

PLSC 502 – Autumn 2022

Probability Distributions

September 29, 2022

Why learn about probability distributions?

- Things we study are both *regular* and *random*.
- Inference: Making claims about the world without getting *all* the data...
- Prediction: Getting things right (on average), but also knowing how precisely we do that.
- For more answer(s) to this question, see [this](#).

What do we need to know?

Distributions are characterized by a combination of their *formulas* and their *moments*...

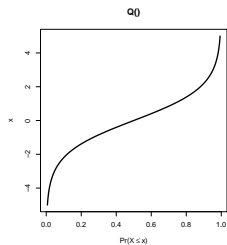
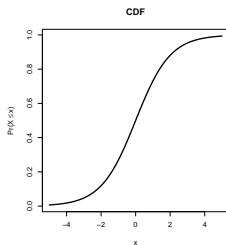
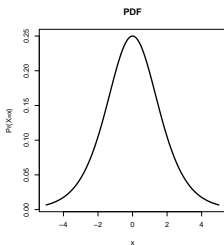
- The *formula* describes the mapping from the outcome space to the probability space.
- The *moments* tell us about the “location,” shape, and other characteristics of a particular instance of a distribution.
 - Most distributions are fully characterized by their formula + one or more (usually a small number of) finite moments (often called “parameters”).
 - A distribution's *moment generating function* provides a bridge between its moments and the formula for the distribution.

Describing Distributions

Attributes of Distributions

Attribute	a/k/a	Meaning	Function	In R
Density	$f(X)$, "PDF" *	$\Pr(X = x)$	Maps from x to $\Pr(X = x)$	<code>ddist</code>
Distribution	$F(X)$, "CDF"	$\Pr(X \leq x) = \int_{-\infty}^x f(X)dX$	Maps from x to $\Pr(X \leq x)$	<code>pdist</code>
Quantile	$Q(p)$, $F_X^{-1}(p)$	Value of x such that $\Pr(X \leq x)$	Maps from $\Pr(X \leq x)$ to x	<code>qdist</code>

* Sometimes called a "probability mass function" ("PMF") when X is discrete.



Discrete Distributions

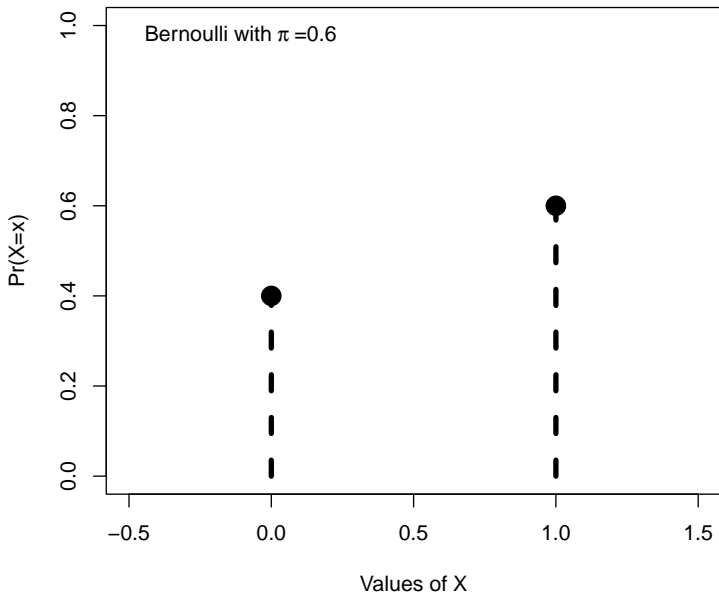
Binary X :

$$\begin{aligned} X &= 0 \text{ with probability } 1 - \pi \\ &= 1 \text{ with probability } \pi. \end{aligned}$$

Density / PDF:

$$\begin{aligned} f(x) &= \begin{cases} 1 - \pi & \text{for } X = 0 \\ \pi & \text{for } X = 1 \end{cases} \\ &= \pi^x (1 - \pi)^{1-x}, \quad x \in \{0, 1\} \end{aligned}$$

$$X \sim \text{Bernoulli}(\pi)$$



CDF:

$$\begin{aligned} F(x) &= \sum_x f(x) \\ &= \begin{cases} 1 - \pi & \text{for } X = 0 \\ 1 & \text{for } X = 1 \end{cases} \end{aligned}$$

Expectation:

$$\begin{aligned} E(X) &= \sum_x xf(x) \\ &= (0)(1 - \pi) + (1)(\pi) \\ &= \pi \end{aligned}$$

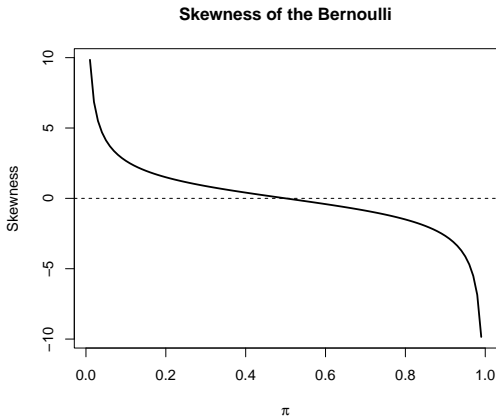
Variance:

$$\begin{aligned}\text{Var}(X) &= \sum_x [X - E(X)]^2 f(x) \\&= \sum_x [X - \pi]^2 f(x) \\&= (0 - \pi)^2(1 - \pi) + (1 - \pi)^2\pi \\&= \pi^2 - \pi^3 + \pi - 2\pi^2 + \pi^3 \\&= \pi^2 - \pi \\&= \pi(1 - \pi)\end{aligned}$$

Even More Bernoulli

Skewness:

$$\text{Skewness} = \frac{(1 - \pi) - \pi}{\sqrt{(1 - \pi)\pi}}$$



Even More Bernoulli

MGF:

$$\begin{aligned}\psi(t) &= \int_{-\infty}^{\infty} \exp(tx) dF(x) \\ &= \sum_{n=0}^1 \exp(tn) \pi^n (1-\pi)^{1-n} \\ &= \exp(0)(1-\pi) + \exp(t)\pi \\ &= (1-\pi) + \pi \exp(t)\end{aligned}$$

Implying:

$$\frac{\partial^k \psi(t)}{\partial^k t} = \pi \exp(t) \quad \forall k$$

and *raw moments*:

$$E(X^k) = \pi \quad \forall k > 0$$

Central moments:

$$M_1 = \pi,$$

$$M_2 = \pi(1 - \pi),$$

$$M_3 = \pi(1 - \pi)(1 - 2\pi),$$

etc.

