PLSC 473: American Judicial Behavior

Christopher Zorn

September 24, 2015

Measuring Association (Bivariate Plots / Statistics)

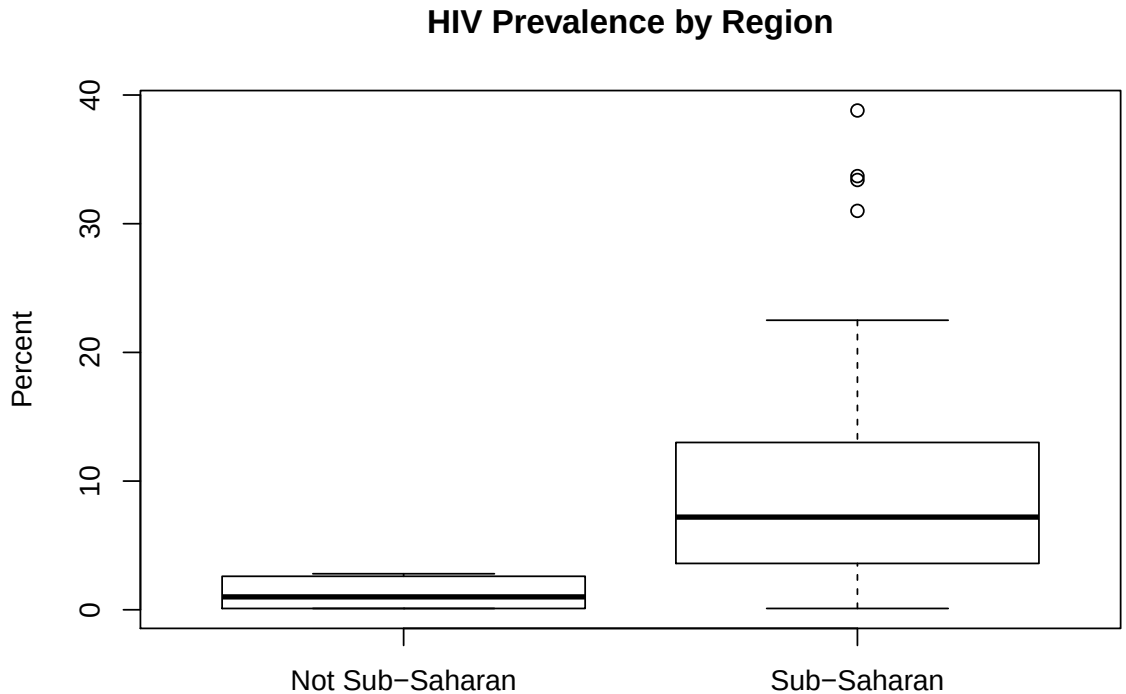
- Pictures
  - Boxplots & Density Plots (redux)
  - Scatterplots
- Numbers
  - Crosstables
  - t-tests
  - Correlation
  - Bivariate regression

Africa Data (Again)

```
> summary(Africa)

     ccode          cabbr                          country     population
 Min.   :404   AGO    : 1   Angola                    : 1   Min.   :   470000
 1st Qu.:452   BDI    : 1   Benin                     : 1   1st Qu.:  3446000
 Median :510   BEN    : 1   Botswana                  : 1   Median :  9662000
 Mean   :510   BWA    : 1   Burundi                   : 1   Mean   : 17388558
 3rd Qu.:556   CAF    : 1   Cameroon                  : 1   3rd Qu.: 19150000
 Max.   :651   CIV    : 1   Central African Republic: 1   Max.   :117000000
               (Other):37   (Other)                   :37
    popthou           popden           polity          gdppppd          tradegdp
 Min.   :   470   Min.   :0.0022   Min.   :-9.000   Min.   : 0.500   Min.   :  4.03
 1st Qu.:  3446   1st Qu.:0.0134   1st Qu.:-4.500   1st Qu.: 0.855   1st Qu.:  7.64
 Median :  9662   Median :0.0357   Median : 0.000   Median : 1.200   Median : 13.56
 Mean   : 17390   Mean   :0.0643   Mean   : 0.512   Mean   : 2.159   Mean   : 30.49
 3rd Qu.: 19189   3rd Qu.:0.0683   3rd Qu.: 5.500   3rd Qu.: 2.040   3rd Qu.: 30.01
 Max.   :116929   Max.   :0.5740   Max.   :10.000   Max.   :10.800   Max.   :272.69

      war             adrate         healthexp              subsaharan    muslperc
 Min.   :0.000   Min.   : 0.10   Min.   :2.00   Not Sub-Saharan: 6   Min.   :  0.0
 1st Qu.:0.000   1st Qu.: 2.70   1st Qu.:3.45   Sub-Saharan    :37   1st Qu.: 10.0
 Median :0.000   Median : 6.00   Median :4.40                        Median : 20.0
 Mean   :0.116   Mean   : 9.37   Mean   :4.60                        Mean   : 36.0
 3rd Qu.:0.000   3rd Qu.:12.90   3rd Qu.:5.80                        3rd Qu.: 55.5
 Max.   :1.000   Max.   :38.80   Max.   :8.60                        Max.   :100.0

    literacy     internalwar      intensity
 Min.   :17.0   Min.   :0.000   Min.   :0.000
 1st Qu.:43.0   1st Qu.:0.000   1st Qu.:0.000
 Median :61.0   Median :0.000   Median :0.000
 Mean   :60.1   Mean   :0.302   Mean   :0.581
 3rd Qu.:78.5   3rd Qu.:1.000   3rd Qu.:1.000
 Max.   :89.0   Max.   :1.000   Max.   :3.000
```

