

GSERM 2023

Regression for Publishing

June 19, 2023

“Regression for Publishing”

- Instructor: Prof. Christopher Zorn
 - Email: zorn@psu.edu
 - Phone: +1-803-553-4077
 - Twitter / Instagram / etc.: [@prisonrodeo](#)
- Class: June 19-23, 2023, 09:15 - 15:15 CET, at the [University of St. Gallen SQUARE](#) building, room 11-0061 Rotmonten.
- The course outline / syllabus is [here](#).
- More important: The syllabus, slides, readings, code, data, etc. are all available on the course [github repo](#) (viewable at <https://github.com/PrisonRodeo/GSERM-RFP-2023>).

Evaluation at GSERM isn't easy... the plan:

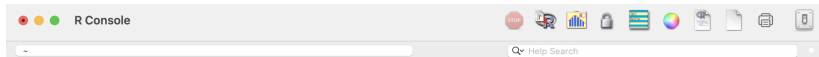
- One “homework exercise”
 - Practical exercise – “real” data analysis and discussion
 - Assigned Tuesday (June 20); due Friday (June 23)
 - Worth 300 possible points
- Final Examination
 - Multiple essay-style questions + “real” data analysis
 - Some choice of questions to answer
 - Assigned Friday (June 23)
 - Due either Friday, June 23, 2023 (“in-class” alternative) or Friday, June 30, 2023 (“take-home” alternative)
 - Worth 700 possible points
- Total course = 1000 possible points
- Grades assessed on Swiss (1 - 6) scale

R

- All examples, plots, etc. are generated using R
- Current version is 4.3.0
- Desktop: Be sure to get the [RStudio](#) / [Posit IDE](#)...
- Alternatively: Can be run in a browser, using [Posit Cloud](#)
- The course Github repo contains a bit of [introductory code](#) for people who may never have used R, and a list of [R resources](#).

Stata

- Current version is 18
- Main linear regression command is `-regress-` (or `reg`)



```
R version 4.3.0 (2023-04-21) -- "Already Tomorrow"  
Copyright (C) 2023 The R Foundation for Statistical Computing  
Platform: aarch64-apple-darwin20 (64-bit)
```

```
R is free software and comes with ABSOLUTELY NO WARRANTY.  
You are welcome to redistribute it under certain conditions.  
Type 'license()' or 'licence()' for distribution details.
```

```
  Natural language support but running in an English locale
```

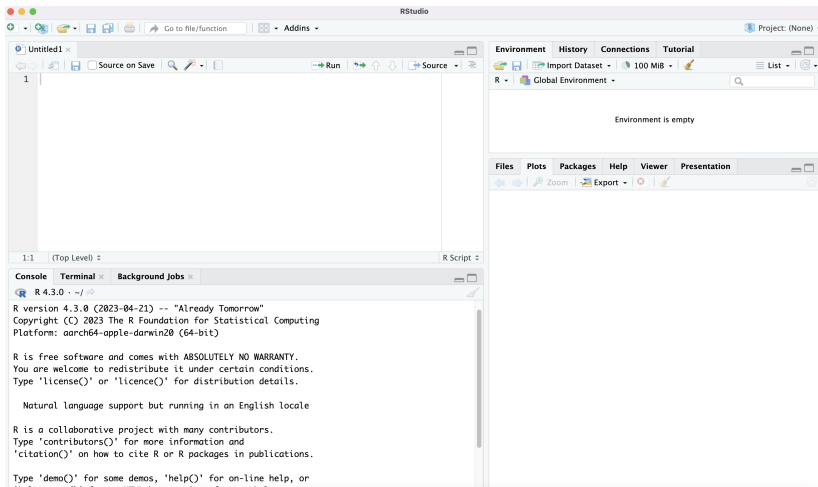
```
R is a collaborative project with many contributors.  
Type 'contributors()' for more information and  
'citation()' on how to cite R or R packages in publications.
```

```
Type 'demo()' for some demos, 'help()' for on-line help, or  
'help.start()' for an HTML browser interface to help.  
Type 'q()' to quit R.
```

```
[R.app GUI 1.79 (8225) aarch64-apple-darwin20]
```

```
[History restored from /Users/cuz10/.Rapp.history]
```

```
> |
```



RStudio (annotated)

This is the "Source" window.

- It's the place where you'll type the code that will then be sent to R.
- It's basically a text editor. You can open text files of any kind here if you want.
- Files that appear here end in (and should be saved with) the extension ".R" (as in "MyCode.R").

You'll spend most of your time working here.

Click here to save your source code. Save often!

Highlight text in the Source window, then click this button to "run" the code.

This is the "Environment" window. It is where you can find all the various "objects" that you create, grouped by object type (data frames, lists, graphs, etc.). Environment is empty

There's also a "History" tab above; switching to that will show what has transpired in the Console window recently.

This is the "working directory." Anything you save will be saved here, unless you tell the program to save it somewhere else.

This is the "Console." When you run the code in the Source window, the results that aren't graphics appear here.

Type 'demo()' for some demos, 'help()' for on-line help, or 'help.start()' for an HTML browser interface to help. Type 'q()' to quit R.

Plots (graphs) that you have created

Packages that are loaded

Help results (obtained by typing "?XXX" in the Console window, e.g. "?table").

Things We Will And Won't Do

Will: "Regression":

$$Y = f(\mathbf{X})$$

Won't: Multivariate regression:

$$\mathbf{Y} = f(\mathbf{X})$$

Won't: Measurement (e.g. PCA, factor analysis, IRT, etc.):

$$\mathbf{Y} = \mathbf{W}^T \mathbf{X}$$

Won't: Classification:

- Cluster Analysis / Network Models / etc.
- Classification and Regression Trees \rightarrow Random Forests.
- Pattern Recognition
- Machine Learning (beyond regression), Support Vector Machines, etc.

Friday = “Participants’ Choice”



PennState

Please rank order these possible choices for topics for Friday's class session (1 = most preferred to 7 = least preferred). When you have finished ranking them, please click on the arrow below.

Extended GLMs

Sample Selection Models

Causal Inference

Survival Analysis

Panel / TSCS Models

Analysis of Text / LLMs

Measurement Models

Why Regression?

	Description	Explanation	Prediction
Task	Summarize data	Correlation/causation	Forecast OOS / future data
Emphasis	Data	Theory / Hypotheses	Outcomes
Focus	Univariate	Multivariate	Multivariate
Typical Application	Summarize / "reduce" data	Discuss marginal associations between predictors and an outcome of interest	Optimize out-of-sample predictive power / minimize prediction error

“Regression,” conceptually:

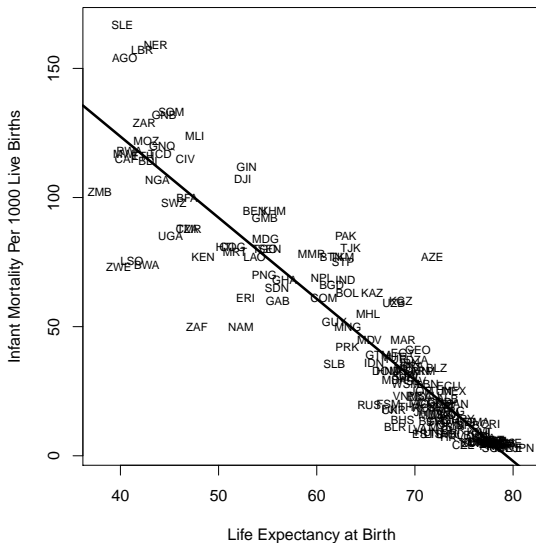
$$\Pr(Y|\mathbf{X}) = f(\mathbf{X})$$

Two important things:

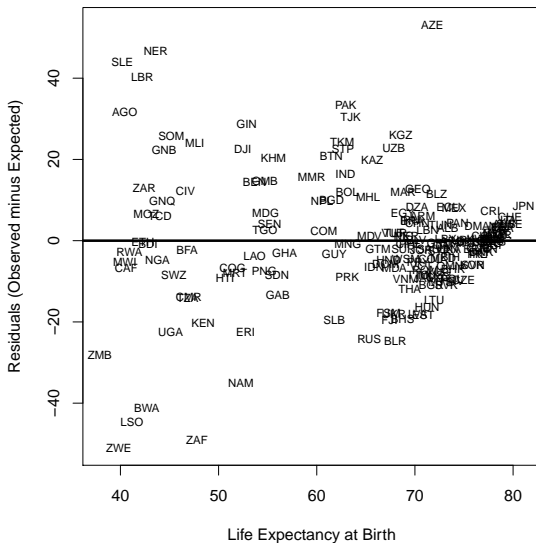
- The distribution of Y is *conditional on all variables in \mathbf{X}* , and
- The conditional distribution of Y is conditional on the *joint distribution* of the elements of \mathbf{X} .

→ Regression is hard...