Assume n independent binary “trials,” each with identical probability of “success” π . Then the number of “successes” in n trials follows a *binomial* distribution:

$$f(x) = \binom{n}{x} \pi^x (1 - \pi)^{n-x}$$

where recall that

$$\binom{n}{x} \equiv \frac{n!}{x!(n-x)!}.$$

$$X \sim \text{binomial}(n, \pi).$$

Why “binomial”?

Binomial Theorem:

$$(a + b)^n = \sum_{k=0}^n \binom{n}{k} a^k b^{n-k}.$$

e.g.

$$(a + b)^2 = a^2 + 2ab + b^2,$$

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3,$$

$$(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4,$$

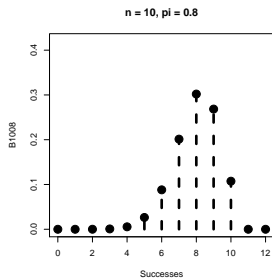
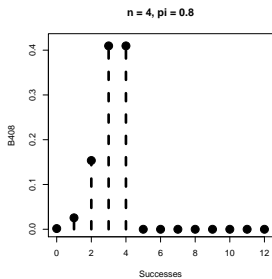
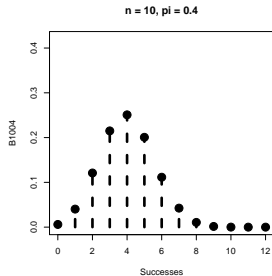
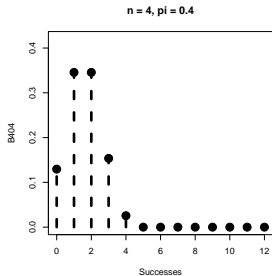
Binomial: Derivation

For $n = 2$:

$$\begin{aligned}\Pr(X = 0) &= \Pr(X_1 = 0, X_2 = 0) \\ &= \Pr(X_1 = 0) \times \Pr(X_2 = 0) \\ &= (1 - \pi)^2\end{aligned}$$

$$\begin{aligned}\Pr(X = 1) &= \Pr(X_1 = 1, X_2 = 0 \text{ or } X_1 = 0, X_2 = 1) \\ &= \Pr(X_1 = 1) \times \Pr(X_2 = 0) + \Pr(X_1 = 0) \times \Pr(X_2 = 1) \\ &= \pi(1 - \pi) + (1 - \pi)\pi \\ &= 2[\pi(1 - \pi)]\end{aligned}$$

$$\begin{aligned}\Pr(X = 2) &= \Pr(X_1 = 1, X_2 = 1) \\ &= \Pr(X_1 = 1) \times \Pr(X_2 = 1) \\ &= \pi^2\end{aligned}$$



CDF:

$$\begin{aligned} F(x) &= \sum_{j=0}^x f(j) \\ &= \sum_{j=0}^x \binom{n}{j} \pi^j (1 - \pi)^{n-j} \end{aligned}$$

Expectation:

$$E(X) = n\pi,$$

Variance:

$$\begin{aligned}\text{Var}(X) &= \sum_x [X - E(X)]^2 f(x) \\ &= \sum_x (X - \pi n)^2 \binom{n}{x} \pi^x (1 - \pi)^{n-x} \\ &= n\pi(1 - \pi).\end{aligned}$$

Skewness:

$$\text{Skewness} = \frac{1 - 2\pi}{\sqrt{n\pi(1 - \pi)}}$$

The Binomial...

- Is *unimodal* (except in certain cases),
- has median $\lceil n\pi \rceil$ or $\lfloor n\pi \rfloor$,
- has mode $\lceil (n+1)\pi \rceil$ or $\lfloor (n+1)\pi \rfloor$,
- has skewness that is:
 - increasing in n , and
 - is largest when $\pi = 0.5$ for a fixed value of n .

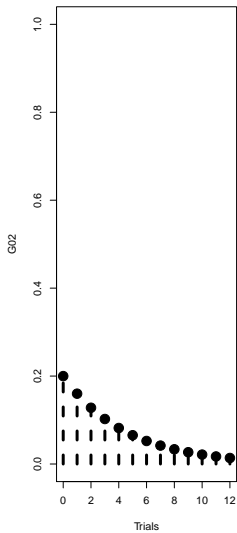
The number of independent Bernoulli trials needed to achieve *one* success is a *geometric* random variable.

