PLSC 502 – Autumn 2022
Probability Distributions

September 29, 2022

Why???

Why learn about probability distributions?

- Things we study are both *regular* and *random*.
- Inference: Making claims about the world without getting *all* the data...
- Prediction: Getting things right (on average), but also knowing how precisely we do that.
- For more answer(s) to this question, see this.

What do we need to know?

Distributions are characterized by a combination of their *formulas* and their *moments*...

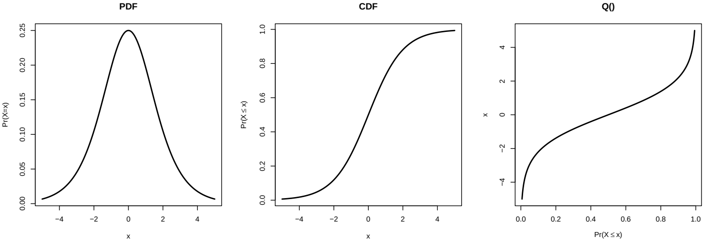
- The *formula* describes the mapping from the outcome space to the probability space.
- The *moments* tell us about the "location," shape, and other characteristics of a particular instance of a distribution.
    - Most distributions are fully characterized by their formula + one or more (usually a small number of) finite moments (often called "parameters").
    - A distribution's *moment generating function* provides a bridge between its moments and the formula for the distribution.

Describing Distributions

Attributes of Distributions

Attribute	a/k/a	Meaning	Function	In R
Density	$f(X)$, "PDF"*	$\Pr(X = x)$	Maps from x to $Pr(X = x)$	`ddist`
Distribution	$F(X)$, "CDF"	$\Pr(X \leq x) = \int_{-\infty}^{x} f(X) dX$	Maps from x to $\Pr(X \leq x)$	`pdist`
Quantile	$Q(p)$, $F_X^{-1}(p)$	Value of x such that $\Pr(X \leq x)$	Maps from $\Pr(X \leq x)$ to x	`qdist`

*Sometimes called a "probability mass function" ("PMF") when X is discrete.



Discrete Distributions

Bernoulli

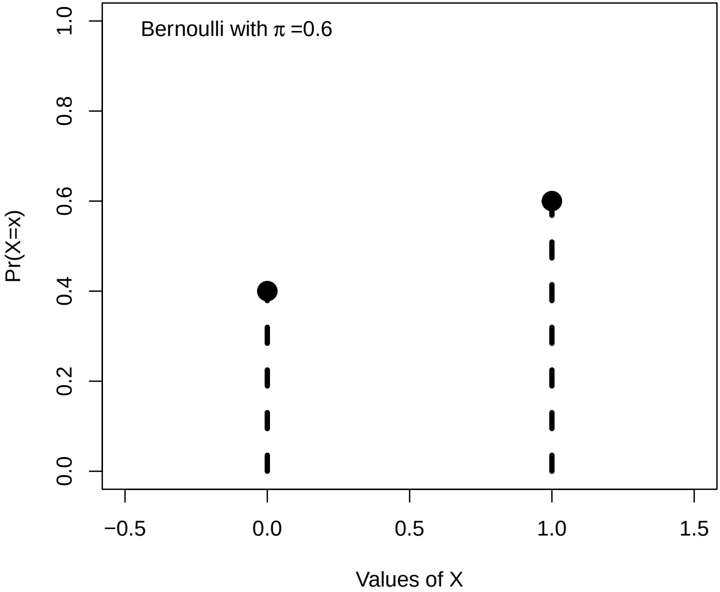
Binary X:

$$
\begin{aligned}
X &= 0 \text{ with probability } 1-\pi \\
&= 1 \text{ with probability } \pi.
\end{aligned}
$$

Density / PDF:

$$
\begin{aligned}
f(x) &= \begin{cases} 1-\pi & \text{for } X=0 \\ \pi & \text{for } X=1 \end{cases} \\
&= \pi^x (1-\pi)^{1-x},\ x \in \{0,1\}
\end{aligned}
$$

$$X \sim \text{Bernoulli}(\pi)$$


Bernoulli with π =0.6
Pr(X=x)
0.0
0.2
0.4
0.6
0.8
1.0
−0.5
0.0
0.5
1.0
1.5
Values of X

Bernoulli, continued

CDF:

$$
\begin{aligned}
F(x) &= \sum_x f(x) \\
&= \begin{cases} 1-\pi & \text{for } X = 0 \\ 1 & \text{for } X = 1 \end{cases}
\end{aligned}
$$

Expectation:

$$
\begin{aligned}
\mathrm{E}(X) &= \sum_x x f(x) \\
&= (0)(1-\pi) + (1)(\pi) \\
&= \pi
\end{aligned}
$$

Bernoulli, continued

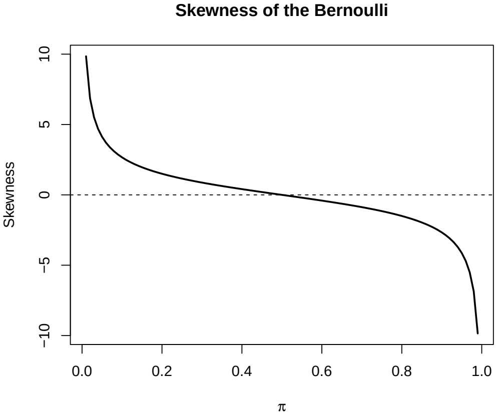
Variance:

$$\begin{aligned}
\mathrm{Var}(X) &= \sum_x [X - \mathrm{E}(X)]^2 f(x) \\
&= \sum_x [X - \pi]^2 f(x) \\
&= (0 - \pi)^2 (1 - \pi) + (1 - \pi)^2 \pi \\
&= \pi^2 - \pi^3 + \pi - 2\pi^2 + \pi^3 \\
&= \pi - \pi^2 \\
&= \pi(1 - \pi)
\end{aligned}$$

Even More Bernoulli

Skewness:

$$\text{Skewness} = \frac{(1-\pi)-\pi}{\sqrt{(1-\pi)\pi}}$$



Even More Bernoulli

MGF:

$$
\begin{aligned}
\psi(t) &= \int_{-\infty}^{\infty} \exp(tx)\, dF(x) \\
&= \sum_{n=0}^{1} \exp(tn)\pi^{n}(1-\pi)^{1-n} \\
&= \exp(0)(1-\pi) + \exp(t)\pi \\
&= (1-\pi) + \pi \exp(t)
\end{aligned}
$$

Implying:

$$\frac{\partial^{k}\psi(t)}{\partial^{k} t} = \pi \exp(t) \;\forall\; k$$

and *raw moments*:

$$\mathrm{E}(X^{k}) = \pi \;\forall\; k > 0$$

Feel the Bern...

