# **GSERM 2023**Regression for Publishing

June 19, 2023

# "Regression for Publishing"

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 Class: June 19-23, 2023, 09:15 - 15:15 CET, at the University of St. Gallen SQUARE building, room 11-0061 Rotmonten.

• The course outline / syllabus is <a href="here">here</a>.

 More important: The syllabus, slides, readings, code, data, etc. are all available on the course github repo (viewable at https://github.com/PrisonRodeo/GSERM-RFP-2023).

#### Assessment

#### Evaluation at GSERM isn't easy... the plan:

- One "homework exercise"
  - · Practical exercise "real" data analysis and discussion
  - · Assigned Tuesday (June 20); due Friday (June 23)
  - · Worth 300 possible points
- Final Examination
  - Multiple essay-style questions + "real" data analysis
  - · Some choice of questions to answer
  - · Assigned Friday (June 23)
  - Due <u>either</u> Friday, June 23, 2023 ("in-class" alternative) or Friday, June 30, 2023 ("take-home" alternative)
  - · Worth 700 possible points
- Total course = 1000 possible points
- Grades assessed on Swiss (1 6) scale

#### Software

#### R

- All examples, plots, etc. are generated using R
- Current version is 4.3.0
- Desktop: Be sure to get the RStudio / Posit IDE...
- Alternatively: Can be run in a browser, using Posit Cloud
- The course Github repo contains a bit of introductory code for people who may never have used R, and a list of R resources.

#### Stata

- Current version is 18
- Main linear regression command is -regress- (or reg)



R version 4.3.0 (2023-04-21) -- "Already Tomorrow"
Copyright (C) 2023 The R Foundation for Statistical Computing
Platform: aarch64-apple-darwin20 (64-bit)

R is free software and comes with ABSOLUTELY NO WARRANTY. You are welcome to redistribute it under certain conditions. Type 'license()' or 'licence()' for distribution details.

Natural language support but running in an English locale

R is a collaborative project with many contributors. Type 'contributors()' for more information and 'citation()' on how to cite R or R packages in publications.

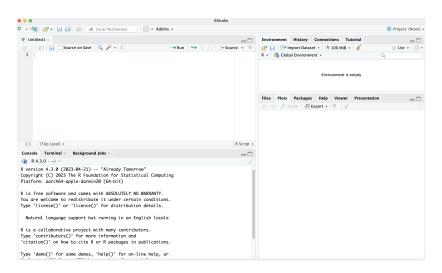
Type 'demo()' for some demos, 'help()' for on-line help, or 'help.start()' for an HTML browser interface to help. Type 'q()' to quit R.

[R.app GUI 1.79 (8225) aarch64-apple-darwin20]

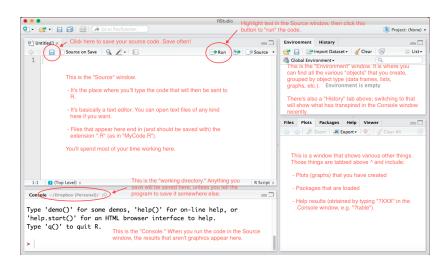
[History restored from /Users/cuz10/.Rapp.history]

>

#### **RStudio**



# RStudio (annotated)



# Things We Will And Won't Do

Will: "Regression":

$$Y = f(\mathbf{X})$$

Won't: Multivariate regression:

$$\mathbf{Y} = f(\mathbf{X})$$

Won't: Measurement (e.g. PCA, factor analysis, IRT, etc.):

$$\mathbf{Y} = \mathbf{W}^{\mathrm{T}} \mathbf{X}$$

Won't: Classification:

- Cluster Analysis / Network Models / etc.
- ullet Classification and Regression Trees o Random Forests.
- Pattern Recognition
- Machine Learning (beyond regression), Support Vector Machines, etc.

# Friday = "Participants' Choice"



Please rank order these possible choices for topics for Friday's class session (1 = most preferred to 7 = least preferred). When you have finished ranking them, please click on the arrow below.

Extended GLMs

Sample Selection Models

Causal Inference

Survival Analysis

Panel / TSCS Models

Analysis of Text / LLMs

Measurement Models

# Why Regression?

	Description	Explanation	Prediction
Task	Summarize data	Correlation/causation	Forecast OOS / future data
Emphasis	Data	Theory / Hypotheses	Outcomes
Focus	Univariate	Multivariate	Multivariate
Typical Application	Summarize / "reduce" data	Discuss marginal associations between predictors and an outcome of interest	Optimize out-of- sample predictive power / minimize prediction error

# Regression

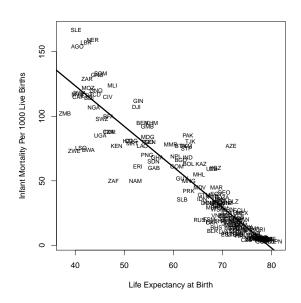
"Regression," conceptually:

$$Pr(Y|X) = f(X)$$

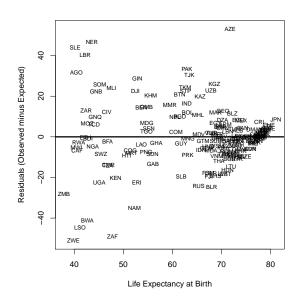
Two important things:

- ullet The distribution of Y is conditional on all variables in  ${\bf X}$ , and
- The conditional distribution of *Y* is conditional on the *joint* distribution of the elements of **X**.
- $\rightarrow$  Regression is <u>hard</u>...

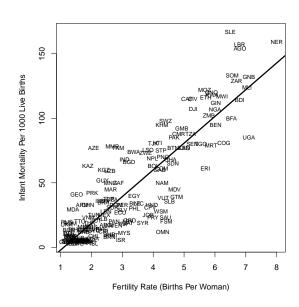
# Example: Infant Mortality and Life Expectancy



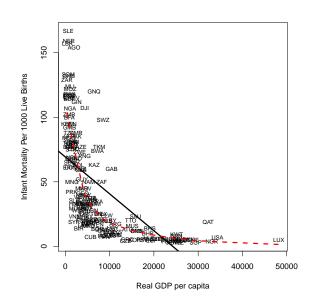
# Infant Mortality and Life Expectancy: "Residuals"



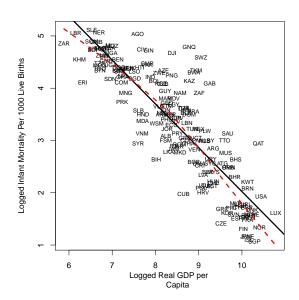
# Infant Mortality and Fertility



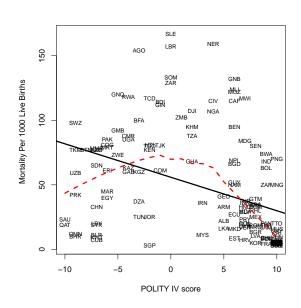
# Infant Mortality and Wealth



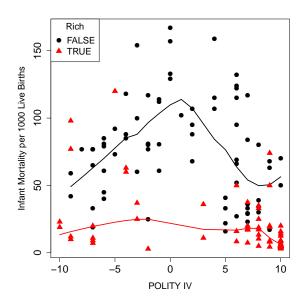
# (Logged) Infant Mortality and (Logged) Wealth