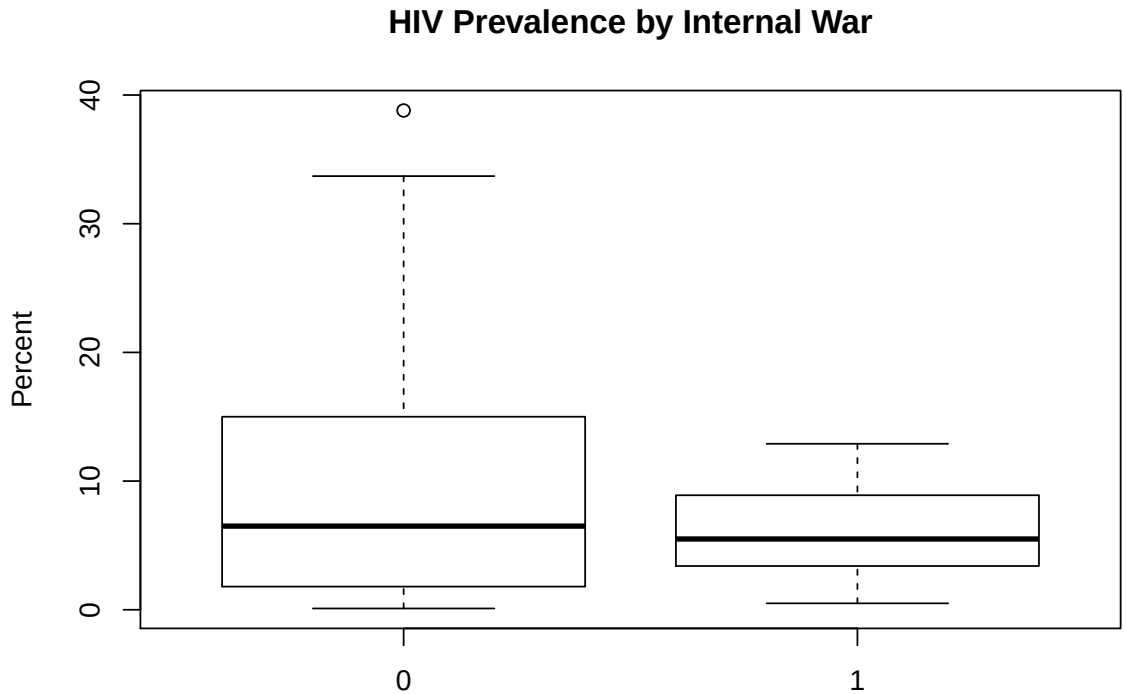
Comparing Boxplots

```
> with(Africa, boxplot(adrate~subsaharan,ylab="Percent",
                       main="HIV Prevalence by Region"))
```