Central moments:

$$M_1 = \pi,$$

$$M_2 = \pi(1 - \pi),$$

$$M_3 = \pi(1 - \pi)(1 - 2\pi),$$

etc.

Assume n independent binary "trials," each with identical probability of "success" π. Then the number of "successes" in n trials follows a *binomial* distribution:

$$f(x) = \binom{n}{x} \pi^x (1 - \pi)^{n-x}$$

where recall that

$$\binom{n}{x} \equiv \frac{n!}{x!(n-x)!}.$$

$$X \sim \text{binomial}(n, \pi).$$

Why “binomial”?

Binomial Theorem:

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}.$$

e.g.

$$\begin{aligned}
(a+b)^2 &= a^2 + 2ab + b^2, \\
(a+b)^3 &= a^3 + 3a^2b + 3ab^2 + y^3, \\
(a+b)^4 &= a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4,
\end{aligned}$$

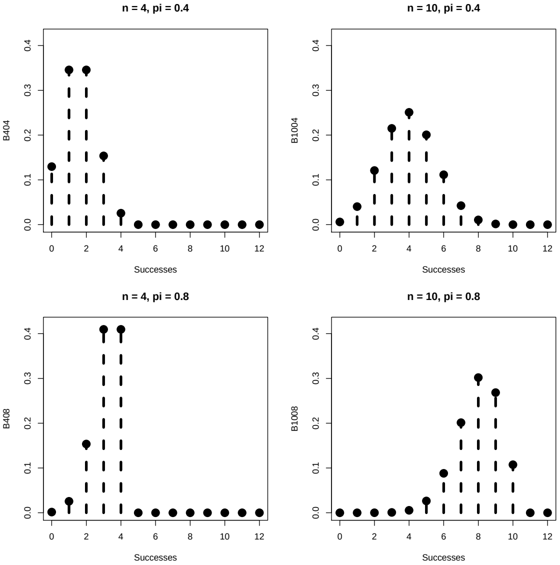
Binomial: Derivation

For $n = 2$:

$$\begin{aligned}\Pr(X=0) &= \Pr(X_1=0, X_2=0)\\ &= \Pr(X_1=0)\times\Pr(X_2=0)\\ &= (1-\pi)^2\end{aligned}$$

$$\begin{aligned}\Pr(X=1) &= \Pr(X_1=1, X_2=0 \textit{ or } X_1=0, X_2=1)\\ &= \Pr(X_1=1)\times\Pr(X_2=0)+\Pr(X_1=0)\times\Pr(X_2=1)\\ &= \pi(1-\pi)+(1-\pi)\pi\\ &= 2[\pi(1-\pi)]\end{aligned}$$

$$\begin{aligned}\Pr(X=2) &= \Pr(X_1=1, X_2=1)\\ &= \Pr(X_1=1)\times\Pr(X_2=1)\\ &= \pi^2\end{aligned}$$


n = 4, pi = 0.4
n = 10, pi = 0.4
n = 4, pi = 0.8
n = 10, pi = 0.8
B404
B1004
B408
B1008
0.0
0.1
0.2
0.3
0.4
0
2
4
6
8
10
12
Successes

CDF:

$$\begin{aligned} F(x) &= \sum_{x} f(x) \\ &= \sum_{j=0}^{x} \binom{n}{j} \pi^{j} (1-\pi)^{n-j} \end{aligned}$$

Expectation:

$$\mathrm{E}(X) = n\pi,$$

More Binomial

Variance:

$$\begin{aligned}
\mathrm{Var}(X) &= \sum_x [X - \mathrm{E}(X)]^2 f(x) \\
&= \sum_x (X - \pi n)^2 \binom{n}{x} \pi^x (1-\pi)^{n-x} \\
&= n\pi(1-\pi).
\end{aligned}$$

Skewness:

$$\text{Skewness} = \frac{1-2\pi}{\sqrt{n\pi(1-\pi)}}$$

The Binomial...

- Is *unimodal* (except in certain cases),
- has median $\lceil n\pi \rceil$ or $\lfloor n\pi \rfloor$,
- has mode $\lceil (n+1)\pi \rceil$ or $\lfloor (n+1)\pi \rfloor$,
- has skewness that is:
  - increasing in n, and
  - is largest when $\pi = 0.5$ for a fixed value of n.

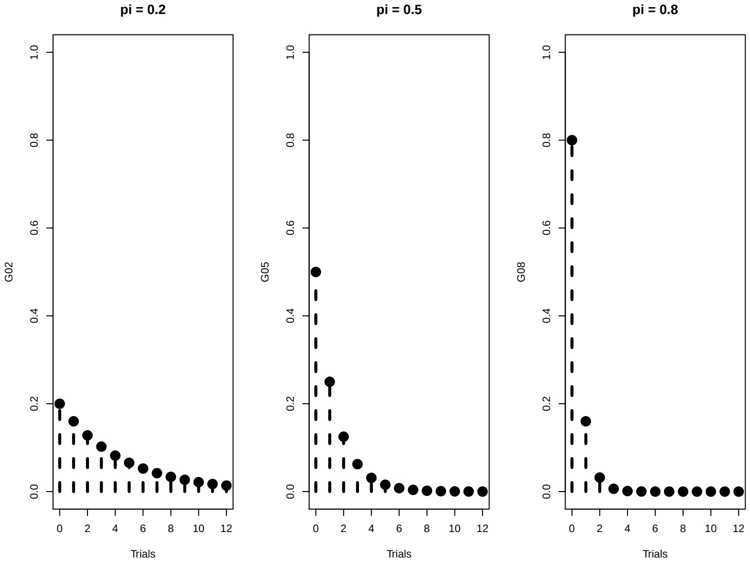
Geometric

The number of independent Bernoulli trials needed to achieve *one* success is a *geometric* random variable.