# Infant Mortality and Democracy



# Infant Mortality, (Dichotomized) Wealth, and Democracy



# Linear Regression

#### Consider random variable Y:

$$Y_i = \mu + u_i \tag{1}$$

$$\mu_i = \beta_0 + \beta_1 X_i$$

SO:

$$Y_i = \beta_0 + \beta_1 X_i + u_i \tag{2}$$

#### Goals:

- Estimate  $\hat{\beta}_0$  and  $\hat{\beta}_1$
- Estimate the *variability*  $\hat{\beta}_0$  and  $\hat{\beta}_1$
- Assess model fit

### Bivariate OLS - Estimation

For a bivariate regression:

$$\hat{\beta}_{1} = \frac{\sum_{i=1}^{N} (X_{i} - \bar{X})(Y_{i} - \bar{Y})}{\sum_{i=1}^{N} (X_{i} - \bar{X})^{2}}$$

$$= \frac{\text{Covariance of } X \text{ and } Y}{\text{Variance of } X}$$
(3)

and

$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X} \tag{4}$$

# $\mathsf{Var}(\hat{eta}_1)$

Assume (for now):

$$u_i \sim \text{i.i.d. } N(0, \sigma^2)$$

meaning:

$$Var(Y|X,\beta) = \sigma^2$$

so:

$$\operatorname{Var}(\hat{\beta}_{1}) = \operatorname{Var}\left[\frac{\sum_{i=1}^{N}(X_{i}-\bar{X})Y_{i}}{\sum_{i=1}^{N}(X_{i}-\bar{X})^{2}}\right]$$

$$= \left[\frac{1}{\sum(X_{i}-\bar{X})^{2}}\right]^{2}\sum(X_{i}-\bar{X})^{2}\operatorname{Var}(Y)$$

$$= \left[\frac{1}{\sum(X_{i}-\bar{X})^{2}}\right]^{2}\sum(X_{i}-\bar{X})^{2}\sigma^{2}$$

$$= \frac{\sigma^{2}}{\sum(X_{i}-\bar{X})^{2}}.$$

# $\mathsf{Var}(\hat{eta}_0)$ and $\mathsf{Cov}(\hat{eta}_0,\hat{eta}_1)$

Similarly:

$$Var(\hat{\beta}_0) = \frac{\sum X_i^2}{N \sum (X_i - \bar{X})^2} \sigma^2$$

and:

$$\mathsf{Cov}(\hat{eta}_0,\hat{eta}_1) = rac{-ar{X}}{\sum (X_i - ar{X})^2} \sigma^2$$

# Important Things

#### Note that:

- $Var(\hat{\beta}_0)$  and  $Var(\hat{\beta}_1) \propto \sigma^2$
- $\operatorname{Var}(\hat{eta}_0)$  and  $\operatorname{Var}(\hat{eta}_1) \propto -\sum (X_i \bar{X})$
- $Var(\hat{\beta}_0)$  and  $Var(\hat{\beta}_1) \propto -N$
- $\operatorname{sign}[\operatorname{Cov}(\hat{eta}_0,\hat{eta}_1)] = -\operatorname{sign}(\bar{X})$

#### Inference

If  $u_i \sim N(0, \sigma^2)$ , then:

$$\hat{eta}_0 \sim \mathcal{N}[eta_0, \mathsf{Var}(\hat{eta}_0)]$$

and

$$\hat{eta}_1 \sim N[eta_1, \mathsf{Var}(\hat{eta}_1)]$$

Means:

$$z_{\hat{\beta}_1} = \frac{(\hat{\beta}_1 - \beta_1)}{\sqrt{\mathsf{Var}(\hat{\beta}_1)}}$$
$$= \frac{(\hat{\beta}_1 - \beta_1)}{\mathsf{s.e.}(\hat{\beta}_1)}$$
$$= \sim \mathcal{N}(0, 1)$$

### A Small Problem...

$$\sigma^2 = ???$$

Solution: use

$$\hat{\sigma}^2 = \frac{\sum \hat{u}_i^2}{N - k}$$

Gives:

$$\widehat{\operatorname{Var}(\hat{eta}_1)} = \frac{\hat{\sigma}^2}{\sum (X_i - \bar{X})^2},$$

and

$$\widehat{\operatorname{Var}(\hat{\beta}_0)} = \frac{\sum X_i^2}{N \sum (X_i - \bar{X})^2} \hat{\sigma}^2$$

# Inference (continued)

So:

$$\widehat{s.e.(\hat{\beta}_1)} = \sqrt{\widehat{\mathsf{Var}(\hat{\beta}_1)}}$$

$$= \sqrt{\frac{\hat{\sigma}^2}{\sum (X_i - \bar{X})^2}}$$

$$= \frac{\hat{\sigma}}{\sqrt{\sum (X_i - \bar{X})^2}}$$

which implies:

$$t_{\hat{\beta}_1} \equiv \frac{(\hat{\beta}_1 - \beta_1)}{\widehat{\text{s.e.}}(\hat{\beta}_1)} = \frac{(\hat{\beta}_1 - \beta_1)}{\frac{\hat{\sigma}}{\sqrt{\sum (X_i - \bar{X})^2}}}$$
$$= \frac{(\hat{\beta}_1 - \beta_1)\sqrt{\sum (X_i - \bar{X})^2}}{\hat{\sigma}}$$
$$\sim t_{N-k}$$

### Predictions and Variance

Point prediction:

$$\hat{Y}_k = \hat{\beta}_0 + \hat{\beta}_1 X_k$$

 $\hat{Y}_k$  is unbiased:

$$\begin{split} \mathsf{E}(\hat{Y}_k) &=& \mathsf{E}(\hat{\beta}_0 + \hat{\beta}_1 X_k) \\ &=& \mathsf{E}(\hat{\beta}_0) + X_k \mathsf{E}(\hat{\beta}_1) \\ &=& \beta_0 + \beta_1 X_k \\ &=& \mathsf{E}(Y_k) \end{split}$$

Variability:

$$\begin{aligned} \operatorname{Var}(\hat{Y}_k) &=& \operatorname{Var}(\hat{\beta}_0 + \hat{\beta}_1 X_k) \\ &=& \frac{\sum X_i^2}{N \sum (X_i - \bar{X})^2} \sigma^2 + \left[ \frac{\sigma^2}{\sum (X_i - \bar{X})^2} \right] X_k^2 + 2 \left[ \frac{-\bar{X}}{\sum (X_i - \bar{X})^2} \sigma^2 \right] X_k \\ &=& \sigma^2 \left[ \frac{1}{N} + \frac{(X_k - \bar{X})^2}{\sum (X_i - \bar{X})^2} \right] \end{aligned}$$

