note: We're rarely substantively interested in the fixed effects $\hat{\alpha}$...

- $\rightarrow$ reparameterize the αs so that they are *information-orthogonal* to the other parameters in the model (including the βs and ϕ)
- Key idea: Transform the αs so that (for example):

$$\mathrm{E}\left(\frac{\partial^2 L_i}{\partial\alpha\partial\beta}\right) = 0$$

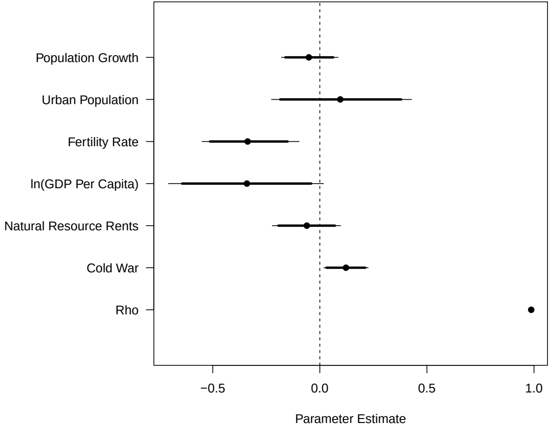
- Can do this via imposition of priors, in a Bayesian framework...
- **In general**, this approach is less assumption-laden and more efficient than the IV/GMM-based approaches discussed above.
- Provides consistent-in-N estimates for T as low as 2...

References:

- Lancaster, T. 2002. "Orthogonal Parameters and Panel Data." *Review of Economic Studies* 69:647-666.
- Pickup et al. (2017) [the "orthogonalized panel model" ("OPM")]

FE + Dynamics Using Orthogonalization

```
> library(OrthoPanels)
> set.seed(7222009)
> OPM.fit <- opm(WomenBusLawIndex~PopGrowth+UrbanPopulation+FertilityRate+
              lnGDPPerCap+NaturalResourceRents+PostColdWar,data=smol,
              index=c("ISO3","Year"),n.samp=1000)
```



OPM Results: Short- and Long-Run Effects

For $\hat{\phi} \approx 0.98$:

Parameter	Short-Run	Long-Run
Population Growth	-0.05	-3.93
Urban Population	0.10	6.87
Fertility Rate	-0.34	-24.90
ln(GDP Per Capita)	-0.34	-25.27
Natural Resource Rents	-0.06	-4.55
Post-Cold War	0.12	9.22

Trends!

<u>What if Y is *trending* over time?</u>

- First Question: Why?
  - Organic growth (e.g., populations)
  - Temporary / short-term factors
  - Covariates...

- Second question: Should we care?
  (A: Yes, usually... $\rightarrow$ "spurious regressions")

- Third question: What to do?
  - Ignore it...
  - Include a counter / trend term...

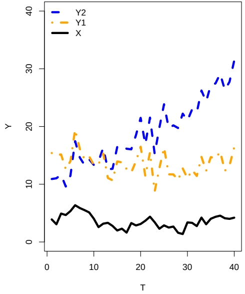
In general, adding a trend term will *decrease* the magnitudes of $\hat{\boldsymbol{\beta}}$...

Trends Matter, Illustrated

Data generating processes:

$$Y_{1t} = 10 + (1 \times X_t) + u_t$$

$$Y_{2t} = 5 + (1 \times X_t) + (0.5 \times T) + u_t$$



	Dependent variable:		
	Y_1	Y_2	
X	0.921*** (0.245)	−0.382 (0.786)	0.874*** (0.255)
T			0.482*** (0.026)
Constant	10.320*** (0.917)	20.220*** (2.948)	5.855*** (1.200)
Observations	40	40	40
R^2	0.272	0.006	0.905
Adjusted R^2	0.253	−0.020	0.900
Residual Std. Error	1.802 (df = 38)	5.793 (df = 38)	1.814 (df = 37)
Note:			*p<0.1; **p<0.05; ***p<0.01