PDF:

$$f(x) = \pi(1-\pi)^{x-1}$$

$$X \sim \text{geometric}(\pi).$$



pi = 0.2
pi = 0.5
pi = 0.8
G02
G05
G08
Trials
0.0
0.2
0.4
0.6
0.8
1.0
0
2
4
6
8
10
12

CDF:

$$
\begin{aligned}
F(x) &= \sum_{j=1}^{x} \pi(1-\pi)^{x-1} \\
&= 1-(1-\pi)^{x}
\end{aligned}
$$

Expectation:

$$\mathrm{E}(X) = \frac{1}{\pi}$$

Variance:

$$\mathrm{Var}(X) = \frac{1-\pi}{\pi^2}$$

Negative Binomial

The number of *failures we observe* (x) before achieving the rth success in a series of independent Bernoulli trials (each with equal probability of success π) is distributed according to a *negative binomial* distribution.

PDF:

$$f(x) = \binom{r + x - 1}{r - 1} \pi^r (1 - \pi)^x$$

More Negative Binomial

CDF:

$$
\begin{aligned}
F(x) &= \sum_{j=0}^{x} \binom{r+j-1}{r-1} \pi^r (1-\pi)^j \\
&= 1 - \mathrm{CDF}_{\text{binomial}}
\end{aligned}
$$

Expected value:

$$
\mathrm{E}(X) = \frac{(1-\pi)r}{\pi}
$$

Even More Negative Binomial

Variance:

$$\mathsf{Var}(X) = \frac{(1-\pi)r}{\pi^2}.$$

Skewness:

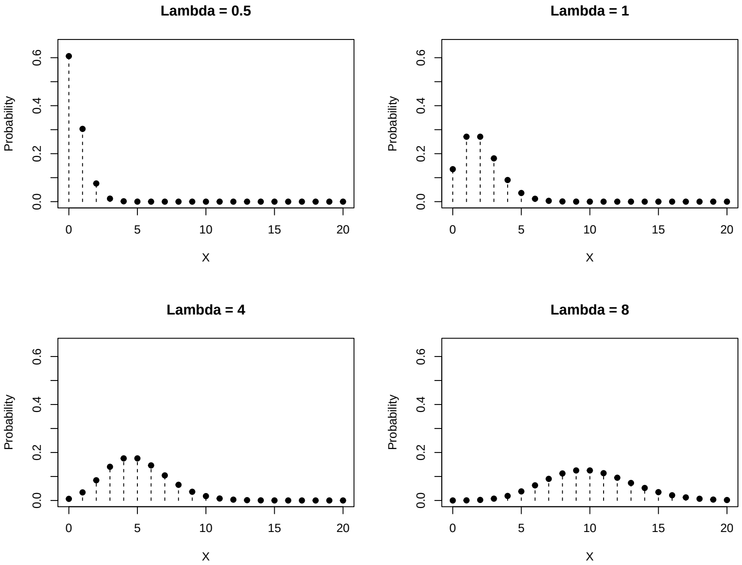
$$\mathsf{Skewness} = \frac{1+\pi}{\sqrt{\pi r}}$$

Poisson

For n independent Bernoulli trials with (sufficiently small) probability of success π and where $n\pi \equiv \lambda > 0$, the probability of observing exactly x total "successes" as the number of trials grows without limit is the *Poisson distribution*.

PDF:

$$
\begin{aligned}
f(x) &= \lim_{n\to\infty}\left[\binom{n}{x}\left(\frac{\lambda}{n}\right)^{x}\left(1-\frac{\lambda}{n}\right)^{n-x}\right] \\
&= \frac{\lambda^{x}\exp(-\lambda)}{x!}.
\end{aligned}
$$


Lambda = 0.5
Lambda = 1
Lambda = 4
Lambda = 8
Probability
X
0.0
0.2
0.4
0.6
0
5
10
15
20

More Poisson

CDF:

$$F(x) = \sum_{j=0}^{x} \frac{\lambda^j \exp(-\lambda)}{x!}.$$

Mean & Variance:

$$E(X) = \mathrm{Var}(X) = \lambda$$

All higher moments are zero...

Alternative Poisson

Independent, *constant-probability* events occurring in time...

$$\text{“Arrival rate”} = \lambda$$

Implies:

$$\begin{aligned} \Pr(\text{Event in } (t, t+h]) &= \lambda \\ \Pr(\text{No event in } (t, t+h]) &= 1 - \lambda \end{aligned}$$

$$N_{\text{Events occurring in } (t, t+h]} = \frac{\exp(-\lambda h)\lambda h^{x}}{x!}$$

If $h = 1 \; \forall \; h$, then:

$$f(x) = \frac{\exp(-\lambda)\lambda^{x}}{x!}$$

Multinomial

Imagine K possible distinct *outcomes* for each "trial," where each possible outcome $k \in \{1, 2, ...K\}$ has π_k and $\sum_{k=1}^{K} \pi_k = 1$.

Define x_k = number of times we observe outcome k out of n trials.