# Variability of Predictions

Prediction variation:

$$\operatorname{Var}(\hat{Y}_k) = \sigma^2 \left[ \frac{1}{N} + \frac{(X_k - \bar{X})^2}{\sum (X_i - \bar{X})^2} \right]$$

means that  $Var(\hat{Y}_k)$ :

- Decreases in N
- Decreases in Var(X)
- Increases in  $|X \bar{X}|$

### Predictions and Inference

Standard error of the prediction:

$$\widehat{\text{s.e.}(\hat{Y}_k)} = \sqrt{\sigma^2 \left[ \frac{1}{N} + \frac{(X_k - \bar{X})^2}{\sum (X_i - \bar{X})^2} \right]}$$

 $\rightarrow$  (e.g.) confidence intervals:

95% c.i.
$$(\hat{Y}_k) = \hat{Y}_k \pm [1.96 \times \widehat{\text{s.e.}(\hat{Y}_k)}]$$

## Variation in Y

We can decompose variation in Y:

$$Var(Y) = Var(\hat{Y} + \hat{u})$$

$$= Var(\hat{Y}) + Var(\hat{u}) + 2 Cov(\hat{Y}, \hat{u})$$

$$= Var(\hat{Y}) + Var(\hat{u})$$

$$\begin{array}{lll} \textbf{TSS} & = & \textbf{MSS} & + & \textbf{RSS} \\ \textbf{("Total")} & & \textbf{("Estimated," or "Model")} & & \textbf{("Residual")} \end{array}$$

# Model Fit: $R^2$

"R-squared":

$$R^{2} = \frac{MSS}{TSS}$$

$$= \frac{\sum (\hat{Y}_{i} - \bar{Y})^{2}}{\sum (Y_{i} - \bar{Y})^{2}}$$

$$= 1 - \frac{RSS}{TSS}$$

$$= 1 - \frac{\sum \hat{u}_{i}^{2}}{\sum (Y_{i} - \bar{Y})^{2}}$$

#### R-squared:

- is "the proportion of variance explained"
- ullet  $\in [0,1]$ 
  - $\cdot R^2 = 1.0 \equiv a$  "perfect (linear) fit"
  - $R^2 = 0 \equiv \text{no (linear) } X Y \text{ association}$

For a single X,

$$R^{2} = \hat{\beta}_{1}^{2} \frac{\sum (X_{i} - \bar{X})^{2}}{\sum (Y_{i} - \bar{Y})^{2}}$$
$$= r_{XY}^{2}$$

"Adjusted"  $R^2$ :

$$R_{adj.}^2 = 1 - \frac{(1 - R^2)(N - c)}{(N - k)}$$

where c=1 if there is a constant in the model and c=0 otherwise.

 $R_{adj.}^2$ :

- $R_{adj.}^2 \to R^2$  as  $N \to \infty$
- $R_{adi.}^2$  can be > 1, or < 0...
- $R_{adi.}^2$  increases with model "fit," but
- The extent of that increase is discounted by a factor proportional to the number of covariates.

# $R^2$ Alternatives

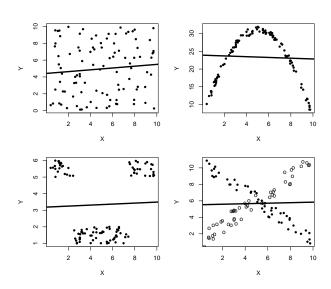
Alternative single measures of model fit include:

• The Standard Error of the Estimate:

$$\mathsf{SEE} = \sqrt{\frac{\mathsf{RSS}}{N-k}}$$

- F-tests
- ROC / AUC
- Graphical methods

# Caution: Different Ways to get $R^2 = 0$



# Linear Regression: K Predictors

Now consider:

$$\mathbf{Y}_{N\times 1} = \mathbf{X}_{N\times K_{K\times 1}} \mathbf{\beta} + \mathbf{u}_{N\times 1}$$

equivalently:

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \dots + \beta_K X_{Ki} + u_i$$

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_N \end{bmatrix} = \begin{bmatrix} 1 & X_{11} & X_{21} & \cdots & X_{K1} \\ 1 & X_{12} & X_{22} & \cdots & X_{K2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & X_{1N} & X_{2N} & \cdots & X_{KN} \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_K \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_N \end{bmatrix}.$$

### Estimating $\beta$

Residuals:

$$\mathbf{u} = \mathbf{Y} - \mathbf{X}\boldsymbol{\beta}$$

The inner product of **u**:

$$\mathbf{u}'\mathbf{u} = \begin{bmatrix} u_1 & u_2 & \cdots & u_N \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_N \end{bmatrix}$$
$$= u_1^2 + u_2^2 + \dots + u_N^2$$
$$= \sum_{i=1}^N u_i^2$$

Sum of squared residuals:

$$u'u = (\mathbf{Y} - \mathbf{X}\beta)'(\mathbf{Y} - \mathbf{X}\beta)$$
$$= \mathbf{Y}'\mathbf{Y} - 2\beta'\mathbf{X}'\mathbf{Y}' + \beta'\mathbf{X}'\mathbf{X}\beta$$

Now get:

$$\frac{\partial \mathbf{u}'\mathbf{u}}{\partial \boldsymbol{\beta}} = -2\mathbf{X}'\mathbf{Y} + 2\mathbf{X}'\mathbf{X}\boldsymbol{\beta}$$

Solve:

$$-2X'Y + 2X'X\beta = 0$$

$$-X'Y + X'X\beta = 0$$

$$X'X\beta = X'Y$$

$$(X'X)^{-1}X'X\beta = (X'X)^{-1}X'Y$$

$$\beta = (X'X)^{-1}X'Y$$

# The Importance of $\mathbf{V}(\hat{eta})$

Variance-covariance matrix:

$$\mathbf{V}(\hat{\beta}) = \mathsf{E}[\hat{\beta} - \mathsf{E}(\hat{\beta})]^{2}$$
$$= \mathsf{E}\{[\hat{\beta} - \mathsf{E}(\hat{\beta})][\hat{\beta} - \mathsf{E}(\hat{\beta})]'\}$$

Rewrite:

$$\begin{aligned} \mathbf{V}(\hat{\boldsymbol{\beta}}) &= & \mathsf{E}(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta})(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta})' \\ &= & \mathsf{E}\{[(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{u}][(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{u}]'\} \\ &= & \mathsf{E}[(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{u}\mathbf{u}'\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}] \end{aligned}$$

# The Importance of $\mathbf{V}(\hat{\boldsymbol{\beta}})$

Taking expectations:

$$\begin{aligned} \mathbf{V}(\hat{\boldsymbol{\beta}}) &= (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathsf{E}(\mathbf{u}\mathbf{u}')\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1} \\ &= (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\sigma^2\mathbf{I}\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1} \\ &= \sigma^2(\mathbf{X}'\mathbf{X})^{-1} \end{aligned}$$