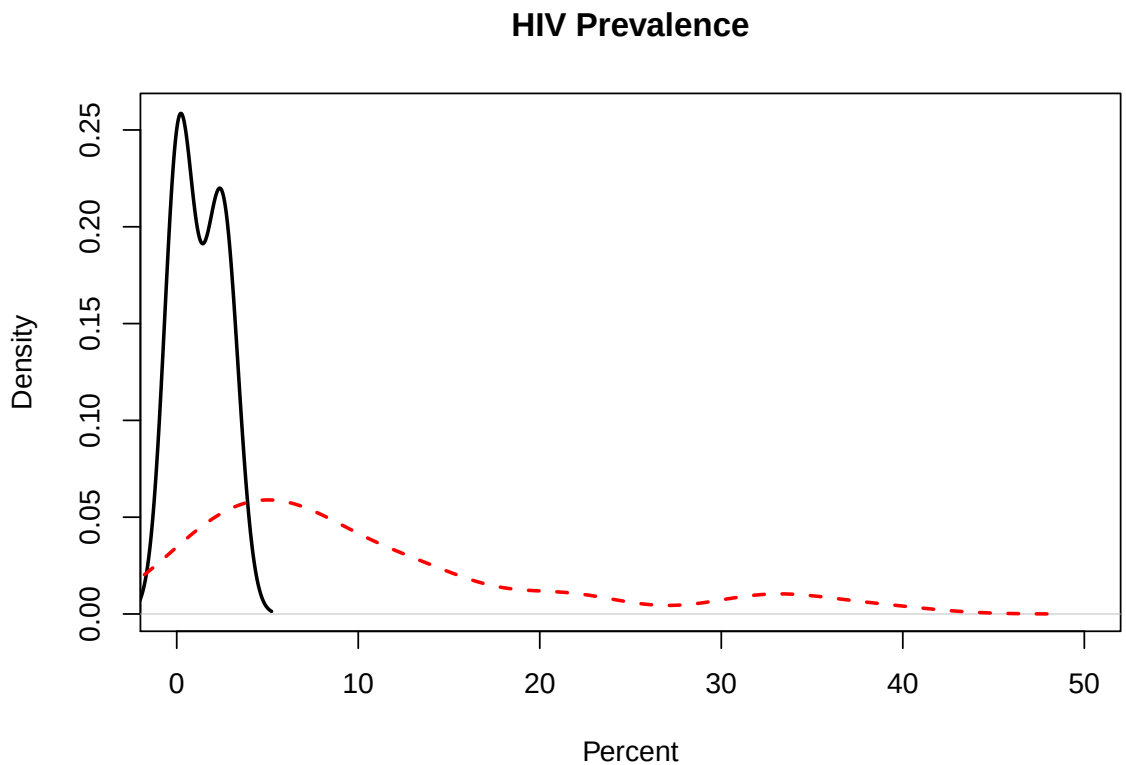
Comparing Boxplots (cont'd)

```
> with(Africa, boxplot(adrate~internalwar,ylab="Percent",
                       main="HIV Prevalence by Internal War"))
```



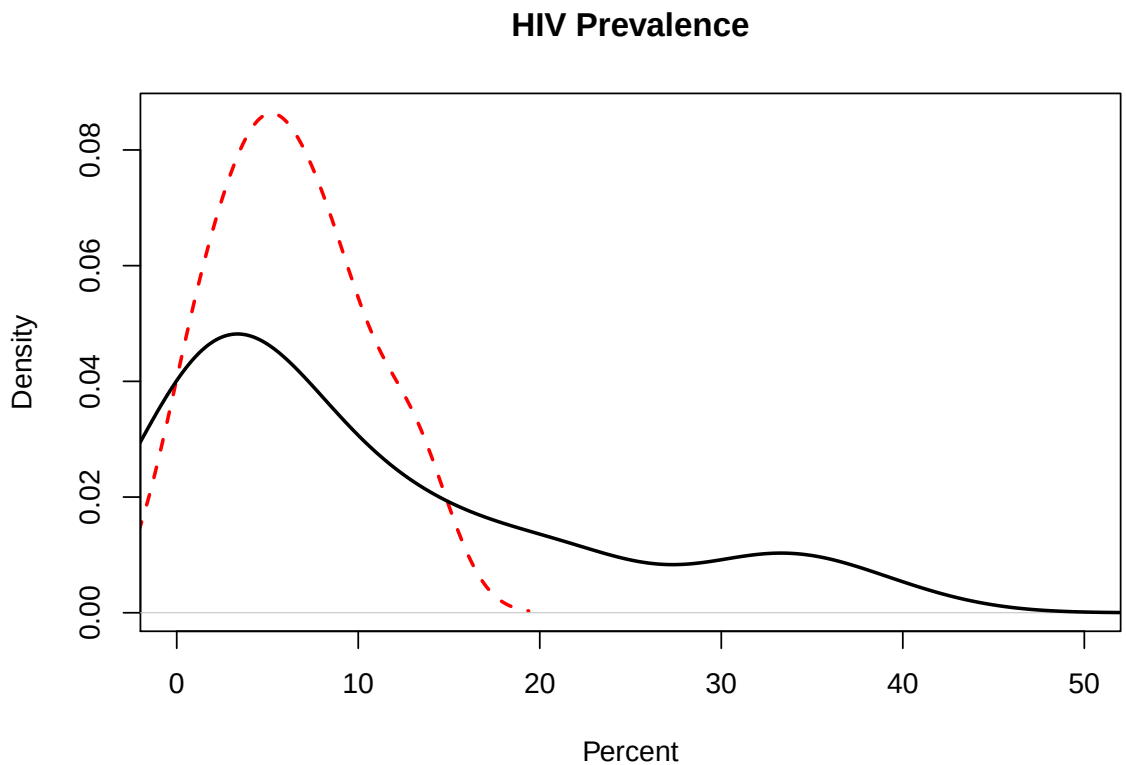
Comparing Density Plots

```
> with(Africa[Africa$subsaharan=="Not Sub-Saharan",],
     plot(density(adrate),lwd=2,col="black",
     main="HIV Prevalence",xlim=c(0,50),xlab="Percent"))
> with(Africa[Africa$subsaharan=="Sub-Saharan",],
     lines(density(adrate),lwd=2,col="red",lty=2))
```