PDF:

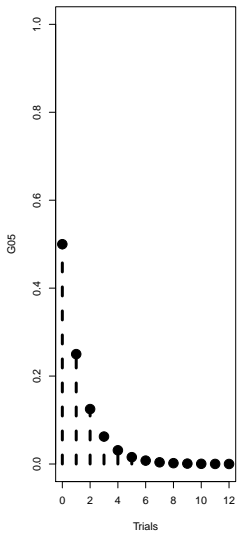
$$f(x) = \pi(1 - \pi)^{x-1}$$

$$X \sim \text{geometric}(\pi).$$

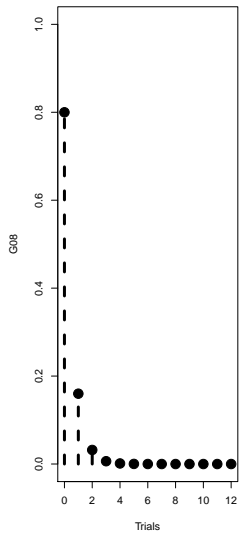
pi = 0.2



pi = 0.5



pi = 0.8



CDF:

$$\begin{aligned} F(x) &= \sum_{j=1}^x \pi(1-\pi)^{x-1} \\ &= 1 - (1-\pi)^x \end{aligned}$$

Expectation:

$$E(X) = \frac{1}{\pi}$$

Variance:

$$\text{Var}(X) = \frac{1-\pi}{\pi^2}$$

Negative Binomial

The number of *failures we observe* (x) before achieving the r th success in a series of independent Bernoulli trials (each with equal probability of success π) is distributed according to a *negative binomial* distribution.

PDF:

$$f(x) = \binom{r+x-1}{r-1} \pi^r (1-\pi)^x$$

More Negative Binomial

CDF:

$$\begin{aligned} F(x) &= \sum_{j=0}^x \binom{r+j-1}{r-1} \pi^r (1-\pi)^j \\ &= 1 - \text{CDF}_{\text{binomial}} \end{aligned}$$

Expected value:

$$E(X) = \frac{(1-\pi)r}{\pi}$$

Even More Negative Binomial

Variance:

$$\text{Var}(X) = \frac{(1 - \pi)r}{\pi^2}.$$

Skewness:

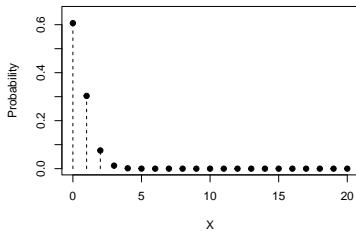
$$\text{Skewness} = \frac{1 + \pi}{\sqrt{\pi r}}$$

For n independent Bernoulli trials with (sufficiently small) probability of success π and where $n\pi \equiv \lambda > 0$, the probability of observing exactly x total “successes” as the number of trials grows without limit is the *Poisson distribution*.

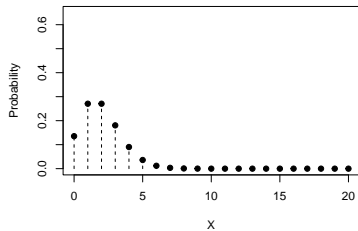
PDF:

$$\begin{aligned} f(x) &= \lim_{n \rightarrow \infty} \left[\binom{n}{x} \left(\frac{\lambda}{n}\right)^x \left(1 - \frac{\lambda}{n}\right)^{n-x} \right] \\ &= \frac{\lambda^x \exp(-\lambda)}{x!}. \end{aligned}$$

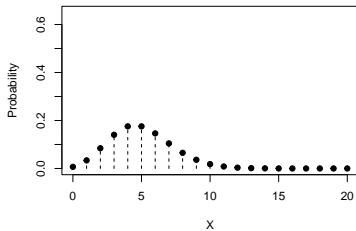
Lambda = 0.5



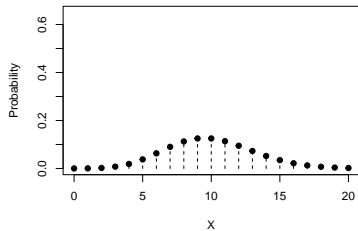
Lambda = 1



Lambda = 4



Lambda = 8



CDF:

$$F(x) = \sum_{j=0}^x \frac{\lambda^j \exp(-\lambda)}{j!}.$$

Mean & Variance:

$$E(X) = \text{Var}(X) = \lambda$$

All higher moments are zero...

Alternative Poisson

Independent, constant-probability events occurring in time...

$$\text{"Arrival rate"} = \lambda$$

Implies:

$$\begin{aligned}\Pr(\text{Event in } (t, t+h]) &= \lambda h \\ \Pr(\text{No event in } (t, t+h]) &= 1 - \lambda h\end{aligned}$$

$$N_{\text{Events occurring in } (t, t+h]} = \frac{\exp(-\lambda h) \lambda h^x}{x!}$$

If $h = 1 \forall h$, then:

$$f(x) = \frac{\exp(-\lambda) \lambda^x}{x!}$$

Multinomial

Imagine K possible distinct *outcomes* for each “trial,” where each possible outcome $k \in \{1, 2, \dots, K\}$ has π_k and $\sum_{k=1}^K \pi_k = 1$.

Define x_k = number of times we observe outcome k out of n trials.

Then for:

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_K \end{pmatrix}$$

the PDF is:

$$f(\mathbf{x}) = \frac{n!}{x_1! x_2! \dots x_K!} \pi_1^{x_1} \pi_2^{x_2} \dots \pi_K^{x_K}$$

Multinomial, continued

Expected value:

$$E(\mathbf{X}) \equiv E \begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ X_K \end{pmatrix} = n \begin{pmatrix} \pi_1 \\ \pi_2 \\ \vdots \\ \pi_K \end{pmatrix}$$

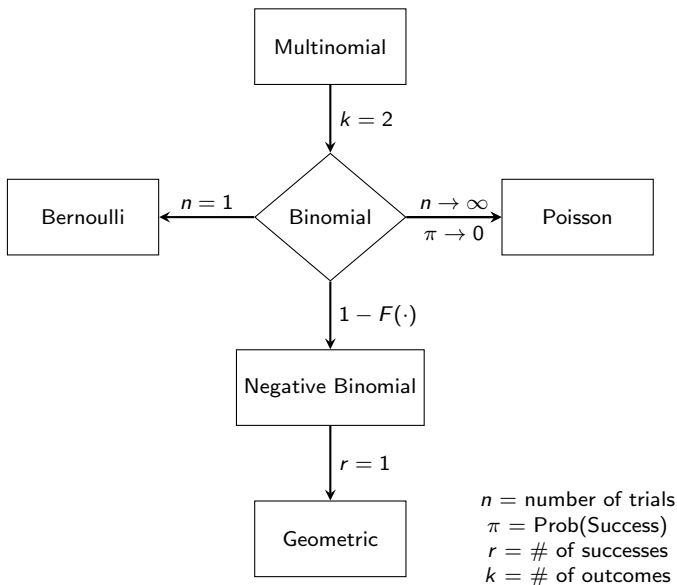
Variance:

$$\text{Var}(\mathbf{X}) \equiv \text{Var} \begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ X_K \end{pmatrix} = n \begin{bmatrix} \pi_1(1 - \pi_1) \\ \pi_2(1 - \pi_2) \\ \vdots \\ \pi_K(1 - \pi_K) \end{bmatrix}$$