# Estimating $\mathbf{V}(\hat{\boldsymbol{\beta}})$

Empirical estimate:

$$\hat{\sigma}^2 = \frac{\hat{\mathbf{u}}'\hat{\mathbf{u}}}{N - K}$$

Yields:

$$\widehat{\mathbf{V}(\hat{\boldsymbol{\beta}})} = \hat{\sigma}^2 (\mathbf{X}'\mathbf{X})^{-1}$$

## **OLS Assumptions**

#### 1. Zero Expectation Disturbances

$$\mathsf{E}(\mathsf{u})=\mathsf{0}$$

#### 2. Homoscedasticity / No Error Correlation

$$\mathsf{E}(\mathbf{u}\mathbf{u}') = \begin{bmatrix} \sigma^2 & 0 & \cdots & 0 \\ 0 & \sigma^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma^2 \end{bmatrix}$$

## OLS Assumptions (continued)

#### 3. "Fixed" X...

- No measurement error in the Xs, and
- Cov(X, u) = 0.

#### 4. X is of full column rank.

#### Means:

- no exact linear relationship among X, and
- K < N.

#### 5. Normal Disturbances

$$\mathbf{u} \sim \mathbf{N}(\mathbf{0}, \sigma^2 \mathbf{I})$$

## **OLS**: Properties

Under these assumptions, the OLS estimate of  $\hat{\beta}$  is:

- Unbiased
- Fully Efficient

```
(i.e., "BLUE")
```

## Example Data: Infant Mortality

```
> # Summary statistics
>
# install.packages("psych") <- Install psych package, if necessary
> library(psych)

> with(IR2000, describe(infantmortalityperK))
  vars n mean sd median trimmed mad min max range skew kurtosis se
1 1179 43.83 40.39 29 38.38 34.26 2.9 167 164.1 1 0.06 3.02

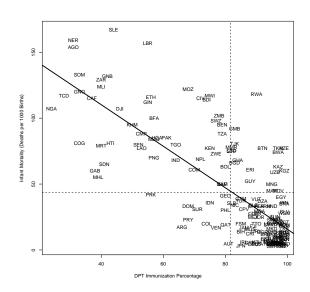
> with(IR2000, describe(DPTpct))
  vars n mean sd median trimmed mad min max range skew kurtosis se
1 1181 81.71 19.77 90 85.23 11.86 24 99 75 -1.31 0.57 1.47
```

## **OLS** Regression

```
> IMDPT<-lm(infantmortalityperK~DPTpct,data=IR2000,na.action=na.exclude)
> summarv.lm(IMDPT)
Call:
lm(formula = infantmortalityperK ~ DPTpct, data = Data)
Residuals:
   Min
            10 Median 30
                                  Max
-56.801 -16.328 -5.105 11.777 86.590
Coefficients:
           Estimate Std. Error t value Pr(>|t|)
(Intercept) 173.2771 8.4893 20.41 <2e-16 ***
DPTpct -1.5763 0.1009 -15.62 <2e-16 ***
___
Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1
Residual standard error: 26.19 on 175 degrees of freedom
  (14 observations deleted due to missingness)
Multiple R-squared: 0.5824, Adjusted R-squared:
F-statistic: 244.1 on 1 and 175 DF, p-value: < 2.2e-16
```

## Analysis of Variance

## Regression of Infant Mortality on DPT Immunization Rates

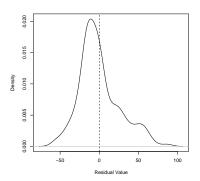


### Fitted Values, Residuals, etc.

```
> IR2000$IMDPTres <- with(IR2000, residuals(IMDPT))
> describe(IR2000$IMDPTres)

var n mean sd median mad min max range skew kurtosis se
1 1 177 0 26.12 -5.1 19.42 -56.8 86.59 143.4 0.75 0.44 1.96
```

> # Residuals (u):

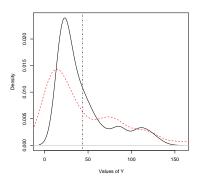


#### Fitted Values

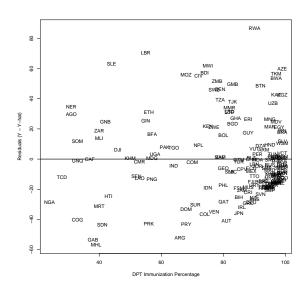
- > # Fitted Values:
- > IR2000\$IMDPThat<-fitted.values(IMDPT)
- > describe(IR2000\$IMDPThat)

var n mean sd median mad min max range skew kurtosis se 1 1 177 44.26 30.84 31.41 18.7 17.22 135.4 118.2 1.3 0.59 2.32

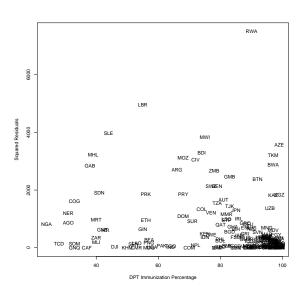
Density Plot: Actual (Y) and Fitted Values  $(\hat{Y})$ 



## Regression Residuals $(\hat{u})$ vs. DPT Percentage



## Squared Residuals vs. DPT Percentage



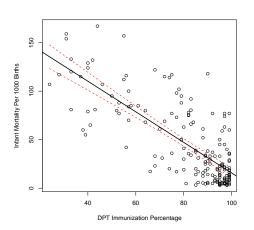
#### Inference

```
Var(\hat{\beta}):
> vcov(IMDPT)
            (Intercept) DPTpct
(Intercept)
            72.0677 -0.83317
DPTpct -0.8332 0.01018
95 percent c.i.s:
> confint(IMDPT)
             2.5 % 97.5 %
(Intercept) 156.523 190.032
DPTpct -1.775 -1.377
```

### **Predictions**

### A Plot, With Cls

Scatterplot of Infant Mortality and DPT Immunizations, along with Least-Squares Line and 95% Prediction Confidence Intervals



## Multivariate Example: Africa Data

- > Data<-read\_csv("https://github.com/PrisonRodeo/GSERM-RFP-2023/raw/main/Data/africa2001.csv")
- > Data<-with(Data, data.frame(adrate,polity,
- + subsaharan=as.numeric(as.factor(subsaharan))-1,
  + muslperc,literacy))