Comparing Density Plots

```
> with(Africa[Africa$internalwar==1,],
     plot(density(adrate),lwd=2,col="red",lty=2,
     main="HIV Prevalence",xlim=c(0,50),xlab="Percent"))
> with(Africa[Africa$internalwar==0,],
     lines(density(adrate),lwd=2,col="black"))
```

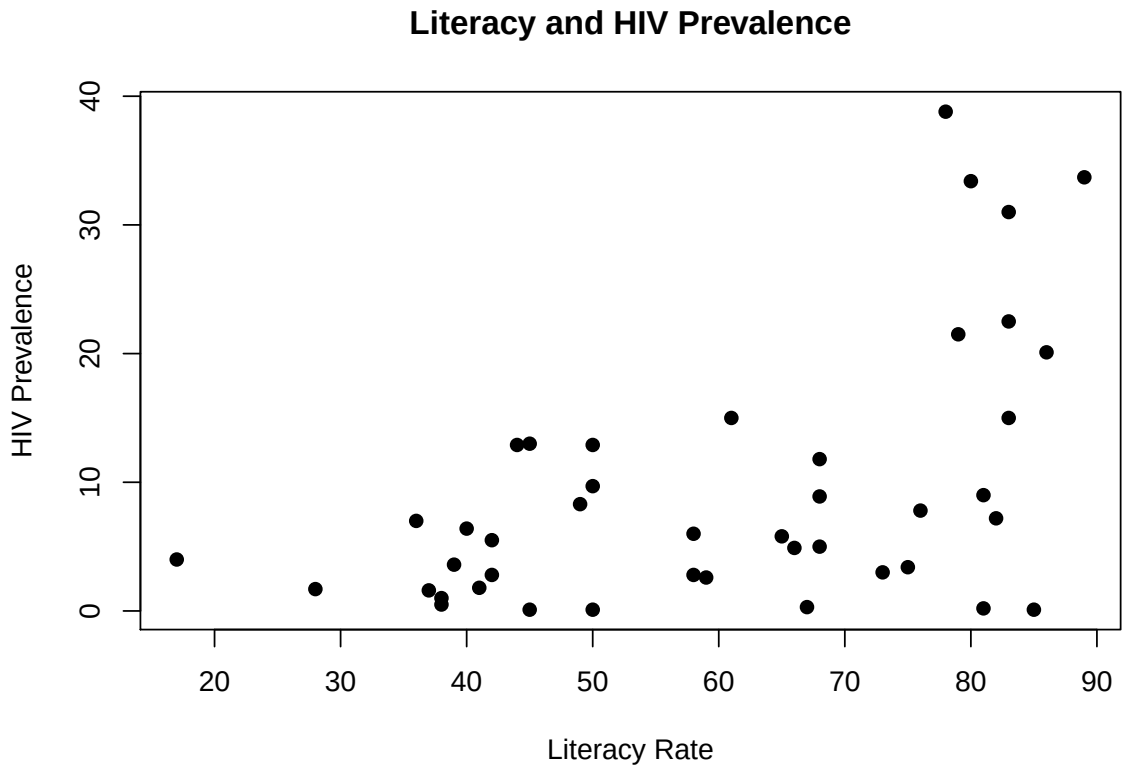


Scatterplots

- Plotting two continuous variables.
- Horizontal ("X") axis = "independent" variable.
- Vertical ("Y") axis = "dependent" variable.
- Illustrates degree of association between X and Y.