Covariance between X_s and X_t , $s \neq t$:

$$\text{Cov}(X_s, X_t) = -n\pi_s\pi_t$$

Schematic



Continuous Distributions

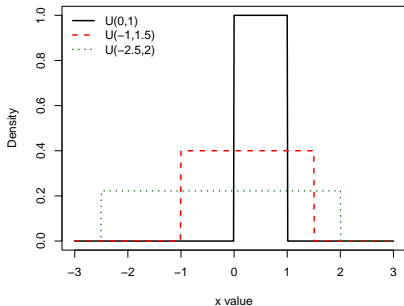
The Uniform Distribution

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{for } a \leq x \leq b, \\ 0 & \text{for } x < a \text{ or } x > b. \end{cases}$$

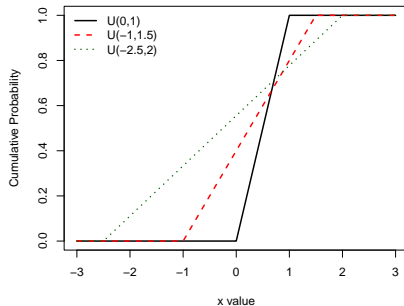
$$F(x) = \int f(x)dx = \begin{cases} 0 & \text{for } x < a \\ \frac{x-a}{b-a} & \text{for } a \leq x < b \\ 1 & \text{for } x \geq b \end{cases}$$

Uniform PDFs and CDFs

Probability Density Functions



Cumulative Distribution Functions



Uniform Characteristics

Expected value / “mean”:

$$E(X) = \check{X} = \frac{a + b}{2}$$

Mode:

$$\text{mode}(X) = [a, b]$$

Variance:

$$\text{Var}(X) = \frac{(b - a)^2}{12}$$

Skewness:

$$\text{Skewness}(X) = 0$$

The Standard Uniform Distribution

Special case of the uniform:

$$X \sim U(0, 1)$$

It has the property that:

$$X \sim 1 - X \sim U(0, 1).$$

In addition, the CDF is also unique:

$$F(x) = x$$

The Normal Distribution

The Normal density is:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

abbreviated:

$$X \sim \phi_{\mu, \sigma^2}.$$

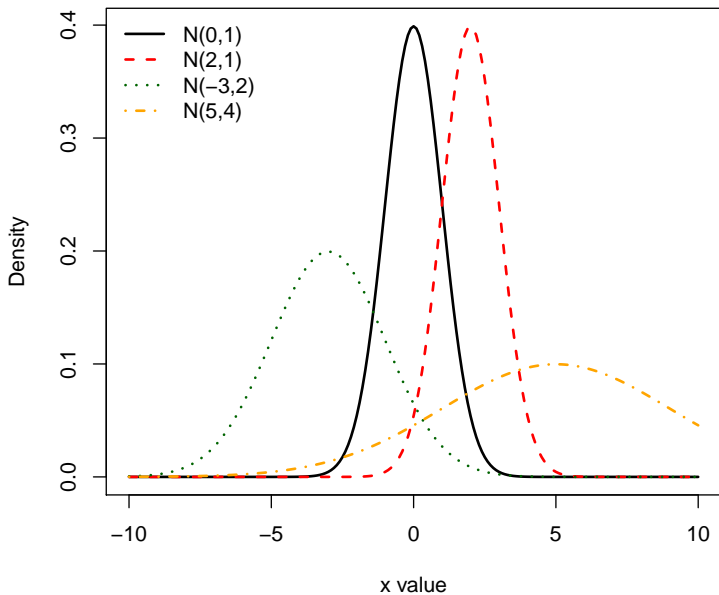
The CDF is:

$$\begin{aligned} F(x) &= \Phi_{\mu, \sigma^2}(x) \\ &= \frac{1}{2} \left[1 + \operatorname{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right) \right], \end{aligned}$$

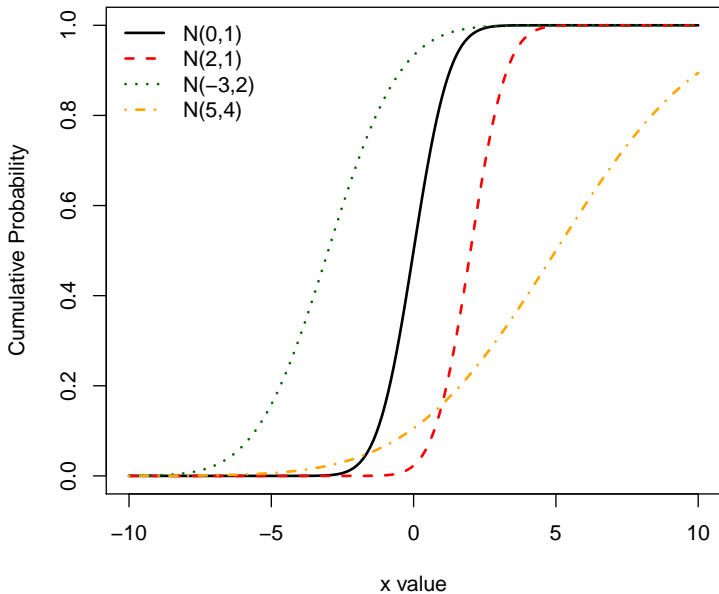
where

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt.$$

Various Normal Densities



Various Normal CDFs



Why Normal?