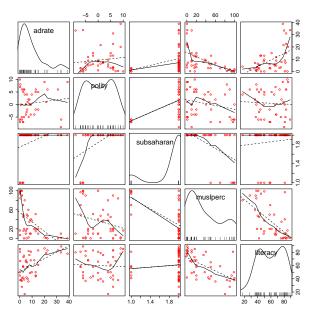
#### > describe(Data)

	vars	n	mean	sd	median	trimmed	mad	min	max	range	skew	kurtosis	se
adrate	1	43	9.37	9.96	6	7.58	6.38	0.1	38.8	38.7	1.44	1.23	1.52
polity	2	43	0.51	5.41	0	0.46	7.41	-9.0	10.0	19.0	0.01	-1.38	0.82
subsaharan	3	43	0.86	0.35	1	0.94	0.00	0.0	1.0	1.0	-2.01	2.08	0.05
muslperc	4	43	35.96	34.58	20	32.87	29.65	0.0	100.0	100.0	0.68	-1.04	5.27
literacy	5	43	60.07	18.94	61	60.63	26.69	17.0	89.0	72.0	-0.20	-1.18	2.89

#### > cor(Data)

	adrate	polity	subsaharan	muslperc	literacy
adrate	1.0000	0.11794	0.33129	-0.5709	0.51489
polity	0.1179	1.00000	0.52820	-0.2392	-0.05079
subsaharan	0.3313	0.52820	1.00000	-0.5773	0.09473
muslperc	-0.5709	-0.23917	-0.57725	1.0000	-0.61960
literacy	0.5149	-0.05079	0.09473	-0.6196	1.00000

### Africa Data



### A Regression

```
> model<-lm(adrate~polity+subsaharan+muslperc+literacy,data=Data)
> summary(model)
Call:
lm(formula = adrate ~ polity + subsaharan + muslperc + literacy,
   data = Data)
Residuals:
   Min
           1Q Median
-15.468 -4.395 -0.525 3.425 22.936
Coefficients:
           Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.6687 10.4113 -0.06 0.949
polity
           -0.0139
                    0.2797
                             -0.05 0.961
subsaharan 3.7297 5.4309
                              0.69 0.496
muslperc -0.0869 0.0628 -1.38 0.175
literacy 0.1657
                      0.0943
                              1.76
                                    0.087 .
Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
Residual standard error: 8.26 on 38 degrees of freedom
Multiple R-squared: 0.377, Adjusted R-squared: 0.312
F-statistic: 5.75 on 4 and 38 DF, p-value: 0.00101
```

## Variance-Covariance Matrix of $\hat{oldsymbol{eta}}$

- > options(digits=4)
  > vcov(model)
- (Intercept) polity subsaharan muslperc literacy (Intercept) 223.4259 1.088030 -72.2628 -0.771309 -1.002421 polity 1.0880 0.078229 -0.6642 -0.000293 0.001968 subsaharan 29.4950 0.206067 0.171765 -72.2628 -0.664212 muslperc -0.7713 -0.000293 0.2061 0.003946 0.004098 literacy -1.0024 0.001968 0.1718 0.004098 0.008898

### Inference: Tests...

```
Test H_0: \beta_{	ext{polity}} = \beta_{	ext{subsaharan}} = 0:
> library(lmtest)
> modelsmall<-lm(adrate~muslperc+literacy,data=Data)
> waldtest(model,modelsmall)

Wald test

Model 1: adrate~polity + subsaharan + muslperc + literacy
Model 2: adrate~muslperc + literacy
Res.Df Df F Pr(>F)
1 38
2 40 -2 0.27 0.76
```

#### More tests...

```
Test H_0: \beta_{\text{muslperc}} = 0.1:
> library(car)
> linearHypothesis(model, "muslperc=0.1")
Linear hypothesis test
Hypothesis:
muslperc = 0.1
Model 1: restricted model
Model 2: adrate ~ polity + subsaharan + muslperc + literacy
  Res.Df RSS Df Sum of Sq F Pr(>F)
1
     39 3200
      38 2595 1 605 8.85 0.0051 **
Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1
```

#### More tests...

```
Test H_0: \beta_{\text{literacy}} = \beta_{\text{muslperc}}:
> linearHypothesis(model,"literacy=muslperc")
Linear hypothesis test
Hypothesis:
- muslperc + literacy = 0
Model 1: restricted model
Model 2: adrate ~ polity + subsaharan + muslperc + literacy
  Res.Df RSS Df Sum of Sq F Pr(>F)
1
      39 3534
      38 2595 1 938 13.7 0.00067 ***
Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1
```

### Reporting

#### The output:

```
> summarv(model)
Call:
lm(formula = adrate ~ polity + subsaharan + muslperc + literacy.
   data = Data)
Residuals:
   Min
           10 Median 30
                               Max
-15.468 -4.395 -0.525 3.425 22.936
Coefficients:
          Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.6687 10.4113 -0.06 0.949
      -0.0139 0.2797 -0.05 0.961
polity
subsaharan 3.7297 5.4309 0.69 0.496
muslperc -0.0869 0.0628 -1.38 0.175
literacy 0.1657 0.0943 1.76 0.087.
Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
Residual standard error: 8.26 on 38 degrees of freedom
Multiple R-squared: 0.377, Adjusted R-squared: 0.312
F-statistic: 5.75 on 4 and 38 DF. p-value: 0.00101
```

### Reporting

#### The table:

Table 1: OLS Regression Model of HIV/AIDS Rates in Africa, 2001

	Model I
(Constant)	-0.67
	(10.41)
POLITY Score	-0.01
	(0.28)
Subsaharan Africa	3.73
	(5.43)
Muslim Percentage of the Population	-0.09
	(0.06)
Literacy Rate	0.17*
	(0.09)
Observations	43
$R^2$	0.38
Adjusted R <sup>2</sup>	0.31
Residual Std. Error	8.26 (df = 38)
F Statistic	$5.75^* (df = 4; 38)$

Note: N=43. Cell entries are coefficient estimates; numbers in parentheses are estimated standard errors. Asterisks indicate p<.05 (one-tailed). See text for details.

## Multiple Models (stargazer defaults)

OLS Regression Models of HIV/AIDS Rates in Africa, 2001

	w/Literacy	w/o Literacy	
(Constant)	-0.67	14.81**	
,	(10.41)	(5.70)	
POLITY Score	-0.01	-0.05	
	(0.28)	(0.29)	
Subsaharan Africa	3.73	0.53	
	(5.43)	(5.25)	
Muslim Percentage of the Population	-0.09	-0.16***	
	(0.06)	(0.05)	
Literacy Rate	0.17*		
	(0.09)		
Observations	43	43	
$R^2$	0.38	0.33	
Adjusted R <sup>2</sup>	0.31	0.27	
Residual Std. Error	8.26 (df = 38)	8.48 (df = 39)	
F Statistic	$5.75^{***} (df = 4; 38)$	$6.30^{***} (df = 3; 39)$	

Note:

\*p<0.1; \*\*p<0.05; \*\*\*p<0.01

### Making Tables

#### R

- LaTeX: texreg, xtable, and stargazer packages
- MS Word: generally cut-and-paste (see, e.g., here: https://sejdemyr.github.io/r-tutorials/basics/tables-in-r/); also KableExtra
- A pretty good summary of many others is here: https://rfortherestofus.com/2019/11/how-to-make-beautiful-tables-in-r/.