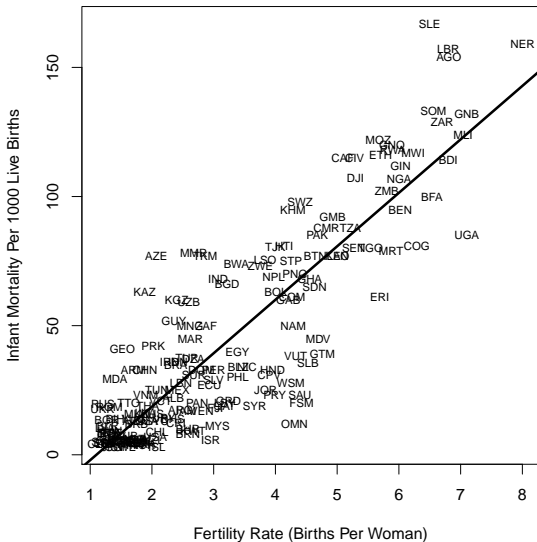
Example: Infant Mortality and Life Expectancy



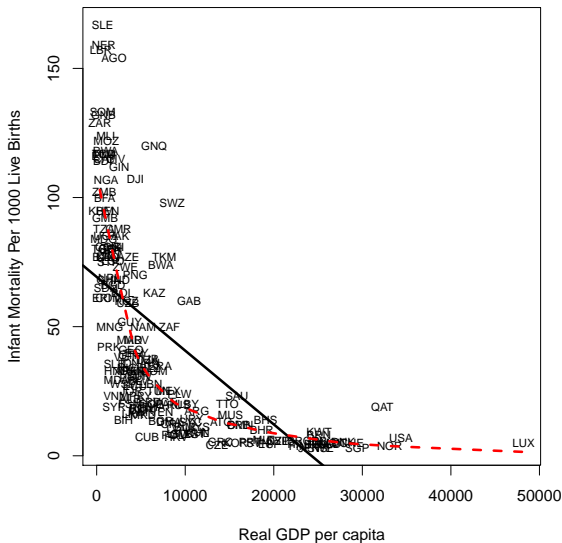
Infant Mortality and Life Expectancy: “Residuals”



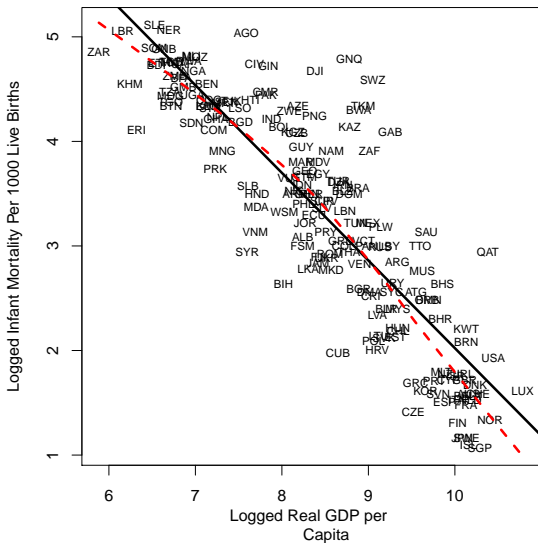
Infant Mortality and Fertility



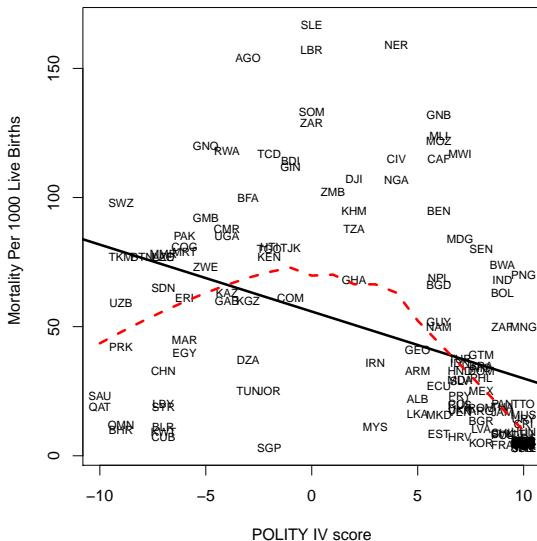
Infant Mortality and Wealth



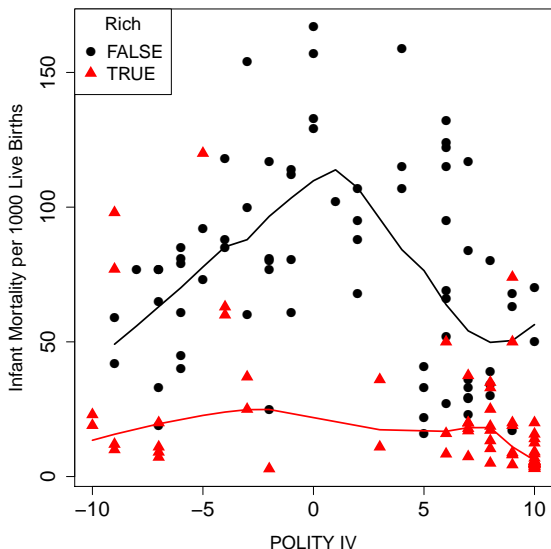
(Logged) Infant Mortality and (Logged) Wealth



Infant Mortality and Democracy



Infant Mortality, (Dichotomized) Wealth, and Democracy



Consider random variable Y :

$$Y_i = \mu + u_i \quad (1)$$

$$\mu_i = \beta_0 + \beta_1 X_i$$

so:

$$Y_i = \beta_0 + \beta_1 X_i + u_i \quad (2)$$

Goals:

- Estimate $\hat{\beta}_0$ and $\hat{\beta}_1$
- Estimate the *variability* $\hat{\beta}_0$ and $\hat{\beta}_1$
- Assess *model fit*

Bivariate OLS - Estimation

For a bivariate regression:

$$\begin{aligned}\hat{\beta}_1 &= \frac{\sum_{i=1}^N (X_i - \bar{X})(Y_i - \bar{Y})}{\sum_{i=1}^N (X_i - \bar{X})^2} \\ &= \frac{\text{Covariance of } X \text{ and } Y}{\text{Variance of } X}\end{aligned}\tag{3}$$

and

$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}\tag{4}$$

$$\text{Var}(\hat{\beta}_1)$$

Assume (for now):

$$u_i \sim \text{i.i.d. } N(0, \sigma^2)$$

meaning:

$$\text{Var}(Y|X, \beta) = \sigma^2$$

so:

$$\begin{aligned} \text{Var}(\hat{\beta}_1) &= \text{Var} \left[\frac{\sum_{i=1}^N (X_i - \bar{X}) Y_i}{\sum_{i=1}^N (X_i - \bar{X})^2} \right] \\ &= \left[\frac{1}{\sum (X_i - \bar{X})^2} \right]^2 \sum (X_i - \bar{X})^2 \text{Var}(Y) \\ &= \left[\frac{1}{\sum (X_i - \bar{X})^2} \right]^2 \sum (X_i - \bar{X})^2 \sigma^2 \\ &= \frac{\sigma^2}{\sum (X_i - \bar{X})^2}. \end{aligned}$$

$$\text{Var}(\hat{\beta}_0) \text{ and } \text{Cov}(\hat{\beta}_0, \hat{\beta}_1)$$

Similarly:

$$\text{Var}(\hat{\beta}_0) = \frac{\sum X_i^2}{N \sum (X_i - \bar{X})^2} \sigma^2$$

and :

$$\text{Cov}(\hat{\beta}_0, \hat{\beta}_1) = \frac{-\bar{X}}{\sum (X_i - \bar{X})^2} \sigma^2$$

Note that:

- $\text{Var}(\hat{\beta}_0)$ and $\text{Var}(\hat{\beta}_1) \propto \sigma^2$
- $\text{Var}(\hat{\beta}_0)$ and $\text{Var}(\hat{\beta}_1) \propto -\sum (X_i - \bar{X})$
- $\text{Var}(\hat{\beta}_0)$ and $\text{Var}(\hat{\beta}_1) \propto -N$
- $\text{sign}[\text{Cov}(\hat{\beta}_0, \hat{\beta}_1)] = -\text{sign}(\bar{X})$

If $u_i \sim N(0, \sigma^2)$, then:

$$\hat{\beta}_0 \sim N[\beta_0, \text{Var}(\hat{\beta}_0)]$$

and

$$\hat{\beta}_1 \sim N[\beta_1, \text{Var}(\hat{\beta}_1)]$$

Means:

$$\begin{aligned} z_{\hat{\beta}_1} &= \frac{(\hat{\beta}_1 - \beta_1)}{\sqrt{\text{Var}(\hat{\beta}_1)}} \\ &= \frac{(\hat{\beta}_1 - \beta_1)}{\text{s.e.}(\hat{\beta}_1)} \\ &= \sim N(0, 1) \end{aligned}$$

A Small Problem...

$$\sigma^2 = ???$$

Solution: use

$$\hat{\sigma}^2 = \frac{\sum \hat{u}_i^2}{N - k}$$

Gives:

$$\widehat{\text{Var}(\hat{\beta}_1)} = \frac{\hat{\sigma}^2}{\sum (X_i - \bar{X})^2},$$

and

$$\widehat{\text{Var}(\hat{\beta}_0)} = \frac{\sum X_i^2}{N \sum (X_i - \bar{X})^2} \hat{\sigma}^2$$