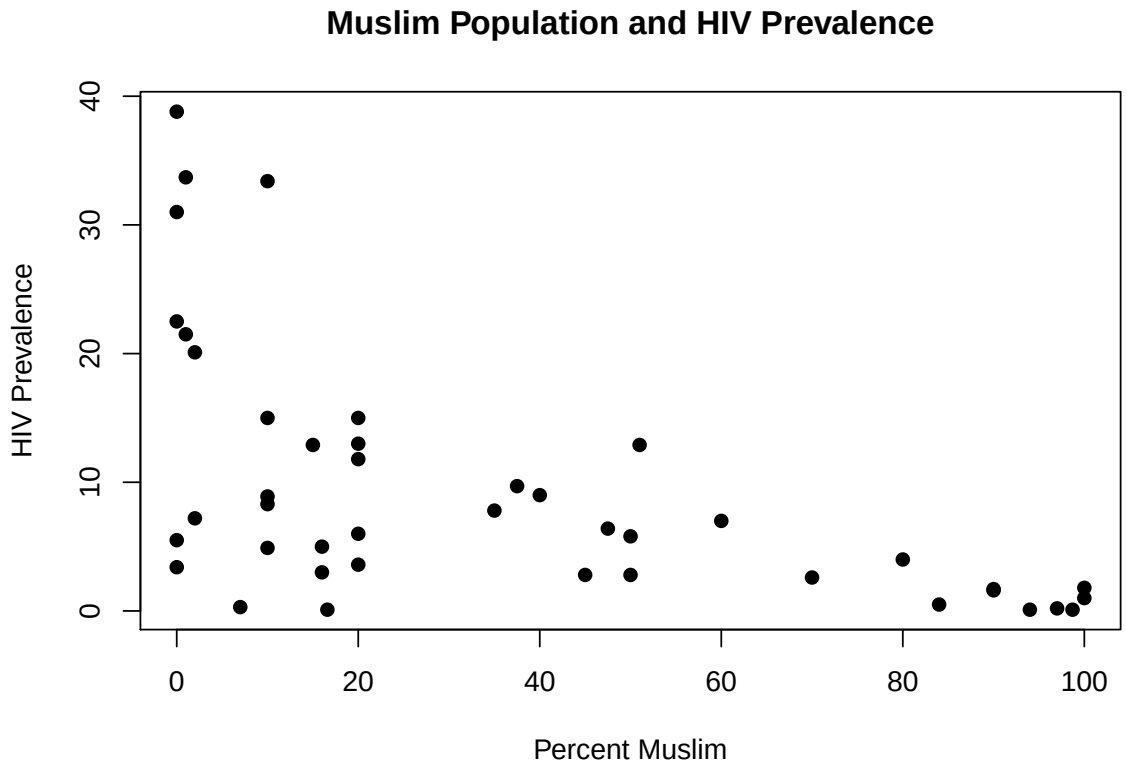
Scatterplots: Examples

```
> with(Africa, plot(literacy,adrate,pch=19,
    xlab="Literacy Rate",ylab="HIV Prevalence",
    main="Literacy and HIV Prevalence"))
```



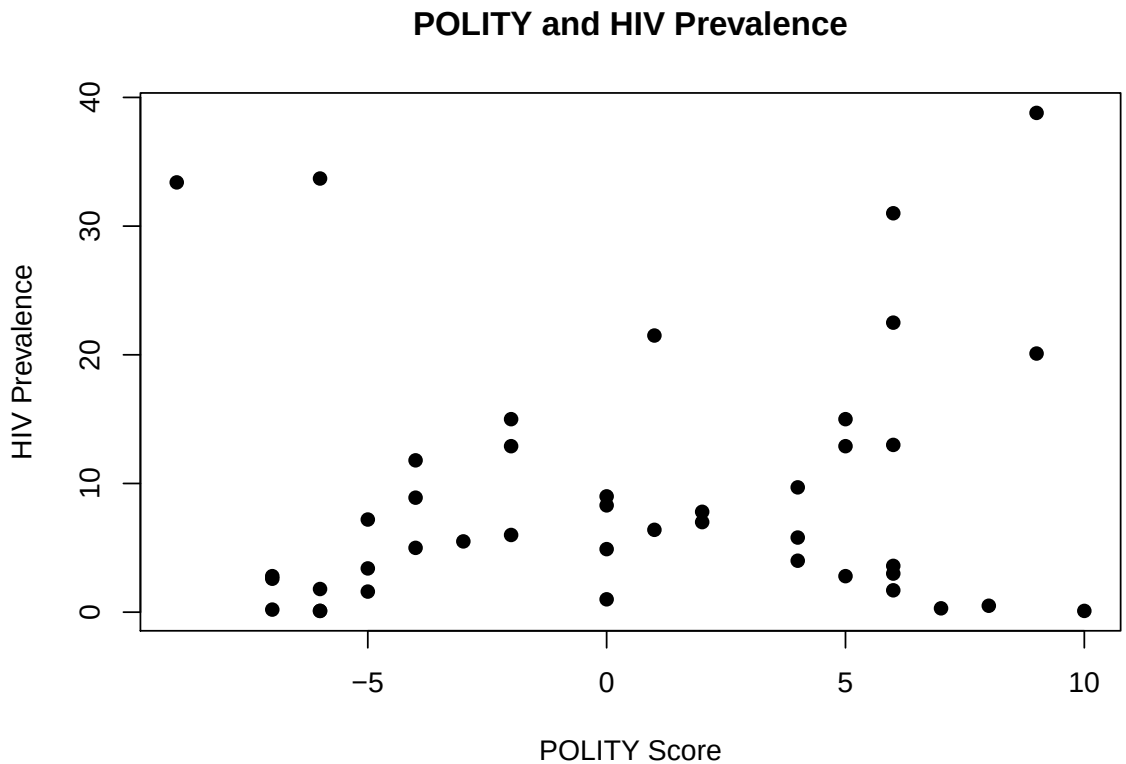
Scatterplots: Examples

```
> with(Africa, plot(muslperc,adrate,pch=19,
    xlab="Percent Muslim",ylab="HIV Prevalence",
    main="Muslim Population and HIV Prevalence"))
```