#### Stata

- estout and esttab commands are standard
- Others: outreg2, tabout, orth\_out, etc. (a summary is here: https://lukestein.github.io/stata-latex-workflows/)
- MS Word: putdocx

## Some Guidelines ("Rules"?)

#### Tables:

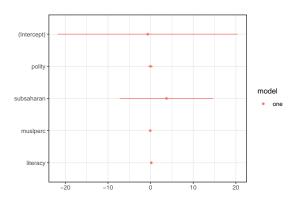
- Use column headings descriptively.
- Use multiple rows / columns rather than multiple tables.
- Learn about significant digits, and don't report more than 4-5 of them (at the most).
- Use a figure to replace a table when you can.
- Be aware of norms about \*s.

#### Figures:

- Report the scale of axes, and label them.
- Use as much "space" as you need, but no more.
- Use color sparingly.

## Plotting Regression Estimates

#### Ladderplot of OLS Results (using dotwhisker)



### Rescaling Covariates

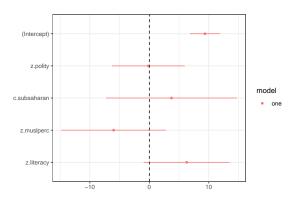
#### A la Gelman (2008):

- Continuous = divide by two standard deviations
- Binary = mean 0, difference of 1 between the two categories

```
> modelS<-standardize(model)
> summary(modelS)
Call:
lm(formula = adrate ~ z.polity + c.subsaharan + z.muslperc +
   z.literacy, data = Data)
Coefficients:
           Estimate Std. Error t value Pr(>|t|)
(Intercept)
              9.37
                       1.26 7.43 0.0000000065 ***
z.polity
            -0.15 3.03 -0.05
                                        0.961
c.subsaharan
             3.73 5.43 0.69 0.496
z.muslperc -6.01 4.34 -1.38 0.175
z.literacy 6.28 3.57 1.76 0.087.
---
Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
Residual standard error: 8.26 on 38 degrees of freedom
Multiple R-squared: 0.377, Adjusted R-squared: 0.312
F-statistic: 5.75 on 4 and 38 DF, p-value: 0.00101
```

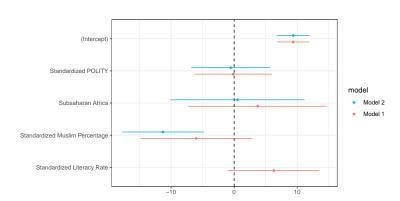
## A Better Regression Plot

#### Ladderplot of Standardized OLS Results



## An Even Better Regression Plot

#### Ladderplot of Standardized OLS Results



### Some Meta-Rules

- Be aware of the norms in your discipline / field, and follow them.
- Ask for advice.
- When in doubt, more information is (probably) better.

## Supplementary Materials

#### Linear Regression

Hypothetically: If we have  $\hat{\beta}_0$  and  $\hat{\beta}_1$ , then:

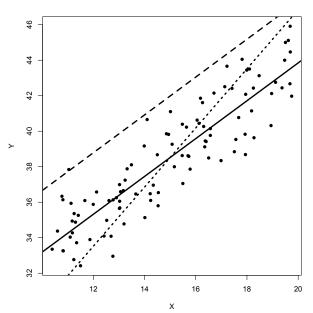
$$\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i$$

and

$$\hat{u}_i = Y_i - \hat{Y}_i 
= Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_i$$

Q: How to estimate  $\hat{\beta}_0$  and  $\hat{\beta}_1$ ?

## Scatterplot: X and Y (with regression lines)



#### **Ordinary Least Squares**

Choose  $\hat{\beta}_0$  and  $\hat{\beta}_1$  to minimize  $\hat{S} = \sum_{i=1}^N \hat{u}_i^2$ .

$$\hat{S} = \sum_{i=1}^{N} (Y_i - \hat{Y}_i)^2$$

$$= \sum_{i=1}^{N} (Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_i)^2$$

$$= \sum_{i=1}^{N} (Y_i^2 - 2Y_i \hat{\beta}_0 - 2Y_i \hat{\beta}_1 X_i + \hat{\beta}_0^2 + 2\hat{\beta}_0 \hat{\beta}_1 X_i + \hat{\beta}_1^2 X_i^2)$$

#### OLS (continued)

Differentiate:

$$\frac{\partial \hat{S}}{\partial \hat{\beta}_{0}} = \sum_{i=1}^{N} (-2Y_{i} + 2\hat{\beta}_{0} + 2\hat{\beta}_{1}X_{i})$$

$$= -2\sum_{i=1}^{N} (Y_{i} - \hat{\beta}_{0} - \hat{\beta}_{1}X_{i})$$

$$= -2\sum_{i=1}^{N} \hat{u}_{i}$$

and

$$\frac{\partial \hat{S}}{\partial \hat{\beta}_1} = \sum_{i=1}^N (-2Y_iX_i + 2\hat{\beta}_0X_i + 2\hat{\beta}_1X_i^2)$$

$$= -2\sum_{i=1}^N (Y_i - \hat{\beta}_0 - \hat{\beta}_1X_i)X_i$$

$$= -2\sum_{i=1}^N \hat{u}_iX_i$$

### OLS (continued)

Yields:

$$\sum_{i=1}^{N} Y_i = N\hat{\beta}_0 + \hat{\beta}_1 \sum_{i=1}^{N} X_i$$

and

$$\sum_{i=1}^{N} Y_i X_i = \hat{\beta}_0 \sum_{i=1}^{N} X_i + \hat{\beta}_1 \sum_{i=1}^{N} X_i^2$$

### OLS (continued)

Solving yields:

$$\hat{\beta}_{1} = \frac{\sum_{i=1}^{N} (X_{i} - \bar{X})(Y_{i} - \bar{Y})}{\sum_{i=1}^{N} (X_{i} - \bar{X})^{2}}$$

$$= \frac{\text{Covariance of } X \text{ and } Y}{\text{Variance of } X}$$

and

$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}$$

# "Stupid Regression Tricks"

### Africa (2001) Data

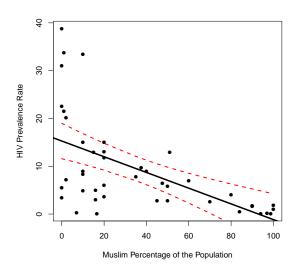
> africa<-read\_csv("https://raw.githubusercontent.com/PrisonRodeo/GSERM-RFP-2023/master/Data/africa2001.csv") > summary(africa)