So:

$$\begin{aligned}
 \widehat{\text{s.e.}}(\hat{\beta}_1) &= \sqrt{\widehat{\text{Var}}(\hat{\beta}_1)} \\
 &= \sqrt{\frac{\hat{\sigma}^2}{\sum (X_i - \bar{X})^2}} \\
 &= \frac{\hat{\sigma}}{\sqrt{\sum (X_i - \bar{X})^2}}
 \end{aligned}$$

which implies:

$$\begin{aligned}
 t_{\hat{\beta}_1} \equiv \frac{(\hat{\beta}_1 - \beta_1)}{\widehat{\text{s.e.}}(\hat{\beta}_1)} &= \frac{(\hat{\beta}_1 - \beta_1)}{\frac{\hat{\sigma}}{\sqrt{\sum (X_i - \bar{X})^2}}} \\
 &= \frac{(\hat{\beta}_1 - \beta_1) \sqrt{\sum (X_i - \bar{X})^2}}{\hat{\sigma}} \\
 &\sim t_{N-k}
 \end{aligned}$$

Predictions and Variance

Point prediction:

$$\hat{Y}_k = \hat{\beta}_0 + \hat{\beta}_1 X_k$$

Y_k is unbiased:

$$\begin{aligned} E(\hat{Y}_k) &= E(\hat{\beta}_0 + \hat{\beta}_1 X_k) \\ &= E(\hat{\beta}_0) + X_k E(\hat{\beta}_1) \\ &= \beta_0 + \beta_1 X_k \\ &= E(Y_k) \end{aligned}$$

Variability:

$$\begin{aligned} \text{Var}(\hat{Y}_k) &= \text{Var}(\hat{\beta}_0 + \hat{\beta}_1 X_k) \\ &= \frac{\sum X_i^2}{N \sum (X_i - \bar{X})^2} \sigma^2 + \left[\frac{\sigma^2}{\sum (X_i - \bar{X})^2} \right] X_k^2 + 2 \left[\frac{-\bar{X}}{\sum (X_i - \bar{X})^2} \sigma^2 \right] X_k \\ &= \sigma^2 \left[\frac{1}{N} + \frac{(X_k - \bar{X})^2}{\sum (X_i - \bar{X})^2} \right] \end{aligned}$$

Variability of Predictions

Prediction variation:

$$\text{Var}(\hat{Y}_k) = \sigma^2 \left[\frac{1}{N} + \frac{(X_k - \bar{X})^2}{\sum (X_i - \bar{X})^2} \right]$$

means that $\text{Var}(\hat{Y}_k)$:

- Decreases in N
- Decreases in $\text{Var}(X)$
- Increases in $|X - \bar{X}|$

Standard error of the prediction:

$$\widehat{\text{s.e.}}(\hat{Y}_k) = \sqrt{\sigma^2 \left[\frac{1}{N} + \frac{(X_k - \bar{X})^2}{\sum (X_i - \bar{X})^2} \right]}$$

→ (e.g.) confidence intervals:

$$95\% \text{ c.i.}(\hat{Y}_k) = \hat{Y}_k \pm [1.96 \times \widehat{\text{s.e.}}(\hat{Y}_k)]$$

We can decompose variation in Y :

$$\begin{aligned}\text{Var}(Y) &= \text{Var}(\hat{Y} + \hat{u}) \\ &= \text{Var}(\hat{Y}) + \text{Var}(\hat{u}) + 2 \text{Cov}(\hat{Y}, \hat{u}) \\ &= \text{Var}(\hat{Y}) + \text{Var}(\hat{u})\end{aligned}$$

$$\begin{array}{ccccc}\mathbf{TSS} & = & \mathbf{MSS} & + & \mathbf{RSS} \\ \text{("Total")} & & \text{("Estimated," or "Model")} & & \text{("Residual")}\end{array}$$

“R-squared” :

$$\begin{aligned} R^2 &= \frac{\text{MSS}}{\text{TSS}} \\ &= \frac{\sum(\hat{Y}_i - \bar{Y})^2}{\sum(Y_i - \bar{Y})^2} \\ &= 1 - \frac{\text{RSS}}{\text{TSS}} \\ &= 1 - \frac{\sum \hat{u}_i^2}{\sum(Y_i - \bar{Y})^2} \end{aligned}$$

R-squared:

- is “the proportion of variance explained”
- $\in [0, 1]$
 - $R^2 = 1.0 \equiv$ a “perfect (linear) fit”
 - $R^2 = 0 \equiv$ no (linear) $X - Y$ association

For a single X ,

$$\begin{aligned} R^2 &= \hat{\beta}_1^2 \frac{\sum (X_i - \bar{X})^2}{\sum (Y_i - \bar{Y})^2} \\ &= r_{XY}^2 \end{aligned}$$

“Adjusted” R^2 :

$$R_{adj.}^2 = 1 - \frac{(1 - R^2)(N - c)}{(N - k)}$$

where $c = 1$ if there is a constant in the model and $c = 0$ otherwise.

$R_{adj.}^2$:

- $R_{adj.}^2 \rightarrow R^2$ as $N \rightarrow \infty$
- $R_{adj.}^2$ can be > 1 , or < 0 ...
- $R_{adj.}^2$ increases with model “fit,” but
- The extent of that increase is discounted by a factor proportional to the number of covariates.

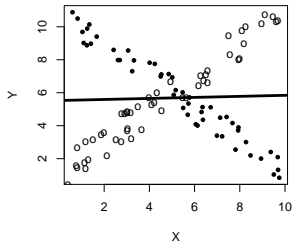
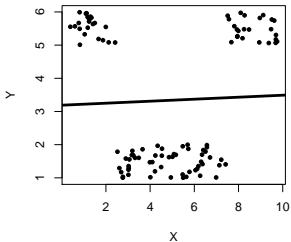
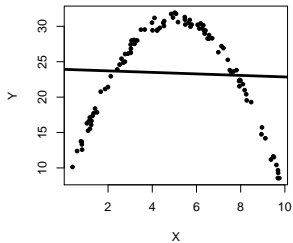
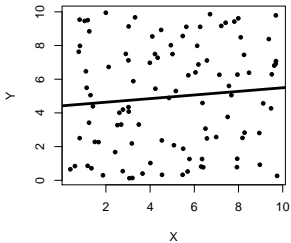
Alternative single measures of model fit include:

- The Standard Error of the Estimate:

$$SEE = \sqrt{\frac{RSS}{N - k}}$$

- F -tests
- ROC / AUC
- Graphical methods

Caution: Different Ways to get $R^2 = 0$



Linear Regression: K Predictors

Now consider:

$$\underset{N \times 1}{\mathbf{Y}} = \underset{N \times K}{\mathbf{X}} \underset{K \times 1}{\boldsymbol{\beta}} + \underset{N \times 1}{\mathbf{u}}$$

equivalently:

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \dots + \beta_K X_{Ki} + u_i$$

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_N \end{bmatrix} = \begin{bmatrix} 1 & X_{11} & X_{21} & \cdots & X_{K1} \\ 1 & X_{12} & X_{22} & \cdots & X_{K2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & X_{1N} & X_{2N} & \cdots & X_{KN} \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_K \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_N \end{bmatrix}.$$

Residuals:

$$\mathbf{u} = \mathbf{Y} - \mathbf{X}\beta$$

The inner product of \mathbf{u} :

$$\begin{aligned} \mathbf{u}'\mathbf{u} &= \begin{bmatrix} u_1 & u_2 & \cdots & u_N \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_N \end{bmatrix} \\ &= u_1^2 + u_2^2 + \dots + u_N^2 \\ &= \sum_{i=1}^N u_i^2 \end{aligned}$$

Sum of squared residuals:

$$\begin{aligned}\mathbf{u}'\mathbf{u} &= (\mathbf{Y} - \mathbf{X}\beta)'(\mathbf{Y} - \mathbf{X}\beta) \\ &= \mathbf{Y}'\mathbf{Y} - 2\beta'\mathbf{X}'\mathbf{Y}' + \beta'\mathbf{X}'\mathbf{X}\beta\end{aligned}$$

Now get:

$$\frac{\partial \mathbf{u}'\mathbf{u}}{\partial \beta} = -2\mathbf{X}'\mathbf{Y} + 2\mathbf{X}'\mathbf{X}\beta$$

Solve:

$$\begin{aligned}-2\mathbf{X}'\mathbf{Y} + 2\mathbf{X}'\mathbf{X}\beta &= 0 \\ -\mathbf{X}'\mathbf{Y} + \mathbf{X}'\mathbf{X}\beta &= 0 \\ \mathbf{X}'\mathbf{X}\beta &= \mathbf{X}'\mathbf{Y} \\ (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{X}\beta &= (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y} \\ \beta &= (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}\end{aligned}$$