Scatterplots: Examples

```
> with(Africa, plot(polity,adrate,pch=19,
    xlab="POLITY Score",ylab="HIV Prevalence",
    main="POLITY and HIV Prevalence"))
```



Crosstables ("Contingency Tables")

- "Two-way" frequency tables
- Reports the number of observations that have a given *pair* of values on two variables (say X and Y).
- Can show frequencies or percentages.
- Best for *discrete* / *categorical* variables

Crosstab Example

Basic crosstab:

```
> with(Africa, xtabs(~internalwar+subsaharan))

            subsaharan
internalwar Not Sub-Saharan Sub-Saharan
          0               5          25
          1               1          12
```

Cell proportions:

```
> with(Africa, prop.table(xtabs(~internalwar+subsaharan)))
            subsaharan
internalwar Not Sub-Saharan Sub-Saharan
          0         0.11628     0.58140
          1         0.02326     0.27907
```

Interpretation: "11.6 percent of all African countries were both not sub-saharan and had no internal conflict in 2001. 58.1 percent of all African countries were both sub-Saharan and had no internal conflict in 2001...." etc.

Crosstab Example (continued)

"Row marginals":

```
> with(Africa, prop.table(xtabs(~internalwar+subsaharan),1))

           subsaharan
internalwar Not Sub-Saharan Sub-Saharan
          0         0.16667     0.83333
          1         0.07692     0.92308
```

Interpretation: "In 2001, 16.7 percent of all African countries that did not have an internal conflict were Saharan, and 83.3 percent were sub-Saharan. In contrast, only 7.7 percent of all African countries that experienced an internal conflict were Saharan, while 92.3 percent were sub-Saharan."

Crosstab Example (continued)

"Column marginals":

```
> with(Africa, prop.table(xtabs(~internalwar+subsaharan),2))

           subsaharan
internalwar Not Sub-Saharan Sub-Saharan
          0          0.8333      0.6757
          1          0.1667      0.3243
```

Interpretation: "In 2001, 16.7 percent of all Saharan African countries experienced an internal conflict. By contrast, 32.4 percent of all sub-Saharan countries experienced an internal conflict. Sub-Saharan countries were therefore about twice as likely to experience an internal conflict than were Saharan countries."

Differences of Means

Suppose Y is our variable of interest, and that there are two groups, $X = 0$ and $X = 1$, with n_0 and n_1 observations in each group. We want to know if $\bar{Y}$ is different for $X = 0$ vs. $X = 1$.

Difference of means:

$$\bar{Y}_1 - \bar{Y}_0 = \frac{1}{n_1} \sum_{i=1}^{n_1} Y_{1i} - \frac{1}{n_0} \sum_{i=1}^{n_0} Y_{0i}$$

Variation

$$s_{\bar{Y}_1 - \bar{Y}_0} = \sqrt{\frac{s_0^2}{n_0} + \frac{s_1^2}{n_1}}.$$

"T-score":

$$t = \frac{(\bar{Y}_1 - \bar{Y}_0)}{s_{\bar{Y}_1 - \bar{Y}_0}}$$

Table: Rough Values of t You'll Want To Get To Know

Absolute Value of t	P-Value
≈ 1.3	0.20
≈ 1.65	0.10
≈ 2	0.05
≈ 2.4	0.02
≈ 2.6	0.01
> 3	< 0.002

T-test: Examples

```
> with(Africa, t.test(adrate~subsaharan))

Welch Two Sample t-test

data:  adrate by subsaharan
t = -5.4, df = 41, p-value = 0.000003
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -12.942  -5.881
sample estimates:
mean in group Not Sub-Saharan     mean in group Sub-Saharan
                        1.267                        10.678
```

Interpretation: "The mean HIV prevalence rate in 2001 was 1.27 percent in Saharan Africa, and 10.68 percent in sub-Saharan Africa. This difference is statistically important / significant ($t = -5.4$)."

T-test: Examples

```
> with(Africa, t.test(internalwar~subsaharan))

Welch Two Sample t-test

data:  internalwar by subsaharan
t = -0.86, df = 7.4, p-value = 0.4
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -0.5883  0.2730
sample estimates:
mean in group Not Sub-Saharan     mean in group Sub-Saharan
                        0.1667                        0.3243
```

Interpretation: "The incidence of internal conflict in 2001 was 16.7 percent in Saharan Africa and 32.4 percent in sub-Saharan Africa. This difference is not statistically important / significant ($t = -0.86$)."