Then for:

$$\mathbf{X} = \begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ X_K \end{pmatrix}$$

the PDF is:

$$f(\mathbf{x}) = \frac{n!}{x_1!x_2!...x_K!}\pi_1^{x_1}\pi_2^{x_2}...\pi_K^{x_K}$$

Multinomial, continued

Expected value:

$$E(\mathbf{X}) \equiv E\begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ X_K \end{pmatrix} = n\begin{pmatrix} \pi_1 \\ \pi_2 \\ \vdots \\ \pi_K \end{pmatrix}$$

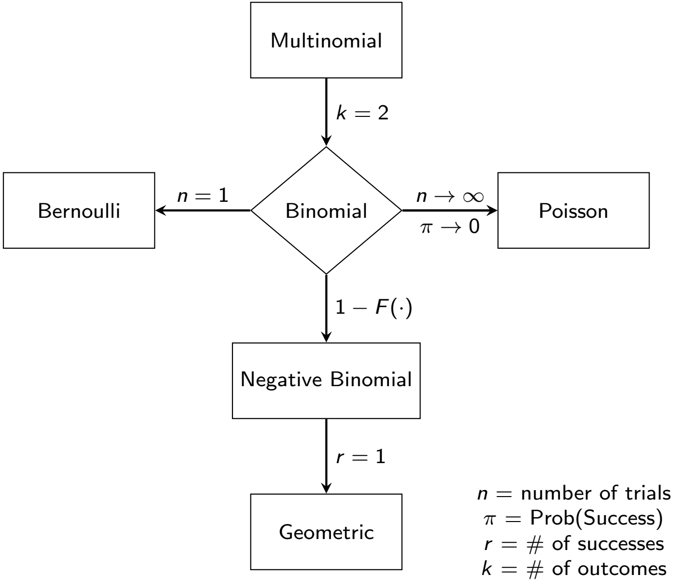
Variance:

$$\mathrm{Var}(\mathbf{X}) \equiv \mathrm{Var}\begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ X_K \end{pmatrix} = n\begin{bmatrix} \pi_1(1-\pi_1) \\ \pi_2(1-\pi_2) \\ \vdots \\ \pi_K(1-\pi_K) \end{bmatrix}$$

Covariance between X_s and X_t, $s \neq t$:

$$\mathrm{Cov}(X_s, X_t) = -n\pi_s\pi_t$$

Schematic


Multinomial
$k = 2$
Bernoulli
$n = 1$
Binomial
$n \to \infty$
$\pi \to 0$
Poisson
$1 - F(\cdot)$
Negative Binomial
$r = 1$
Geometric
n = number of trials
π = Prob(Success)
r = # of successes
k = # of outcomes

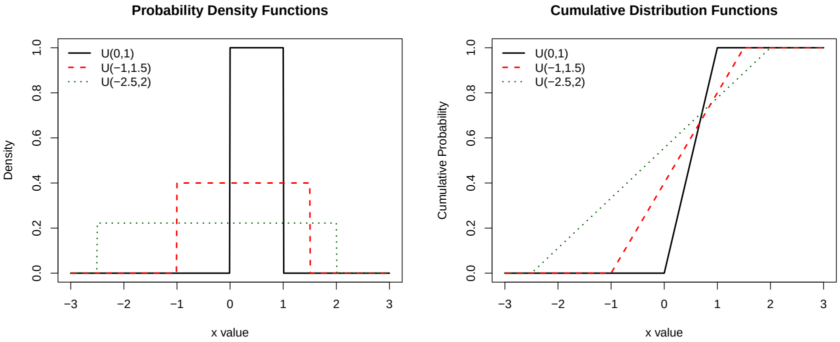
Continuous Distributions

The Uniform Distribution

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{for } a \leq x \leq b, \\ 0 & \text{for } x < a \text{ or } x > b. \end{cases}$$

$$F(x) = \int f(x)dx = \begin{cases} 0 & \text{for } x < a \\ \frac{x-a}{b-a} & \text{for } a \leq x < b \\ 1 & \text{for } x \geq b \end{cases}$$

Uniform PDFs and CDFs



Uniform Characteristics

Expected value / "mean":

$$E(X) = \check{X} = \frac{a+b}{2}$$

Mode:

$$\text{mode}(X) = [a, b]$$

Variance:

$$\text{Var}(X) = \frac{(b-a)^2}{12}$$

Skewness:

$$\text{Skewness}(X) = 0$$

The Standard Uniform Distribution

Special case of the uniform:

$$X \sim U(0,1)$$

It has the property that:

$$X \sim 1 - X \sim U(0,1).$$

In addition, the CDF is also unique:

$$F(x) = x$$

The Normal Distribution

The Normal density is:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

abbreviated:

$$X \sim \phi_{\mu,\sigma^2}.$$

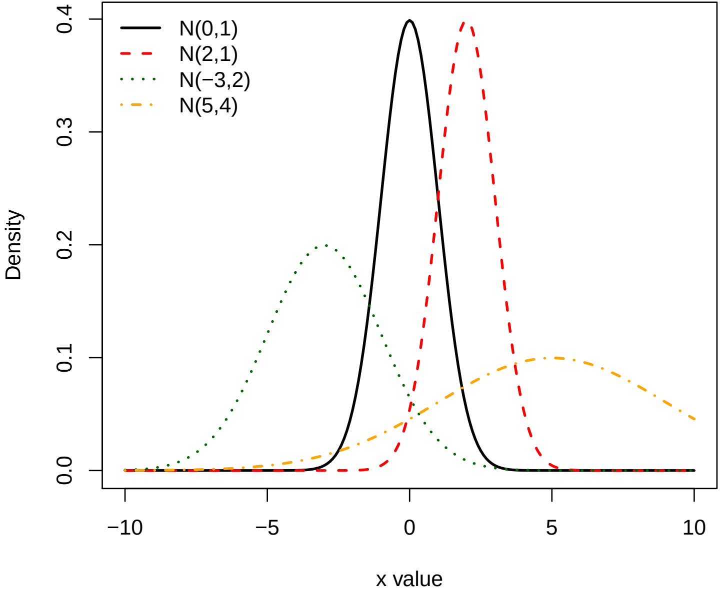
The CDF is:

$$\begin{aligned} F(x) &= \Phi_{\mu,\sigma^2}(x) \\ &= \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right], \end{aligned}$$