The Importance of $\mathbf{V}(\hat{\beta})$

Variance-covariance matrix:

$$\begin{aligned}\mathbf{V}(\hat{\beta}) &= E[\hat{\beta} - E(\hat{\beta})]^2 \\ &= E\{[\hat{\beta} - E(\hat{\beta})][\hat{\beta} - E(\hat{\beta})]'\}\end{aligned}$$

Rewrite:

$$\begin{aligned}\mathbf{V}(\hat{\beta}) &= E(\hat{\beta} - \beta)(\hat{\beta} - \beta)' \\ &= E\{[(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{u}][(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{u}]'\} \\ &= E[(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{u}\mathbf{u}'\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}]\end{aligned}$$

The Importance of $\mathbf{V}(\hat{\beta})$

Taking expectations:

$$\begin{aligned}\mathbf{V}(\hat{\beta}) &= (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{E}(\mathbf{u}\mathbf{u}')\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1} \\ &= (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\sigma^2\mathbf{I}\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1} \\ &= \sigma^2(\mathbf{X}'\mathbf{X})^{-1}\end{aligned}$$

Estimating $\mathbf{V}(\hat{\beta})$

Empirical estimate:

$$\hat{\sigma}^2 = \frac{\hat{\mathbf{u}}' \hat{\mathbf{u}}}{N - K}$$

Yields:

$$\widehat{\mathbf{V}(\hat{\beta})} = \hat{\sigma}^2 (\mathbf{X}' \mathbf{X})^{-1}$$

OLS Assumptions

1. Zero Expectation Disturbances

$$E(\mathbf{u}) = \mathbf{0}$$

2. Homoscedasticity / No Error Correlation

$$E(\mathbf{u}\mathbf{u}') = \begin{bmatrix} \sigma^2 & 0 & \cdots & 0 \\ 0 & \sigma^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma^2 \end{bmatrix}$$

OLS Assumptions (continued)

3. "Fixed" \mathbf{X} ...

- No *measurement error* in the \mathbf{X} s, and
- $\text{Cov}(\mathbf{X}, \mathbf{u}) = \mathbf{0}$.

4. \mathbf{X} is of full column rank.

Means:

- no exact linear relationship among \mathbf{X} , and
- $K < N$.

5. Normal Disturbances

$$\mathbf{u} \sim \mathbf{N}(\mathbf{0}, \sigma^2 \mathbf{I})$$

Under these assumptions, the OLS estimate of $\hat{\beta}$ is:

- **Unbiased**
- **Fully Efficient**

(i.e., **“BLUE”**)

Example Data: Infant Mortality

```
> # Summary statistics
>
> # install.packages("psych") <- Install psych package, if necessary
> library(psych)

> with(IR2000, describe(infantmortalityperK))
  vars   n mean    sd median trimmed   mad min max range skew kurtosis   se
1     1 179 43.83 40.39    29  38.38 34.26 2.9 167 164.1    1    0.06 3.02

> with(IR2000, describe(DPTpct))
  vars   n mean    sd median trimmed   mad min max range skew kurtosis   se
1     1 181 81.71 19.77    90  85.23 11.86 24  99   75 -1.31    0.57 1.47
```

OLS Regression

```
> IMDPT<-lm(infantmortalityperK~DTPtct,data=IR2000,na.action=na.exclude)
> summary.lm(IMDPT)
```

Call:

```
lm(formula = infantmortalityperK ~ DTPtct, data = Data)
```

Residuals:

Min	1Q	Median	3Q	Max
-56.801	-16.328	-5.105	11.777	86.590

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	173.2771	8.4893	20.41	<2e-16 ***
DTPtct	-1.5763	0.1009	-15.62	<2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 26.19 on 175 degrees of freedom
(14 observations deleted due to missingness)

Multiple R-squared: 0.5824, Adjusted R-squared: 0.58

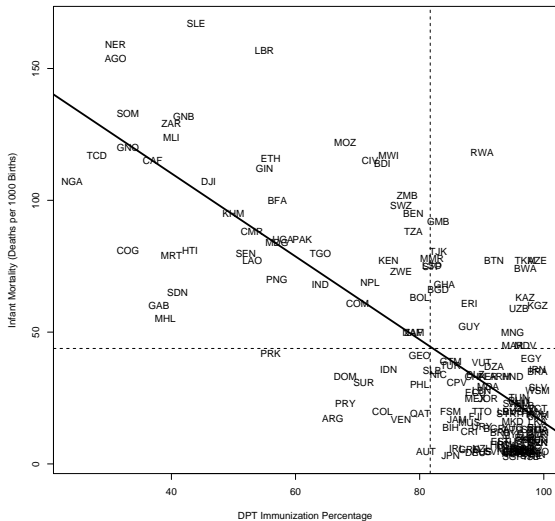
F-statistic: 244.1 on 1 and 175 DF, p-value: < 2.2e-16

Analysis of Variance

```
> anova(IMDPT)
Analysis of Variance Table

Response: infantmortalityperK
          Df Sum Sq Mean Sq F value    Pr(>F)
DPTpct      1 167423  167423   244.09 < 2.2e-16 ***
Residuals 175 120033      686
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

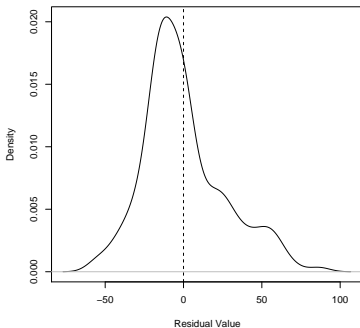
Regression of Infant Mortality on DPT Immunization Rates



Fitted Values, Residuals, etc.

```
> # Residuals (u):  
> IR2000$IMDPTres <- with(IR2000, residuals(IMDPT))  
> describe(IR2000$IMDPTres)
```

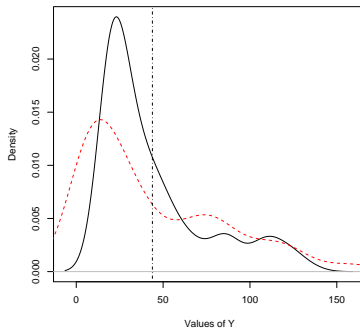
	var	n	mean	sd	median	mad	min	max	range	skew	kurtosis	se
1	1	177	0	26.12	-5.1	19.42	-56.8	86.59	143.4	0.75	0.44	1.96



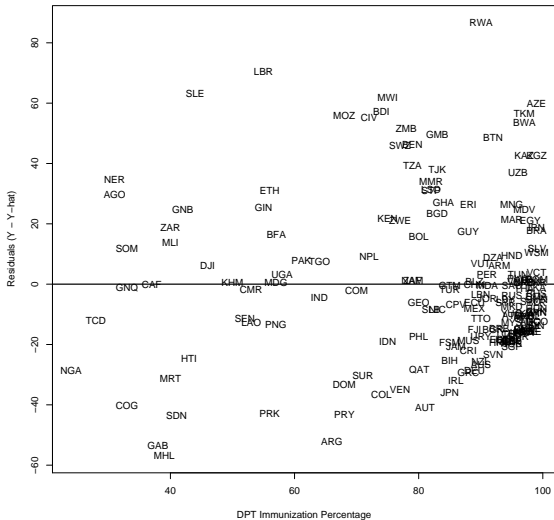
```
> # Fitted Values:  
> IR2000$IMDPThat<-fitted.values(IMDPT)  
> describe(IR2000$IMDPThat)
```

	var	n	mean	sd	median	mad	min	max	range	skew	kurtosis	se
1	1	177	44.26	30.84	31.41	18.7	17.22	135.4	118.2	1.3	0.59	2.32

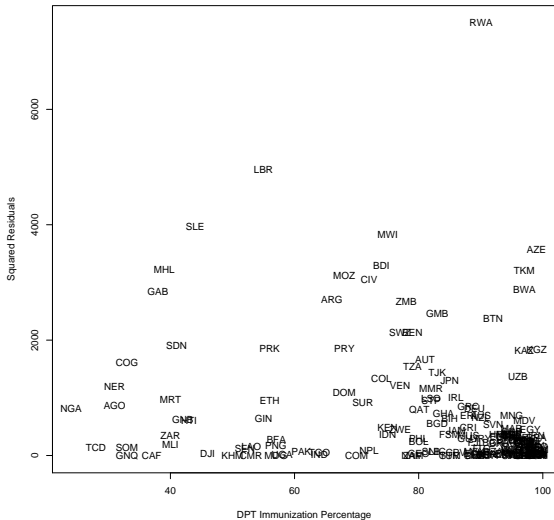
Density Plot: Actual (Y) and Fitted Values (\hat{Y})



Regression Residuals (\hat{u}) vs. DPT Percentage



Squared Residuals vs. DPT Percentage



$\text{Var}(\hat{\beta})$:

```
> vcov(IMDPT)
```

	(Intercept)	DPTpct
(Intercept)	72.0677	-0.83317
DPTpct	-0.8332	0.01018

95 percent c.i.s:

```
> confint(IMDPT)
```

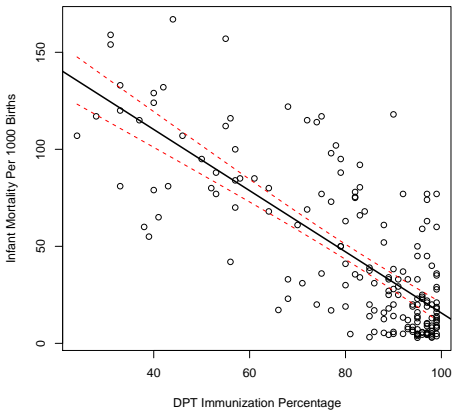
	2.5 %	97.5 %
(Intercept)	156.523	190.032
DPTpct	-1.775	-1.377

```
> SEs<-predict(IMDPT,interval="confidence")
> SEs
```

	fit	lwr	upr
1	25.10	20.53	29.68
3	17.22	12.05	22.40
4	23.53	18.84	28.21
.			
.			
<rows omitted>			
.			
.			
189	21.95	17.15	26.75
190	39.29	35.36	43.23
191	17.22	12.05	22.40

A Plot, With CIs

Scatterplot of Infant Mortality and DPT Immunizations, along with Least-Squares Line and 95% Prediction Confidence Intervals



Multivariate Example: Africa Data

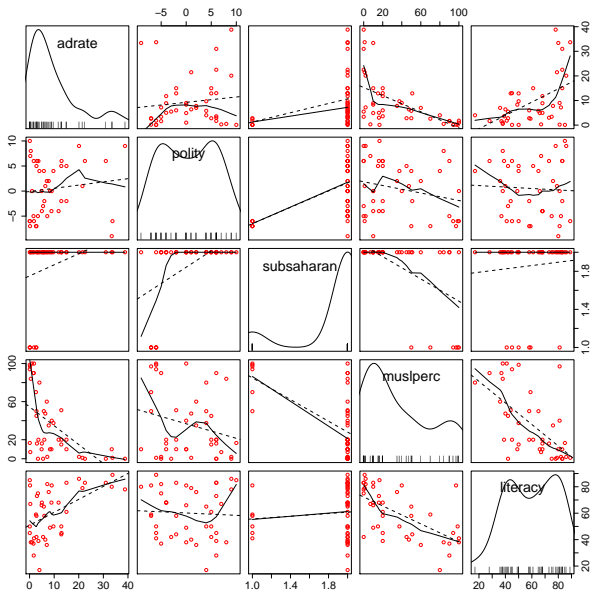
```
> Data<-read_csv("https://github.com/PrisonRodeo/GSERM-RFP-2023/raw/main/Data/africa2001.csv")

> Data<-with(Data, data.frame(adrate, polity,
+                             subsaharan=as.numeric(as.factor(subsaharan))-1,
+                             muslperc, literacy))

> describe(Data)
      vars  n  mean   sd median trimmed  mad  min   max range  skew kurtosis   se
adrate    1 43  9.37  9.96     6    7.58  6.38  0.1  38.8  38.7  1.44    1.23 1.52
polity    2 43  0.51  5.41     0    0.46  7.41 -9.0  10.0  19.0  0.01   -1.38 0.82
subsaharan 3 43  0.86  0.35     1    0.94  0.00  0.0   1.0   1.0 -2.01    2.08 0.05
muslperc   4 43 35.96 34.58    20   32.87 29.65  0.0 100.0 100.0  0.68   -1.04 5.27
literacy   5 43 60.07 18.94    61   60.63 26.69 17.0  89.0  72.0 -0.20   -1.18 2.89

> cor(Data)
      adrate  polity subsaharan muslperc literacy
adrate  1.0000  0.11794   0.33129  -0.5709  0.51489
polity   0.1179  1.00000   0.52820  -0.2392 -0.05079
subsaharan 0.3313  0.52820   1.00000  -0.5773  0.09473
muslperc -0.5709 -0.23917  -0.57725   1.0000 -0.61960
literacy  0.5149 -0.05079   0.09473  -0.6196  1.00000
```


Africa Data



A Regression

```
> model<-lm(adrate~polity+subsaharan+muslperc+literacy,data=Data)
```

```
> summary(model)
```

Call:

```
lm(formula = adrate ~ polity + subsaharan + muslperc + literacy,
    data = Data)
```

Residuals:

Min	1Q	Median	3Q	Max
-15.468	-4.395	-0.525	3.425	22.936

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-0.6687	10.4113	-0.06	0.949
polity	-0.0139	0.2797	-0.05	0.961
subsaharan	3.7297	5.4309	0.69	0.496
muslperc	-0.0869	0.0628	-1.38	0.175
literacy	0.1657	0.0943	1.76	0.087

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 8.26 on 38 degrees of freedom

Multiple R-squared: 0.377, Adjusted R-squared: 0.312

F-statistic: 5.75 on 4 and 38 DF, p-value: 0.00101

Variance-Covariance Matrix of $\hat{\beta}$

```
> options(digits=4)
> vcov(model)
```

	(Intercept)	polity	subsaharan	muslperc	literacy
(Intercept)	223.4259	1.088030	-72.2628	-0.771309	-1.002421
polity	1.0880	0.078229	-0.6642	-0.000293	0.001968
subsaharan	-72.2628	-0.664212	29.4950	0.206067	0.171765
muslperc	-0.7713	-0.000293	0.2061	0.003946	0.004098
literacy	-1.0024	0.001968	0.1718	0.004098	0.008898

Test $H_0 : \beta_{\text{polity}} = \beta_{\text{subsaharan}} = 0$:

```
> library(lmtest)
> modelsmall<-lm(adrate~muslperc+literacy,data=Data)
> waldtest(model,modelsmall)
```

Wald test

Model 1: adrate ~ polity + subsaharan + muslperc + literacy

Model 2: adrate ~ muslperc + literacy

	Res.Df	Df	F	Pr(>F)
1	38			
2	40	-2	0.27	0.76

Test $H_0 : \beta_{\text{muslperc}} = 0.1$:

```
> library(car)
> linearHypothesis(model,"muslperc=0.1")
```

Linear hypothesis test

Hypothesis:
muslperc = 0.1

Model 1: restricted model

Model 2: adrate ~ polity + subsaharan + muslperc + literacy

	Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
1	39	3200				
2	38	2595	1	605	8.85	0.0051 **

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Test $H_0 : \beta_{\text{literacy}} = \beta_{\text{muslperc}}$:

```
> linearHypothesis(model,"literacy=muslperc")
```

Linear hypothesis test

Hypothesis:

- muslperc + literacy = 0

Model 1: restricted model

Model 2: adrate ~ polity + subsaharan + muslperc + literacy

	Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
1	39	3534				
2	38	2595	1	938	13.7	0.00067 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

The output:

```
> summary(model)
```

```
Call:
```

```
lm(formula = adrate ~ polity + subsaharan + muslperc + literacy,
    data = Data)
```

```
Residuals:
```

	Min	1Q	Median	3Q	Max
	-15.468	-4.395	-0.525	3.425	22.936

```
Coefficients:
```

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-0.6687	10.4113	-0.06	0.949
polity	-0.0139	0.2797	-0.05	0.961
subsaharan	3.7297	5.4309	0.69	0.496
muslperc	-0.0869	0.0628	-1.38	0.175
literacy	0.1657	0.0943	1.76	0.087

```
---
```

```
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
Residual standard error: 8.26 on 38 degrees of freedom
```