Continuous Variables: Correlation

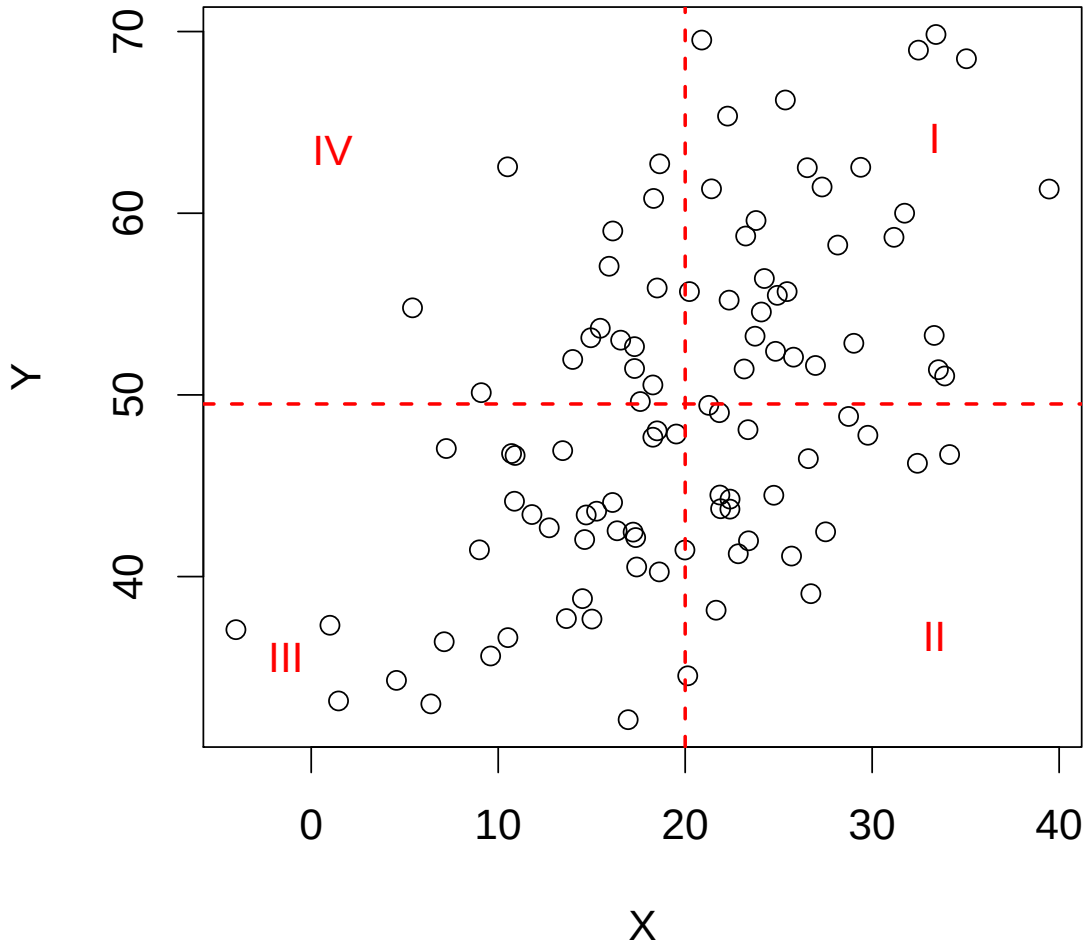
Linear relationship:

$$\frac{\partial Y}{\partial X} = m;$$

"Pearson's r":

$$r = \frac{\sum_{i=1}^{N} \left(\frac{X_i - \bar{X}}{s_X}\right) \left(\frac{Y_i - \bar{Y}}{s_Y}\right)}{N - 1}$$

Pearson's r: Intuition



The Meaning of r

Table: Rough Values of r You'll Want To Get To Know

Absolute Value of t	Meaning
$r = -1.0$	*Perfect negative* linear association
$r \approx -0.7$	*Strong negative* linear association
$r \approx -0.3$	*Weak negative* linear association
$r \approx 0$	No linear association
$r \approx 0.3$	*Weak positive* linear association
$r \approx 0.7$	*Strong positive* linear association
$r = 1.0$	*Perfect positive* linear association

Correlation: Examples

¿ with(Africa, cor(adrate,literacy)) [1] 0.5149
¿ with(Africa, cor(adrate,muslperc)) [1] -0.5709
¿ with(Africa, cor(adrate,popden)) [1] -0.1681

Interpretation: "In 2001, there was a moderate, positive correlation between HIV prevalence rates and literacy rates in Africa. That same year, there was a moderate, negative correlation between HIV prevalence and the percentage of the population who self-identified as Muslim, and a small, negative correlation between HIV prevalence rates and population density."