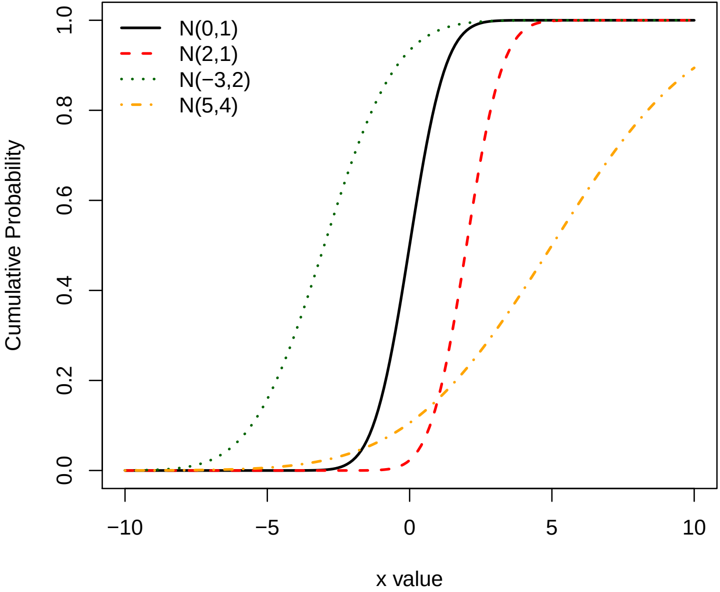
where

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt.$$

Various Normal Densities



Various Normal CDFs



Why Normal?

For $i = \{1, 2, ...N\}$ i.i.d. X_i with $\mu_i < \infty$ and $\sigma_i^2 > 0$, define:

$$X = \sum_{i=1}^{N} X_i.$$

Then

$$\begin{aligned} \mathrm{E}(X) &= \sum_{i=1}^{N} \mu_i \\ &= \mu < \infty \end{aligned}$$

and

$$\begin{aligned} \mathrm{Var}(X) &= \sum_{i=1}^{N} \sigma_i^2 \\ &= \sigma^2 < \infty. \end{aligned}$$

Central Limit Theorem

The "CLT":

$$\lim_{N\to\infty} X = \lim_{N\to\infty} \sum_{i=1}^{N} X_i \xrightarrow{D} N(\cdot)$$

"...we often think of a normal distribution as being appropriate when the observed variable X can take on a range of continuous values, and when the observed value of X can be thought of as the product of a large number of relatively small, independent "shocks" or perturbations."

Properties of the Normal Distribution

The normal is a two-parameter distribution, where

$$\mu \in (-\infty, \infty)$$

and

$$\sigma^2 \in (0, \infty).$$

For $X \sim N(\mu, \sigma^2)$:

- X has support in $\mathfrak{R}$
- $\text{Skewness}(X) = 0$
- X is *mesokurtic*
- If $X \sim N(\mu, \sigma^2)$, then $aX + b \sim N(a\mu + b, a^2\sigma^2)$

The Standard Normal Distribution

Linear transformation:

- $b = \frac{-\mu}{\sigma}$,
- $a = \frac{1}{\sigma}$.

Yields:

$$\begin{aligned} ax + b &\sim N(a\mu + b, a^2\sigma^2) \\ &\sim N(0,1) \end{aligned}$$

In other words:

- If $X \sim N(\mu, \sigma^2)$, then $Z = \frac{(x-\mu)}{\sigma} \sim N(0,1)$.
- The PDF is:

$$f(z) \equiv \phi(z) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{(z)^2}{2}\right]$$

Similarly, we often write the CDF for the standard normal as $\Phi(\cdot)$.

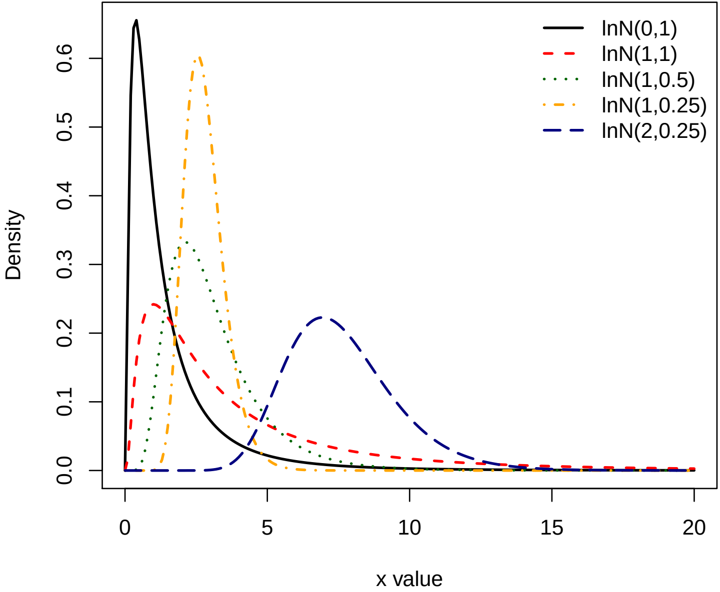
The Log-Normal Distribution

$$Y = \exp(X) \sim \mathrm{LogN}(m, s^2)$$

PDF:

$$f(y) = \frac{1}{ys\sqrt{2\pi}} \times \exp\left[\frac{-(\ln y - m)^2}{2s^2}\right].$$

Log-Normal PDFs



The χ^2 Distribution

PDF:

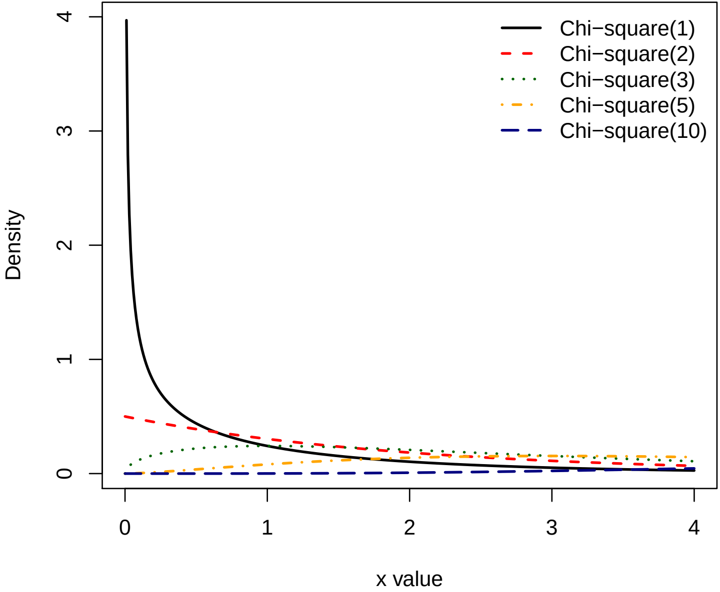
$$\begin{aligned} f(w) &= \frac{1}{2^k\Gamma(k)} w^k \exp\left[\frac{-w}{2}\right] \\ &= \frac{w^{\frac{k-2}{2}} \exp(\frac{-w}{2})}{2^{\frac{k}{2}}\Gamma(\frac{k}{2})} \end{aligned}$$