For $i = \{1, 2, \dots, N\}$ i.i.d. X_i with $\mu_i < \infty$ and $\sigma_i^2 > 0$, define:

$$X = \sum_{i=1}^N X_i.$$

Then

$$\begin{aligned} E(X) &= \sum_{i=1}^N \mu_i \\ &= \mu < \infty \end{aligned}$$

and

$$\begin{aligned} \text{Var}(X) &= \sum_{i=1}^N \sigma_i^2 \\ &= \sigma^2 < \infty. \end{aligned}$$

Central Limit Theorem

The “CLT”:

$$\lim_{N \rightarrow \infty} X = \lim_{N \rightarrow \infty} \sum_{i=1}^N X_i \xrightarrow{D} N(\cdot)$$

“...we often think of a normal distribution as being appropriate when the observed variable X can take on a range of continuous values, and when the observed value of X can be thought of as the product of a large number of relatively small, independent “shocks” or perturbations.”

Properties of the Normal Distribution

The normal is a two-parameter distribution, where

$$\mu \in (-\infty, \infty)$$

and

$$\sigma^2 \in (0, \infty).$$

For $X \sim N(\mu, \sigma^2)$:

- X has support in \Re
- $\text{Skewness}(X) = 0$
- X is *mesokurtic*
- If $X \sim N(\mu, \sigma^2)$, then $aX + b \sim N(a\mu + b, a^2\sigma^2)$

The Standard Normal Distribution

Linear transformation:

- $b = \frac{-\mu}{\sigma},$
- $a = \frac{1}{\sigma}.$

Yields:

$$\begin{aligned}ax + b &\sim N(a\mu + b, a^2\sigma^2) \\ &\sim N(0, 1)\end{aligned}$$

In other words:

- If $X \sim N(\mu, \sigma^2)$, then $Z = \frac{(x-\mu)}{\sigma} \sim N(0, 1).$
- The PDF is:

$$f(z) \equiv \phi(z) = \frac{1}{\sqrt{2\pi}} \exp \left[-\frac{(z)^2}{2} \right]$$

Similarly, we often write the CDF for the standard normal as $\Phi(\cdot).$

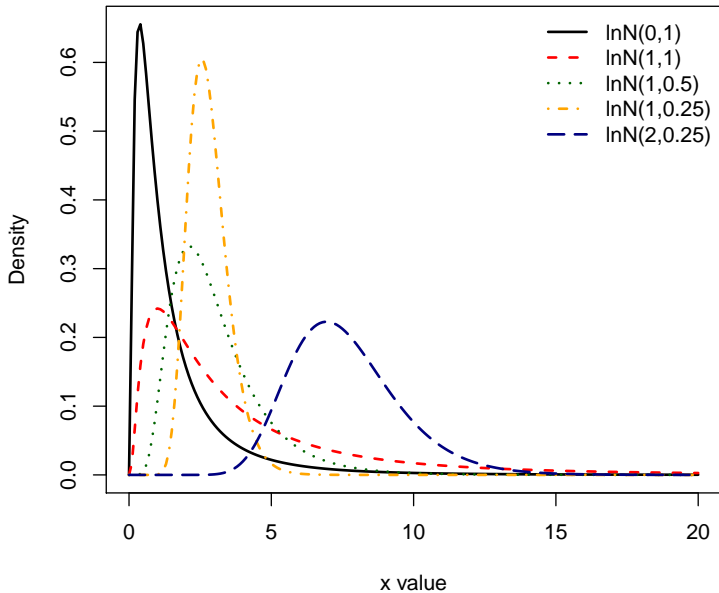
The Log-Normal Distribution

$$Y = \exp(X) \sim \text{LogN}(m, s^2)$$

PDF:

$$f(y) = \frac{1}{ys\sqrt{2\pi}} \times \exp \left[\frac{-(\ln y - m)^2}{2s^2} \right].$$

Log-Normal PDFs



The χ^2 Distribution

PDF:

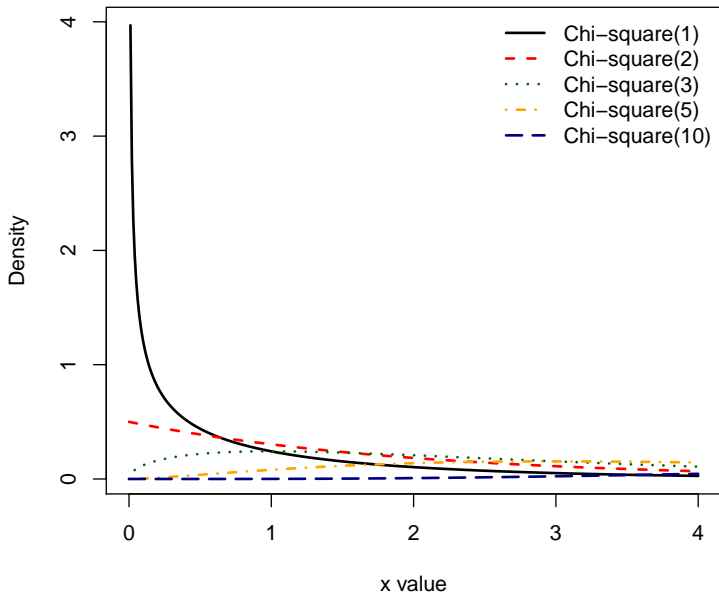
$$\begin{aligned} f(w) &= \frac{1}{2^k \Gamma(k)} w^k \exp\left[-\frac{w}{2}\right] \\ &= \frac{w^{\frac{k-2}{2}} \exp\left(-\frac{w}{2}\right)}{2^{\frac{k}{2}} \Gamma\left(\frac{k}{2}\right)} \end{aligned}$$

where $\Gamma(k) = \int_0^\infty t^{k-1} \exp(-t) dt$.