(Bivariate) Linear Regression

- Related to correlation: *linear* association.
- "Fit" a (straight) line through the points of a scatterplot:

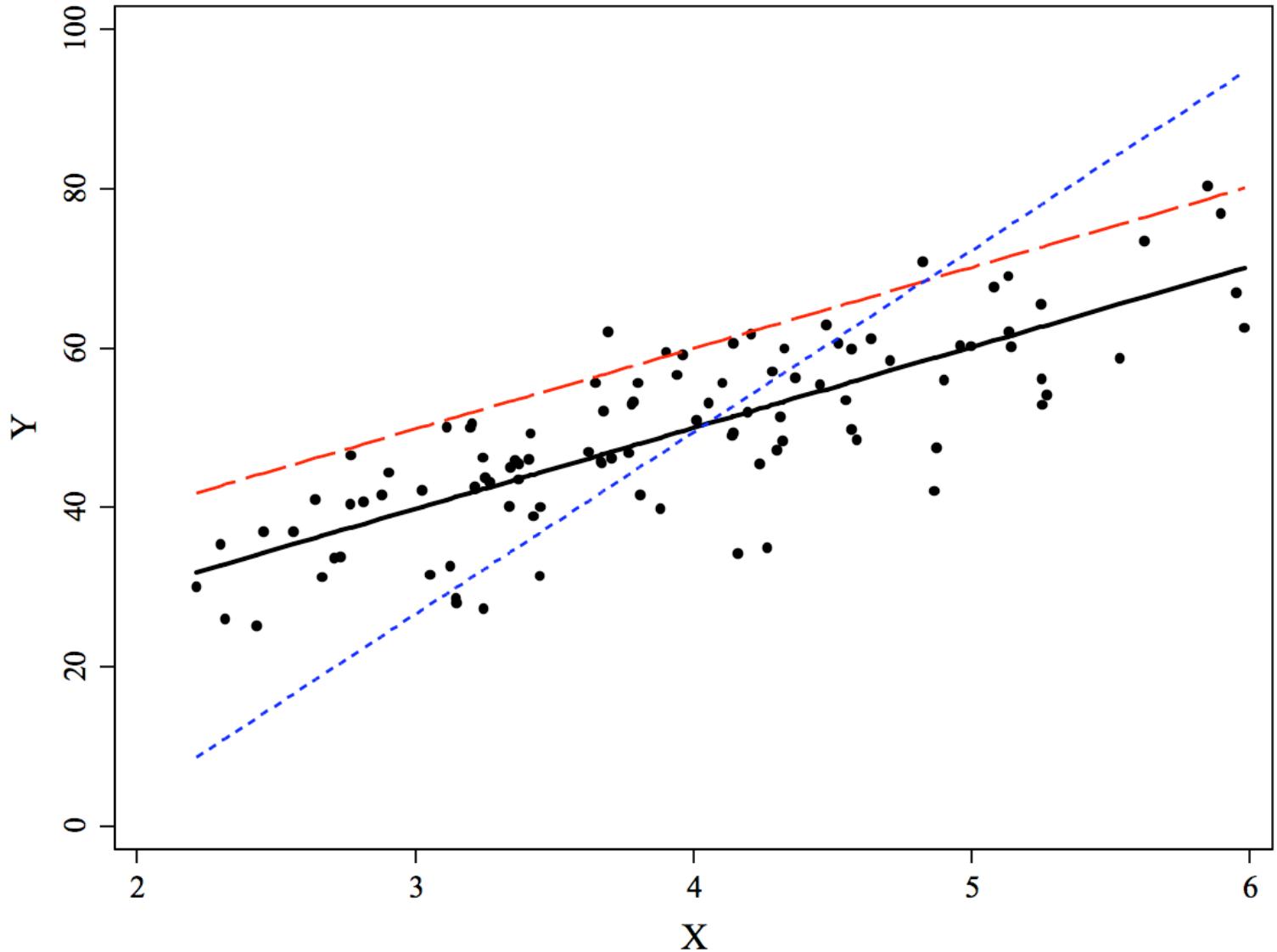
$$Y_i = \beta_0 + \beta_1 X_i + u_i \tag{1}$$

  - β_0 is the "intercept"
  - β_1 is the "slope"

- Based on principle of "least squares":

Choose β_0 and β_1 so that they make $u_i^2 = (Y_i - \beta_0 - \beta_1 X_i)^2$ as small as possible.

Linear Regression: Intuition



Linear Regression: Examples

```
> fit<-with(Africa, lm(adrate~literacy))
> summary(fit)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  -6.9012     4.4301   -1.56  0.12696
literacy      0.2708     0.0704    3.85  0.00041 ***
---
Signif. codes:
0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 8.64 on 41 degrees of freedom
Multiple R-squared:  0.265, Adjusted R-squared:  0.247
F-statistic: 14.8 on 1 and 41 DF,  p-value: 0.000411
```

Interpretation: "In the 2001 Africa data, the intercept is -6.9 and the slope is 0.27. This means that a one percentage point increase in literacy is associated with an increase in the HIV prevalence rate of 0.27 percentage points."

Linear Regression Illustrated

```
> with(Africa, plot(literacy,adrate,pch=19,
     xlab="Literacy Rate",ylab="HIV Prevalence",
     main="Literacy and HIV Prevalence"))
> abline(fit,lwd=3,col="red")
```