```
Multiple R-squared:  0.377, Adjusted R-squared:  0.312
```

```
F-statistic: 5.75 on 4 and 38 DF,  p-value: 0.00101
```

The table:

Table 1: OLS Regression Model of HIV/AIDS Rates in Africa, 2001

	Model I
(Constant)	-0.67 (10.41)
POLITY Score	-0.01 (0.28)
Subsaharan Africa	3.73 (5.43)
Muslim Percentage of the Population	-0.09 (0.06)
Literacy Rate	0.17* (0.09)
Observations	43
R ²	0.38
Adjusted R ²	0.31
Residual Std. Error	8.26 (df = 38)
F Statistic	5.75* (df = 4; 38)

Note: N = 43. Cell entries are coefficient estimates; numbers in parentheses are estimated standard errors. Asterisks indicate $p < .05$ (one-tailed). See text for details.

Multiple Models (stargazer defaults)

OLS Regression Models of HIV/AIDS Rates in Africa, 2001

	w/Literacy	w/o Literacy
(Constant)	-0.67 (10.41)	14.81** (5.70)
POLITY Score	-0.01 (0.28)	-0.05 (0.29)
Subsaharan Africa	3.73 (5.43)	0.53 (5.25)
Muslim Percentage of the Population	-0.09 (0.06)	-0.16*** (0.05)
Literacy Rate	0.17* (0.09)	
Observations	43	43
R ²	0.38	0.33
Adjusted R ²	0.31	0.27
Residual Std. Error	8.26 (df = 38)	8.48 (df = 39)
F Statistic	5.75*** (df = 4; 38)	6.30*** (df = 3; 39)

Note:

* p<0.1; ** p<0.05; *** p<0.01

R

- LaTeX: texreg, xtable, and stargazer packages
- MS Word: generally cut-and-paste (see, e.g., here: <https://sejdemyr.github.io/r-tutorials/basics/tables-in-r/>); also KableExtra
- A pretty good summary of many others is here: <https://rfortherestofus.com/2019/11/how-to-make-beautiful-tables-in-r/>.

Stata

- estout and esttab commands are standard
- Others: outreg2, tabout, orth_out, etc. (a summary is here: <https://lukestein.github.io/stata-latex-workflows/>)
- MS Word: putdocx

Some Guidelines (“Rules”?)

Tables:

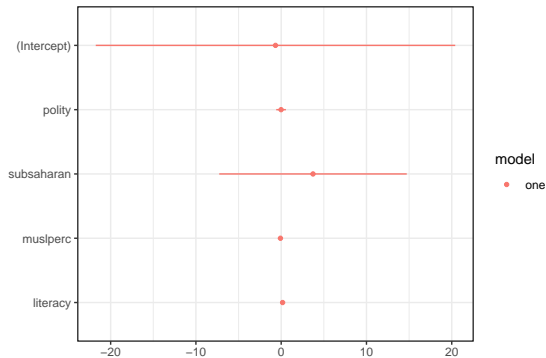
- *Use column headings descriptively.*
- *Use multiple rows / columns rather than multiple tables.*
- *Learn about significant digits, and don't report more than 4-5 of them (at the most).*
- *Use a figure to replace a table when you can.*
- *Be aware of norms about *s.*

Figures:

- *Report the scale of axes, and label them.*
- *Use as much “space” as you need, but no more.*
- *Use color sparingly.*

Plotting Regression Estimates

Ladderplot of OLS Results (using dotwhisker)



A la Gelman (2008):

- Continuous = divide by two standard deviations
- Binary = mean 0, difference of 1 between the two categories

```
> modelS<-standardize(model)
> summary(modelS)
```

Call:

```
lm(formula = adrate ~ z.polity + c.subsaharan + z.muslperc +
    z.literacy, data = Data)
```

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	9.37	1.26	7.43	0.0000000065 ***
z.polity	-0.15	3.03	-0.05	0.961
c.subsaharan	3.73	5.43	0.69	0.496
z.muslperc	-6.01	4.34	-1.38	0.175
z.literacy	6.28	3.57	1.76	0.087 .

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

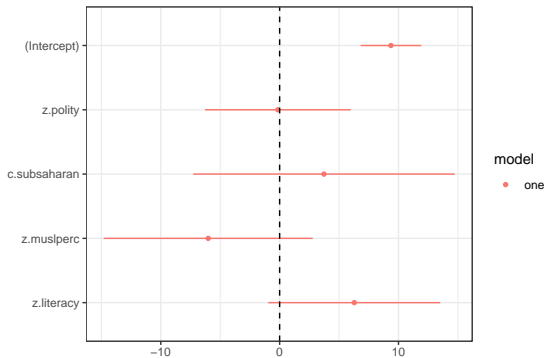
Residual standard error: 8.26 on 38 degrees of freedom

Multiple R-squared: 0.377, Adjusted R-squared: 0.312

F-statistic: 5.75 on 4 and 38 DF, p-value: 0.00101

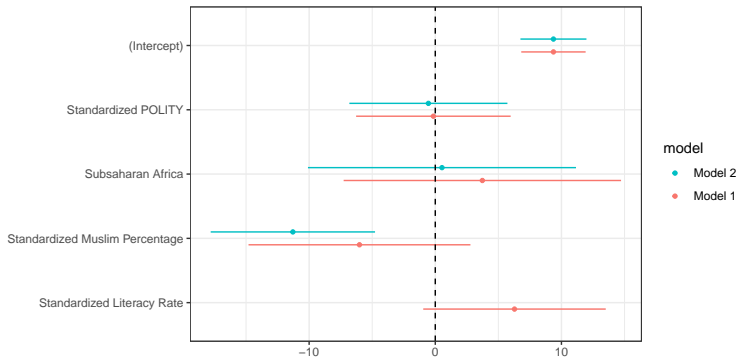
A Better Regression Plot

Ladderplot of Standardized OLS Results



An Even Better Regression Plot

Ladderplot of Standardized OLS Results



- *Be aware of the norms in your discipline / field, and follow them.*
- *Ask for advice.*
- *When in doubt, more information is (probably) better.*

Supplementary Materials

Hypothetically: If we have $\hat{\beta}_0$ and $\hat{\beta}_1$, then:

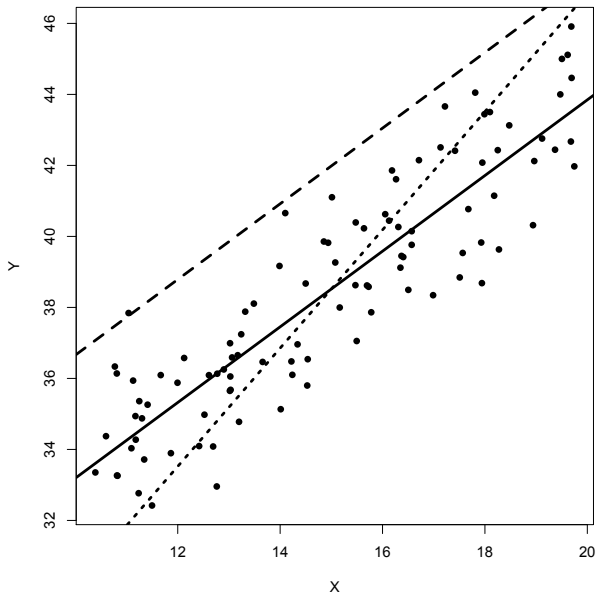
$$\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i$$

and

$$\begin{aligned}\hat{u}_i &= Y_i - \hat{Y}_i \\ &= Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_i\end{aligned}$$

Q: How to estimate $\hat{\beta}_0$ and $\hat{\beta}_1$?

Scatterplot: X and Y (with regression lines)



Ordinary Least Squares

Choose $\hat{\beta}_0$ and $\hat{\beta}_1$ to minimize $\hat{S} = \sum_{i=1}^N \hat{u}_i^2$.

$$\begin{aligned}\hat{S} &= \sum_{i=1}^N (Y_i - \hat{Y}_i)^2 \\ &= \sum_{i=1}^N (Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_i)^2 \\ &= \sum_{i=1}^N (Y_i^2 - 2Y_i\hat{\beta}_0 - 2Y_i\hat{\beta}_1 X_i + \hat{\beta}_0^2 + 2\hat{\beta}_0\hat{\beta}_1 X_i + \hat{\beta}_1^2 X_i^2)\end{aligned}$$

Differentiate:

$$\begin{aligned}\frac{\partial \hat{S}}{\partial \hat{\beta}_0} &= \sum_{i=1}^N (-2Y_i + 2\hat{\beta}_0 + 2\hat{\beta}_1 X_i) \\ &= -2 \sum_{i=1}^N (Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_i) \\ &= -2 \sum_{i=1}^N \hat{u}_i\end{aligned}$$

and

$$\begin{aligned}\frac{\partial \hat{S}}{\partial \hat{\beta}_1} &= \sum_{i=1}^N (-2Y_i X_i + 2\hat{\beta}_0 X_i + 2\hat{\beta}_1 X_i^2) \\ &= -2 \sum_{i=1}^N (Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_i) X_i \\ &= -2 \sum_{i=1}^N \hat{u}_i X_i\end{aligned}$$

Yields:

$$\sum_{i=1}^N Y_i = N\hat{\beta}_0 + \hat{\beta}_1 \sum_{i=1}^N X_i$$

and

$$\sum_{i=1}^N Y_i X_i = \hat{\beta}_0 \sum_{i=1}^N X_i + \hat{\beta}_1 \sum_{i=1}^N X_i^2$$

Solving yields:

$$\begin{aligned}\hat{\beta}_1 &= \frac{\sum_{i=1}^N (X_i - \bar{X})(Y_i - \bar{Y})}{\sum_{i=1}^N (X_i - \bar{X})^2} \\ &= \frac{\text{Covariance of } X \text{ and } Y}{\text{Variance of } X}\end{aligned}$$

and

$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}$$

“Stupid Regression Tricks”

Africa (2001) Data

