# PLSC 502: "Statistical Methods for Political Research"

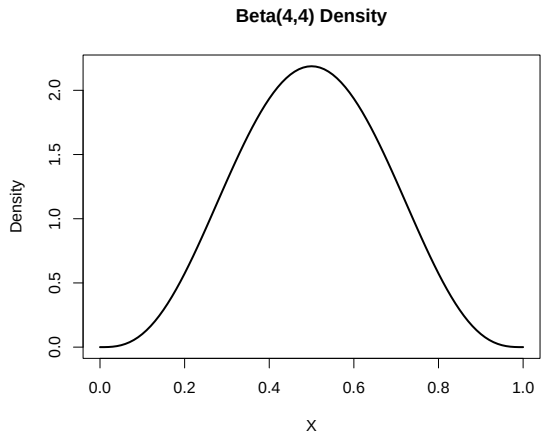
**Exercise Five**

October 29, 2024

## Part I

We'll start with a simulation-based exercise. The beta distribution is a two-parameter probability density with support on the unit [0,1] interval. It's parameters are "shape" parameters that define the shape of the probability density; they are typically denoted $a$ and $b$, and the distribution is sometimes written as $\mathcal{B}(a, b)$. Those two parameters $a$ and $b$ completely characterize the shape of the density of a beta-distributed variate.

For this part of the exercise, we're going to work with a beta distribution with $a = 4$ and $b = 4$ [so, $\mathcal{B}(4, 4)$]; that specific density is illustrated in the figure here:




The general goal of this part of the simulation is to show how Normal-based confidence intervals provide poor coverage when sample sizes are small. To that end, do the following:

1. Choose some different sample sizes; for example, you might consider $N \in \{4, 10, 40, 100, 500\}$. Make sure that at least a couple of them are pretty small (as in, $N < 20$).
2. For a given sample size, simulate samples of size $N$ by drawing $N$ values from a $\mathcal{B}(4, 4)$ distribution;[1] do this $K$ times, where $K$ is large.
3. For each of the $K$ samples of size $N$, estimate $\hat{a}$ and $\hat{b}$, as well as $\hat{a}$'s and $\hat{b}$'s standard errors (that is, $\sigma_{\hat{a}}$ and $\sigma_{\hat{b}}$).[2]
4. Using the estimates $\hat{a}$ and $\hat{b}$ and their standard errors $\sigma_{\hat{a}}$ and $\sigma_{\hat{b}}$, construct $K$ 95-percent two-tailed confidence intervals for $\hat{a}$ and $\hat{b}$, on the assumption that $\hat{a} \sim \mathcal{N}(a, \sigma^2_{\hat{a}})$ and $\hat{b} \sim \mathcal{N}(b, \sigma^2_{\hat{b}})$.
5. Repeat steps (2-4) for each of the values of $N$ in (1).
6. Conclude by discussing:
   (a) ...how the confidence intervals around $\hat{a}$ and $\hat{b}$ change with $N$, and
   (b) ...how well the Normal-based intervals perform in terms of "coverage." Specifically, for each different value of $N$, does the empirical reality (the number of those $K$ confidence intervals for $a$ and $b$ that contain the true values $a = 4$ and $b = 4$) match the theoretical expectation associated with a 95-percent confidence interval? Explain any differences you find.

[1] This can be done using the `rbeta` command with options `shape1=4` and `shape2=4`.

[2] This step is probably most easily done using the `fitdistr` command in the `MASS` package; using that command, you can specify (e.g.):

```
> foo <- fitdistr(X,"beta",start=list(shape1=1,shape2=1))
```

and the resulting object will have estimates $\hat{a}$ and $\hat{b}$ in `foo$estimate` and estimated standard errors in `foo$sd`.