CDF:

$$F(w) = \frac{\gamma(k/2, w/2)}{\Gamma(k/2)}$$

χ^2 Densities



χ^2 Characteristics

For $W \sim \chi_k^2$:

$$E(W) = k$$

and:

$$\text{Var}(W) = 2k.$$

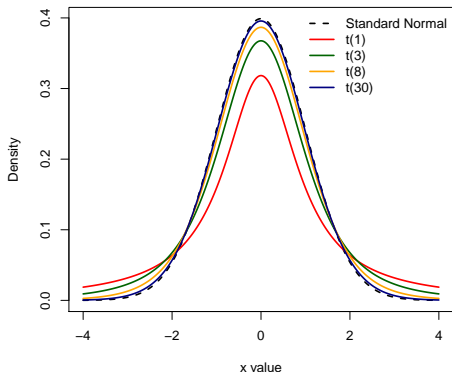
Also:

- If $Z \sim N(0, 1)$, then $Z^2 \sim \chi_1^2$
- If $W_j \sim \chi_j^2$ and $W_k \sim \chi_k^2$ and independent, then
 - $W_j + W_k$ is $\sim \chi_{j+k}^2$ and more generally
 - $\sum_{i=1}^k W_i \sim \chi_k^2$.

Student's t Distribution

PDF:

$$f(x) = \frac{\Gamma(\frac{k+1}{2})}{\sqrt{k\pi} \Gamma(\frac{k}{2})} \left(1 + \frac{x^2}{k}\right)^{-\left(\frac{k+1}{2}\right)}$$



Student's t Characteristics

- Mean: $\mu = 0$
- Variance: $\sigma^2 = \frac{k}{k-2}$
- Skewness is 0 for $k > 3$; otherwise undefined
- $t_k \rightarrow N(0, 1)$ as $k \rightarrow \infty$
- If if $Z \sim N(0, 1)$, $W \sim \chi_k^2$, and Z and W are independent, then

$$\frac{Z}{\sqrt{W/k}} \sim t_k$$

and

$$\frac{Z^2}{W/k} \sim t_k.$$

The F Distribution

PDF:

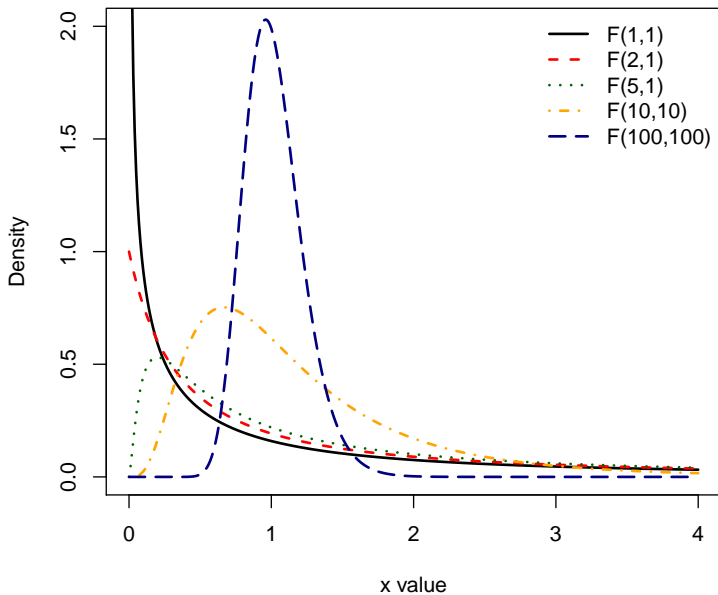
$$f(x) = \frac{\left(\frac{kx}{kx+\ell}\right)^{k/2} \left(1 - \frac{kx}{kx+\ell}\right)^{\ell/2}}{x B(k/2, \ell/2)}$$

where $B(x, y) = \int_0^1 t^{x-1}(1-t)^{y-1} dt$.

And we say:

$$X \sim F_{k,\ell}$$

F Densities

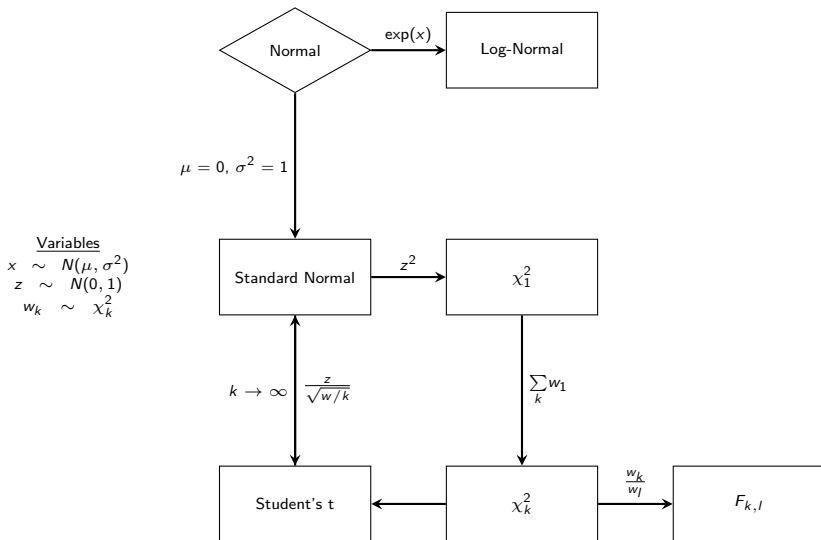


For an $F_{k,\ell}$ distribution:

- $E(X) = \frac{\ell}{\ell-2}$
- $\text{Var}(X) = \frac{2\ell^2(k+\ell-2)}{k(\ell-2)^2(\ell-4)}$
- $\text{Skewness} = \frac{(2k+\ell-2)\sqrt{8(\ell-4)}}{(\ell-6)\sqrt{k(k+\ell-2)}}$
- For independent $W_1 \sim \chi_k^2$ and $W_2 \sim \chi_\ell^2$:

$$\frac{W_1}{W_2} \sim F_{k,\ell}$$

Relationships Among Continuous Distributions



Summary

A Few Distributions

Distribution	First Parameter	Meaning	Second Parameter	Meaning
Bernoulli	π	Pr(Success)		
Binomial	n	N of trials	π	Pr(Success)
Geometric	π	Pr(Success)		
Negative Binomial	r	Rank of "success"	π	Pr(Success)
Poisson	λ	"Arrival rate"		
Uniform	a	Minimum	b	Maximum
Normal	μ	Mean	σ^2	Variance
Lognormal	m	Mean	s^2	Variance
Student's t	k	Degrees of freedom		
Chi-Square	k	Degrees of freedom		
F	k	Degrees of freedom	ℓ	Degrees of freedom

Other Useful Distributions

There are a lot; here, in no particular order:

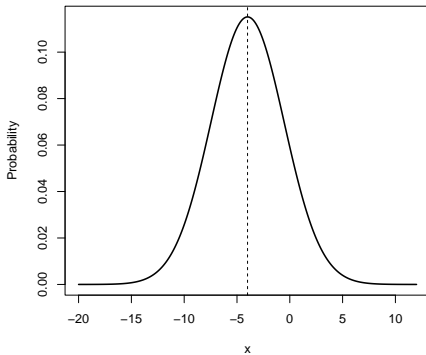
- The **beta distribution** – a continuous two-parameter distribution defined on $[0, 1]$; widely used to model percentages / proportions, and as a “prior” in Bayesian analysis.
- The **exponential distribution** – a continuous one-parameter distribution defined on $[0, \infty)$ that is related to the Poisson distribution (and others).
- The **Weibull distribution** – a continuous two parameter distribution defined on $[0, \infty)$ widely used in survival analysis.
- The **gamma distribution** – a continuous two parameter distribution defined on $[0, \infty)$ that encompasses the exponential, chi-square, and other distributions as special cases.
- The **logistic distribution** – a continuous two parameter distribution defined on \mathbb{R} that resembles the Normal and is widely used in statistics (e.g., logistic regression) and machine learning.
- The **Cauchy distribution** – a continuous two parameter distribution defined on \mathbb{R} and that is interesting mainly because it has no defined MGF.

Practical Things

How To Plot A Density / PDF

"Plot a Normal distribution with a mean of -4 and a variance of 12."

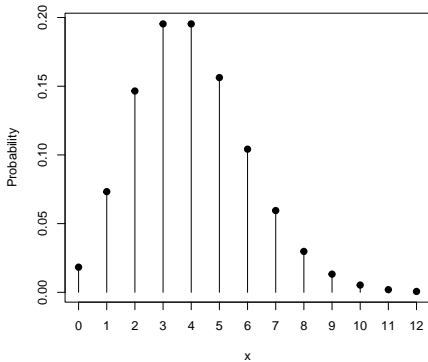
```
> x<-seq(-20,12,by=0.1)  
> PlotNorm<-dnorm(x,-4,sqrt(12))
```



Plotting A Discrete Density

"Plot a Poisson distribution with $\lambda = 4$."

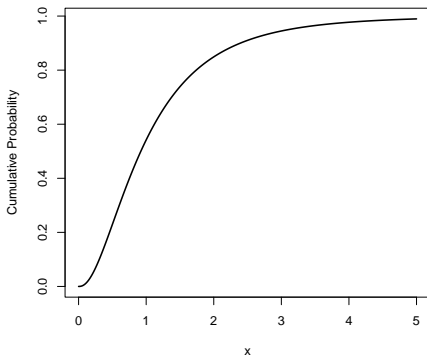
```
> x<-seq(0,12)
> PlotPois<-dpois(x,4)
```



How To Plot A CDF

"Plot the CDF of an F distribution with $k = 5$ and $\ell = 12$."

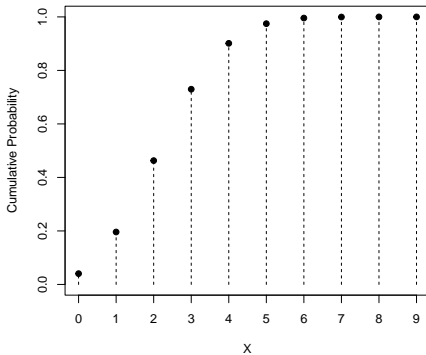
```
> x<-seq(0,5,by=0.01)  
> PlotF<-pf(x,5,12)
```



CDF: Discrete Distribution

"Plot the CDF of a binomial distribution with $\pi = 0.3$ and $n = 9$."

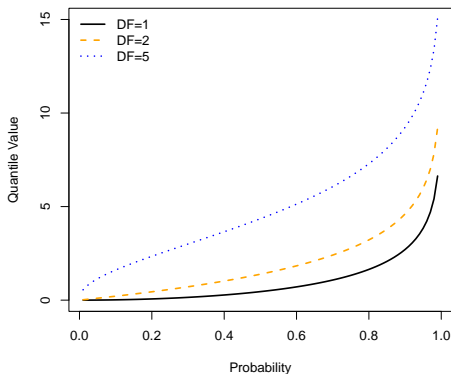
```
> x<-seq(0,9)
> PlotBinom9<-pbinom(x,9,0.3)
```



Quantiles of a Distribution

“Plot the quantiles of three χ^2 distributions with one, two, and five degrees of freedom.”

```
> P<-seq(0.01,0.99,by=0.01) # probabilities  
> ChiSq1<-qchisq(P,1) # df=1  
> ChiSq2<-qchisq(P,2) # df=2  
> ChiSq5<-qchisq(P,5) # df=5
```