Trends Matter, Part II

Table: FE Models of WBLI

	FE	FE.trend	FE.intx
Population Growth	-0.316^{**}	-0.488^{***}	-0.458^{***}
	(0.135)	(0.113)	(0.113)
Urban Population	7.459^{***}	0.393	0.762^{*}
	(0.471)	(0.412)	(0.414)
Fertility Rate	-3.350^{***}	2.573^{***}	2.488^{***}
	(0.293)	(0.265)	(0.264)
ln(GDP Per Capita)	12.790^{***}	3.486^{***}	3.179^{***}
	(0.428)	(0.391)	(0.392)
Natural Resource Rents	0.800^{***}	0.500^{***}	0.514^{***}
	(0.185)	(0.155)	(0.154)
Post-Cold War	3.472^{***}	-0.865^{***}	-3.943^{***}
	(0.144)	(0.141)	(0.454)
Trend (1950=0)		0.732^{***}	0.666^{***}
		(0.012)	(0.015)
Post-Cold War × Trend			0.090^{***}
			(0.013)
Observations	8,318	8,318	8,318
R^2	0.528	0.670	0.672
Adjusted R^2	0.517	0.662	0.664

$^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Another Approach: FEIS

"Fixed Effects Individual Slope" models

- Cite: Bruederl, Josef, and Volker Ludwig. 2015. "Fixed-Effects Panel Regression." In *The Sage Handbook of Regression Analysis and Causal Inference*, Eds. Henning Best and Christof Wolf. Los Angeles: Sage, pp. 327-357.
- FE + unit-level slopes for (some / all) predictor variables
- Equivalent to including $N-1$ interactions between a predictor $\mathbf{X}$ and each of the α_is
- Also can test for homogeneity of estimated slopes (Hausman-like test)
- See the `feisr` R package, and its accompanying vignette, or `xtfeis` in Stata

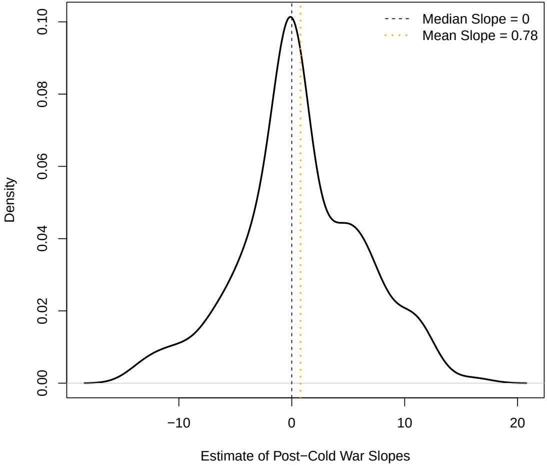
FEIS Example: Post-Cold War

```
> FEIS<-feis(WomenBusLawIndex~PopGrowth+UrbanPopulation+FertilityRate+lnGDPPerCap+NaturalResourceRents | PostColdWar,
+            data=(smol),id="ID",robust=FALSE)

> summary(FEIS)

Call:
feis(formula = WomenBusLawIndex ~ PopGrowth + UrbanPopulation +
    FertilityRate + lnGDPPerCap + NaturalResourceRents | PostColdWar,
    data = (smol), id = "ID", robust = FALSE)


Residuals:
   Min. 1st Qu.  Median 3rd Qu.    Max.
-26.445  -3.265   0.027   3.214  48.843

Coefficients:
                     Estimate Std. Error t-value   Pr(>|t|)
PopGrowth              -0.551      0.109   -5.08 0.00000039 ***
UrbanPopulation         9.151      0.554   16.52    < 2e-16 ***
FertilityRate          -7.165      0.321  -22.31    < 2e-16 ***
lnGDPPerCap            16.151      0.476   33.95    < 2e-16 ***
NaturalResourceRents    0.130      0.161    0.81       0.42
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Normal standard errors
Slope parameters:  PostColdWar
Total Sum of Squares:    513000
Residual Sum of Squares: 297000
R-Squared:      0.421
Adj. R-Squared: 0.421
```

```
> FEIS.test<-feistest(FEIS)
> summary(FEIS.test)


Call:
feis(formula = WomenBusLawIndex ~ PopGrowth + UrbanPopulation +
    FertilityRate + lnGDPPerCap + NaturalResourceRents | PostColdWar,
    data = (smol), id = "ID", robust = FALSE)

Artificial Regression Test

FEIS vs. FE:
------------
H0: FEIS and FE estimates consistent
Alternative H1: FE inconsistent
Model constraints: PopGrowth_hat, UrbanPopulation_hat, FertilityRate_hat,
lnGDPPerCap_hat, NaturalResourceRents_hat = 0

Chi-squared test:
Chisq = 1403.0, df = 5, P(> X2) = 0.0


FE vs. RE:
------------
H0: FE and RE estimates consistent
Alternative H1: RE inconsistent
Model constraints: PopGrowth_mean, UrbanPopulation_mean, FertilityRate_mean,
lnGDPPerCap_mean, NaturalResourceRents_mean, PostColdWar_mean = 0

Chi-squared test:
Chisq = 420.9, df = 6, P(> X2) = 0.0


FEIS vs. RE:
------------
H0: FEIS and RE estimates consistent
Alternative H1: RE inconsistent
Model constraints: PopGrowth_hat, UrbanPopulation_hat, FertilityRate_hat,
lnGDPPerCap_hat, NaturalResourceRents_hat = 0

Chi-squared test:
Chisq = 1566.7, df = 5, P(> X2) = 0.0
```