**Part II**

The "real" data for this exercise are drawn from Washington University's *The American Panel Study* (TAPS), which collected data on a panel of respondents over several monthly waves from 2012-2017. (Here, we will be looking only at single-response questions, so the data are cross-sectional.) The data have $N \approx 1100$, and are a probability sample of adults living in the U.S. in 2016; details on the survey's sampling design are available here. The data are available on the course github repository, in the "Exercises" folder, in a file named `PLSC502-2024-ExerciseFive.csv`. In addition to a respondent identifier variable (`WUSTLID`), the pool of independent variables in those data are:[3]

- Political party identification indicators – binary variables for `Democrat` (Democratic Party) and `GOP` (Republican Party), with independents serving as a reference category;
- `Ideology` – a seven-point Likert-type indicator variable, where higher values indicate greater political conservatism (right-wing) and lower values indicating greater progressivism (left-wing);
- `Education` – measured as a twelve-category ordinal variable with values ranging from 3 to 15, where the lowest value corresponds to a 5th-6th grade level of education and the highest reflects a doctoral degree;
- `Income` – a 15-category ordinal variable, where higher values indicate higher income levels (where each unit roughly corresponds to an increase of $10,000 in annual income);
- `Age2016` – the respondent's age in years, as of 2016;
- `Female` – a binary indicator of sex, naturally-coded;
- Racial classifications – binary indicator variables for `White`, `Black`, and `Asian` identification (with "other" as the reference category);
- `FT.Communists` is the respondent's placement of "communists" on a 0-100 "feeling thermometer" scale;
- `InterviewDuration` records the number of seconds that each respondent took to complete the (on-line) survey;
- `HowManyKids` indicates the number of children (under the age of 18) that each respondent has in their household;
- Finally, the TAPS survey also asked a bonkers series of yes-no / binary-response questions related to respondents' specific behaviors and preferences, including:
    - Have you ever taken the shampoo and conditioner bottles from a hotel or motel? (`StealShampoo`; 0 = no, 1 = yes)
    - During the past year, have you ever run out of gas while driving a car or other vehicle? (`RunOutOfGas`; 0 = no, 1 = yes)
    - Have you ever looked directly at the sun to see an eclipse without using a filter? (`LookedAtEclipse`; 0 = no, 1 = yes)
    - Have you ever stolen a street sign? (`StolenStreetSign`; 0 = no, 1 = yes)
    - Would you rather be attacked by a big bear or a swarm of bees? (`BeesOrBear`; 0 = bees, 1 = bear)

Note that several of the variables in these data have missing values, some of them in substantial numbers.

[3]The data also contain a set of survey `weights` that reflect the sampling scheme; you can ignore those, for now.

Using these data, do the following:

1. Calculate and report the 95 percent confidence intervals for:
   (a) The mean of `Age2016`;
   (b) The mean of `FT.Communists`;
   (c) The mean of `InterviewDuration`;
   (d) The proportion of "bear" responses to `BeesOrBear`.
2. Plot the 80, 95, and 99-percent confidence intervals for each of the four variables in (1).
3. Discuss, in words, two of the four confidence intervals in (1). What do they *mean* in substantive terms?
4. Use both significance tests with $\alpha = 0.10$ and $P$-values to examine the following hypotheses:
   (a) $\overline{\texttt{Ideology}} = 5$.
   (b) $\overline{\texttt{HowManyKids}} = 0.5$.
   (c) $\overline{\texttt{Female}} = 0.5$.
   (d) $\overline{\texttt{StealShampoo}} = 0.9$.

As usual, use plots, words, or combinations thereof to complete this exercise. In discussing your "findings," refer to the population from which the sample was drawn (i.e., adults over the age of 18 living in the U.S.).

Submit your answers **in PDF format**. In addition to your answers, please include a copy of all computer code used to conduct your simulations, generate your figures, etc. This can be in any form – a separate `.R` or `.do` file, an appendix in the PDF, or as a `.Rmd` or similar format containing both content and code. This homework exercise is due by 11:59 p.m. ET on Thursday, November 7, 2024; submit your materials in electronic format – via e-mail attachment – to Morrgan (`mth5492@psu.edu`) *and* to me (`zorn@psu.edu`). This exercise is worth 50 possible points.