```
> africa<-read_csv("https://raw.githubusercontent.com/PrisonRodeo/GSERM-RFP-2023/master/Data/africa2001.csv")
> summary(africa)
```

cocode	cabbr	country	population	pophou
Min. :404	AGO : 1	Angola	: 1 Min. : 470000	Min. : 470
1st Qu.:452	BDI : 1	Benin	: 1 1st Qu.: 3446000	1st Qu.: 3446
Median :510	BEN : 1	Botswana	: 1 Median : 9662000	Median : 9662
Mean :510	BWA : 1	Burundi	: 1 Mean : 17388558	Mean : 17390
3rd Qu.:556	CAF : 1	Cameroon	: 1 3rd Qu.: 19150000	3rd Qu.: 19189
Max. :651	CIV : 1	Central African Republic:	1 Max. :117000000	Max. :116929
	(Other):37	(Other)	:37	

popden	polity	gdppppd	tradegdp	war	adrate
Min. :0.0022	Min. :~9.000	Min. : 0.500	Min. : 4.03	Min. :0.000	Min. : 0.10
1st Qu.:0.0134	1st Qu.:~4.500	1st Qu.: 0.855	1st Qu.: 7.64	1st Qu.:0.000	1st Qu.: 2.70
Median :0.0357	Median : 0.000	Median : 1.200	Median : 13.56	Median :0.000	Median : 6.00
Mean :0.0643	Mean : 0.512	Mean : 2.159	Mean : 30.49	Mean :0.116	Mean : 9.37
3rd Qu.:0.0683	3rd Qu.: 5.500	3rd Qu.: 2.040	3rd Qu.: 30.01	3rd Qu.:0.000	3rd Qu.:12.90
Max. :0.5740	Max. :10.000	Max. :10.800	Max. :272.69	Max. :1.000	Max. :38.80

healthexp	subсахaran	muslperc	literacy	internalwar	intensity
Min. :2.00	Not Sub-Saharan: 6	Min. : 0.0	Min. :17.0	Min. :0.000	Min. :0.000
1st Qu.:3.45	Sub-Saharan :37	1st Qu.: 10.0	1st Qu.:43.0	1st Qu.:0.000	1st Qu.:0.000
Median :4.40		Median : 20.0	Median :61.0	Median :0.000	Median :0.000
Mean :4.60		Mean : 36.0	Mean :60.1	Mean :0.302	Mean :0.581
3rd Qu.:5.80		3rd Qu.: 55.5	3rd Qu.:78.5	3rd Qu.:1.000	3rd Qu.:1.000
Max. :8.60		Max. :100.0	Max. :89.0	Max. :1.000	Max. :3.000

A Simple Regression

```
> fit<-with(africa, lm(adrates~muslperc))  
> summary(fit)
```

Call:

```
lm(formula = adrate ~ muslperc)
```

Residuals:

Min	1Q	Median	3Q	Max
-13.828	-5.206	0.279	2.022	23.521

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	15.2787	1.8322	8.34	0.00000000023 ***
muslperc	-0.1644	0.0369	-4.45	0.00006390853 ***

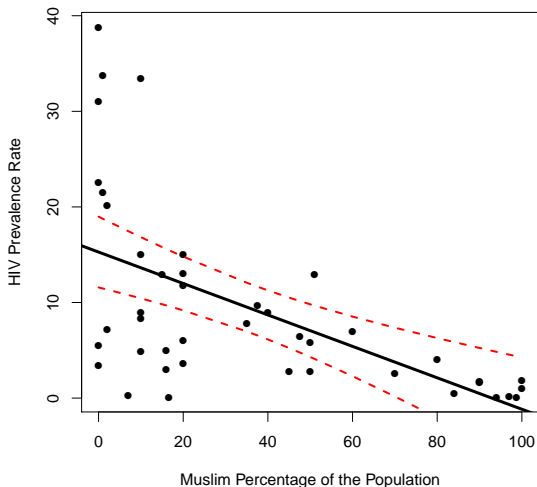
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 8.28 on 41 degrees of freedom

Multiple R-squared: 0.326, Adjusted R-squared: 0.31

F-statistic: 19.8 on 1 and 41 DF, p-value: 0.0000639

Scatterplot of HIV/AIDS Rates on Muslim Population Percentage, Africa 2001



Adding a Constant to X

```
> africa$muslplusten<-africa$muslperc+10
> fit2<-with(africa, lm(adrate~muslplusten,data=africa))
> summary(fit2)
```

Call:

```
lm(formula = adrate ~ muslplusten, data = africa)
```

Residuals:

Min	1Q	Median	3Q	Max
-13.828	-5.206	0.279	2.022	23.521

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	16.9232	2.1152	8.00	0.00000000066 ***
muslplusten	-0.1644	0.0369	-4.45	0.00006390853 ***

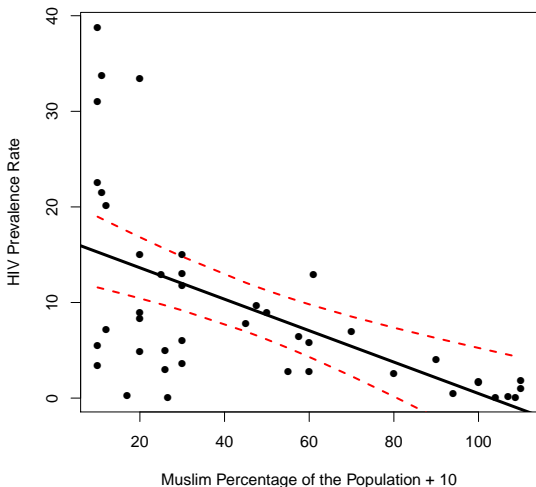
Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1

Residual standard error: 8.28 on 41 degrees of freedom

Multiple R-squared: 0.326, Adjusted R-squared: 0.31

F-statistic: 19.8 on 1 and 41 DF, p-value: 0.0000639

Scatterplot of HIV/AIDS Rates on Rescaled Muslim Population Percentage



Multiplying Y by a Constant

```
> africa$screwrate<-africa$adrate*(-314)
> fit3<-with(africa, lm(screwrate~muslperc))
> summary(fit3)
```

Call:

```
lm(formula = screwrate ~ muslperc)
```

Residuals:

Min	1Q	Median	3Q	Max
-7386	-635	-88	1635	4342

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-4797.5	575.3	-8.34	0.00000000023 ***
muslperc	51.6	11.6	4.45	0.00006390853 ***

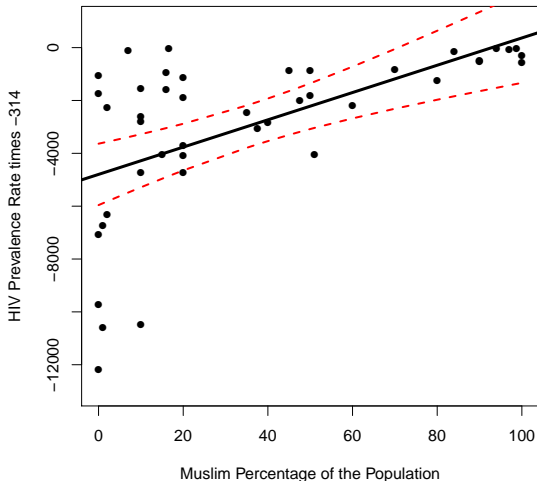
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2600 on 41 degrees of freedom

Multiple R-squared: 0.326, Adjusted R-squared: 0.31

F-statistic: 19.8 on 1 and 41 DF, p-value: 0.0000639

Scatterplot of Rescaled HIV/AIDS Rates on Muslim Population Percentage



Reversing the scales of X and Y

```
> africa$nonmuslimpct <- 100 - africa$muslperc  
> africa$noninfected <- 100 - africa$adrate  
> fit4<-lm(noninfected~nonmuslimpct,data=africa)  
> summary(fit4)
```

Call:

```
lm(formula = noninfected ~ nonmuslimpct, data = africa)
```

Residuals:

Min	1Q	Median	3Q	Max
-23.521	-2.022	-0.279	5.206	13.828

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	101.1660	2.6808	37.74	< 2e-16 ***
nonmuslimpct	-0.1644	0.0369	-4.45	0.000064 ***

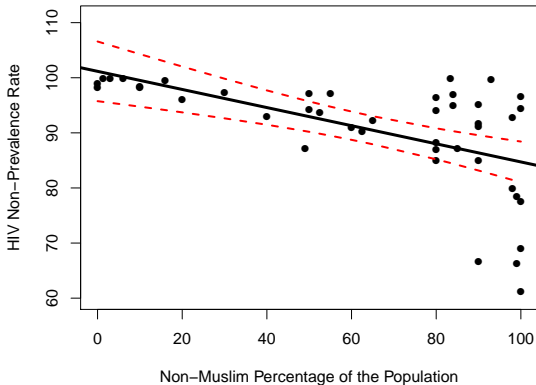
Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1

Residual standard error: 8.28 on 41 degrees of freedom

Multiple R-squared: 0.326, Adjusted R-squared: 0.31

F-statistic: 19.8 on 1 and 41 DF, p-value: 0.0000639

Scatterplot of HIV/AIDS Non-Infection Rates on Non-Muslim Population Percentage



Linear Transformations

- Adding (subtracting) a positive constant to X shifts the X -axis to the left (right).
- Adding (subtracting) a positive constant to Y shifts the Y -axis downwards (upwards).
- Multiplying X (Y) times a positive constant greater than 1.0 stretches the X (Y) axis.
- Multiplying X (Y) times a positive constant less than 1.0 shrinks the X (Y) axis.
- Multiplying X (Y) times a negative constant inverts the X (Y) axis, and stretches / shrinks it as above.

Use: “Centering” a Variable