FEIS: Unit-Specific Slopes

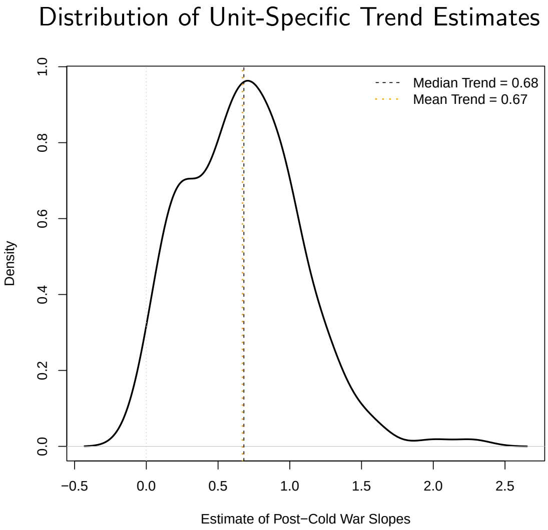
Distribution of Unit-Specific Slopes for *Post-Cold War*



FEIS: Unit-Specific Trends

```
> FEIS2<-feis(WomenBusLawIndex~PopGrowth+UrbanPopulation+FertilityRate+lnGDPPerCap+NaturalResourceRents+PostColdWar | Year,
+             data=(smol),id="ID",robust=FALSE)

> summary(FEIS2)

Call:
feis(formula = WomenBusLawIndex ~ PopGrowth + UrbanPopulation +
    FertilityRate + lnGDPPerCap + NaturalResourceRents + PostColdWar |
    Year, data = (smol), id = "ID", robust = FALSE)


Residuals:
     Min.   1st Qu.    Median   3rd Qu.      Max.
-18.35028  -2.44970  -0.00667   2.41306  40.80105

Coefficients:
                     Estimate Std. Error t-value Pr(>|t|)
PopGrowth             -0.3977     0.0817   -4.87  1.1e-06 ***
UrbanPopulation        0.6252     0.7412    0.84     0.40
FertilityRate          0.1747     0.3341    0.52     0.60
lnGDPPerCap            4.5056     0.5445    8.27  < 2e-16 ***
NaturalResourceRents  -0.1474     0.1214   -1.21     0.22
PostColdWar           -0.7071     0.1032   -6.85  7.8e-12 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Normal standard errors
Slope parameters:  Year
Total Sum of Squares:    164000
Residual Sum of Squares: 161000
R-Squared:      0.0183
Adj. R-Squared: 0.0176
```

FEIS: Unit-Specific Trends



Dynamic Models: Software

R :

- the `plm` package (`purtest` for unit roots; `plm` for first-difference models; `pgmm` for Arellano-Bond)
- the `panelvar` package (FD, A&B; Blundell and Bond (FOD))
- the `panelAR` package (GLS-ARMA models)
- the `gls` package (GLS)
- the `pdynmc` package (GMM models via moment conditions)
- the `dynpanel` package (A&H, A&B; minimal...)

Stata :

- `xtgls` (GLS)
- `xtpcse` (PCSEs)
- `xtabond` / `xtabond2` / `xtdpd` / `xtdpdsys` (A&H + A&B + etc. + dynamic models)
- Others...

Things to consider:

- N vs. T...
- Are dynamics nuisance or substance?
- What problem(s) do you *really* care about?