where $\Gamma(k) = \int_0^\infty t^{k-1}\exp(-t)\, dt$.

CDF:

$$F(w) = \frac{\gamma(k/2, w/2)}{\Gamma(k/2)}$$

χ^2 Densities



χ^2 *Characteristics*

For $W \sim \chi^2_k$:

$$E(W) = k$$

and:

$$\text{Var}(W) = 2k.$$

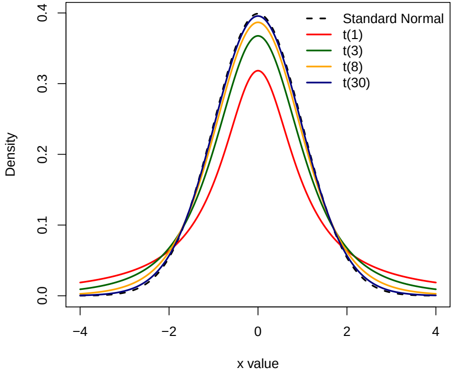
Also:

- If $Z \sim N(0,1)$, then $Z^2 \sim \chi^2_1$
- If $W_j \sim \chi^2_j$ and $W_k \sim \chi^2_k$ and independent, then
  - $W_j + W_k$ is $\sim \chi^2_{j+k}$ and more generally
  - $\sum_{i=1}^{k} W_i \sim \chi^2_k$.

Student's t Distribution

PDF:

$$f(x) = \frac{\Gamma\left(\frac{k+1}{2}\right)}{\sqrt{k\pi}\,\Gamma\left(\frac{k}{2}\right)} \left(1 + \frac{x^2}{k}\right)^{-\left(\frac{k+1}{2}\right)}$$



Student's t Characteristics

- Mean: $\mu = 0$
- Variance: $\sigma^2 = \frac{k}{k-2}$
- Skewness is 0 for $k > 3$; otherwise undefined
- $t_k \to N(0,1)$ as $k \to \infty$
- If if $Z \sim N(0,1)$, $W \sim \chi^2_k$, and Z and W are independent, then

$$\frac{Z}{\sqrt{W/k}} \sim t_k$$

and

$$\frac{Z^2}{W/k} \sim t_k.$$

The F Distribution

PDF:

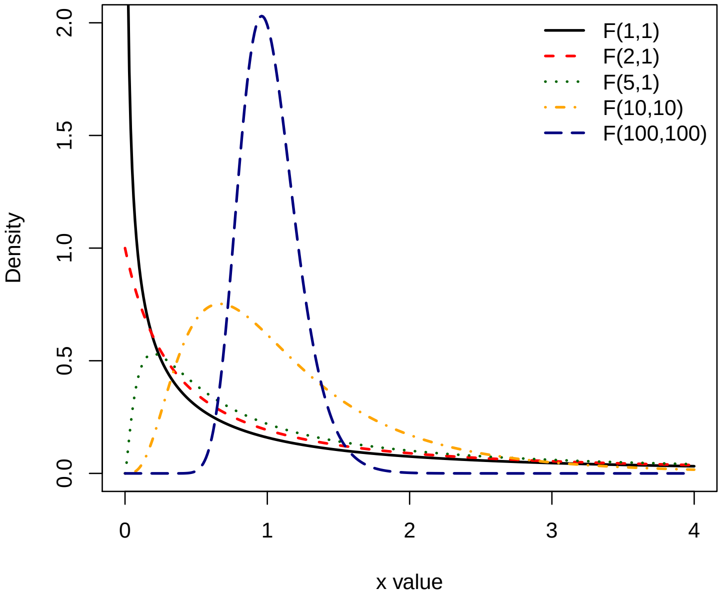
$$f(x) = \frac{\left(\frac{k\,x}{k\,x+\ell}\right)^{k/2} \left(1 - \frac{k\,x}{k\,x+\ell}\right)^{\ell/2}}{x\,\mathrm{B}(k/2, \ell/2)}$$

where $\mathrm{B}(x, y) = \int_0^1 t^{x-1}(1-t)^{y-1}\,dt$.

And we say:

$$X \sim F_{k,\ell}$$

F Densities



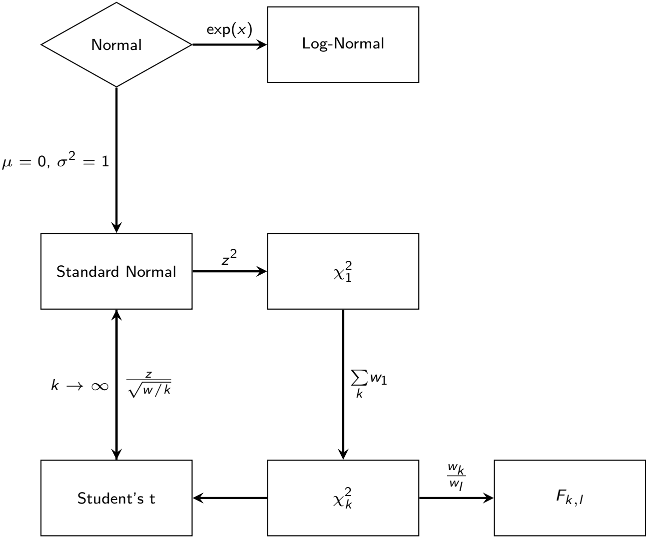
F Characteristics

For an $F_{k,\ell}$ distribution:

- $\mathrm{E}(X) = \frac{\ell}{\ell-2}$
- $\mathrm{Var}(X) = \frac{2\,\ell^2\,(k+\ell-2)}{k(\ell-2)^2(\ell-4)}$
- $\text{Skewness} = \frac{(2k+\ell-2)\sqrt{8(\ell-4)}}{(\ell-6)\sqrt{k(k+\ell-2)}}$
- For independent $W_1 \sim \chi^2_k$ and $W_2 \sim \chi^2_\ell$:

$$\frac{W_1}{W_2} \sim F_{k,\ell}$$

Relationships Among Continuous Distributions



Variables

$$x \sim N(\mu, \sigma^2)$$
$$z \sim N(0, 1)$$
$$w_k \sim \chi^2_k$$

Summary

A Few Distributions

Distribution	First Parameter	Meaning	Second Parameter	Meaning
Bernoulli	π	Pr(Success)		
Binomial	n	N of trials	π	Pr(Success)
Geometric	π	Pr(Success)		
Negative Binomial	r	Rank of "success"	π	Pr(Success)
Poisson	λ	"Arrival rate"		
Uniform	a	Minimum	b	Maximum
Normal	μ	Mean	σ^2	Variance
Lognormal	m	Mean	s^2	Variance
Student's t	k	Degrees of freedom		
Chi-Square	k	Degrees of freedom		
F	k	Degrees of freedom	ℓ	Degrees of freedom

Other Useful Distributions

There are a lot; here, in no particular order:

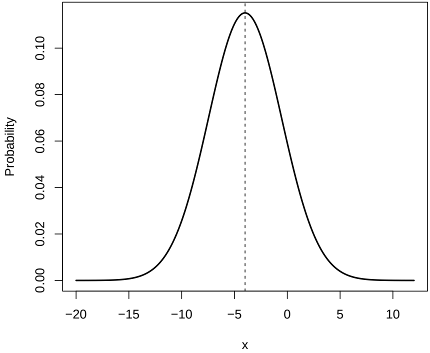
- The beta distribution – a continuous two-parameter distribution defined on $[0, 1]$; widely used to model percentages / proportions, and as a "prior" in Bayesian analysis.
- The exponential distribution – a continuous one-parameter distribution defined on $[0, \infty)$ that is related to the Poisson distribution (and others).
- The Weibull distribution – a continuous two parameter distribution defined on $[0, \infty)$ widely used in survival analysis.
- The gamma distribution – a continuous two parameter distribution defined on $[0, \infty)$ that encompasses the exponential, chi-square, and other distributions as special cases.
- The logistic distribution – a continuous two parameter distribution defined on $\mathbb{R}$ that resembles the Normal and is widely used in statistics (e.g., logistic regression) and machine learning.
- The Cauchy distribution – a continuous two parameter distribution defined on $\mathbb{R}$ and that is interesting mainly because it has no defined MGF.

Practical Things

How To Plot A Density / PDF

"Plot a Normal distribution with a mean of -4 and a variance of 12."

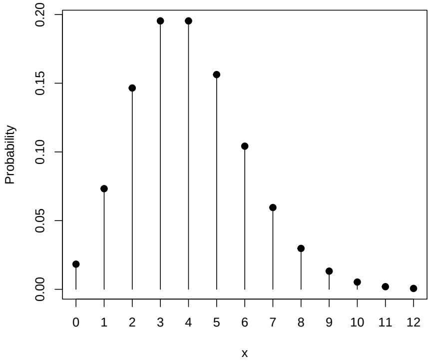
```
> x<-seq(-20,12,by=0.1)
> PlotNorm<-dnorm(x,-4,sqrt(12))
```



Plotting A Discrete Density

"Plot a Poisson distribution with $\lambda = 4$."

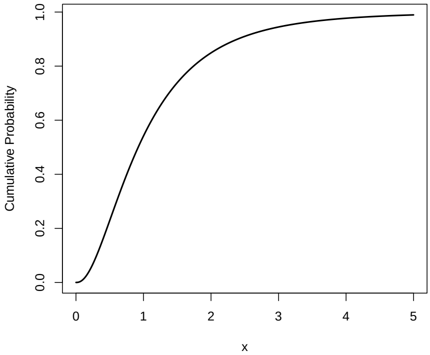
```
> x<-seq(0,12)
> PlotPois<-dpois(x,4)
```



How To Plot A CDF

"Plot the CDF of an F distribution with $k = 5$ and $\ell = 12$."

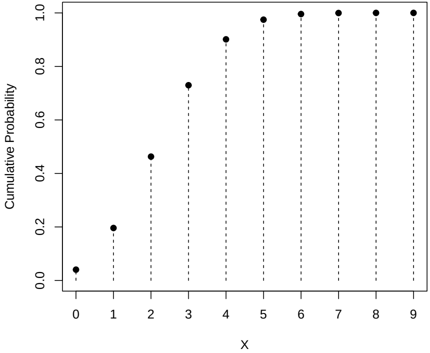
```
> x<-seq(0,5,by=0.01)
> PlotF<-pf(x,5,12)
```



CDF: Discrete Distribution

"Plot the CDF of a binomial distribution with $\pi = 0.3$ and $n = 9$."