```
> africa$muslcenter<-africa$muslperc - mean(africa$muslperc, na.rm=TRUE)
> fit5<-lm(adrate~muslcenter,data=africa)
> summary(fit5)
```

Call:

```
lm(formula = adrate ~ muslcenter, data = africa)
```

Residuals:

Min	1Q	Median	3Q	Max
-13.828	-5.206	0.279	2.022	23.521

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	9.3651	1.2622	7.42	0.0000000042 ***
muslcenter	-0.1644	0.0369	-4.45	0.0000639085 ***

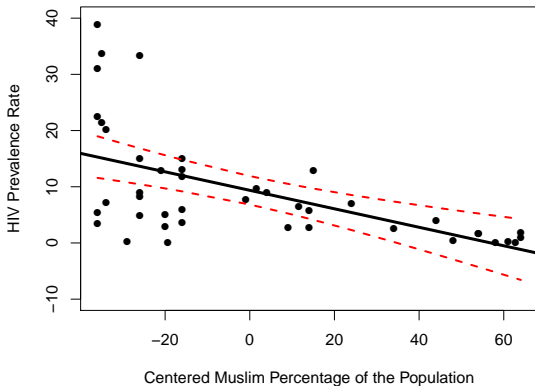
Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1

Residual standard error: 8.28 on 41 degrees of freedom

Multiple R-squared: 0.326, Adjusted R-squared: 0.31

F-statistic: 19.8 on 1 and 41 DF, p-value: 0.0000639

Scatterplot of HIV/AIDS Infection Rates on (Centered) Muslim Population Percentage



Use: Rescaling X for Interpretability

```
> fit6<-lm(adrate~population,data=africa)
> summary(fit6)
```

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	10.5883163475	1.9140361989	5.53	0.000002 ***
population	-0.0000000703	0.0000000671	-1.05	0.3

```
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1 1

Residual standard error: 9.95 on 41 degrees of freedom
Multiple R-squared:  0.0261, Adjusted R-squared:  0.00234
F-statistic: 1.1 on 1 and 41 DF, p-value: 0.301
```

```
> africa$popmil<-africa$population / 1000000
> fit7<-lm(adrate~popmil,data=africa)
> summary(fit7)
```

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	10.5883	1.9140	5.53	0.000002 ***
popmil	-0.0703	0.0671	-1.05	0.3

```
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1 1

Residual standard error: 9.95 on 41 degrees of freedom
Multiple R-squared:  0.0261, Adjusted R-squared:  0.00234
F-statistic: 1.1 on 1 and 41 DF, p-value: 0.301
```

Dichotomous Xs: Bivariate Regression \equiv *t*-test

```
> fit8<-lm(adrate~subsaharan,data=africa)
> summary(fit8)
```

Residuals:

Min	1Q	Median	3Q	Max
-10.58	-6.23	-1.78	2.22	28.12

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	1.27	3.88	0.33	0.75
subsaharanSub-Saharan	9.41	4.19	2.25	0.03 *

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 9.51 on 41 degrees of freedom

Multiple R-squared: 0.11, Adjusted R-squared: 0.088

F-statistic: 5.05 on 1 and 41 DF, p-value: 0.03

```
> with(africa,
+       t.test(adrate~subsaharan, var.equal=TRUE))
```

Two Sample t-test

data: adrate by subsaharan

t = -2.2, df = 41, p-value = 0.03

alternative hypothesis: true difference in means is not equal to 0

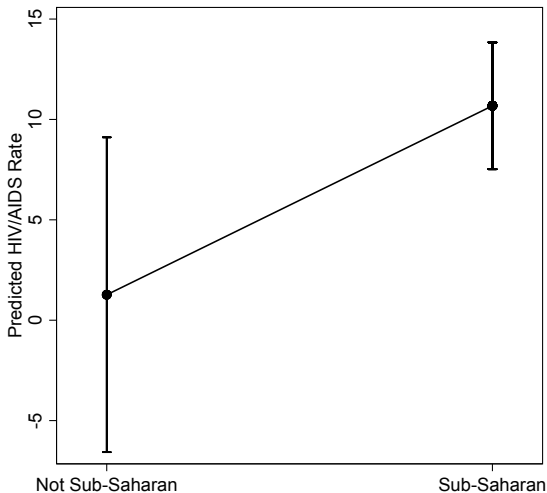
95 percent confidence interval:

-17.8659 -0.9576

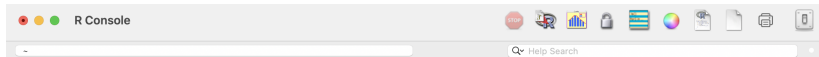
sample estimates:

mean in group Not Sub-Saharan	mean in group Sub-Saharan
1.267	10.678

Expected Values of HIV/AIDS Infection Rates in Saharan and Sub-Saharan Africa



R Things



```
R version 4.3.0 (2023-04-21) -- "Already Tomorrow"  
Copyright (C) 2023 The R Foundation for Statistical Computing  
Platform: aarch64-apple-darwin20 (64-bit)
```

```
R is free software and comes with ABSOLUTELY NO WARRANTY.  
You are welcome to redistribute it under certain conditions.  
Type 'license()' or 'licence()' for distribution details.
```

```
  Natural language support but running in an English locale
```

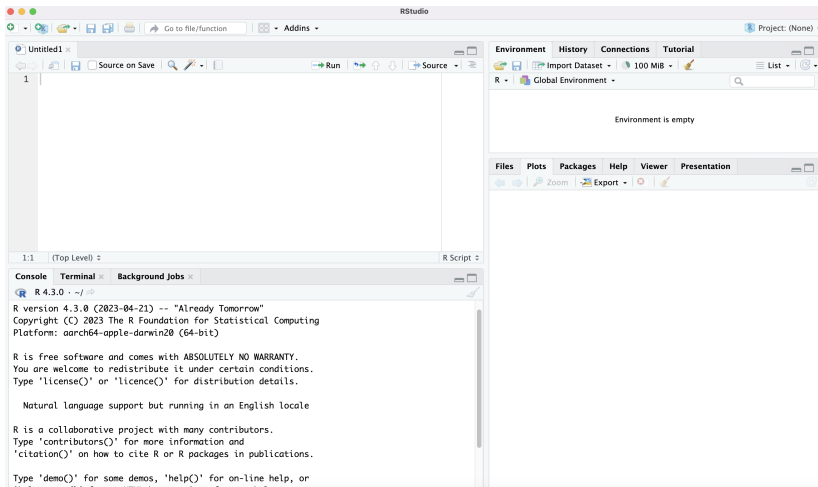
```
R is a collaborative project with many contributors.  
Type 'contributors()' for more information and  
'citation()' on how to cite R or R packages in publications.
```

```
Type 'demo()' for some demos, 'help()' for on-line help, or  
'help.start()' for an HTML browser interface to help.  
Type 'q()' to quit R.
```

```
[R.app GUI 1.79 (8225) aarch64-apple-darwin20]
```

```
[History restored from /Users/cuz10/.Rapp.history]
```

```
> |
```



RStudio (annotated)

This is the "Source" window.

- It's the place where you'll type the code that will then be sent to R.
- It's basically a text editor. You can open text files of any kind here if you want.
- Files that appear here end in (and should be saved with) the extension ".R" (as in "MyCode.R").

You'll spend most of your time working here.

Click here to save your source code. Save often!

Highlight text in the Source window, then click this button to "run" the code.

This is the "Environment" window. It is where you can find all the various "objects" that you create, grouped by object type (data frames, lists, graphs, etc.). Environment is empty

There's also a "History" tab above; switching to that will show what has transpired in the Console window recently.

This is the "working directory." Anything you save will be saved here, unless you tell the program to save it somewhere else.

This is the "Console." When you run the code in the Source window, the results that aren't graphics appear here.

Type 'demo()' for some demos, 'help()' for on-line help, or 'help.start()' for an HTML browser interface to help. Type 'q()' to quit R.

Plots (graphs) that you have created

Packages that are loaded

Help results (obtained by typing "?XXX" in the Console window, e.g. "?table").

This:

```
> table(df$X)
```

... means “Type the phrase ‘table(df\$X)’ on the command line,” or – equivalently – “Type the phrase ‘table(df\$X)’ into your Source code, and then run it.”

More often, you'll see:

```
with(df, plot(Y~X,pch=19,col="red")) # draw a scatterplot  
abline(h=0,lty=2) # add a horizontal line at zero  
abline(v=0,lty=2) # add a vertical line at zero  
text(df$X,df$Y,labels=df$names,pos=1) # add labels
```

... which means “Put this block of text into your Source code, and then run it.”

Note:

- R / RStudio ignores line breaks
- Anything to the right of a “#” is a comment

Very basic R examples...
(see `GSERM-2023-R-Intro.R` in the github repo)

Help For Learning R(Studio)

In rough order of preference:

- Quick-R (<http://www.statmethods.net/>)
- The “Level-Zero” R Tutorial (doesn't integrate RStudio, but is otherwise very good)
- Statistics with R
- The Do It Yourself Introduction to R
- Also be sure to consult the Regression for Publishing “Useful R Resources” guide (on GitHub).