ccode	cabbr		intry popula	tion po	pthou
Min. :404	AGO : 1 Angol	a	: 1 Min. :	470000 Min.	: 470
1st Qu.:452	BDI : 1 Benin		: 1 1st Qu.:	3446000 1st 0	u.: 3446
Median :510	BEN : 1 Botswana		: 1 Median :	9662000 Media	n: 9662
Mean :510	BWA : 1 Burun	di	: 1 Mean :	17388558 Mean	: 17390
3rd Qu.:556	CAF : 1 Camer	oon	: 1 3rd Qu.:	19150000 3rd 0	u.: 19189
Max. :651	CIV : 1 Centr	al African Republi	.c: 1 Max. :	117000000 Max.	:116929
(Other):37 (Other) :37					
popden	polity	gdppppd	tradegdp	war	adrate
Min. :0.0022			Min. : 4.03		Min. : 0.10
1st Qu.:0.0134	1 st Qu.:-4.500	1st Qu.: 0.855	1st Qu.: 7.64	1st Qu.:0.000	1st Qu.: 2.70
Median :0.035	7 Median : 0.000	Median : 1.200	Median : 13.56	Median:0.000	Median: 6.00
Mean :0.0643	3 Mean : 0.512	Mean : 2.159	Mean : 30.49	Mean :0.116	Mean : 9.37
3rd Qu.:0.0683	3 3rd Qu.: 5.500	3rd Qu.: 2.040	3rd Qu.: 30.01	3rd Qu.:0.000	3rd Qu.:12.90
Max. :0.5740	Max. :10.000	Max. :10.800	Max. :272.69	Max. :1.000	Max. :38.80
healthexp subsaharan muslperc		literacy	internalwar	intensity	
Min. :2.00	Not Sub-Saharan:	6 Min. : 0.0	Min. :17.0	Min. :0.000	Min. :0.000
1st Qu.:3.45	Sub-Saharan :3	7 1st Qu.: 10.0	1st Qu.:43.0	1st Qu.:0.000	1st Qu.:0.000
Median :4.40		Median: 20.0	Median :61.0	Median:0.000	Median:0.000
Mean :4.60		Mean : 36.0	Mean :60.1	Mean :0.302	Mean :0.581
3rd Qu.:5.80 3rd Qu.: 55.5			3rd Qu.:78.5	3rd Qu.:1.000	3rd Qu.:1.000
Max. :8.60		Max. :100.0	Max. :89.0	Max. :1.000	Max. :3.000

#### A Simple Regression

```
> fit<-with(africa, lm(adrate~muslperc))
> summary(fit)
Call:
lm(formula = adrate ~ muslperc)
Residuals:
   Min
            10 Median
                           30
                                  Max
-13.828 -5.206 0.279 2.022 23.521
Coefficients:
           Estimate Std. Error t value
                                          Pr(>|t|)
                       1.8322 8.34 0.00000000023 ***
(Intercept) 15.2787
muslperc -0.1644 0.0369 -4.45 0.00006390853 ***
Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1
Residual standard error: 8.28 on 41 degrees of freedom
Multiple R-squared: 0.326, Adjusted R-squared: 0.31
F-statistic: 19.8 on 1 and 41 DF, p-value: 0.0000639
```

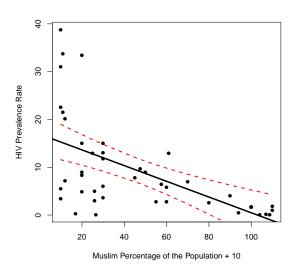
# Scatterplot of HIV/AIDS Rates on Muslim Population Percentage, Africa 2001



#### Adding a Constant to X

```
> africa$muslplusten<-africa$muslperc+10
> fit2<-with(africa, lm(adrate~muslplusten,data=africa))</pre>
> summary(fit2)
Call:
lm(formula = adrate ~ muslplusten, data = africa)
Residuals:
   Min
            10 Median
                            3Q
                                  Max
-13.828 -5.206 0.279 2.022 23.521
Coefficients:
           Estimate Std. Error t value
                                           Pr(>|t|)
(Intercept) 16.9232 2.1152 8.00 0.00000000066 ***
muslplusten -0.1644 0.0369 -4.45 0.00006390853 ***
---
Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1
Residual standard error: 8.28 on 41 degrees of freedom
Multiple R-squared: 0.326, Adjusted R-squared: 0.31
F-statistic: 19.8 on 1 and 41 DF, p-value: 0.0000639
```

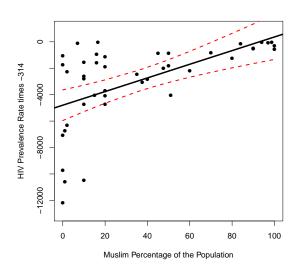
# Scatterplot of HIV/AIDS Rates on Rescaled Muslim Population Percentage



#### Multiplying Y by a Constant

```
> africa$screwyrate<-africa$adrate*(-314)</pre>
> fit3<-with(africa, lm(screwyrate~muslperc))</pre>
> summary(fit3)
Call:
lm(formula = screwyrate ~ muslperc)
Residuals:
  Min 10 Median
                        30
                             Max
                -88 1635 4342
-7386 -635
Coefficients:
           Estimate Std. Error t value
                                           Pr(>|t|)
(Intercept) -4797.5
                         575.3 -8.34 0.00000000023 ***
muslperc
               51.6
                         11.6 4.45 0.00006390853 ***
Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1
Residual standard error: 2600 on 41 degrees of freedom
Multiple R-squared: 0.326, Adjusted R-squared: 0.31
F-statistic: 19.8 on 1 and 41 DF, p-value: 0.0000639
```

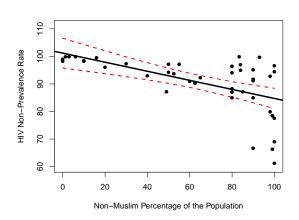
# Scatterplot of Rescaled HIV/AIDS Rates on Muslim Population Percentage



### Reversing the scales of X and Y

```
> africa$nonmuslimpct <- 100 - africa$muslperc
> africa$noninfected <- 100 - africa$adrate
> fit4<-lm(noninfected~nonmuslimpct,data=africa)
> summarv(fit4)
Call:
lm(formula = noninfected ~ nonmuslimpct, data = africa)
Residuals:
   Min
           10 Median
                         30
                                Max
-23.521 -2.022 -0.279 5.206 13.828
Coefficients:
           Estimate Std. Error t value Pr(>|t|)
(Intercept) 101.1660
                     2.6808 37.74 < 2e-16 ***
Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1
Residual standard error: 8.28 on 41 degrees of freedom
Multiple R-squared: 0.326.Adjusted R-squared: 0.31
F-statistic: 19.8 on 1 and 41 DF, p-value: 0.0000639
```

# Scatterplot of HIV/AIDS Non-Infection Rates on Non-Muslim Population Percentage



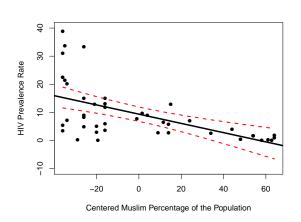
#### Linear Transformations

- Adding (subtracting) a positive constant to *X* shifts the *X*-axis to the <u>left</u> (right).
- Adding (subtracting) a positive constant to *Y* shifts the *Y*-axis downwards (upwards).
- Multiplying X(Y) times a positive constant greater than 1.0 stretches the X(Y) axis.
- Multiplying X (Y) times a positive constant less than 1.0 shrinks the X (Y) axis.
- Multiplying X (Y) times a negative constant <u>inverts</u> the X
  (Y) axis, and stretches / shrinks it as above.