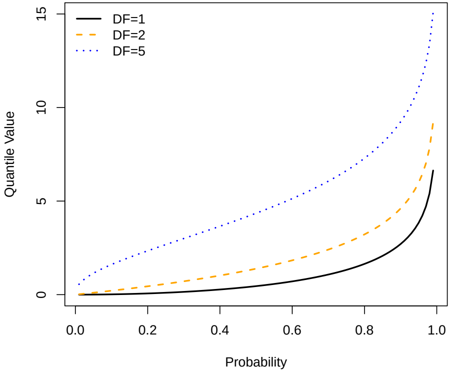
```
> x<-seq(0,9)
> PlotBinom9<-pbinom(x,9,0.3)
```



Quantiles of a Distribution

"Plot the quantiles of three χ^2 distributions with one, two, and five degrees of freedom."

```
> P<-seq(0.01,0.99,by=0.01) # probabilities
> ChiSq1<-qchisq(P,1) # df=1
> ChiSq2<-qchisq(P,2) # df=2
> ChiSq5<-qchisq(P,5) # df=5
```



Simulating Random Variables

Commands for Generating Random Variates

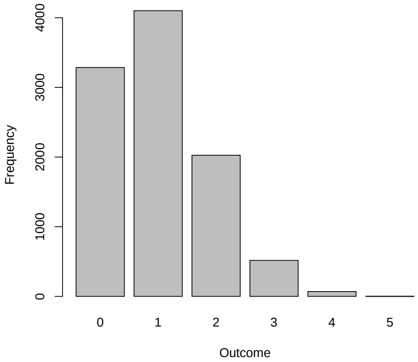
Distribution	R	Stata
Binomial(n, π)	`rbinom()`	`rbinomial`
Geometric(π)	`rgeom()`	?
Negative Binomial(n, π)	`rnbinom()`	`rnbinomial`
Poisson(λ)	`rpois()`	`rpoisson`
Uniform($0, 1$)	`runif()`	`runiform`
Normal($0, 1$)	`rnorm()`	`rnormal`
Lognormal($0, 1$)	`rlnorm()`	`xlgn`*
Student's $t(k)$	`rt()`	`rt`
Chi-Square(k)	`rchisq()`	`rchi2`
$F(k, \ell)$	`rf()`	`rndf`*

Note: Stata commands marked with an asterisk are from Hilbe's `-rnd-` group of commands. "?" indicates that I'm not aware of any "canned" way of doing this, though one can always generate them "by hand" using the appropriate PDF function.

Drawing From A Distribution

"Draw 10,000 random draws from a binomial distribution with $n = 5$ and $\pi = 0.2$."

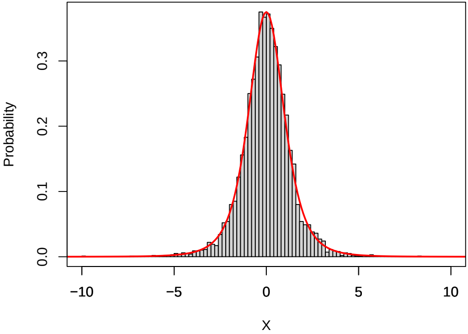
```
> Xbinom5point2<-rbinom(10000,5,0.2)
```



More Draws From A Distribution

"Draw 5000 random draws from a *t* distribution with 4 degrees of freedom, and compare the distribution of values to the theoretical density."

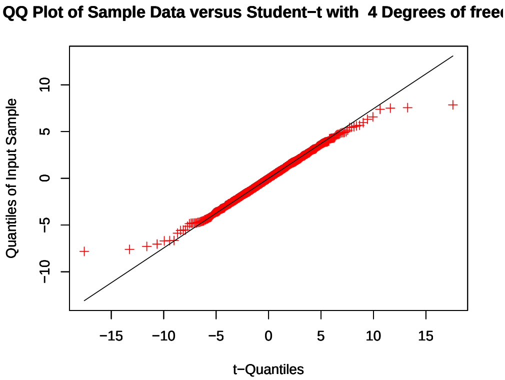
```
> TDraws<-rt(5000,4)
```



(continued...)

(Same comparison, using a Q-Q plot...)

```
> library(Dowd)
> TQQPlot(TDraws,4)
```



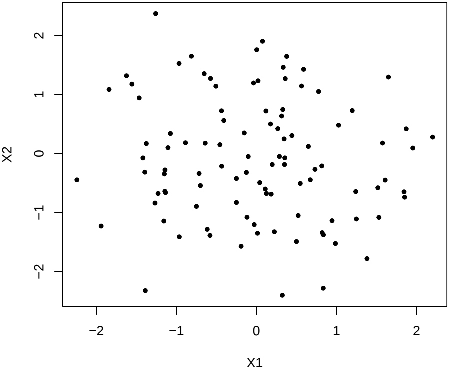
Pseudo-Random Numbers and “Seeds”

```
> seed<-3229 # calling "seed" some thing
> set.seed(seed) # setting the system seed
> rt(3,1) # three draws from a t distrib. w/1 d.f.
[1] -0.1113 -0.7306  1.9839

> seed<-1077
> set.seed(seed) # resetting the seed
> rt(3,1) # different values for the draws
[1]   -0.5211    7.9161 -155.3186

> seed<-3229 # original seed
> set.seed(seed)
> rt(3,1) # identical values of the draws
[1] -0.1113 -0.7306  1.9839
```

Seeds and Simulations

The right way:

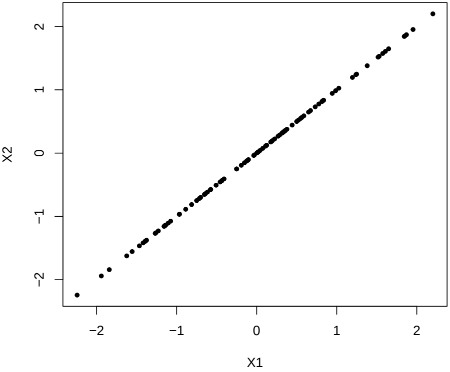
```
> X<-matrix(NA,nrow=100,ncol=2)
> set.seed(7222009)
> for(i in 1:2){
+   X[,i]<-rnorm(100)}
```



Seeds and Simulations (continued)

The wrong way:

```
> X<-matrix(NA,nrow=100,ncol=2)
> for(i in 1:2){
+   set.seed(7222009)
+   X[,i]<-rnorm(100)}
```



Best Practices

In general:

1. **Always** set a seed value.
2. Try to use a single/consistent seed value over time/projects.
3. Keep seeds *outside* loops/`apply` statements.
6. Use `ddists` and `pdists` for theoretical quantities, `rdists` for generating simulated "data" / variates.
5. Plot discrete distributions discretely....
6. ...and continuous distributions continuously.