Simulating Random Variables

Commands for Generating Random Variates

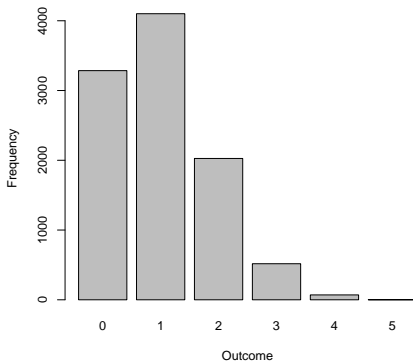
Distribution	R	Stata
Binomial(n, π)	<code>rbinom()</code>	<code>rbinomial</code>
Geometric(π)	<code>rgeom()</code>	<code>?</code>
Negative Binomial(n, π)	<code>rnbinom()</code>	<code>rnbinomial</code>
Poisson(λ)	<code>rpois()</code>	<code>rpoisson</code>
Uniform(0, 1)	<code>runif()</code>	<code>runiform</code>
Normal(0, 1)	<code>rnorm()</code>	<code>rnormal</code>
Lognormal(0, 1)	<code>rlnorm()</code>	<code>xlgn*</code>
Student's $t(k)$	<code>rt()</code>	<code>rt</code>
Chi-Square(k)	<code>rchisq()</code>	<code>rchi2</code>
$F(k, \ell)$	<code>rf()</code>	<code>rndf*</code>

Note: Stata commands marked with an asterisk are from Hilbe's `-rnd-` group of commands. “?” indicates that I’m not aware of any “canned” way of doing this, though one can always generate them “by hand” using the appropriate PDF function.

Drawing From A Distribution

"Draw 10,000 random draws from a binomial distribution with $n = 5$ and $\pi = 0.2$."

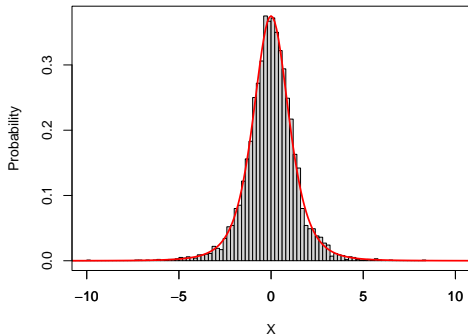
```
> Xbinom5point2<-rbinom(10000,5,0.2)
```



More Draws From A Distribution

“Draw 5000 random draws from a t distribution with 4 degrees of freedom, and compare the distribution of values to the theoretical density.”

```
> TDraws<-rt(5000,4)
```

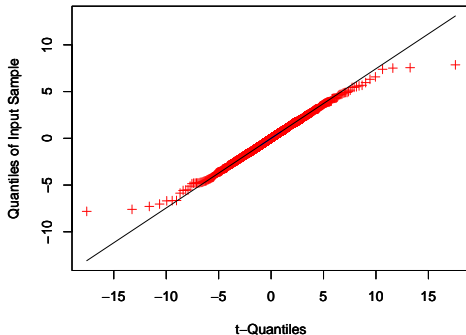


(continued...)

(Same comparison, using a Q-Q plot...)

```
> library(Dowd)
> TQQPlot(TDraws,4)
```

QQ Plot of Sample Data versus Student-t with 4 Degrees of freedom



Pseudo-Random Numbers and “Seeds”

```
> seed<-3229 # calling "seed" some thing  
> set.seed(seed) # setting the system seed  
> rt(3,1) # three draws from a t distrib. w/1 d.f.  
[1] -0.1113 -0.7306  1.9839
```

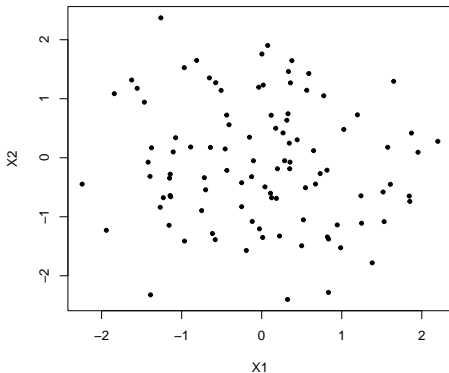
```
> seed<-1077  
> set.seed(seed) # resetting the seed  
> rt(3,1) # different values for the draws  
[1] -0.5211  7.9161 -155.3186
```

```
> seed<-3229 # original seed  
> set.seed(seed)  
> rt(3,1) # identical values of the draws  
[1] -0.1113 -0.7306  1.9839
```

Seeds and Simulations

The right way:

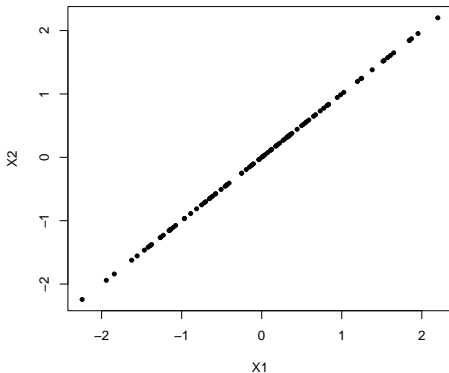
```
> X<-matrix(NA,nrow=100,ncol=2)
> set.seed(7222009)
> for(i in 1:2){
+   X[,i]<-rnorm(100)}
```



Seeds and Simulations (continued)

The wrong way:

```
> X<-matrix(NA,nrow=100,ncol=2)
> for(i in 1:2){
+   set.seed(7222009)
+   X[,i]<-rnorm(100)}
```



In general:

1. **Always** set a seed value.
2. Try to use a single/consistent seed value over time/projects.
3. Keep seeds *outside* loops/apply statements.
6. Use `ddists` and `pdists` for theoretical quantities, `rdists` for generating simulated “data” / variates.
5. Plot discrete distributions discretely....
6. ...and continuous distributions continuously.