### Use: "Centering" a Variable

```
> africa$muslcenter<-africa$muslperc - mean(africa$muslperc, na.rm=TRUE)
> fit5<-lm(adrate~muslcenter.data=africa)</pre>
> summary(fit5)
Call:
lm(formula = adrate ~ muslcenter, data = africa)
Residuals:
   Min
            10 Median
                          3Q
                                  Max
-13.828 -5.206 0.279 2.022 23.521
Coefficients:
           Estimate Std. Error t value Pr(>|t|)
(Intercept) 9.3651 1.2622 7.42 0.0000000042 ***
muslcenter -0.1644 0.0369 -4.45 0.0000639085 ***
---
Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1
Residual standard error: 8.28 on 41 degrees of freedom
Multiple R-squared: 0.326, Adjusted R-squared: 0.31
F-statistic: 19.8 on 1 and 41 DF, p-value: 0.0000639
```

## Scatterplot of HIV/AIDS Infection Rates on (Centered) Muslim Population Percentage



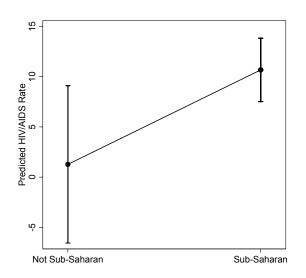
#### Use: Rescaling X for Interpretability

```
> fit6<-lm(adrate~population,data=africa)</pre>
> summary(fit6)
                           Std. Error t value Pr(>|t|)
                Estimate
(Intercept) 10.5883163475 1.9140361989 5.53 0.000002 ***
population -0.0000000703 0.0000000671 -1.05
                                                   0.3
Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1
Residual standard error: 9.95 on 41 degrees of freedom
Multiple R-squared: 0.0261, Adjusted R-squared: 0.00234
F-statistic: 1.1 on 1 and 41 DF, p-value: 0.301
> africa$popmil<-africa$population / 1000000
> fit7<-lm(adrate~popmil,data=africa)</pre>
> summary(fit7)
           Estimate Std. Error t value Pr(>|t|)
(Intercept) 10.5883 1.9140 5.53 0.000002 ***
popmil
           -0.0703 0.0671 -1.05 0.3
---
Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1
Residual standard error: 9.95 on 41 degrees of freedom
Multiple R-squared: 0.0261, Adjusted R-squared: 0.00234
F-statistic: 1.1 on 1 and 41 DF, p-value: 0.301
```

#### Dichotomous Xs: Bivariate Regression $\equiv t$ -test

```
> fit8<-lm(adrate~subsaharan.data=africa)
> summarv(fit8)
Residuals:
  Min
           10 Median
-10.58 -6.23 -1.78 2.22 28.12
Coefficients:
                      Estimate Std. Error t value Pr(>|t|)
(Intercept)
                         1.27
                                    3.88
                                            0.33
                                                     0.75
                                            2.25
subsaharanSub-Saharan
                         9.41
                                    4.19
                                                     0.03 *
Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1
Residual standard error: 9.51 on 41 degrees of freedom
Multiple R-squared: 0.11, Adjusted R-squared: 0.088
F-statistic: 5.05 on 1 and 41 DF, p-value: 0.03
> with(africa,
       t.test(adrate~subsaharan, var.equal=TRUE))
Two Sample t-test
data: adrate by subsaharan
t = -2.2, df = 41, p-value = 0.03
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -17.8659 -0.9576
sample estimates:
mean in group Not Sub-Saharan
                                 mean in group Sub-Saharan
                       1.267
                                                    10.678
```

# Expected Values of HIV/AIDS Infection Rates in Saharan and Sub-Saharan Africa



# R Things



R version 4.3.0 (2023-04-21) -- "Already Tomorrow"
Copyright (C) 2023 The R Foundation for Statistical Computing
Platform: aarch64-apple-darwin20 (64-bit)

R is free software and comes with ABSOLUTELY NO WARRANTY. You are welcome to redistribute it under certain conditions. Type 'license()' or 'licence()' for distribution details.

Natural language support but running in an English locale

R is a collaborative project with many contributors.

Type 'contributors()' for more information and

'citation()' on how to cite R or R packages in publications.

Type 'demo()' for some demos, 'help()' for on-line help, or 'help.start()' for an HTML browser interface to help.

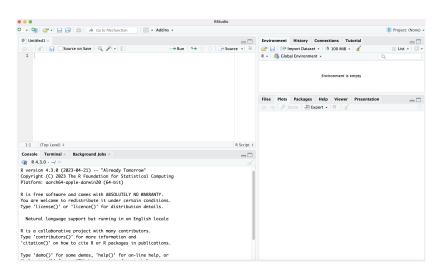
Type 'q()' to quit R.

[R.app GUI 1.79 (8225) aarch64-apple-darwin20]

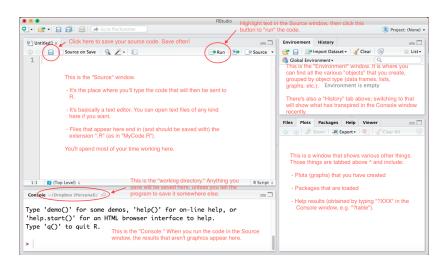
[History restored from /Users/cuz10/.Rapp.history]

>

#### **RStudio**



### RStudio (annotated)



#### Inside the Source Window

#### This:

> table(df\$X)

... means "Type the phrase 'table(dfX)' on the command line," or – equivalently – "Type the phrase 'table(dfX)' into your Source code, and then run it."

#### Inside the Source Window

#### More often, you'll see:

```
with(df, plot(Y~X,pch=19,col="red")) # draw a scatterplot abline(h=0,lty=2) # add a horizontal line at zero abline(v=0,lty=2) # add a vertical line at zero text(df$X,df$Y,labels=df$names,pos=1) # add labels
```

... which means "Put this block of text into your Source code, and then run it."

#### Note:

- R / RStudio ignores line breaks
- Anything to the right of a "#" is a comment

Very basic R examples...

(see GSERM-2023-R-Intro.R in the github repo)

### Help For Learning R(Studio)

#### In rough order of preference:

- Quick-R (http://www.statmethods.net/)
- The "Level-Zero" R Tutorial (doesn't integrate RStudio, but is otherwise very good)
- Statistics with R
- The Do It Yourself Introduction to R
- Also be sure to consult the Regression for Publishing "Useful R Resources" guide (on GitHub).